Testing the limits of collinear factorization with a diquark spectator model

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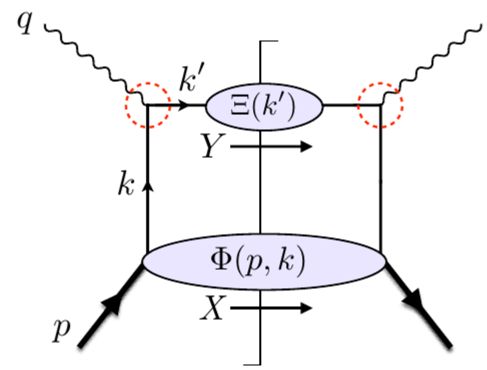
GHP 2021

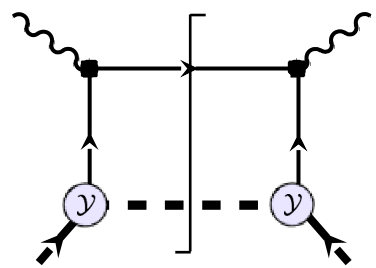
(based on: J. Guerrero, AA, 2010.07339)

Overview

JLab 12: extensive PDF, TMD, GPD program
HERMES, COMPASS: kaons
Heavier particles, ...



- **Revisiting Collinear Factorization at sub-asymptotic energy**
  - Freedom in parton virtuality choices
  - Target and quark mass correction scheme
- **Testing the kinematic approximations**
- **Testing the limits of Collinear Factorization**
  - Expose role of transverse momentum corrections
- **Conclusions and outlook**



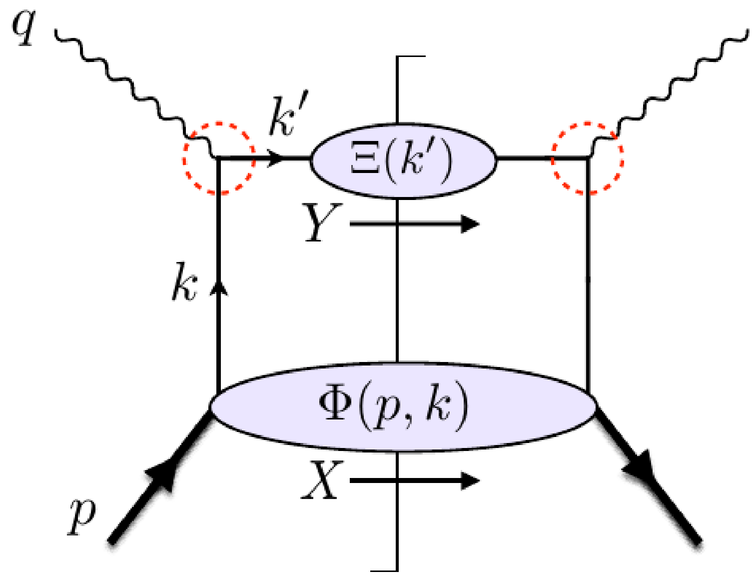
DIS

Collinear Factorization at sub-asymptotic energy

DIS at subasymptotic energies

- **Cannot neglect:**
  - Target, hadron masses vs. W^2, Q^2
  - Mismatch between final state partonic, hadronic masses

$$\Xi(k') = \int \frac{d^4\eta}{(2\pi)^4}\, e^{ik'\cdot\eta}\, \langle\Omega|\, \psi_q(\eta)\, \overline{\psi}_q(0)\, |\Omega\rangle$$



$$\Phi_q(p,k) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\cdot\xi}\, \langle N(p)|\, \bar{\psi}_q(0)\, \psi_q(\xi)\, |N(p)\rangle$$

- **Idea:** *(see also Collins, Rogers, Stasto 2007)*
  - Respect "external" kinematics
  - Localize, mimize approximations
  - Carefully distinguish:
    - Pure kinematic approximations
    - Expansions around "collinear" momenta (sytematically improvable)

= "in line with" the target's 3D momentum (p^+, 0_T)

