The Ohio State University

Center for Frontiers in Nuclear Science

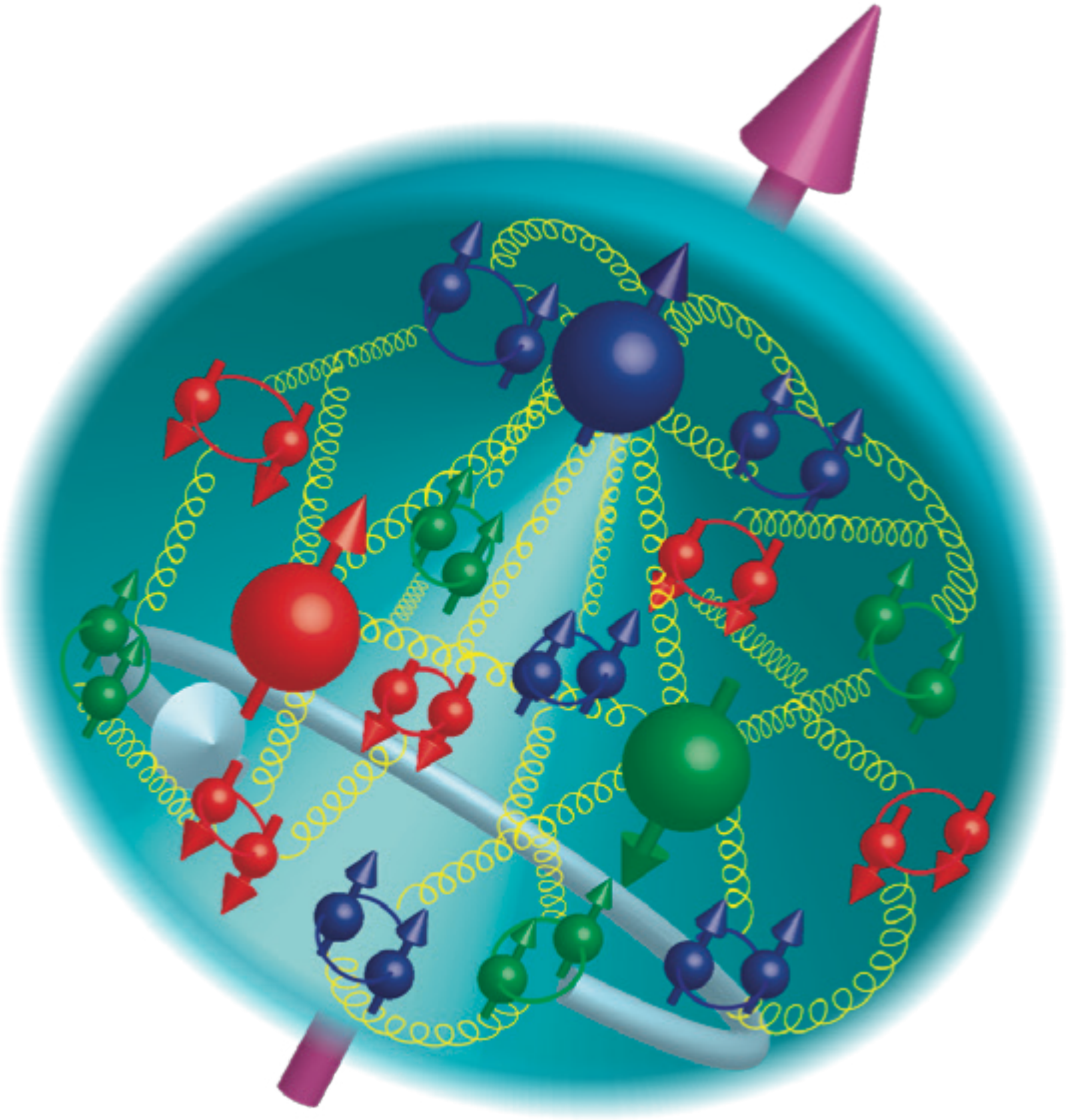
The role of the axial anomaly in polarized DIS: Emergent axion-like dynamics and the small x effective action

Andrey Tarasov

Based on Andrey Tarasov and Raju Venugopalan
Phys. Rev. D 102 (2020) 11, 114022 (arXiv:2008.08104), and in preparation

GHP 2021

The proton's spin puzzle



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Spin

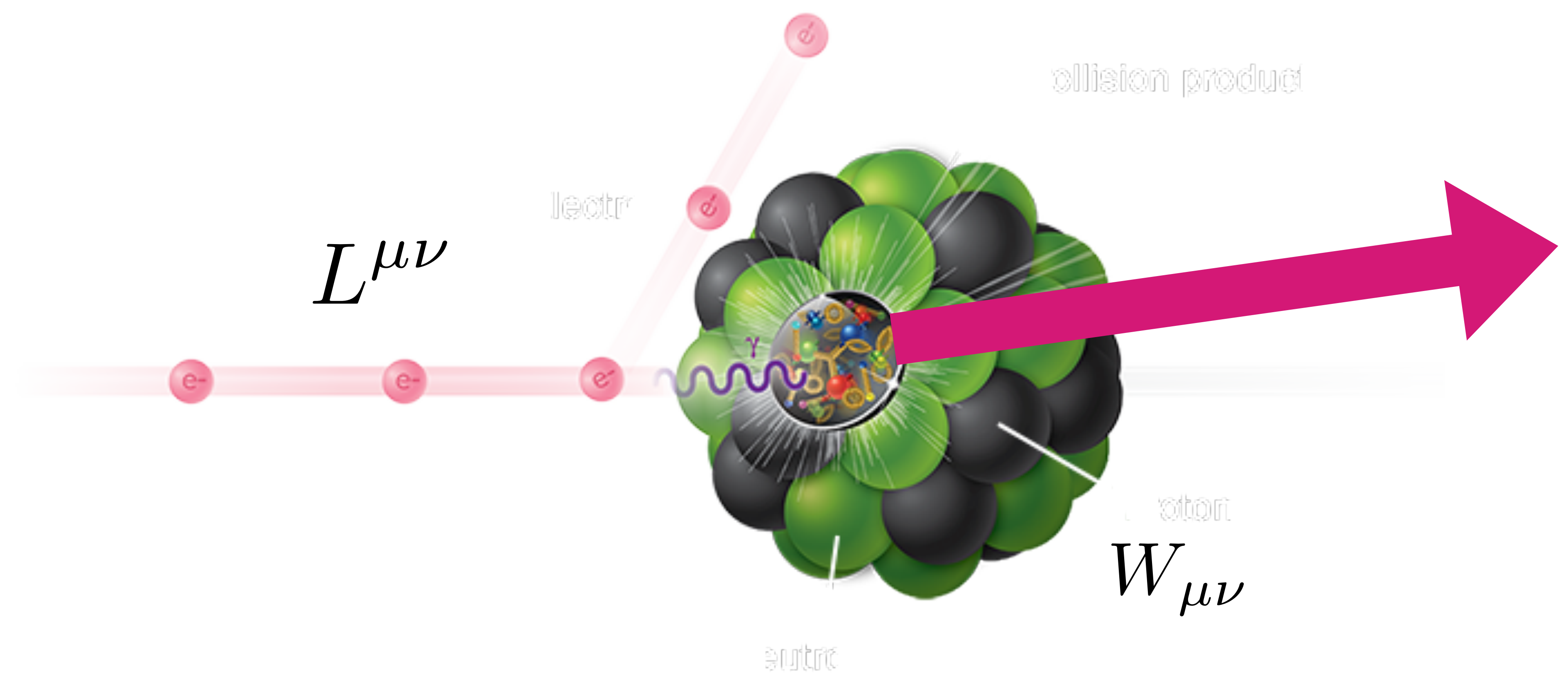
Quark helicity

Gluon helicity

Orbital angular momentum

- DIS experiments showed that quarks carry only about 30% of the proton's spin $\Delta\Sigma = 0.25 \sim 0.3$
- Failure of the constituent quark model to explain spin of the proton - *spin crisis*

