The Ohio State University

Center for Frontiers in Nuclear Science

Rapidity evolution of the gluon helicity at small-x

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In collaboration with F. Cougoulic, Y. Kovchegov, Y. Tawabutr

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Spin at small-x

A lot of progress in perturbative approach to spin at small-x

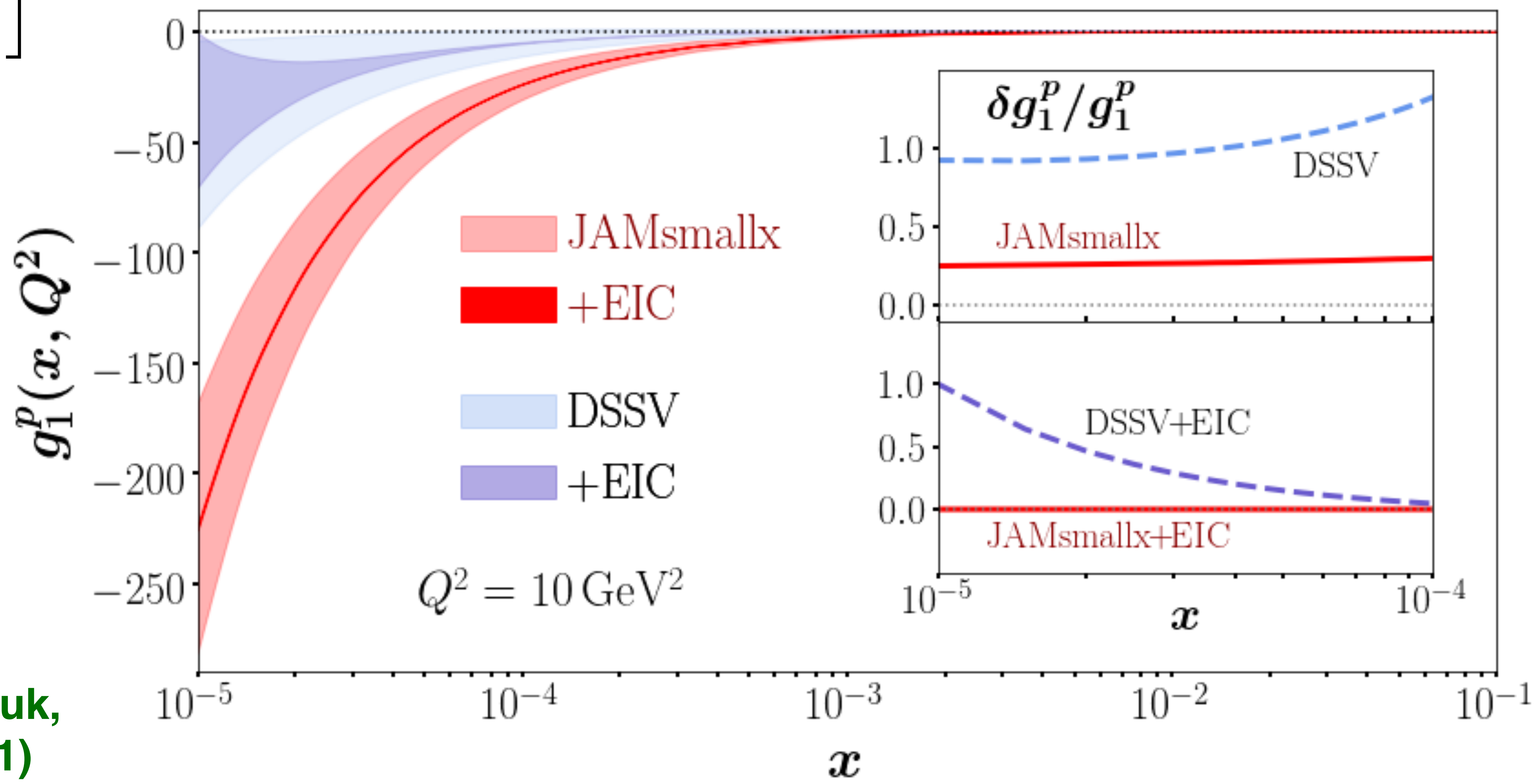
Y. V. Kovchegov, D. Pitonyak and M. D. Sievert (16-19); T. Altinoluk, N. Armesto, G. Beuf, M. Martinez and C. A. Salgado (2014);Y. Hatta, Y. Nakagawa, F. Yuan, Y. Zhao and B. Xiao (2017); G. A. Chirilli (19-20); R. Boussarie, Y. Hatta and F. Yuan (2019); F. Cougoulic and Y. V. Kovchegov (19-20); T. Altinoluk, G. Beuf, A. Czajka and A. Tymowska (2012)

Oscillations in g_1 at small-x:

$$\Delta\Sigma(x,Q^2)\Big|_{\text{large-}N_c\&N_f} \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \cos\left[\omega_q \ln\left(\frac{1}{x}\right) + \varphi_q\right]$$

Y. V. Kovchegov and Y. Tawabutr (2020)

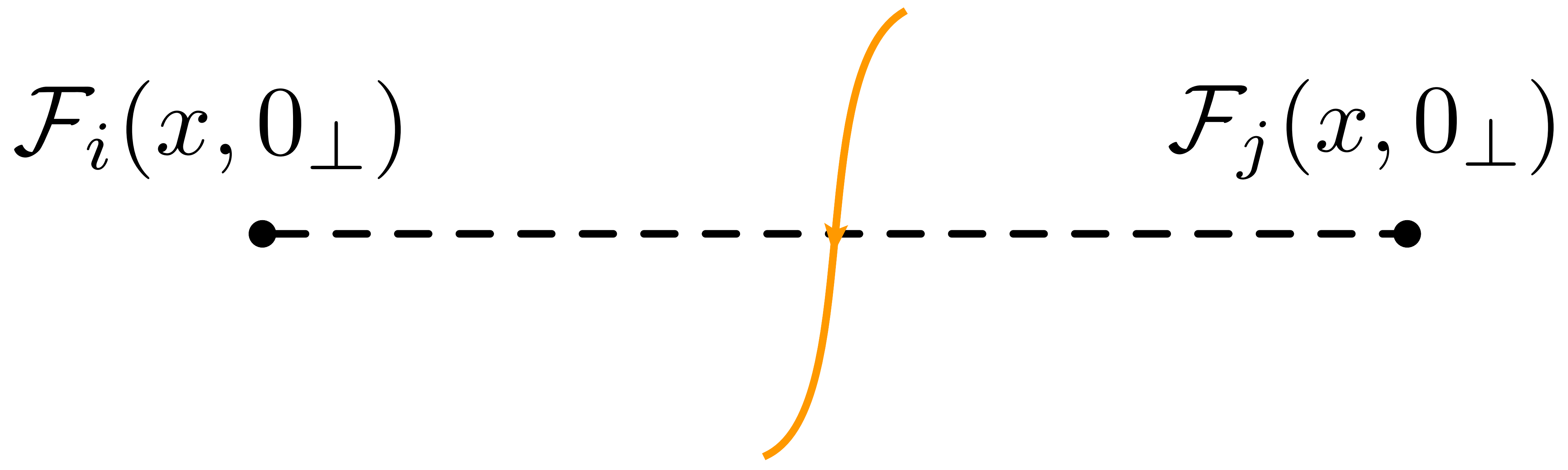
There is still a lot of questions. How does the anomaly appear in this calculations (see Tarasov and Venugopalan, 2020)? What about connection to DGLAP evolution at small-x?