Factorization in DIS revisited

(1) Minimal approx. overall 4-momentum conservation:

full momenta

approximated

$\widetilde{k}$ and $\widetilde{k}'$

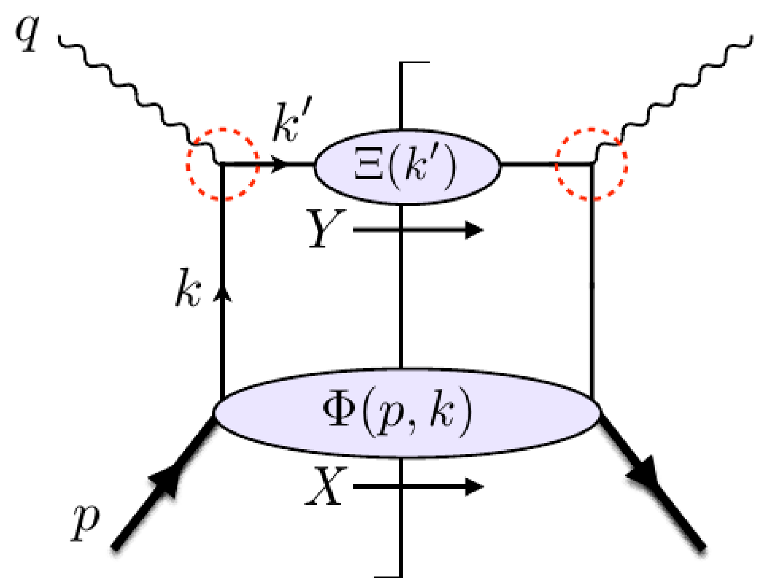
$$2MW^{\mu\nu} \approx (2\pi)^3 \int d^4k \; d^4k' \; \mathrm{Tr}\left[\Phi_q(p,k)\, \gamma^\mu \, \Xi(k')\gamma^\nu\right] \; \delta^{(4)}(\widetilde{k} + q - \widetilde{k}')$$

only the parton light-cone "virtualities"
are approximated (and choice postponed)

$$\widetilde{k}^\mu = \left(xp^+, \frac{v^2}{2xp^+}, \boldsymbol{k_T} \right)$$

$$\widetilde{k}'^\mu = \left(\frac{v'^2}{2k'^-}, k'^-, \boldsymbol{k'_T} \right)$$

unapproximated
leading and transv. momenta



Factorization in DIS revisited

(2) Let 6 integrations out of 8 act on correlators

$\boldsymbol{k'_T} = \boldsymbol{k_T}$

because of the delta-functi

approximation (step 1)

$$2MW^{\mu\nu} = 2\int dx\, dk'^{-}\,\delta\Big(x-\xi-\frac{v'^{2}}{2p^{+}k'^{-}}\Big)\,\delta\Big(\frac{v^{2}}{2xp^{+}}+q^{-}-k'^{-}\Big)$$

$$\int d^2\boldsymbol{k_T}\ \mathrm{Tr}\left[\Phi_q(x,\boldsymbol{k_T})\,\gamma^{\mu}\,J(k'^{-},\boldsymbol{k_T})\gamma^{\nu}\right]$$

$$\Phi_q(x,\boldsymbol{k_T}) \equiv \int dk^{-}\,\Phi_q(k)$$ ← **Quark-distribution TMD correlator**

$$J(k'^{-},\boldsymbol{k'_T}) = \frac{1}{2}\int dk'^{+}\,\Xi(k')$$ ← **TMD-inclusive jet correlator**

Accardi & Signori,
PLB 798 (2019) 134993
2005.11310

REMARK:

- **k_T in the trace is not neglected but integrated over**
- k_T dynamics is retained also in ``collinear'' observables

Factorization in DIS revisited

(3) Expand the correaltors in "twists"   ($k^+ \gg k_T \gg k^-$ $k'^- \gg k'_T \gg k'^+$)

LT: as if quarks were on-shell, massless, and collinear to proton, photon

$$\Phi_q(x, \boldsymbol{k_T}) = \frac{1}{2} q(x, k_T^2)\, \not{n} + \frac{M}{2P^+}\Big[e(x, k_T^2)\, \mathbb{I} + q^\perp(x, k_T^2) \frac{\not{k}_T}{M} \Big] + \mathcal{O}\Big(\frac{\mu^2}{(p^+)^2} \Big)$$

$$xe = x\,\widetilde{e} + \frac{m_q}{M} q$$

$$J(k'^-, \boldsymbol{k'_T}) = \frac{1}{2} \alpha(k'^-)\, \not{n} + \frac{\Lambda_{\rm tw}}{2k'^-}\Big[\zeta(k'^-)\, \mathbb{I} + \alpha(k'^-) \frac{\not{k}'_T}{\Lambda_{\rm tw}} \Big] + \mathcal{O}\Big(\frac{\mu^2}{(k'^-)^2} \Big)$$