First moment of g_1



$$\int_0^1 dx_B\, g_1(x_B, Q^2) = \frac{1}{18}\left(3F + D + 2\,\Sigma(Q^2)\right)$$

Combination of F and D can be measured in β-decay and hyperon decay experiments

In term of helicity PDFs iso-singlet quark helicity is

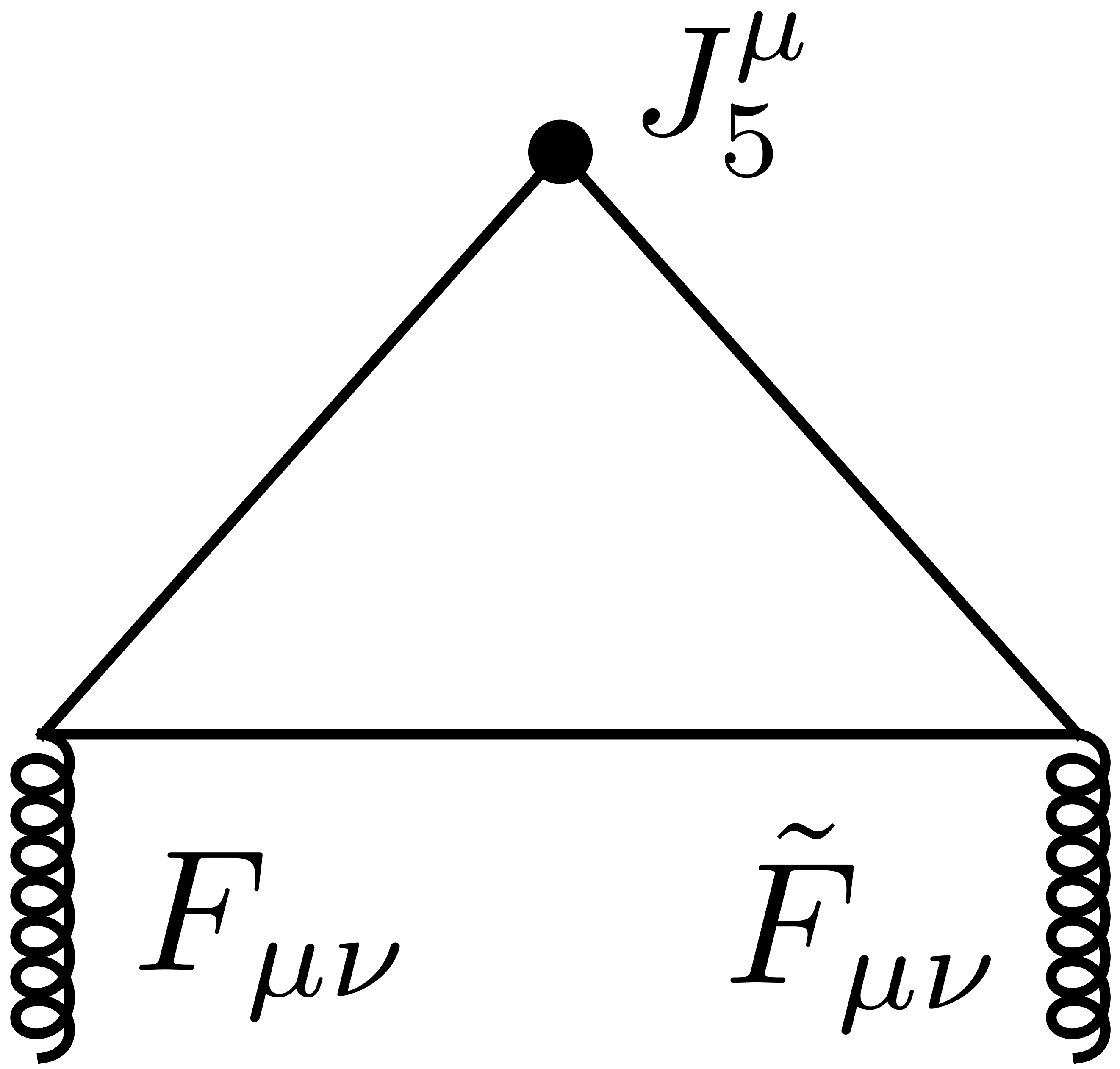
$$\Sigma(Q^2) = \sum_f \int_0^1 dx_B\, \left(\Delta q_f(x_B, Q^2) + \Delta\bar{q}_f(x_B, Q^2)\right)$$

Quark spin contribution is defined by iso-singlet axial vector current

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N} \sum_f \langle P, S|\bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f|P, S\rangle \equiv \frac{1}{M_N}\langle P, S|J_5^\mu(0)|P, S\rangle$$

Iso-singlet axial vector current

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N} \langle P, S | J_5^\mu(0) | P, S \rangle$$



The current is not conserved due to presence of the chiral anomaly.

The famous anomaly equation:

$$\partial^\mu J_\mu^5(x) = \frac{n_f \alpha_s}{2\pi} \, \mathrm{Tr} \left(F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x) \right) = 2\, n_f \, \partial_\mu K^\mu$$

Chern-Simons current:

$$K_\mu = \frac{\alpha_S}{4\pi} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu \left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma \right) \right]$$

Perturbative and non-perturbative interplay

The key role of the anomaly is seen from the structure of the triangle graph in the off-forward limit

$$\frac{1}{M_N}\langle P', S|J_5^\mu(0)|P, S\rangle = \Sigma(Q^2, t)\, S^\mu + h(Q^2, t)\, l\cdot S\, l^\mu$$