D. Adamiak, Y. V. Kovchegov, W. Melnitchouk, D. Pitonyak, N. Sato and M. D. Sievert (2021)

Two types of operators

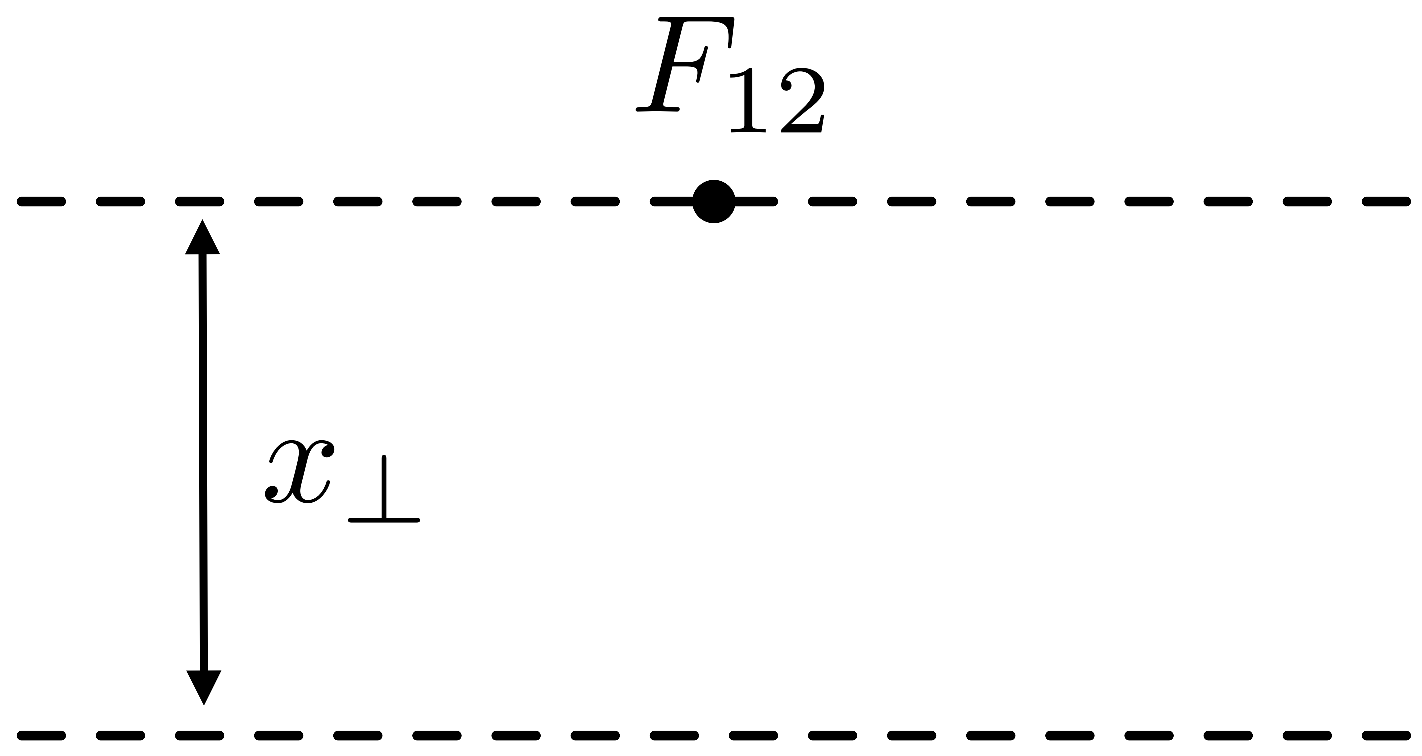
Two different types of operators generating gluon helicity can be found in large- and small-x literature
Gluon helicity at large-x is defined as a matrix element of a collinear operator:



$$\epsilon^{ij} \mathcal{F}_i^a(x, 0_\perp) \mathcal{F}_j^a(x, 0_\perp)$$

$$\mathcal{F}_i^a(x, z_\perp) \equiv \int dz^- e^{ixP^+z^-} [\infty, z^-]_z^{am} F_{-i}^m(z^-, z_\perp)$$

at the same time at small-x it is associated with a polarized dipole operator:

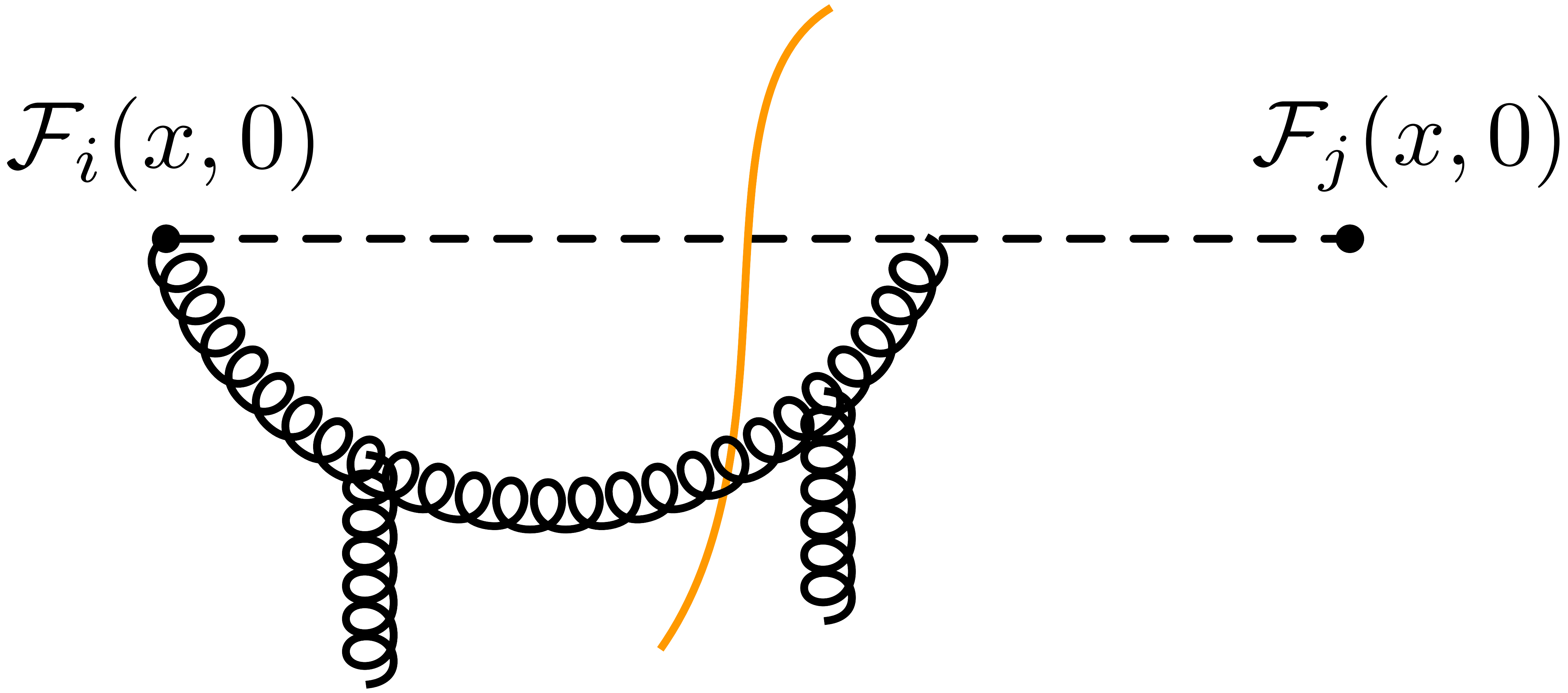


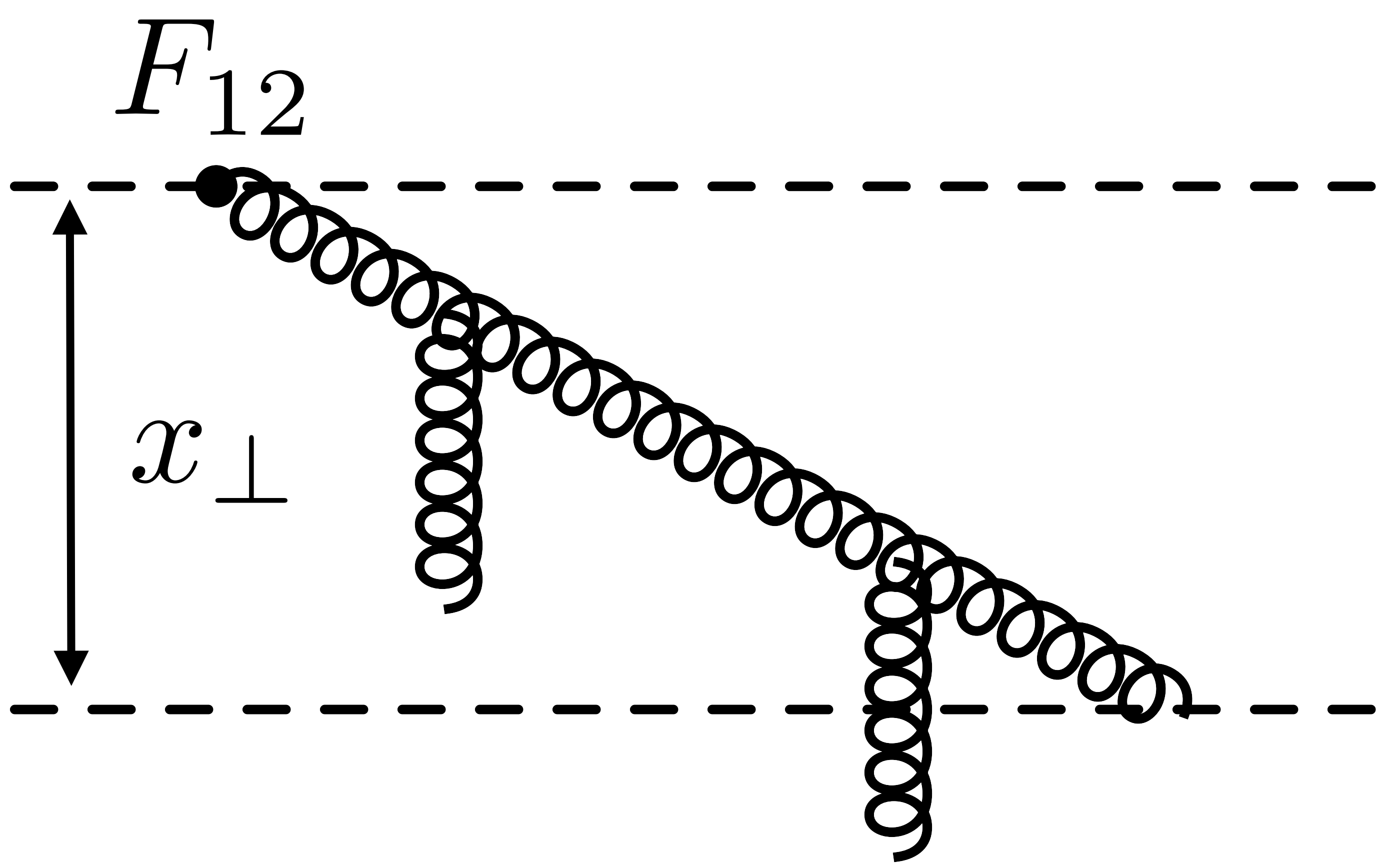
$$\mathrm{Tr_c}\big(U^{pol}(x_\perp) U^\dagger(0_\perp)\big)$$
$$\equiv \frac{iP^+\epsilon^{ij}}{s} \int dz^- \big([\infty, z^-]_x F_{ij}(z^-, z_\perp)[z^-, -\infty]\big)^{ab} [-\infty, \infty]_0^{ba}$$

Both operators are antisymmetric in transverse Lorentz indexes.
We want to better understand relation between two structure

NLO correction

One can expect a mixing between two operators at the NLO order at small-x





The result of calculation of a diagram has a structure $D = C \otimes \mathcal{O}(B)$, where B is the external field, and C is a coefficient. We want to find the form of operator $\mathcal{O}$, and check whether an initial operator mixes with another one.

Background field method

Background field method is a powerful framework for calculation of the Feynman diagrams which allows to understand the operator structure of the external (background) fields

Matrix element of a product of operators:

$$\langle p_1|\mathcal{O}_1(A)\dots\mathcal{O}_n(A)|p_2\rangle = \int \mathcal{D}A\ \Psi^*_{p_1}(A)\mathcal{O}_1(A)\dots\mathcal{O}_n(A)\Psi_{p_2}(A)e^{iS_{QCD}(A)}$$

Separate different modes of the system $A = C + B$:

$$\langle p_1|\mathcal{O}_1(A)\dots\mathcal{O}_n(A)|p_2\rangle$$

$$= \int \mathcal{D}B\ \Psi^*_{p_1}(B)\left(\int \mathcal{D}C\ \mathcal{O}_1(C{+}B)\dots\mathcal{O}_n(C{+}B)e^{iS_{QCD}(C+B)-iS_{QCD}(B)}\right)\Psi_{p_2}(B)e^{iS_{QCD}(B)}$$

Abbott (1982)

We calculate the integral over C fields in the fixed background of B fields

QCD Lagrangian in the background B field:

$$S_{bQCD}(C,B) = S_{QCD}(C+B) - S_{QCD}(B)$$

Gluon propagator in the background field

QCD Lagrangian in the background B field:

$$S_{bQCD}(C, B) = S_{QCD}(C + B) - S_{QCD}(B) - \frac{1}{2}(\mathcal{D}^\mu C_\mu)^2$$

↓ background-Feynman gauge

gauge-fixing term

$$i\langle A^a_\mu(x) A^b_\nu(y)\rangle = (x|\frac{1}{\mathcal{P}^2 + 2iF_B + i\epsilon}|y)^{ab}_{\mu\nu}$$

$\mathcal{P}_\mu = p_\mu + gB_\mu$

scalar interaction (scalar QED)

"spin" term

One can use different gauges for the "quantum" and background fields. For the background field we choose $B_+ = 0$. Otherwise, it's perfectly arbitrary.

The result of computation of a diagram D has a form

$$D = C \otimes \mathcal{O}(B)$$

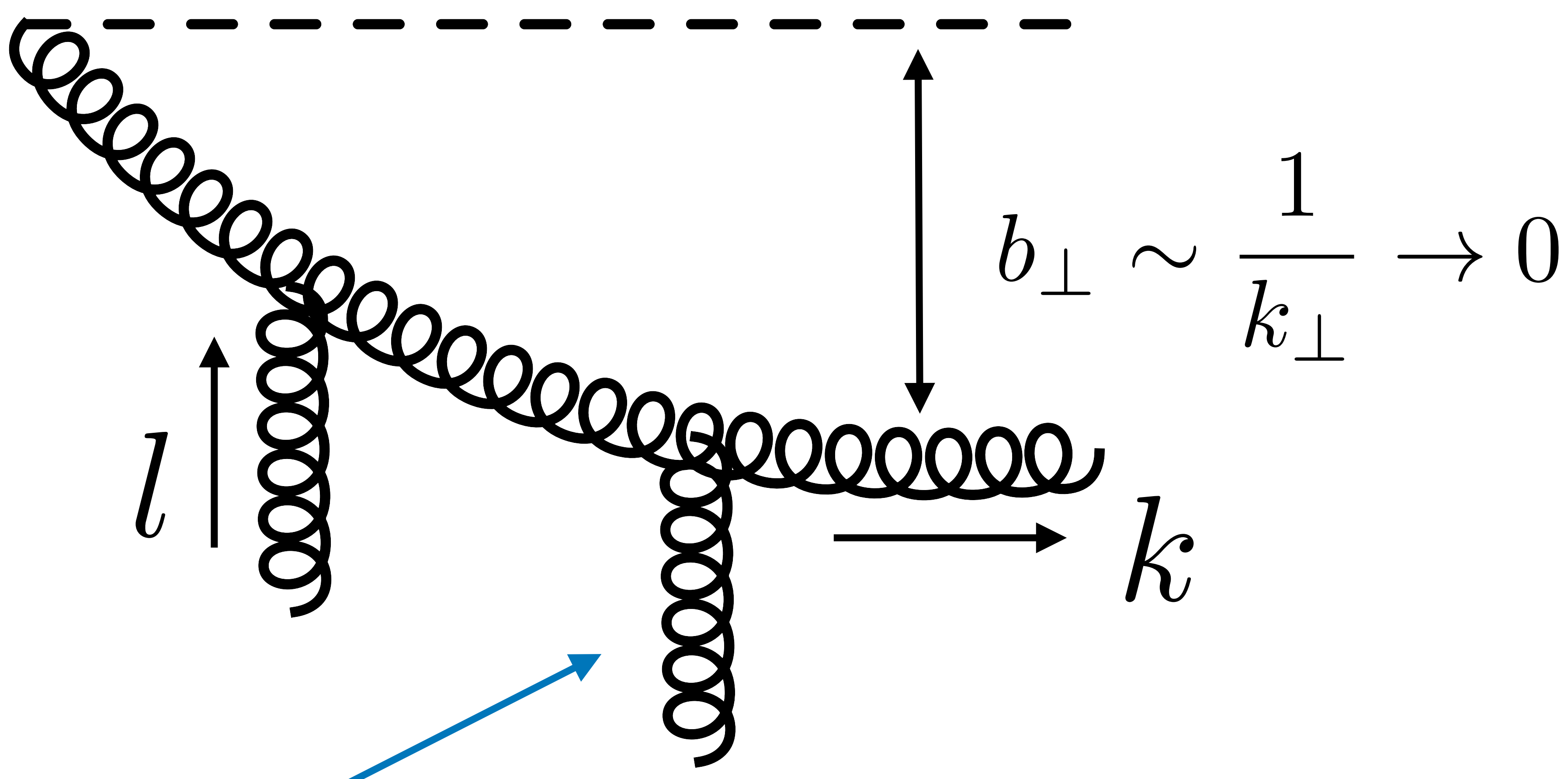
The structure is similar to the one of the worldline approach