$\boxed{\text{Accardi \& Signori, PLB 798 (2019) 134993, 2005.11310}}$

$$\alpha(k'^-) = \frac{1}{2(2\pi)^3}$$

analogous to unpolarized FF

$$\zeta(k'^-) = \frac{1}{2(2\pi)^3} \frac{M_j}{\Lambda_{\rm tw}}, \quad M_j = \mathcal{O}(\Lambda_{\rm tw})$$: non-perturbative dressed quark mass

Factorization in DIS revisited

(4) Integrate over kT (after all this is inclusive DIS!)

$$2MW^{\mu\nu} = \int \frac{dx}{x} \mathcal{H}^{\mu\nu}(x,\bar{x})\, q(x)$$

$$q(x) = \int d^2\boldsymbol{k_T}\, q(x, k_T^2) = \int d^2\boldsymbol{k_T}\, dk^- \mathrm{Tr}\left[\frac{\gamma^+}{2}\Phi_q(p,k)\right]_{k^+=xp^+}$$ ⟵ Collinear PDF

Hard-scattering tensor:

as if quarks were on-shell, massless, and collinear to proton, photon….

$$\mathcal{H}^{\mu\nu}(x,\bar{x}) = x\,\frac{1}{4}\delta(x-\bar{x})\,\mathrm{Tr}\left[\bar{\not{n}}\gamma^\mu \not{n}\gamma^\nu\right]$$ ⟵ gauge invariant: $q_\mu \mathcal{H}^{\mu\nu} = 0$

Light-cone fraction $\bar{x}$ from δ^+, δ^-:

$$\bar{x} = \frac{\xi}{2}\left(1 + \frac{v'^2 - v^2}{Q^2} + \sqrt{\left(1+\frac{v'^2-v^2}{Q^2}\right)^2 + 4\frac{v^2}{Q^2}}\right)$$

$$= \xi\left(1 + \frac{v'^2}{Q^2} - \frac{v^2 v'^2}{Q^4} + \mathcal{O}\left(\frac{\mu^6}{Q^6}\right)\right)$$

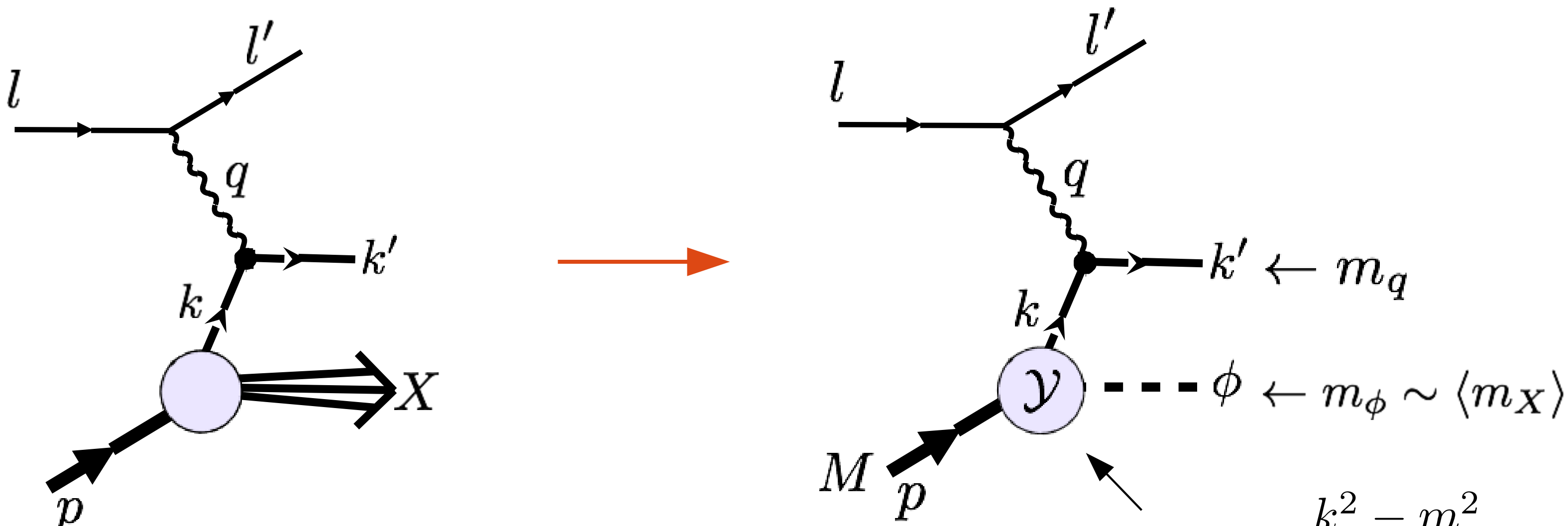
…but evaluated at a generalized Nachtamann scaling variable