R. L. Jaffe

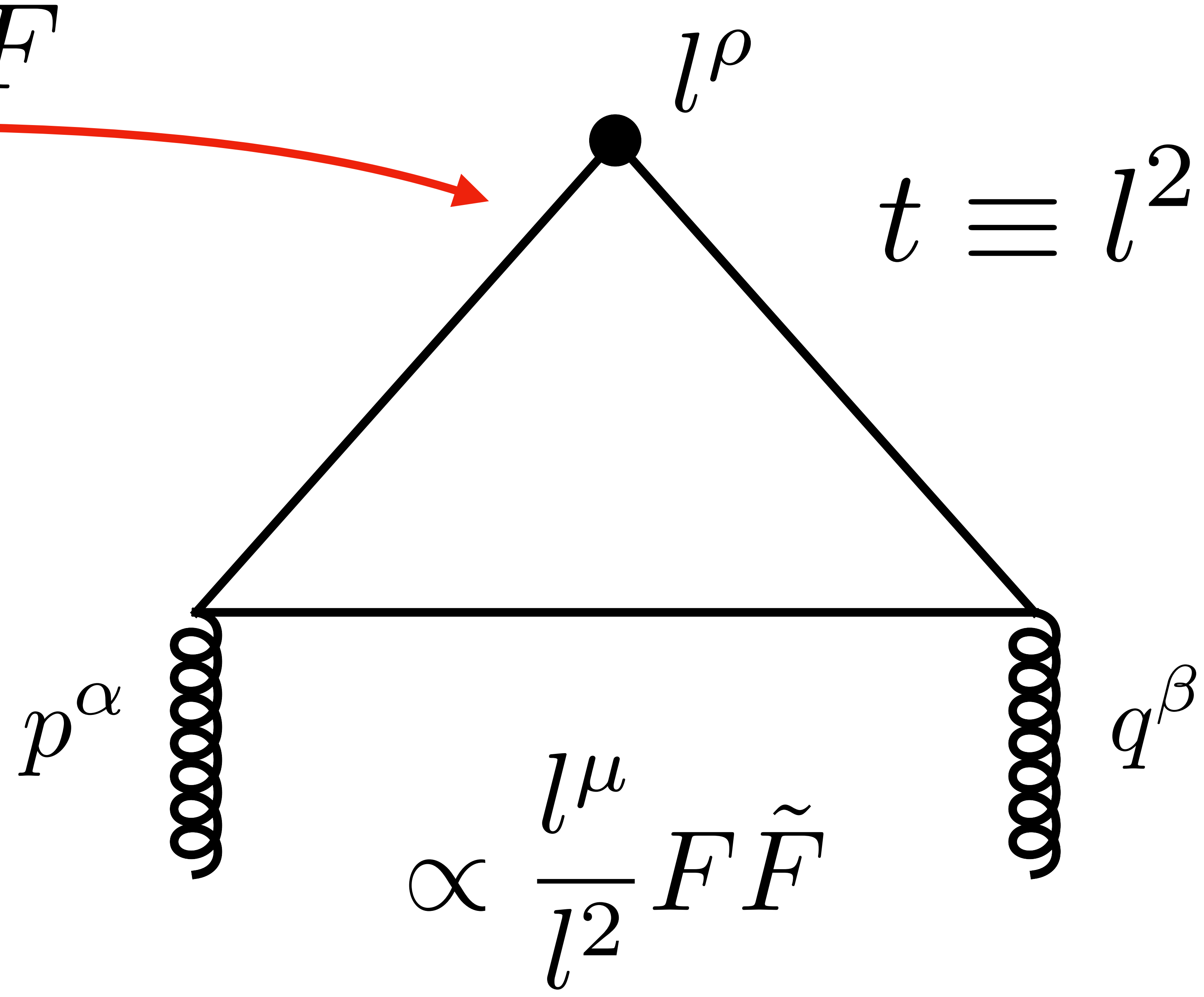
A. Manohar

In the forward limit only Σ contribute, h doesn't have a pole

The triangle diagram has an infrared pole and $\kappa(t) \propto F\tilde{F}$

$$\frac{1}{M_N}\langle P', S|J_5^\mu(0)|P, S\rangle = \boxed{\frac{l\cdot S\, l^\mu}{l^2}\kappa(Q^2, t)}$$

Exact result!



The existence of the infrared pole of the triangle diagram has not been addressed in pQCD calculations.

Perturbative and non-perturbative interplay

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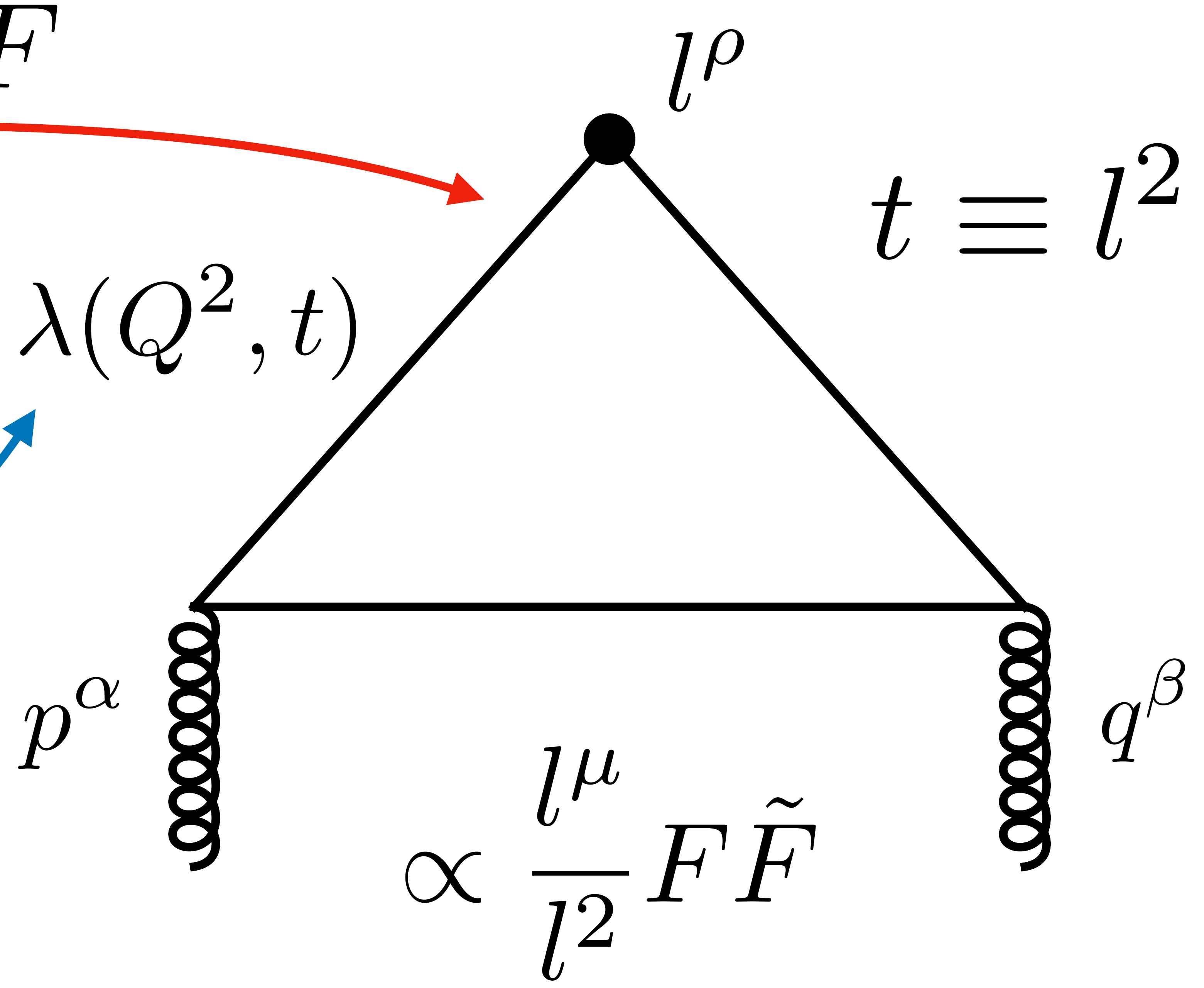
The triangle diagram has an infrared pole and $\kappa(t) \propto F\tilde{F}$

$$\frac{1}{M_N}\langle P', S|J_5^\mu(0)|P, S\rangle = \boxed{\frac{l \cdot S\, l^\mu}{l^2}\kappa(Q^2, t)} + \left(S^\mu - \frac{l \cdot S\, l^\mu}{l^2}\right)\lambda(Q^2, t)$$