$$W[A] = -\frac{1}{2}\mathrm{Tr}_c \int_0^\infty \frac{dT}{T} \int \mathcal{D}x \int \mathcal{D}\psi \exp\left\{-\int_0^T d\tau \left(\frac{1}{4}\dot{x}^2 + \frac{1}{2}\psi_\mu \dot{\psi}^\mu + ig\dot{x}^\mu A_\mu - ig\psi^\mu\psi^\nu F_{\mu\nu}\right)\right\}$$

scalar interaction
Wilson line

"spin" term

Collinear expansion of the gluon propagator



In the collinear approximation, $k_\perp \gg l_\perp$, a typical deviation from the light-cone direction is small: $b_\perp \to 0$

One can expand the background fields onto the light-cone:

$$B_\mu(x) = B_\mu(y) + (x-y)^\alpha \partial_\alpha B_\mu(y) + \ldots$$

Balitsky, Braun (88-89)

In the case of a general field B_μ, the leading terms of the expansion contains three types of operators of the background field B_μ: Wilson lines, F_{-i}, and F_{ij}

see Balitsky, Tarasov (15-16)

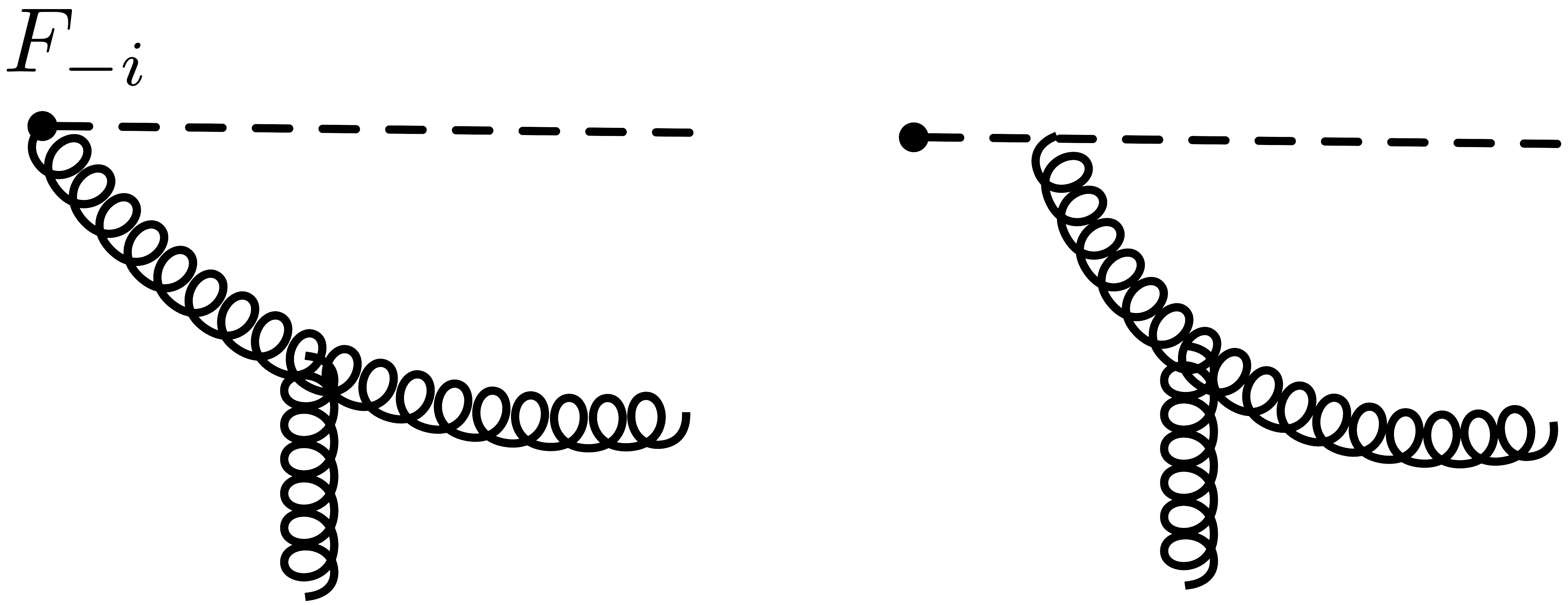
$$\lim_{k^2 \to 0} k^2 \langle A^a_\mu(k) A^b_\nu(y) \rangle$$

$$= -ie^{i\frac{k_\perp^2}{2k^-}y^-} e^{-ik_\perp y_\perp} \Big\{ g_{\mu\nu}[\infty, y^-]^{ab}_{y_\perp} + \frac{1}{k^-}\int_{y^-}^{\infty} dz^- \Big(-ik^j (z-y)^- g_{\mu\nu}[\infty, z^-]_{y_\perp} F_{-j}[z^-, y^-]^{ab}_{y_\perp}$$

$$+(p_{2\nu}\delta^j_\mu - p_{2\mu}\delta^j_\nu)[\infty, z^-]_{y_\perp} F_{-j}[z^-, y^-]^{ab}_{y_\perp} \Big) - \boxed{\frac{1}{k^-}\int_{y^-}^{\infty} dz^- [\infty, z^-]_{y_\perp} F^\perp_{\mu\nu}[z^-, y^-]^{ab}_{y_\perp}} \Big\}$$

New term, gives rise to the small-x helicity evolution

NLO correction to the large-x gluon helicity operator



To calculate the NLO correction to the operator

$$\epsilon^{ij}\mathcal{F}_i^a(x,0_\perp)\mathcal{F}_j^a(x,0_\perp)$$

let's start with a correlator

$$L^{ab}_{\mu i}(k,y_\perp,x) = i\lim_{k^2\to 0} k^2\langle A^a_\mu(k)\mathcal{F}^b_i(x,y_\perp)\rangle$$