v^2 only at order $1/Q^4 \implies$ can safely approximate $v^2 \approx 0$

Testing CF
in a di-quark spectator model

Testing massive CF in a spectator model

- **Use spectator model:**
  - Captures essential elements of QCD
  - Known parameters, analytical calculations
  - Full vs. factorized cross section; PDFs: calculated vs. fitted
- **Test the proposed extended CF scheme:**
  - Validity of kinematic approximations
  - How far can we push in kinematics (large-*x*, small Q^2)



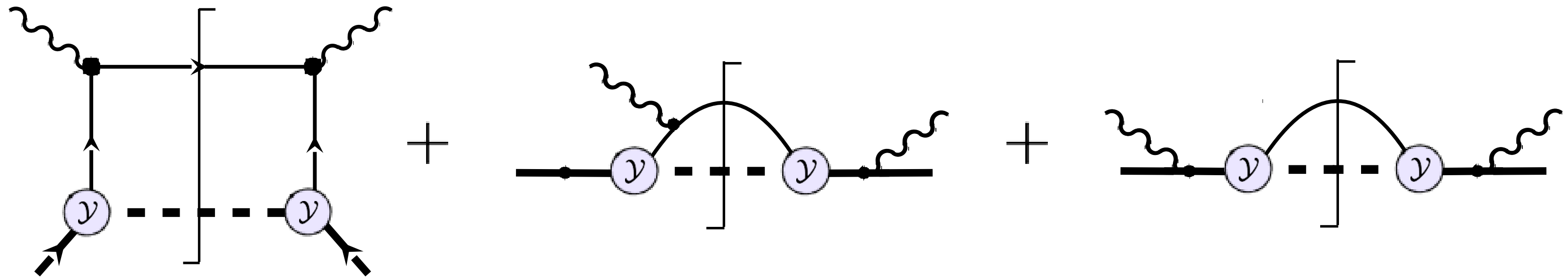
$$\mathcal{Y} = ig\frac{k^2 - m_q^2}{(k^2 - \Lambda^2)^2}\mathbb{I}$$

simulates confinement

Full cross section at LO

- **Gauge invariance:** need also quasi-elastic photon-proton scattering

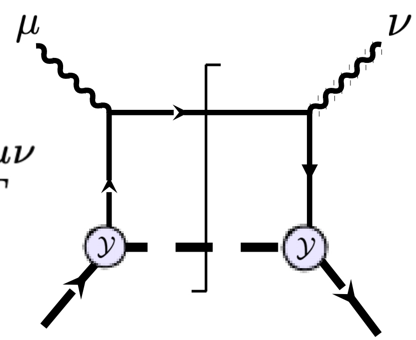
Moffat et al, PRD 95 (2017)



DIS — INT: interference (+ h.c.) — RES: "excited" proton decay

- **Gauge invariant structure function decomposition** *Guerrero, AA, 2010.07339*
  - For example, transverse DIS str.fn. (similarly for others)

$$F_T^{\rm DIS} = P_T^{\mu\nu}$$



$$P_T^{\mu\nu}(p,q) = -\hat{g}^{\mu\nu} + \frac{\hat{p}^\mu \hat{p}^\nu}{\hat{p}^2}$$