Pseudoscalar contribution

R. L. Jaffe, A. Manohar
Nucl. Phys., B337:509–546, 1990

Perturbative and non-perturbative interplay

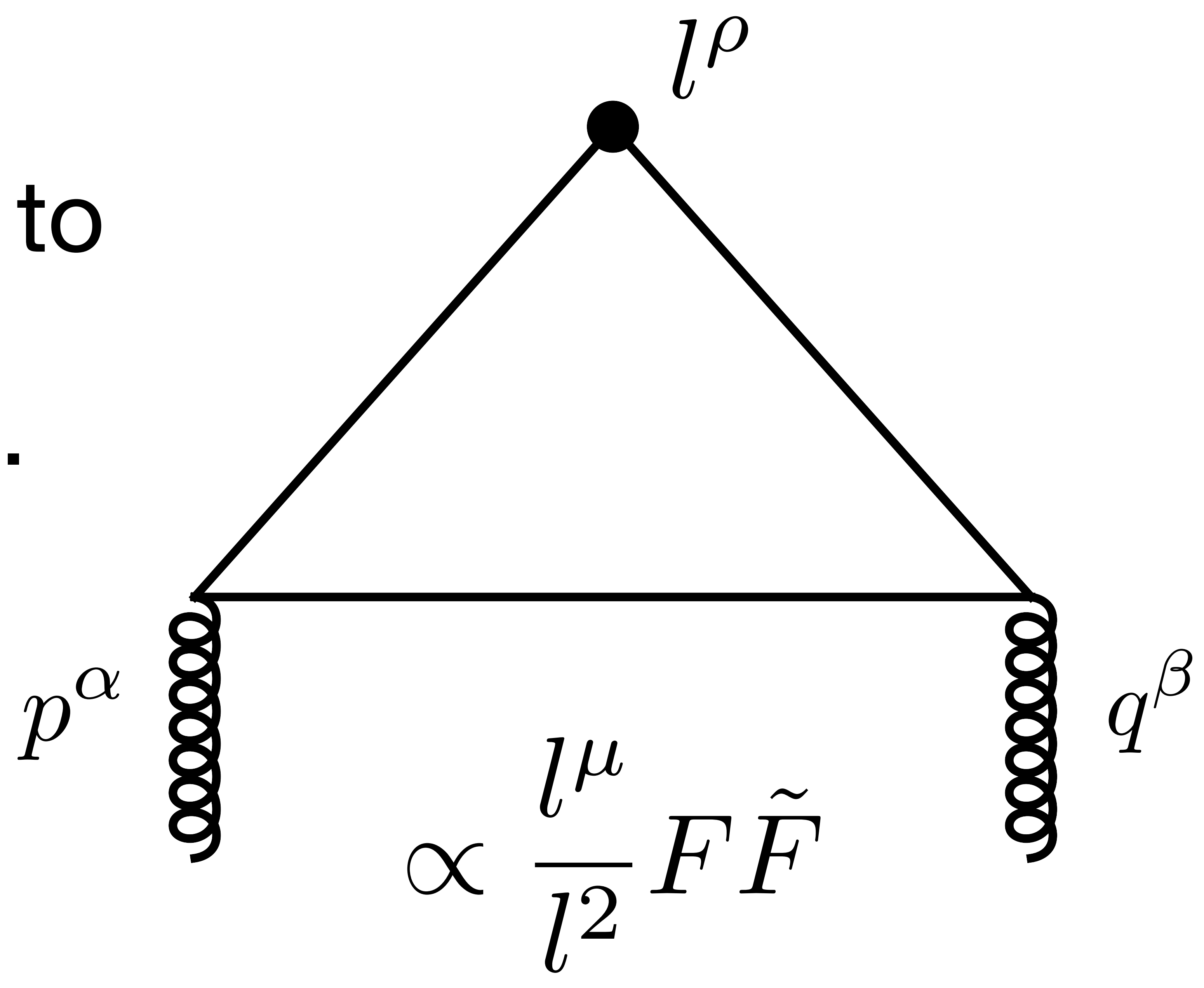
The infrared pole of the triangle diagram must be cancelled by a pole in the non-perturbative contribution:

$$\Sigma(Q^2) = \frac{n_f\,\alpha_s}{2\pi\,M_N} \lim_{l_\mu \to 0} \langle P', S| \frac{1}{il \cdot s} \mathrm{Tr}\left(F\tilde{F}\right)(0)|P, S\rangle$$

The result is manifestly gauge invariant!

The presence of the pole in the triangle diagram is related to topological properties of QCD (measure of the QCD path integral), which are described by the chiral Ward identities. The triangle diagram is not local!

Generalization of this result to $g_1(x, Q^2)$, and interplay with non-perturbative physics can be explored efficiently in a worldline framework



The triangle anomaly in the worldline formalism

Imaginary part of the QCD effective action:

Functional integrals over trajectories of a point-like particle

$$\Gamma[A, A_5] = -\frac{1}{2}\mathrm{Tr_c}\int_0^\infty \frac{dT}{T}\int \mathcal{D}x \int_{AP}\mathcal{D}\psi$$

$$\times \exp\left\{ -\int_0^T d\tau \Big(\frac{1}{4}\dot{x}^2 + \frac{1}{2}\psi_\mu\dot{\psi}^\mu + ig\dot{x}^\mu A_\mu - ig\psi^\mu\psi^\nu F_{\mu\nu} - 2i\psi_5\dot{x}^\mu\psi_\mu\psi_\nu A_5^\nu + i\psi_5\partial_\mu A_5^\mu + (D-2)A_5^2 \Big) \right\}$$

Free "propagation" Vector coupling Axial coupling

D.G.C. McKeon, C. Schubert, Phys. Lett. B 440 (1998) 101

The triangle anomaly:

$$\langle P', S|J_5^\kappa|P, S\rangle = \int d^4y \frac{\partial}{\partial A_{5\kappa}(y)}\Gamma[A, A_5]\Big|_{A_5=0} e^{ily} \equiv \Gamma_5^\kappa[l]$$

The triangle anomaly in the worldline formalism

$$\Gamma_5^\kappa[l] = \frac{1}{4\pi^2}\boxed{\frac{l^\kappa}{l^2}}\int \frac{d^4k_2}{(2\pi)^4}\int \frac{d^4k_4}{(2\pi)^4}\ \mathrm{Tr_c} F_{\alpha\beta}(k_2)\tilde{F}^{\alpha\beta}(k_4)\ (2\pi)^4\delta^4(l+k_2+k_4)$$

Steven Adler

John S. Bell

We reproduce famous infrared pole of the anomaly

$$S^\mu \int_0^1 dx_B\, g_1(x_B,Q^2)\Big|_{Q^2\to\infty}$$

$$= \sum_f e_f^2 \frac{\alpha_S}{2i\pi M_N} \lim_{l_\mu\to 0} \frac{l^\mu}{l^2}\langle P',S|\mathrm{Tr_c} F_{\alpha\beta}(0)\tilde{F}^{\alpha\beta}(0)|P,S\rangle$$

Roman Jackiw

The infrared pole must be cancelled by a pseudoscalar contribution

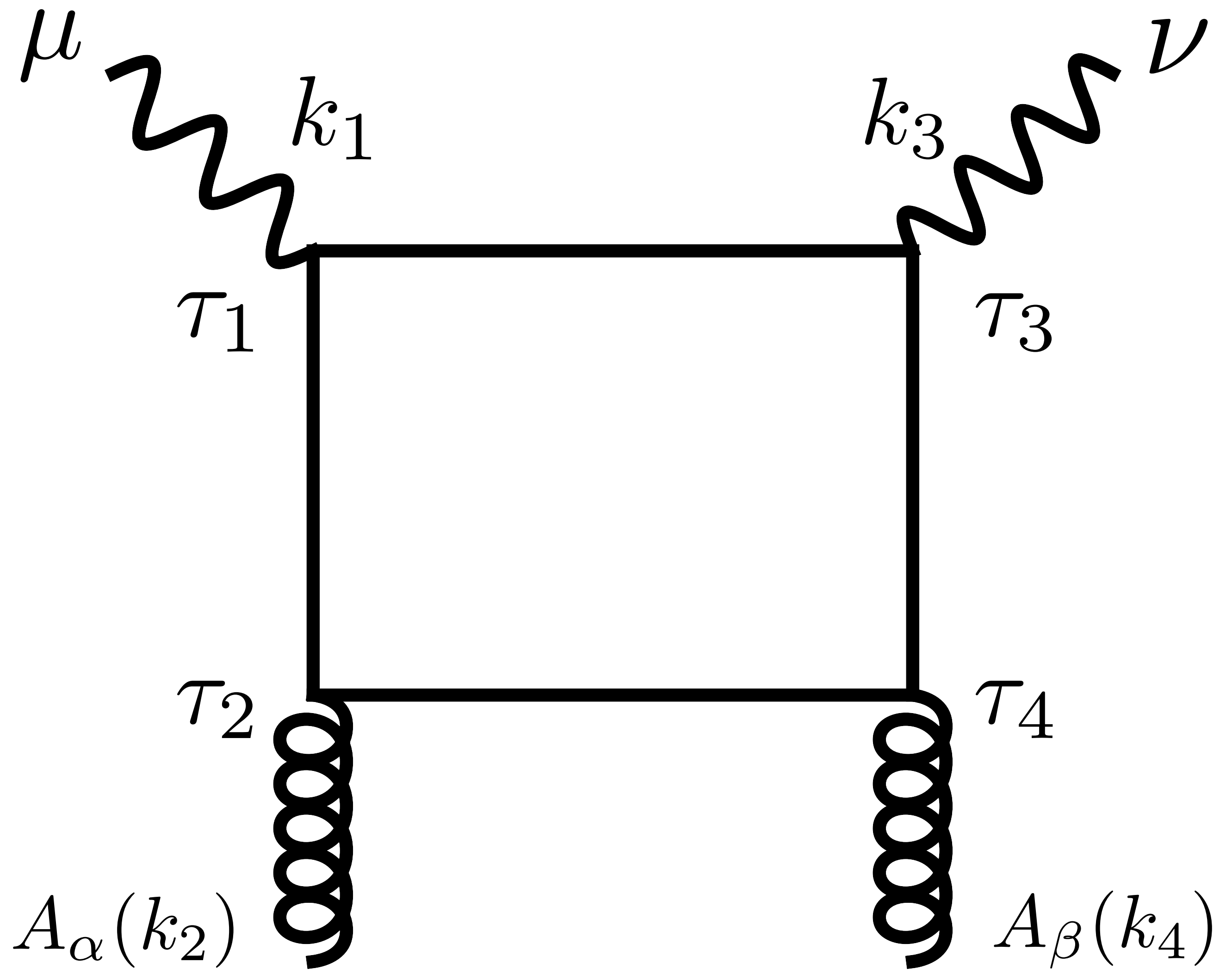
Finding the axial anomaly in the box diagram