To get the NLO correction we need to take a product of two such correlates

$$L^{ab}_{\mu i}(k,y_\perp,x) = \frac{2e^{-ik_\perp y_\perp}}{2xP^+k^- + k_\perp^2}\Big\{\frac{2xP^+k^-}{k_\perp^2}\Big(\frac{k_\perp^2}{2k^-}p_{2\mu} - k^-p_{1\mu}\Big)\delta^j_i - k_i\delta^j_\mu + \frac{2xP^+k^-g_{\mu i}k^j}{2xP^+k^-+k_\perp^2}$$
$$+\frac{2k^-k_ik^j}{2xP^+k^-+k_\perp^2}p_{1\mu}\Big\}\int_{-\infty}^{\infty}dz^- e^{i(xP^+ + \frac{k_\perp^2}{2k^-})z^-}[\infty,z^-]_{y_\perp}F_{-j}[z^-,\infty]^{ab}_{y_\perp}$$
$$+\frac{2xP^+e^{-ik_\perp y_\perp}}{2xP^+k^-+k_\perp^2}\int_{-\infty}^{\infty}dy^- e^{i(xP^+ + \frac{k_\perp^2}{2k^-})y^-}[\infty,y^-]_{y_\perp}F^\perp_{\mu i}[y^-,\infty]^{ab}_{y_\perp}$$

The gluon interacts with the background field via operators with F_{-j} or F_{ij}. At this level the new structure is still there →

Polarized DGLAP

$$\mathcal{F}^a_i(x, z_\perp) \equiv \int dz^- e^{ixP^+z^-} [\infty, z^-]^{am}_z F^m_{-i}(z^-, z_\perp)$$

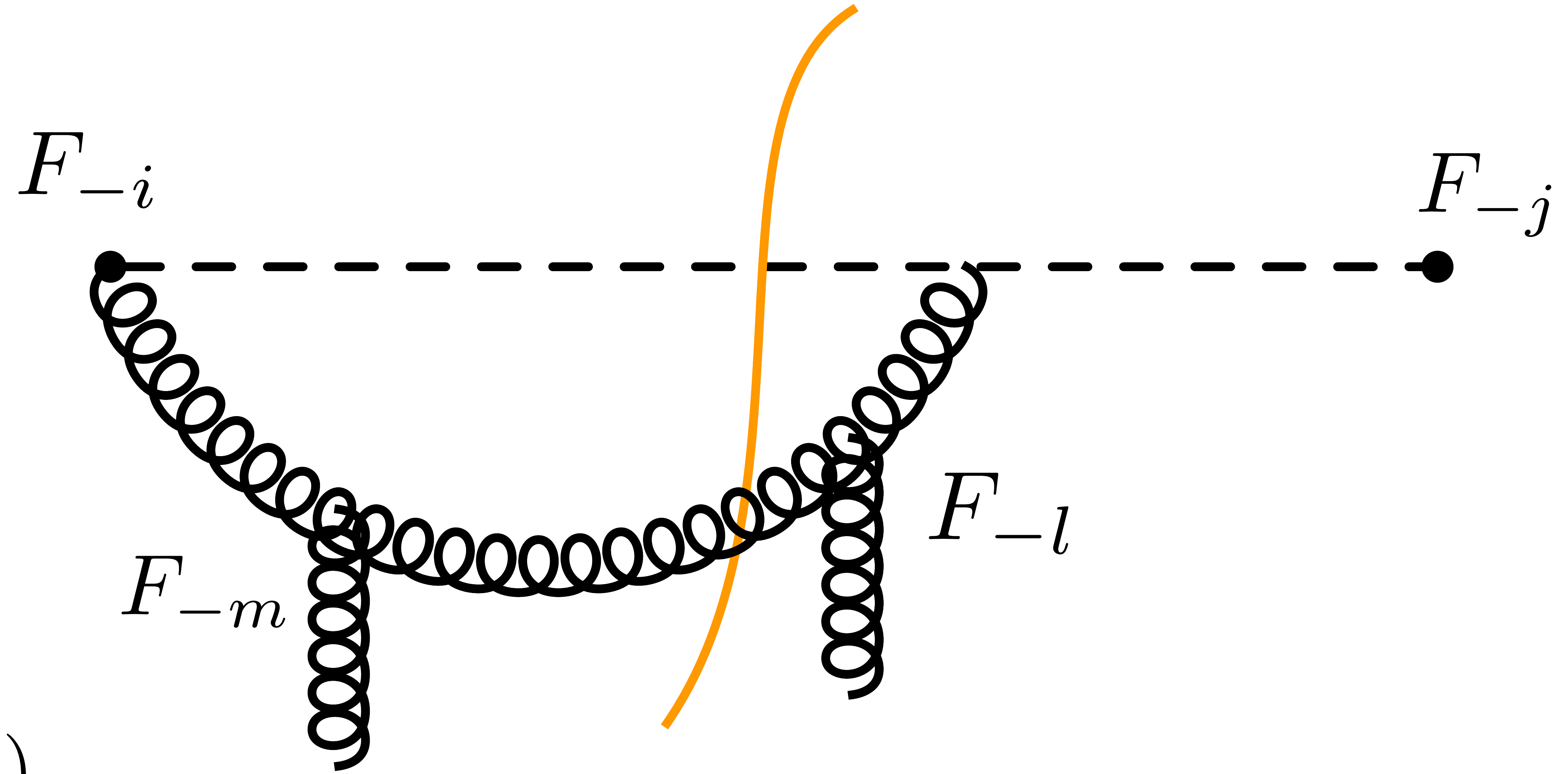
Taking product of operators we find that only F_{-m} operator survives

$$\epsilon_{ij} \langle \tilde{\mathcal{F}}^{ai}(x, 0_\perp) \mathcal{F}^{aj}(x, 0_\perp) \rangle$$

$$= \frac{g^2 N_c}{8\pi^3} \int \frac{dk^-}{k^-} \int d^2 k_\perp \mathcal{K}(k^-, k_\perp, x) \epsilon_{ml} \tilde{\mathcal{F}}^{am}(x + \frac{k_\perp^2}{2P^+k^-}, 0_\perp) \mathcal{F}^{al}(x + \frac{k_\perp^2}{2P^+k^-}, 0_\perp)$$

where $\mathscr{K}$ is a kernel. Introducing variable

$$z \equiv \frac{xP^+}{xP^+ + \frac{k_\perp^2}{2k^-}}$$

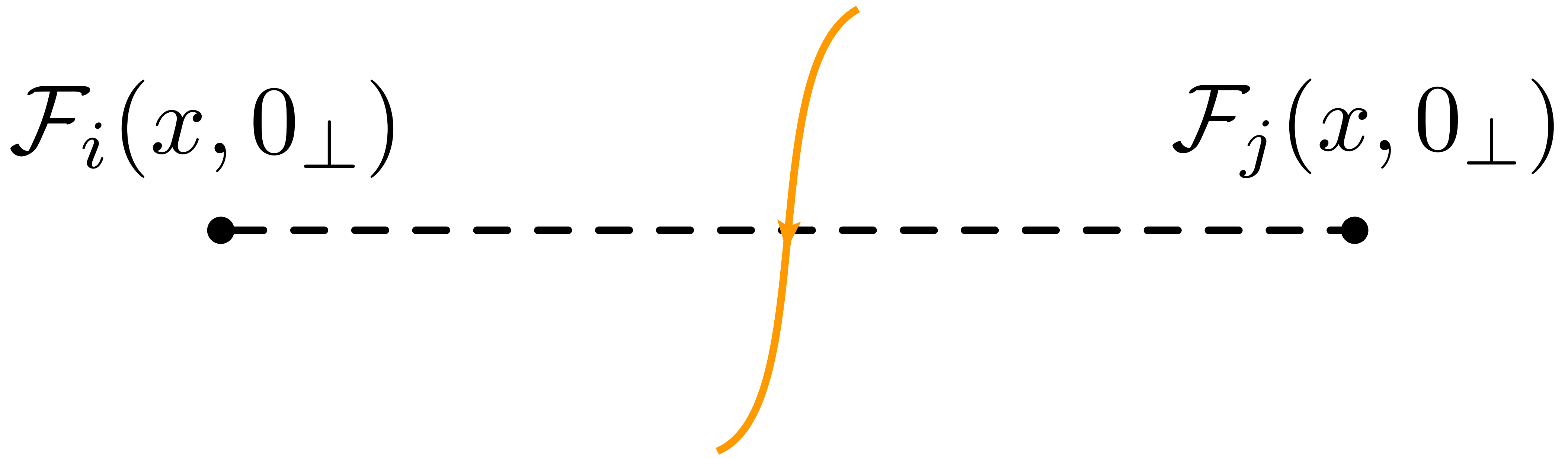


we can rewrite the equation as the polarized DGLAP:

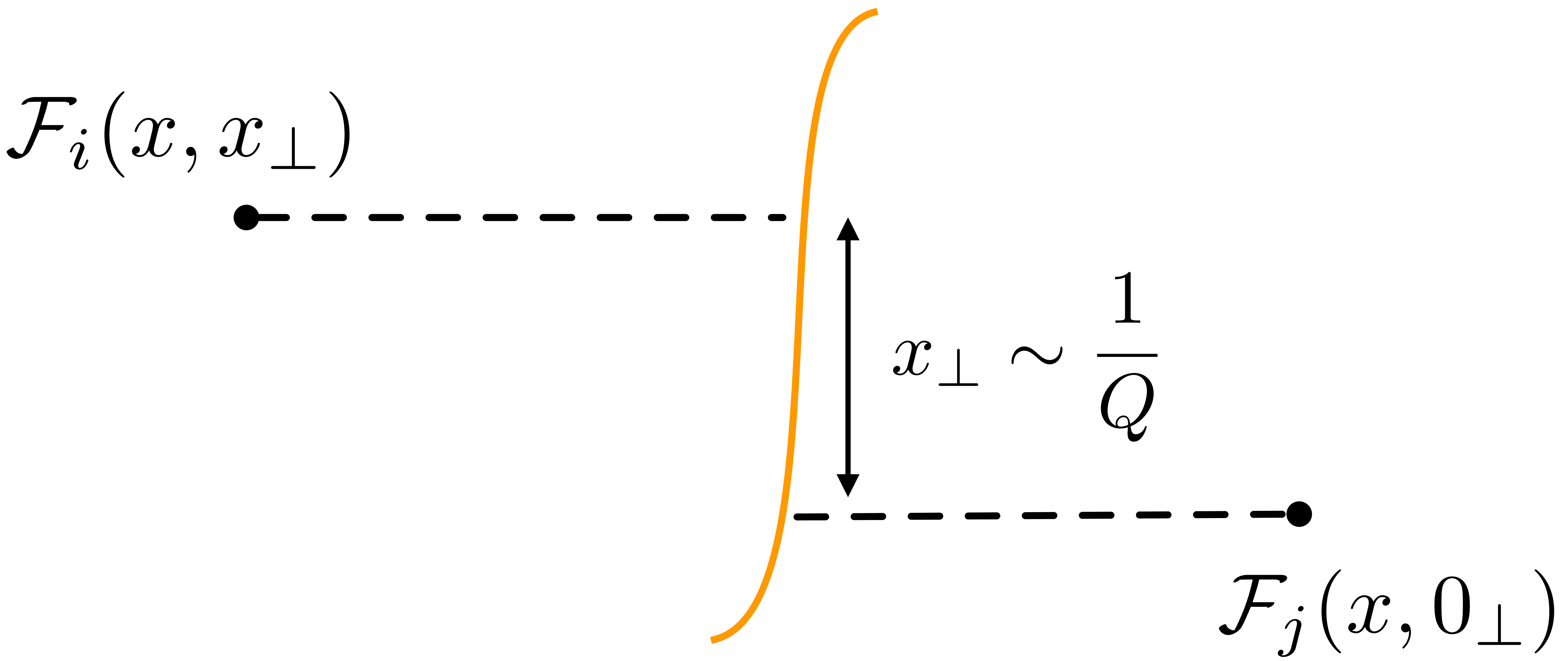
$$\frac{d}{d \ln \mu^2} \Delta g(x) = \frac{g^2 N_c}{4\pi^2} \int_x^1 \frac{dz}{z} \left[\frac{2z^2 - 3z + 2}{1 - z} \right] \Delta g(\frac{x}{z})$$

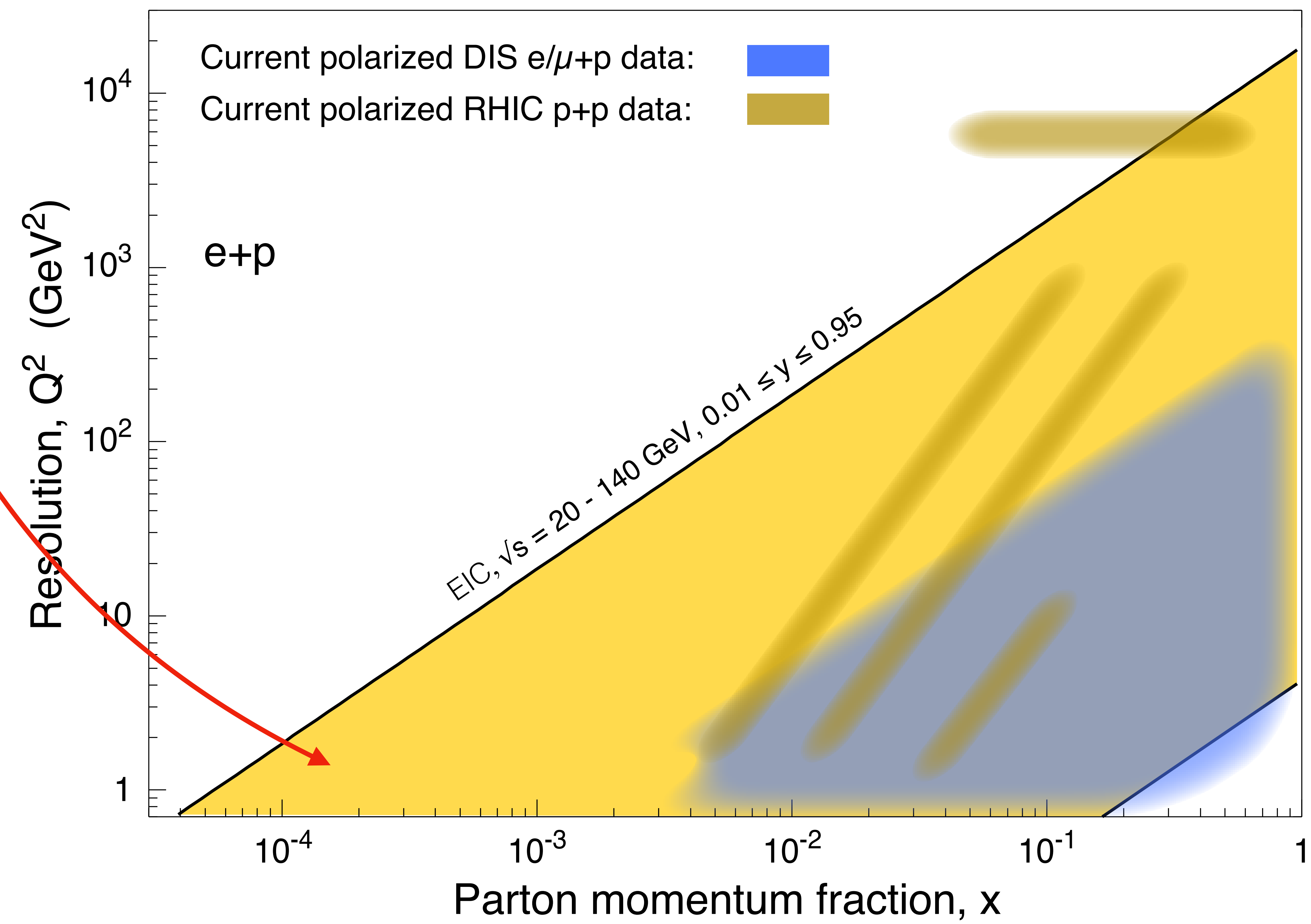
Dipole structure at small-x

It's not a surprise that F_{ij} doesn't contribute in the large-x regime (collinear emission). It's assumed that there is a large scale $Q^2 \to \infty$, which is a reasonable assumption at large-x