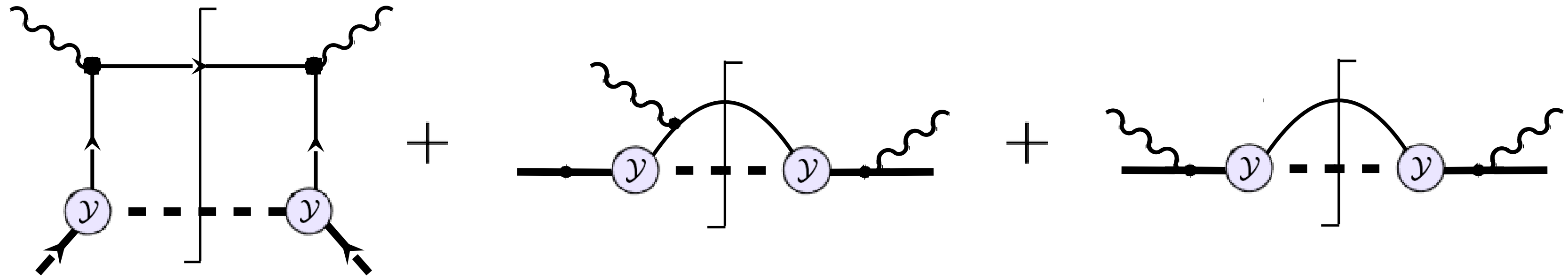
$$\hat{p}^\mu = p^\mu - \frac{p\cdot q}{q^2} q^\mu$$

$$\hat{g}^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$$

Full cross section at LO

- **Gauge invariance:** need also quasi-elastic photon-proton scattering

Moffat et al, PRD 95 (2017)



DIS + INT: interference (+ h.c.) + RES: "excited" proton decay

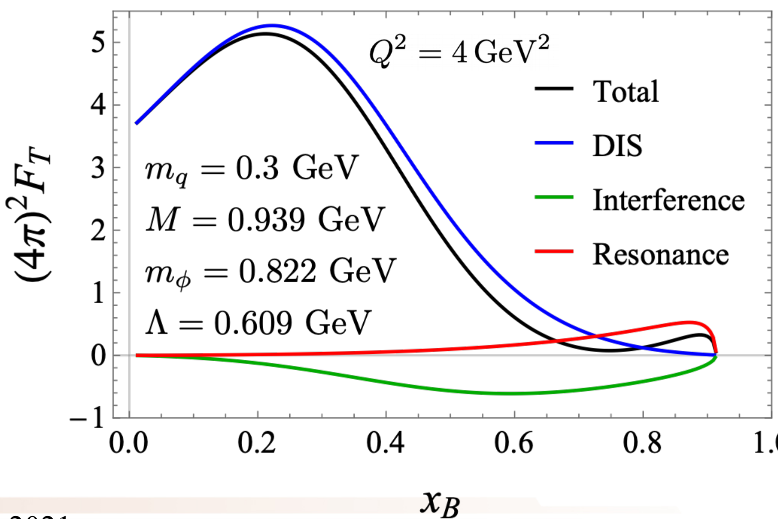
- **Gauge invariant structure function decomposition**

Guerrero, AA, 2010.07339

  - Resonance contribution at large *x* cut by phase space, but:

    $$F_T^{\rm INT} \propto \frac{1}{Q^2}$$ can be fitted away

  - Non negligible, negative INT contribution also at smaller x

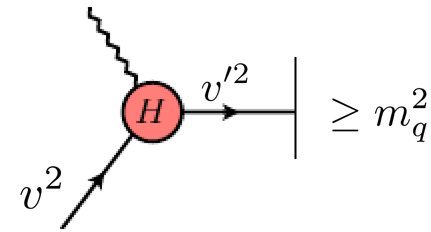


Factorization and kinematic approximations (reprisal)

- Factorized structure function:

$$F_T^{\rm CF}(x_B,Q^2) = q(\bar{x}) \qquad \text{with} \qquad \bar{x} = \xi\left(1+\frac{\bar{v}'^2}{Q^2}-\frac{\bar{v}^2\bar{v}'^2}{Q^4}+O\left(\frac{\mu^6}{Q^6}\right)\right)$$

- Choose initial state quark $v^2 = 0$
  - Equivalent to limiting oneself to $O(1/Q^2)$



- Consider a sequence of approximations for $\bar{x}$:

$\bar{x}$	M	m_q	v'^2
x_B	0	0	0
ξ	✓	0	0
$\xi_q \equiv \xi\left(1+\frac{m_q^2}{Q^2}\right)$	✓	✓	m_q^2
$\xi_q^{(T)} \equiv \xi\left(1+\frac{m_q^2+\langle k_T^2\rangle}{Q^2}\right)$	✓	✓	$m_q^2+\langle k_T^2\rangle$