Tarasov, Venugopalan (2019)

$$\Gamma_A^{\mu\nu}[k_1,k_3] = \int \frac{d^4k_2}{(2\pi)^4}\int \frac{d^4k_4}{(2\pi)^4}\ \Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4]\ \mathrm{Tr}_c(\tilde{A}_\alpha(k_2)\tilde{A}_\beta(k_4))$$

Hadronic tensor

Box diagram



Worldline representation of the box diagram
(one loop exact!):

$$\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4] \equiv -\frac{g^2e^2e_f^2}{2}\int_0^\infty \frac{dT}{T}\ \int \mathcal{D}x\int\mathcal{D}\psi\ \exp\Big\{-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi\cdot\dot{\psi}\Big)\Big\}$$
$$\times\Big[V_1^\mu(k_1)V_3^\nu(k_3)V_2^\alpha(k_2)V_4^\beta(k_4)-(\mu\leftrightarrow\nu)\Big]$$

where the vector current $V_i^\mu(k_i) \equiv \int_0^T d\tau_i(\dot{x}_i^\mu + 2i\psi_i^\mu k_j\cdot\psi_j)e^{ik_i\cdot x_i}$

The box diagram in the Bjorken limit

In the Bjorken limit $Q^2 \to \infty$ and x_B is fixed

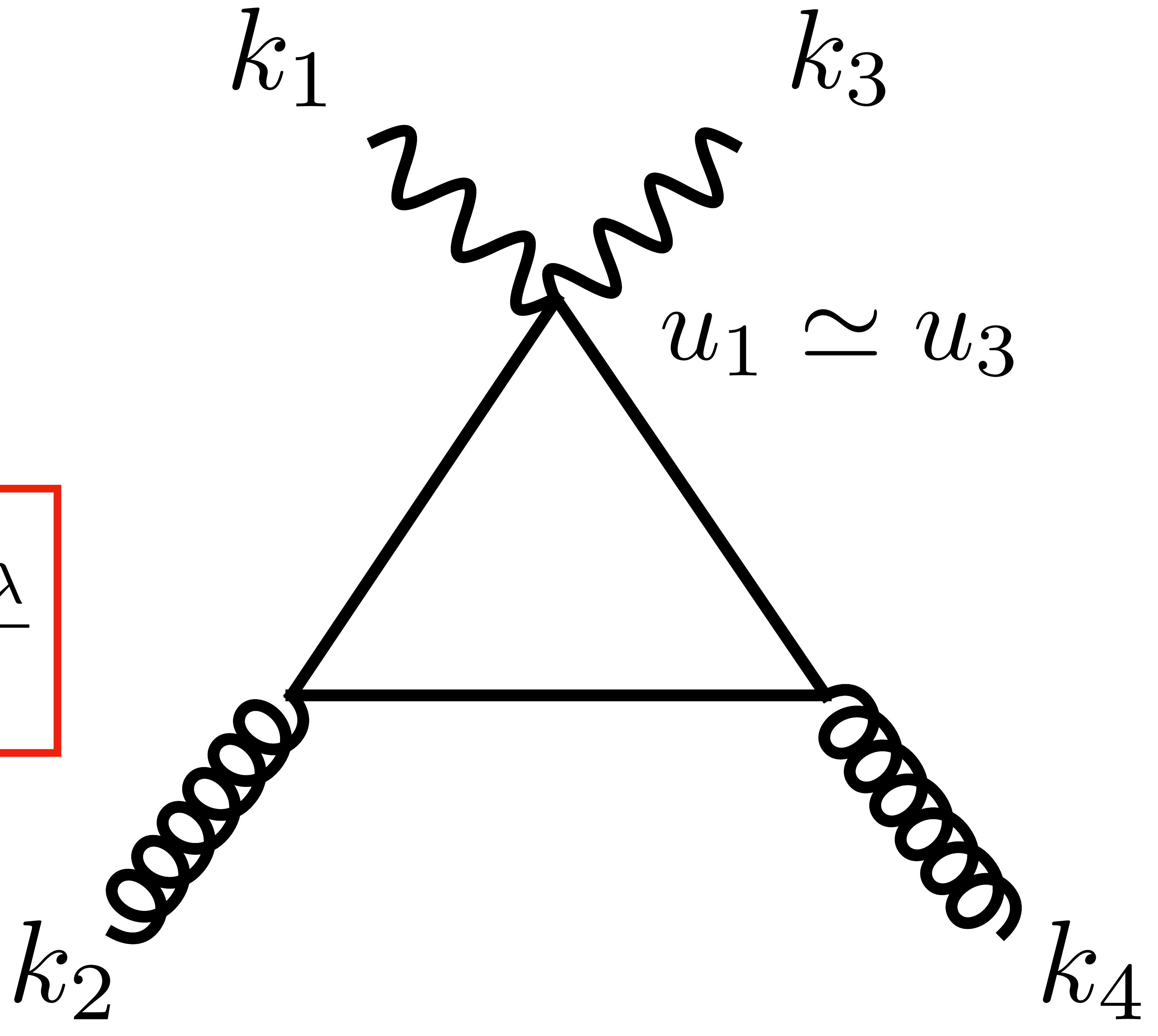
$$\Gamma_A^{\mu\nu\alpha\beta}[k_1, k_3, k_2, k_4]\Big|_{Q^2\to\infty}$$

$$= -2\frac{g^2e^2e_f^2}{\pi^2}\epsilon^{\mu\nu\eta}{}_{\kappa}(k_{1\eta} - k_{3\eta})(2\pi)^4\delta^{(4)}(\sum_{i=1}^{4} k_i)\boxed{\frac{(k_2^\kappa + k_4^\kappa)\epsilon^{\alpha\beta\sigma\lambda}k_{2\sigma}k_{4\lambda}}{(k_2+k_4)^2}}$$

$$\times \prod_{k=2}^{4}\int_0^1 d\beta_k\ \delta(1 - \sum_{k=2,3,4}\beta_k)\frac{1}{(k_1+k_2)^2\beta_3 + k_1^2\beta_4 + k_3^2\beta_2}$$

triangle diagram

Kinematic factor, dependence on x_B and Q^2



$$u_1 - u_3 \sim \Lambda^2_{\mathrm{QCD}}/Q^2$$

The structure function g_1 in the Bjorken limit

infrared pole — non-local operator — non-pole contribution

$$S^\mu g_1(x_B,Q^2)\Big|_{Q^2\to\infty}$$

$$= \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P',S|\mathrm{Tr_c} F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{Q^2}\right)$$

The structure function g_1 is dominated by the triangle anomaly, hence g_1 is topological.