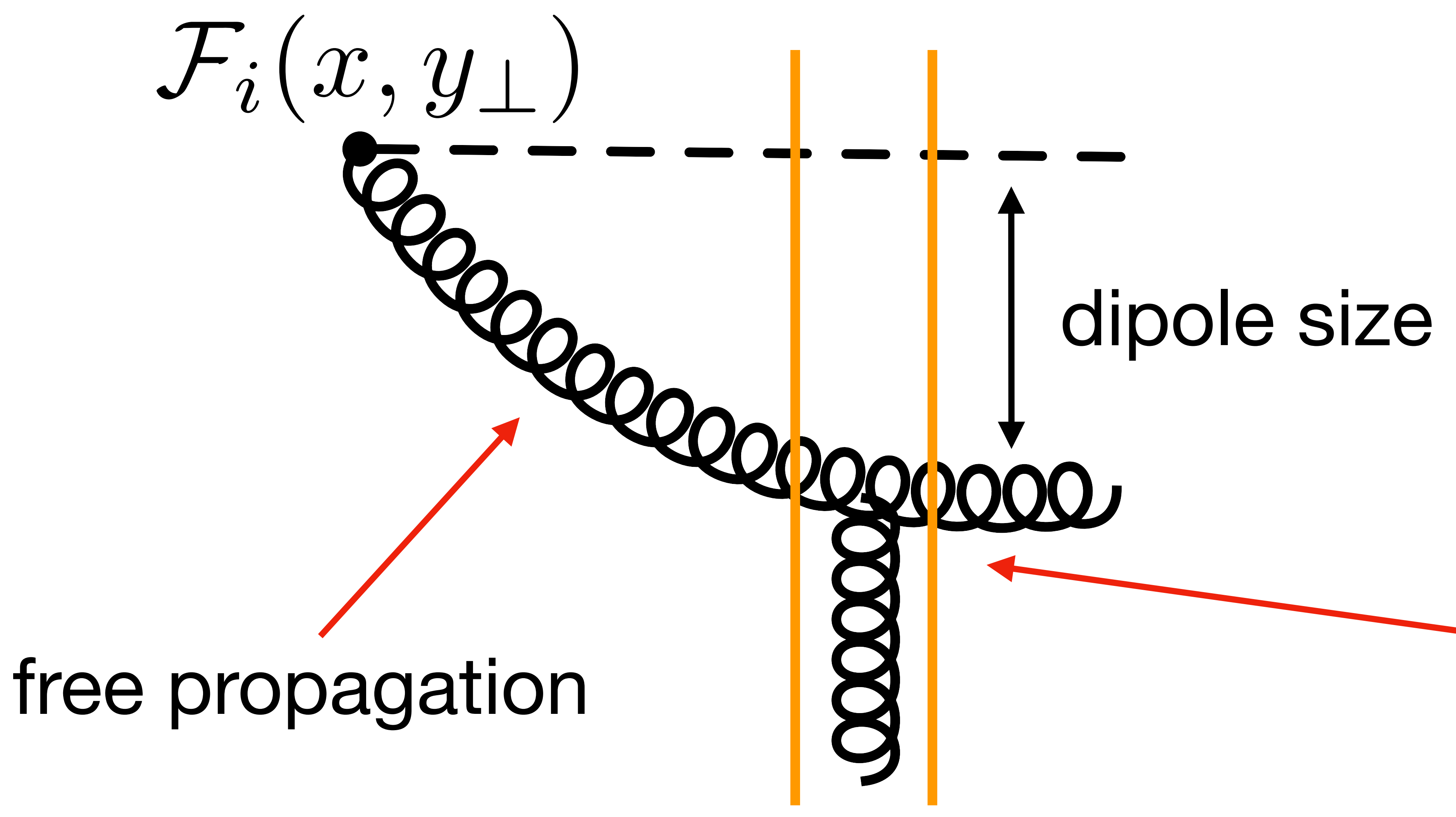


However, in the small-x limit the scale Q^2 is finite. For this reason contribution of higher order twists become important, which can be described by dipole structures





NLO correction ar small-x



Since Q^2 is finite, the collinear expansion breaks down and should be substituted with a resummation in a localized background field (shock-wave).

The expansion contains both DGLAP-type terms, constructed from F_{-k}, and new terms with F_{kl}

interaction with the background field

free propagation before interaction with the background field

$$L^{ab}_{\mu i}(k, y_\perp, x) = i \lim_{k^2 \to 0} k^2 \langle A^a_\mu(k) \mathcal{F}^b_i(x, y_\perp) \rangle$$

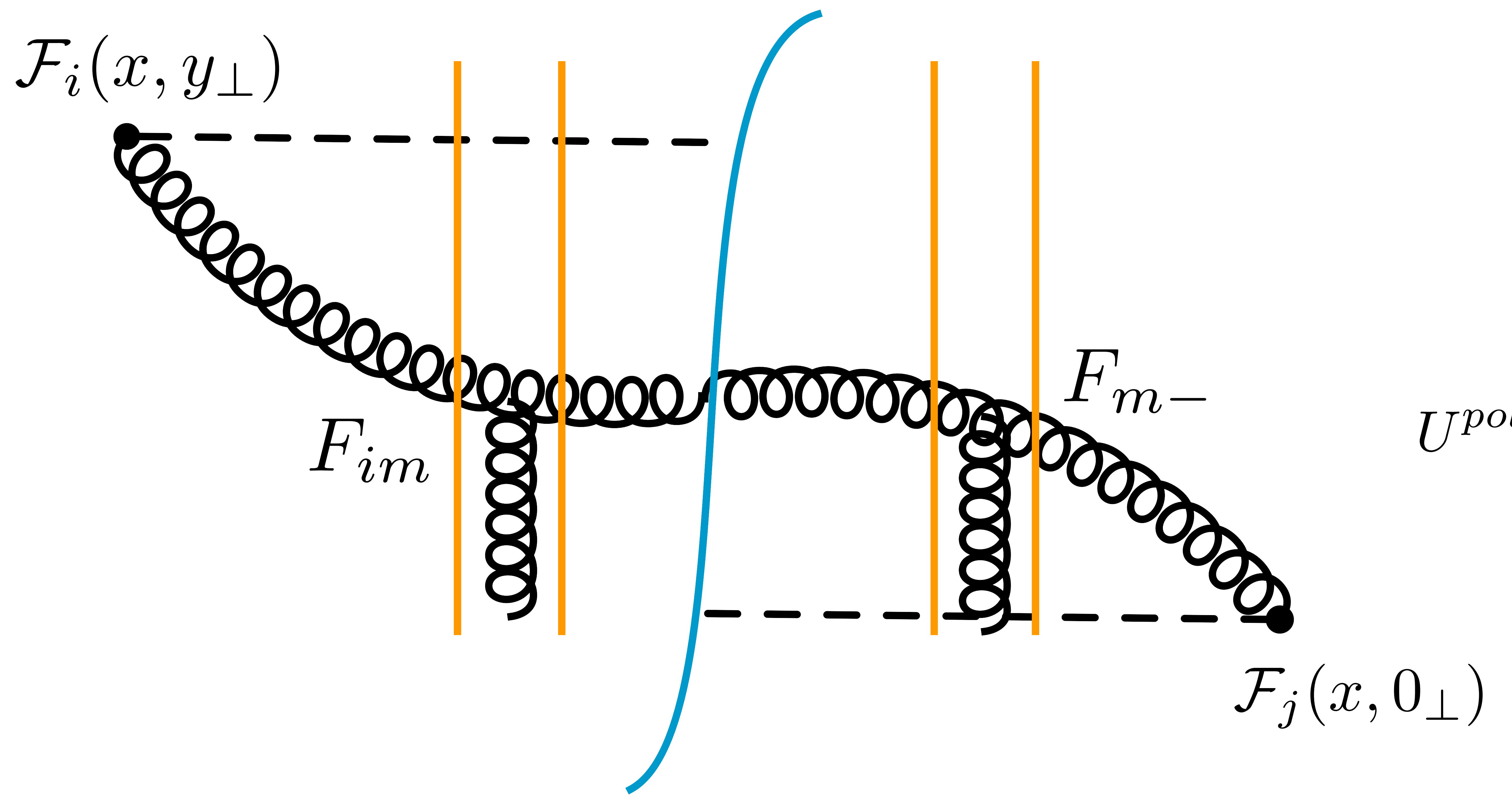
$$= (k_\perp| - 2i\partial^\perp_\mu U \frac{p_i}{2xP^+k^- + p_\perp^2} U^\dagger + \ldots |y_\perp)^{ab} + (k_\perp| \frac{1}{k^-} \int_{-\infty}^{\infty} dz^- [\infty, z^-] F^\perp_{\mu i} [z^-, \infty] \frac{2xP^+k^-}{2xP^+k^- + p_\perp^2} |y_\perp)^{ab}$$

$$U \equiv [\infty, -\infty]$$

DGLAP-type terms

new terms with F_{kl}

Mixing of operators at small-x



Taking product of operators we find that F_{kl} do survive. In the small-x regime we see a mixing between two operators