"external" experimental variables

unobserved but calculable in the diquark model:
- relevance of HT
- need for TMD formalism

Test #1: average internal kinematics

- Full diagram:

$$\langle\mathcal{O}\rangle(x_B,Q^2) = \frac{\int_0^{k^2_{T,\max}} dk_T^2\, \mathcal{O}(x_B,Q^2,k_T)\mathcal{F}_T^{\rm DIS}(x_B,Q^2,k_T)}{\int_0^{k^2_{T,\max}} dk_T^2\, \mathcal{F}_T^{\rm DIS}(x_B,Q^2,k_T)}$$

where $k^2_{T,\max} = \dfrac{\big(W^2-(m_\phi+m_q)^2\big)\big(W^2-(m_\phi-m_q)^2\big)}{4W^2}$

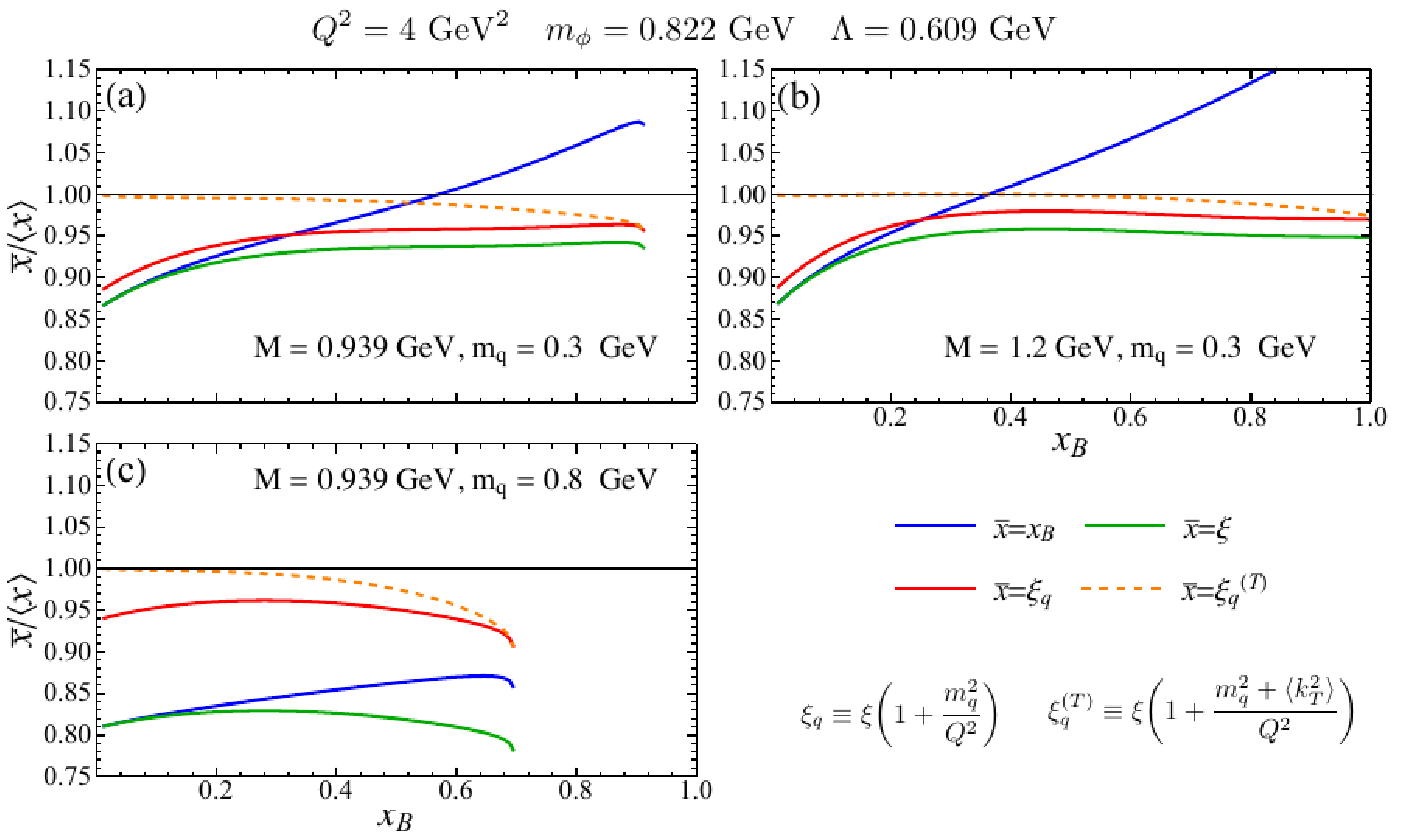
- Factorized diagram:

$$\langle\mathcal{O}\rangle_{\rm CF}(x_B,Q^2) = \frac{\int_0^{\infty} dk_T^2\, \mathcal{O}(x_B,Q^2,k_T)\mathcal{F}_T^{\rm CF}(x_B,Q^2,k_T)}{\int_0^{\infty} dk_T^2\, \mathcal{F}_T^{\rm CF}(x_B,Q^2,k_T)}$$

  - Momentum conservation broken in T direction: k_T ranges to infinity
  - (Vital to put quark field on light-cone in PDF defintions!)

Test #1: average internal kinematics

- **Average parton light-cone momentum fraction *x*:**
  - Mass corrections necessary for stable ~95% approximation
  - Transverse momentum accounts for remaining ~5%

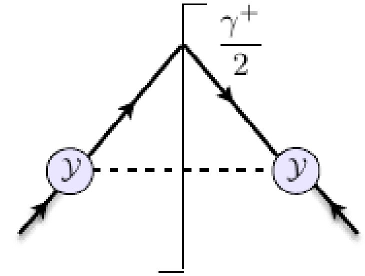


$$\xi_q \equiv \xi\left(1+\frac{m_q^2}{Q^2}\right) \qquad \xi_q^{(T)} \equiv \xi\left(1+\frac{m_q^2+\langle k_T^2\rangle}{Q^2}\right)$$

Test #2: testing the CF limits

- **The PDF is analytically calculable in the model:**

$$q(x) = \int_0 d^2\boldsymbol{k_T}\, dk^- \,\mathrm{Tr}\left[\frac{\gamma^+}{2}\Phi(p,k)\right]_{k^+=xp^+} = \int \frac{d^2\boldsymbol{k_T}\, dk^-}{(2\pi)^4}\,\mathrm{Tr}\Big[$$



$$\Big]_{k^+=xp^+}$$

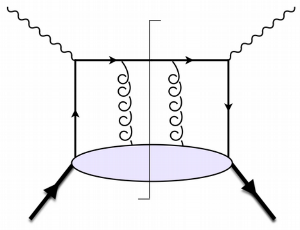
$$= \frac{g^2}{(2\pi)^2}\,\frac{\left[2(m_q + xM)^2 + L^2(\Lambda^2)\right](1-x)^3}{24L^6(\Lambda^2)}$$