First moment of g_1:

$$S^\mu \int_0^1 dx_B\, g_1(x_B,Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_S}{2i\pi M_N}\lim_{l_\mu\to 0}\frac{l^\mu}{l^2}\langle P',S|\mathrm{Tr_c} F_{\alpha\beta}(0)\tilde{F}^{\alpha\beta}(0)|P,S\rangle$$

matches calculation of the triangle diagram

The pole must be removed by contribution of the pseudo scalar sector

The structure function g_1 in the Regge limit

Tarasov, Venugopalan (arXiv:2008.08104)

In the Regge limit $x_B \to 0$ and Q^2 is fixed

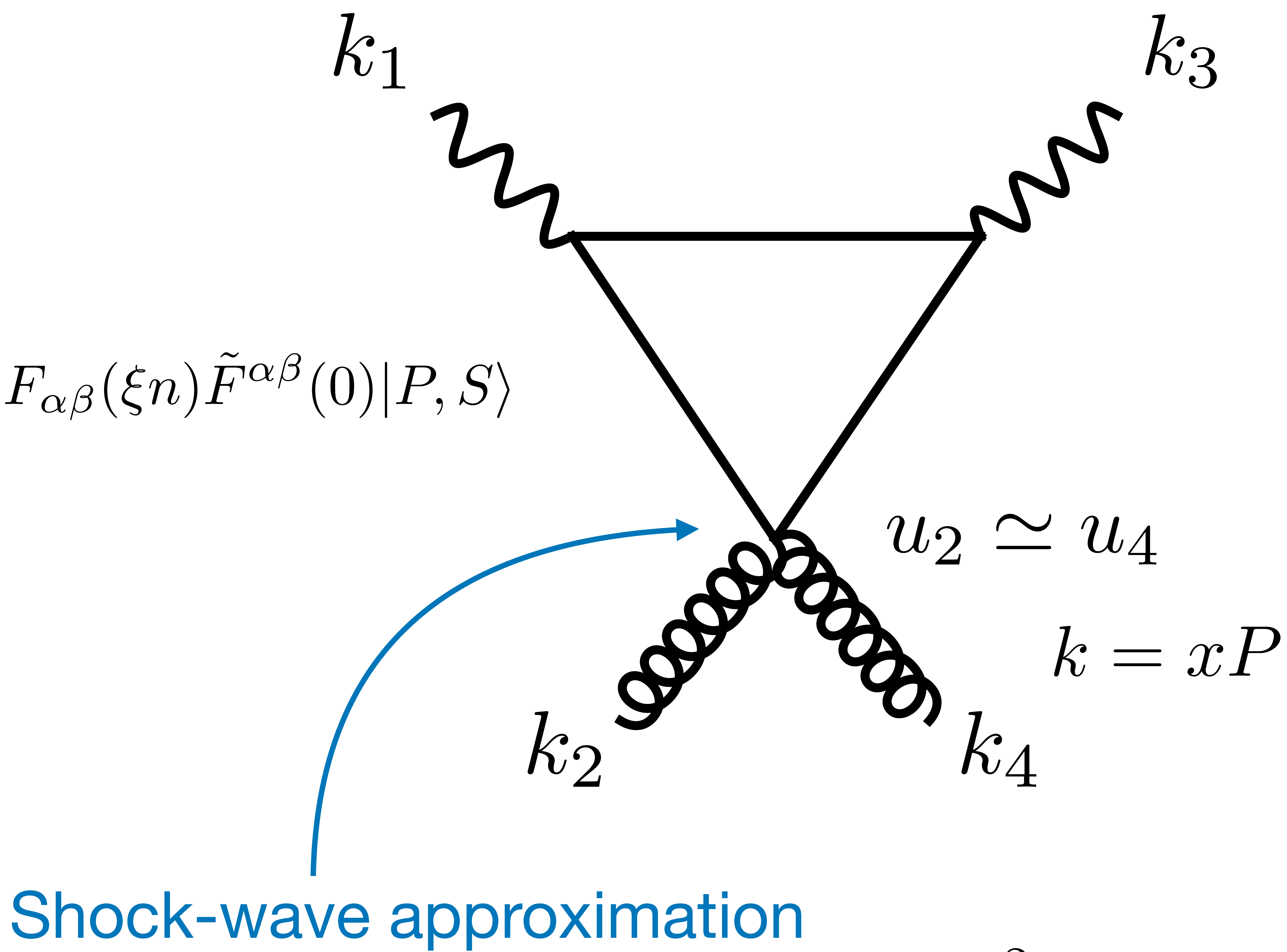
$$S^\mu g_1(x_B, Q^2)\Big|_{Q^2\to\infty}$$

$$= \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x} \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu \to 0} \frac{l^\mu}{l^2} \langle P', S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P, S\rangle$$

In Regge asymptotics: $x > x_B$ and $x_B \to 0$

The result for $g_1(x_B, Q^2)$ is formally identical in Bjorken and Regge limits

$g_1(x_B, Q^2)$ is topological in both asymptotic limits of QCD



$$u_2 - u_4 \sim \frac{Q_s^2}{2xP\cdot q} \sim x_B$$

The structure function g_1 in the Regge limit

Tarasov, Venugopalan (arXiv:2008.08104)

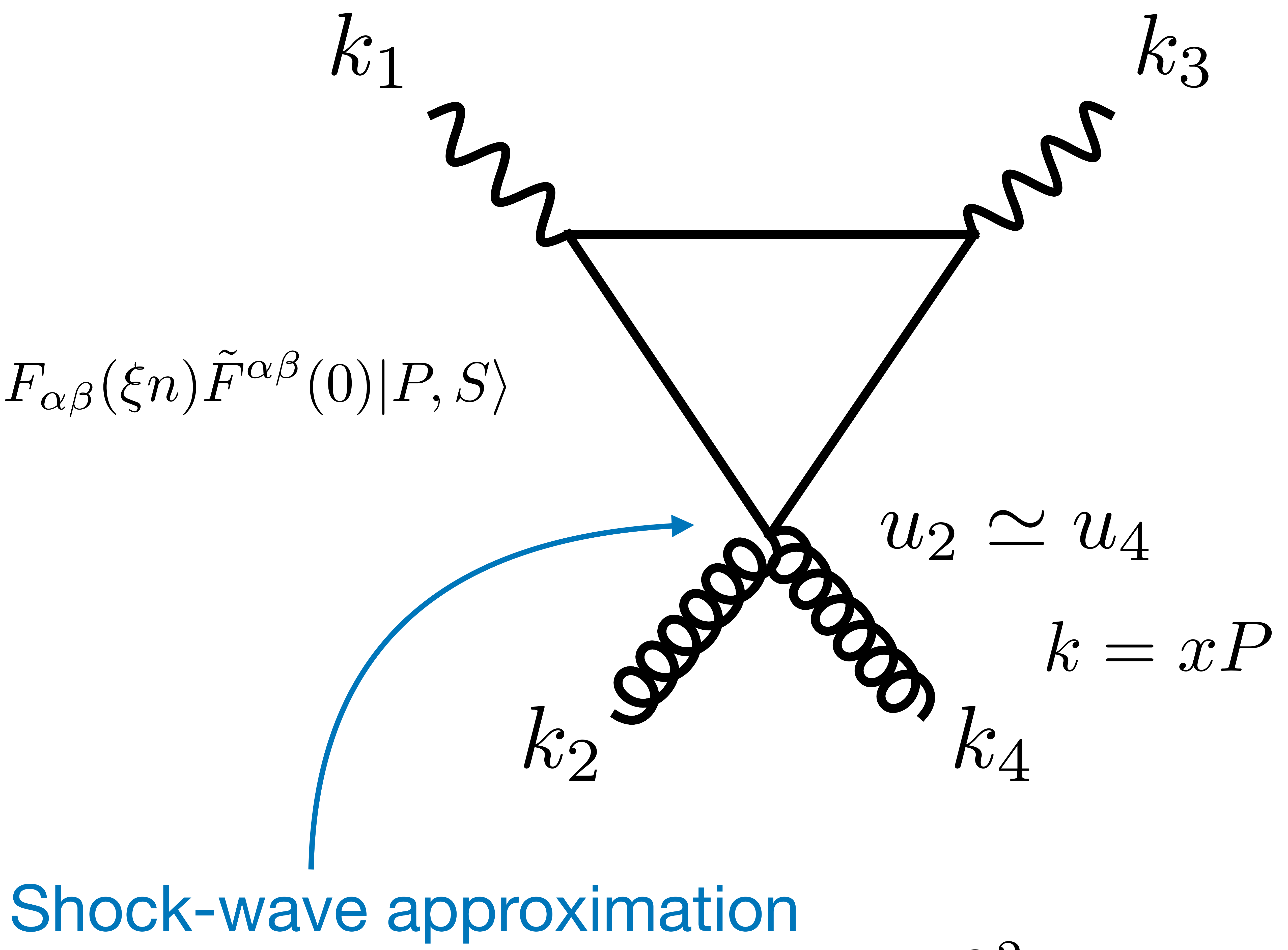
In the Regge limit $x_B \to 0$ and Q^2 is fixed