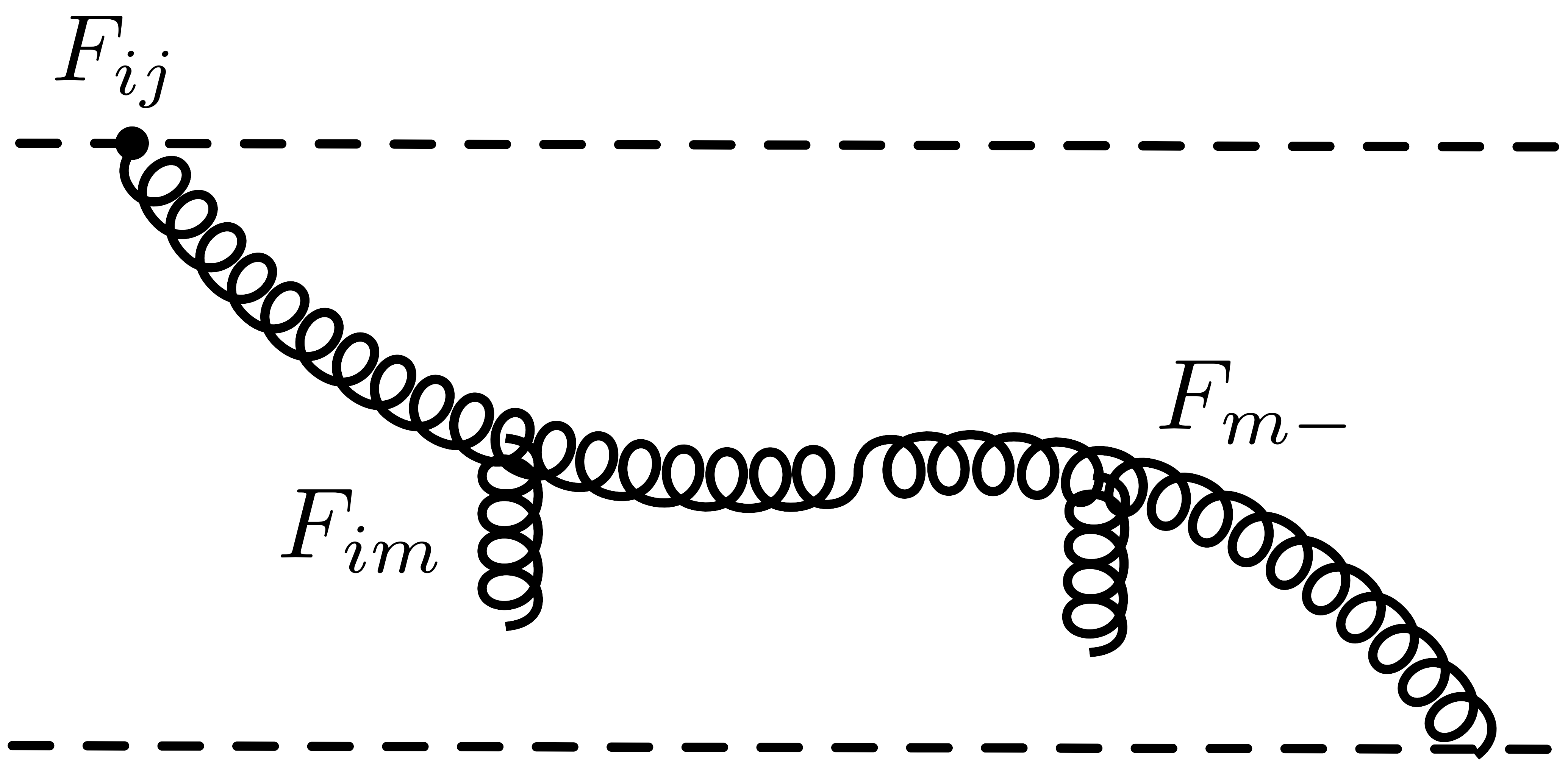
$$U^{pol}(z_\perp) \equiv \frac{iP^+\epsilon^{ij}}{s} \int dz^-[\infty,z^-]_z F_{ij}(z^-, z_\perp)[z^-, -\infty]_z$$

polarized Wilson line

$$\epsilon^{ij}\langle \mathcal{F}_i^a(x, 0_\perp)\mathcal{F}_j^a(x, 0_\perp)\rangle \propto \int_0^\infty \frac{dp^-}{p^-}\Big[\, 2(x_\perp|\frac{p_k}{p_\perp^2}U^{pol}\frac{p_k}{p_\perp^2}|0_\perp) + (x_\perp|\frac{1}{p_\perp^2}|0_\perp)U^{pol}(x_\perp) + (x_\perp|\frac{1}{p_\perp^2}|0_\perp)U^{pol}(0_\perp)\Big]$$

+ DGLAP-type terms

Small-x helicity evolution



Evolution of the polarized dipole. The F_{kl} term in the gluon propagator reproduces the known result

$$G = G^{(0)} + \alpha_s(\mathcal{K}_{DLA} + \mathcal{K}_{SLA_L}) \otimes G$$

Y. V. Kovchegov, D. Pitonyak and M. D. Sievert (16-19); see also G. A. Chirilli (2020)

However, there is a mixing with the DGLAP-type evolution, i.e. transverse logs.

Single-Logarithmic Contribution to the small-x helicity evolution

Helicity evolution at small-x:

$$G = G^{(0)} + \alpha_s(\mathcal{K}_{DLA} + \mathcal{K}_{SLA_L} + \mathcal{K}_{SLA_T}) \otimes G$$

F_{12}

$$\alpha_s\, \mathcal{K}_{\mathrm{DLA}} = \frac{\alpha_s}{2\pi} \int \frac{dz'}{z'} \int \frac{dx_{21}^2}{x_{21}^2}$$

Double logarithm $\alpha_s \log^2(1/x)$, dominates at small-x

$$\alpha_s\, \mathcal{K}_{\mathrm{SLA}_T} = \frac{\alpha_s}{2\pi^2} \int\limits_0^z dz'\, \Delta P\left(\frac{z'}{z}\right) \int \frac{dx_{21}^2}{x_{21}^2}$$

Y. Kovchegov, A. Tarasov, Y. Tawabutr, in preparation

see next talk by Yossathorn (Josh) Tawabutr!

$$\alpha_s\, \mathcal{K}_{\mathrm{SLA}_L} = \frac{\alpha_s}{2\pi^2} \int \frac{dz'}{z'} \int d^2x_2\, \Delta P_L(\underline{x}_{20}, \underline{x}_{21})$$

Y. V. Kovchegov, D. Pitonyak and M. D. Sievert (16-19)

Conclusions

- We study mixing between large-x and small-x gluon helicity operators using the background field method
- We calculate the NLO correction to the operators and find that at large-x there is no such mixing and the evolution is solely described by the polarized DGLAP equation
- However, we emphasize that in the small-x regime, when Q^2 is small, the twist expansion breaks down and different types of dipole-like operators appear in the polarized scattering. We find that those operators do mix with each other
- We find that both DGLAP evolution and the small-x helicity evolution contribute at small-x. However, the latter describes resummation of $\alpha_s \log^2(1/x)$ and should dominate