$$S^\mu g_1(x_B, Q^2)\Big|_{Q^2\to\infty}$$

$$= \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x} \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu \to 0} \frac{l^\mu}{l^2} \langle P', S | \mathrm{Tr_c} F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) | P, S \rangle$$

In Regge asymptotics: $x > x_B$ and $x_B \to 0$

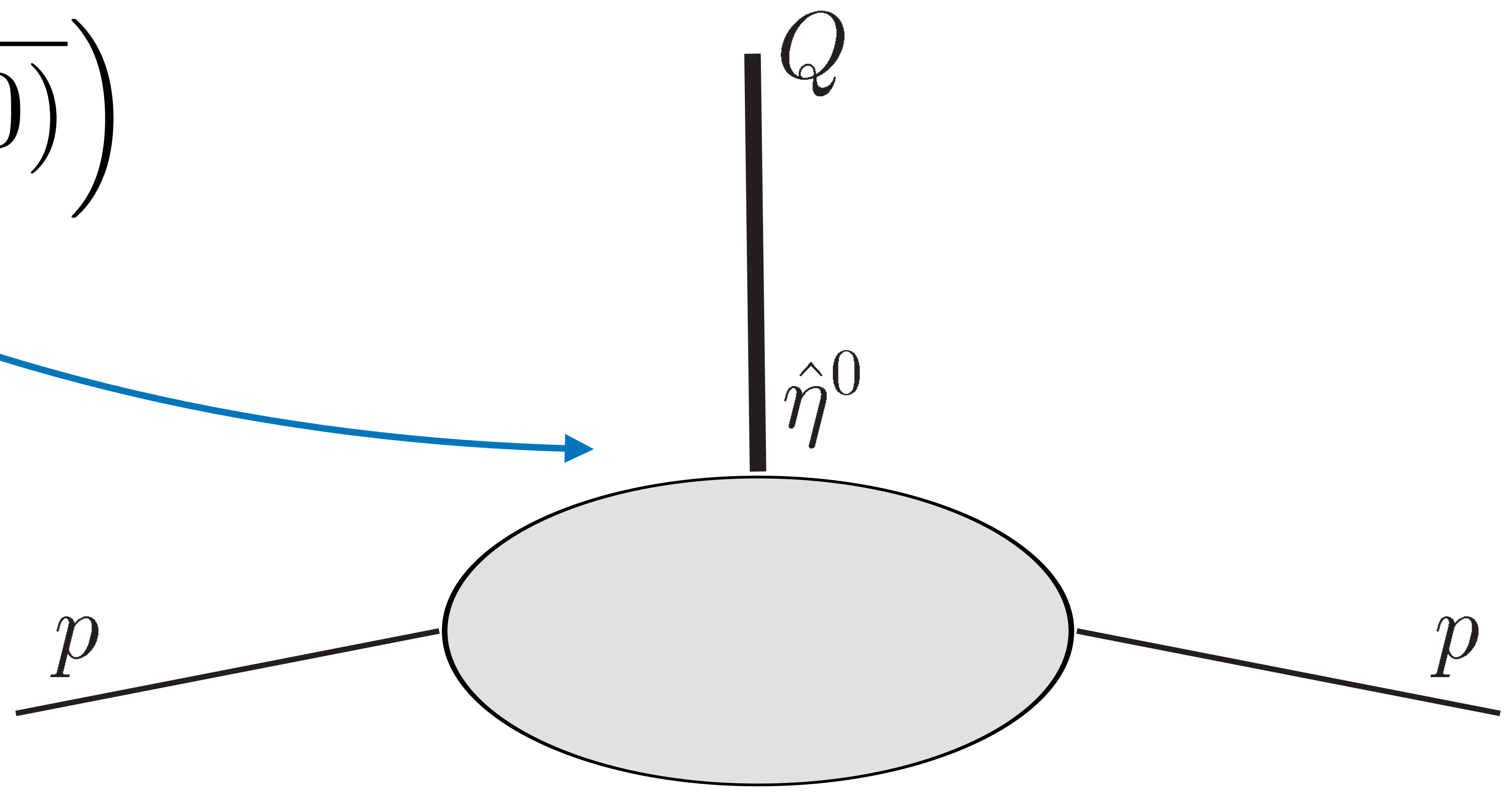
To get this result in high-energy OPE one has to include sub-sub-eikonal corrections. Eikonal expansion breaks down!



$$u_2 - u_4 \sim \frac{Q_s^2}{2xP \cdot q} \sim x_B$$

Anomalous chiral Ward identities

$$\Sigma(Q^2) = \frac{1}{3m_N}\Delta C_1^S(\alpha_S)\left(g_{QNN}\chi(0) + g_{\eta' NN}\sqrt{\chi'(0)}\right)$$
$$\approx 0$$



Topological charge density:

$$Q(x) = \frac{\alpha_S}{8\pi} Tr\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

$$a^0|_{Q^2=10GeV^2} = 0.33 \pm 0.05$$

Topological susceptibility:

$$\chi_{\rm YM}(p^2) = i\int dx\, e^{iP\cdot x}\langle 0|T(Q(x)Q(0)|0\rangle$$

In agreement with COMPASS and HERMES data

In the chiral limit $\chi(0) \to 0$, the slope $\chi'(0)$ is estimated to be small. Topological charge screening of spin.

Shore, Veneziano (1990)
Narison, Shore, Veneziano, hep-ph/9812333
G. M. Shore, hep-ph/0701171
S. D. Bass, hep-ph/0411005

Imaginary part of the effective action

QCD effective action:

$$W = W_R + iW_I$$

unpolarized scattering

anomaly effects,
polarized scattering

Since the anomaly comes from the measure of the QCD path integral, its effect can be extracted from the most general extension of the imaginary part of the QCD effective action

D'Hoker, Gagne,hep-th/9508131

$$W_{\Im}[\Phi, \Pi, A] = \frac{1}{8} \int_{-1}^{1} d\alpha \int_{0}^{\infty} dT\, \mathcal{N} \int_{\mathrm{PBC}} \mathcal{D}x\, \mathcal{D}\psi\, \mathrm{tr}_c\, \mathcal{J}(0)\, \mathcal{P}\, e^{-\int_0^T d\tau \mathcal{L}_\alpha}$$

(pseudo-)scalar fields
cancel the infrared pole

Pseudoscalar contribution

To take into account contribution of the pseudoscalar sector we use the most general form of the imaginary part of the effective action

$$W_{\Im}[\Phi, \Pi, A] = \frac{1}{8}\int_{-1}^{1} d\alpha \int_{0}^{\infty} dT\, \mathcal{N} \int_{\mathrm{PBC}} \mathcal{D}x\, \mathcal{D}\psi\, \mathrm{tr}_c\, \mathcal{J}(0)\, \mathcal{P}\, e^{-\int_0^T d\tau \mathcal{L}_\alpha}$$

D'Hoker, Gagne,hep-th/9508131

where the worldline Lagrangian

scalar field

pseudoscalar field

$$\mathcal{L}_\alpha = \frac{\dot{x}^2}{2\mathcal{E}} + \frac{1}{2}\psi_A\dot{\psi}_A - i\dot{x}\cdot A + \frac{i}{2}\mathcal{E}\psi_\mu F_{\mu\nu}\psi_\nu + \frac{1}{2}\mathcal{E}\alpha^2\Phi^2 + \frac{1}{2}\mathcal{E}\Pi^2$$

$$+ i\alpha\mathcal{E}\psi_\mu\psi_6 D_\mu\Phi + \boxed{i\mathcal{E}\psi_\mu\psi_5 D_\mu\Pi} + \alpha\mathcal{E}\psi_5\psi_6[\Pi,\Phi] + \text{axial vector coupling}$$

Generates the iso-singlet Wess-Zumino-Witten term $\propto \eta_0 F\tilde{F}$

Leutwyler (1996); Herrara-Sikody et al (1997); Leutwyler-Kaiser (2000)

With this form of the effective action, we see explicitly how the pole of the anomaly is canceled by η_0 exchange

Axion-like effective action

At small-x, the gluon field couples to a large number of quark (world-line trajectories), one can construct from their density matrix the effective action for an ensemble of spinning, colored partons

$$g_1(x_B, Q^2) \propto \int \mathcal{D}\rho W_Y^P[\rho] \int \mathcal{D}\eta_0 \tilde{W}_Y^{P,S}[\eta_0]$$

$$\times \int \mathcal{D}A\, Q\, \exp\left(iS_{YM}[A] + \frac{i}{N_c}\mathrm{Tr}_c(\rho U_{[-\infty,\infty]})\right)$$

$$\times \exp\left(\int d^4x\left(-\frac{Q^2}{2\chi_{YM}} - \sqrt{\frac{N_c}{2}}Q\eta_0 + \frac{1}{2}F^2\eta_0\partial^2\eta_0\right)\right)$$

proton state

MV effective action

McLerran, Venugopalan (1994)

axion-like effective action

Veneziano, Mod. Phys. Lett. (1989)
Hatsuda, PLB (1990)

Tarasov, Venugopalan, in preparation

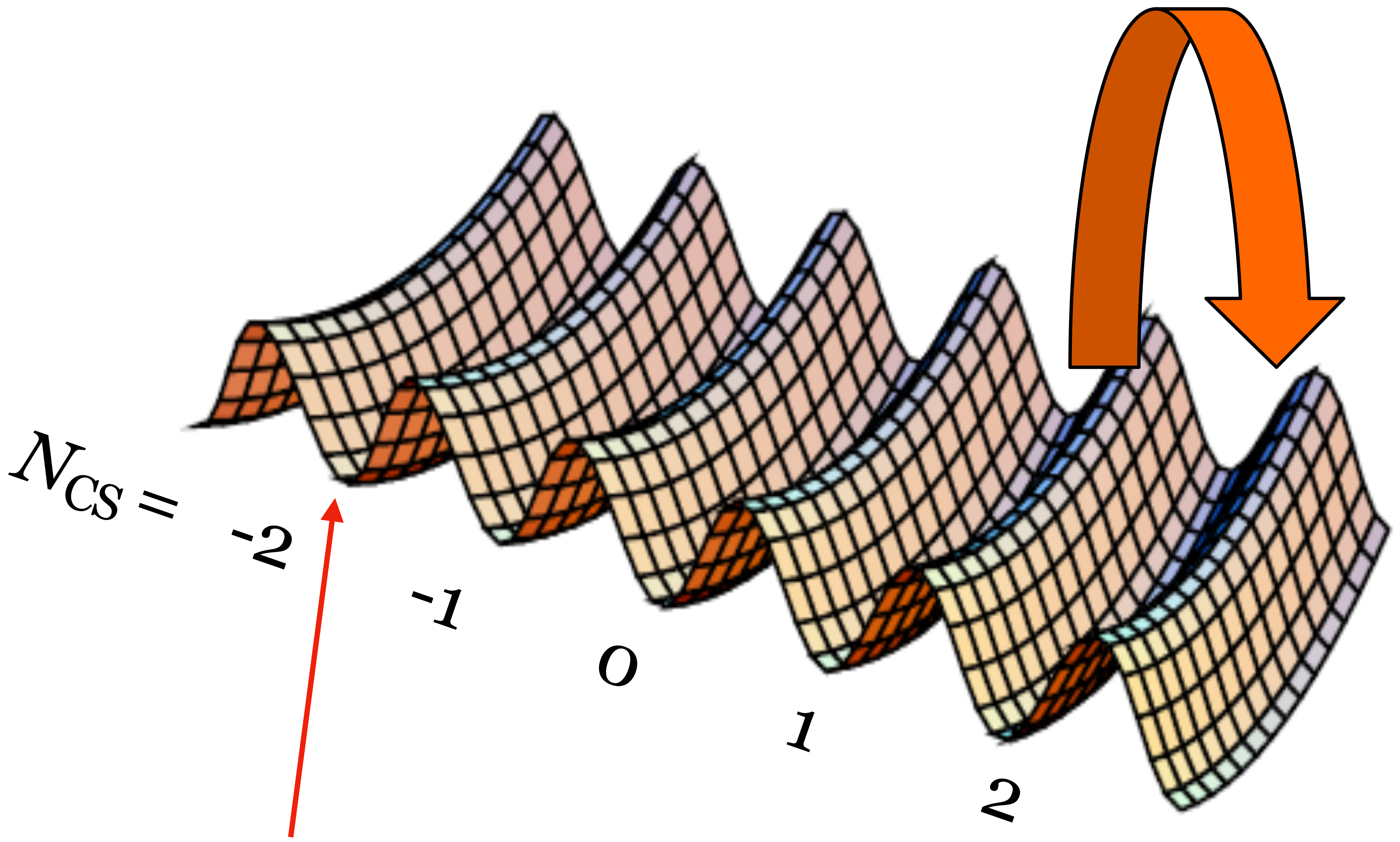
$$m_{\eta'}^2 = 2n_f \frac{\chi_{YM}}{F^2}$$

Topological transitions at small-x

Because of the presence of two scales - mass of the η' and saturation scale, g_1 can be sensitive to real-time topological transitions (sphaleron transitions)

$$m_{\eta'}^2 = 2n_f \frac{\chi_{YM}}{F^2}$$

$$Q_s^2 > m_{\eta'}^2$$



Over the barrier transition between different topological sectors of QCD vacuum can lead to oscillation in g_1 at small-x

Summary

- We show that the anomaly appears in both the Bjorken limit of large Q^2 and in the Regge limit of small x_B. We find that the infrared pole in the anomaly arises in both limits

- The cancellation of the pole involves a subtle interplay of perturbative and nonperturbative physics that is deeply related to the $U_A(1)$ problem in QCD

- The cancelation of the pole gives the leading contribution to g_1 given by the expectation value of the topological susceptibility and its derivative

- We introduce a small x effective action that follows from the cancellation of the infrared pole in the matrix element of the anomaly. This effective action, consistent with anomalous chiral Ward identities, is controlled by two dimensionful scales in Regge asymptotics. The first is the color charge squared per unit area, while the second is the pure Yang-Mills topological susceptibility

- The spin diffusion at small-x, because of the presence of two scales, can be sensitive to real-time topological transitions (sphaleron transitions) which open possibilities to study it at EIC