where $L^2(\Lambda^2) = x m_\phi^2 + (1-x)\Lambda^2 - x(1-x)M^2$

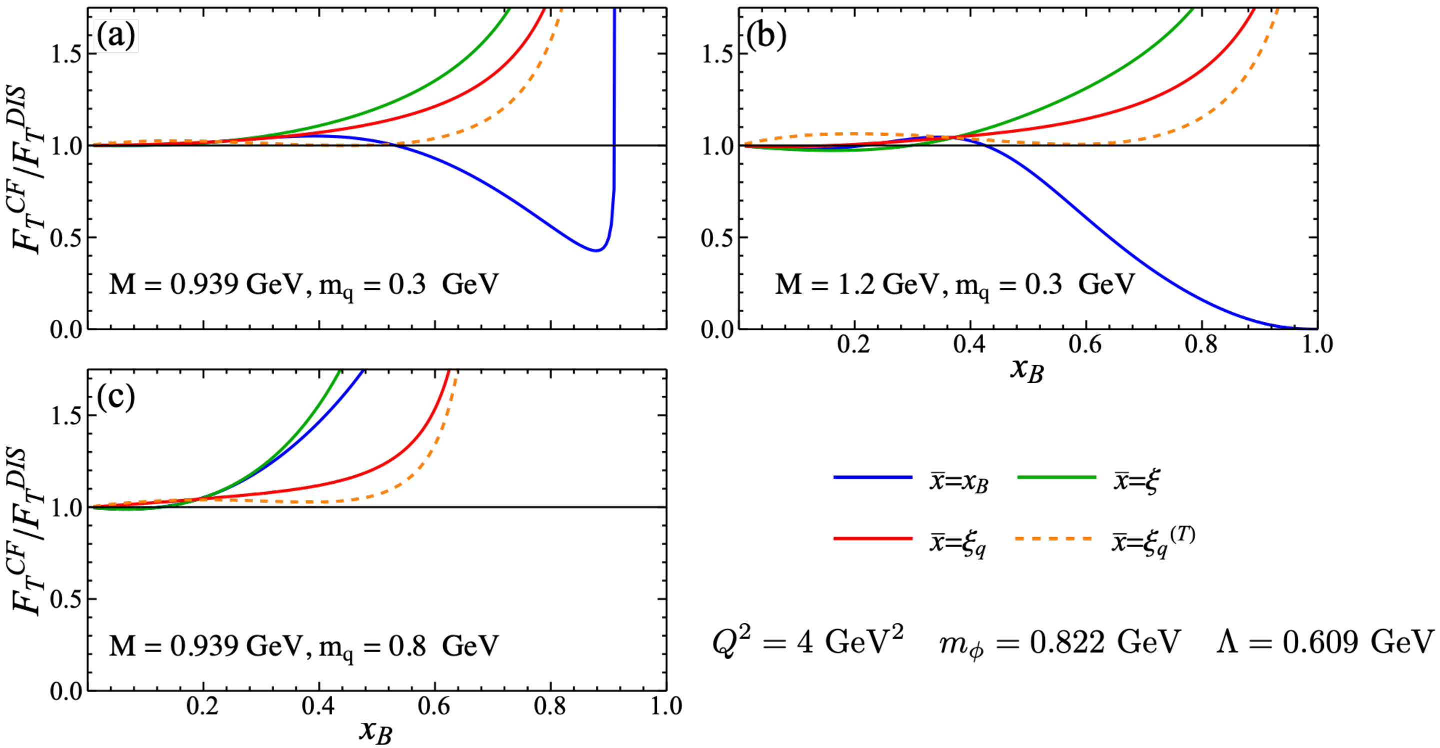
- **Can compare factorized F_T^{CF} vs. full F_T^{DIS}**
  - Verify efficacy of kinematic approximations
  - Determine magnitude of transverse momentum corrections

Test #2: testing the CF limits

- **Mass corrections** are essential, as before
- k_T **corrections** needed for max reach in x_B
  - Medium x_B : controllable in CF at Higher Twist
  - Large x_B : irreducible breaking of coll. factorization

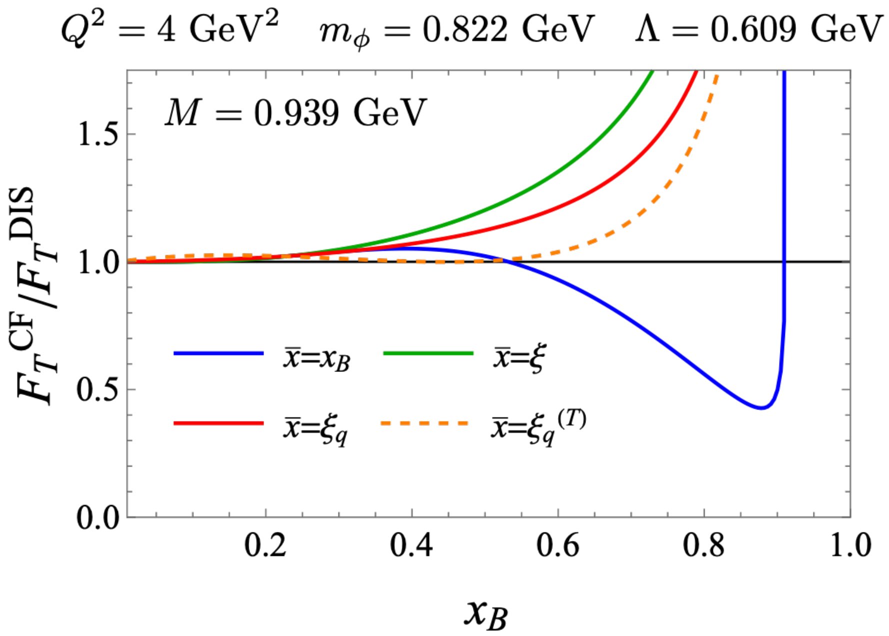


Qiu, PRD 42 (1990) 30

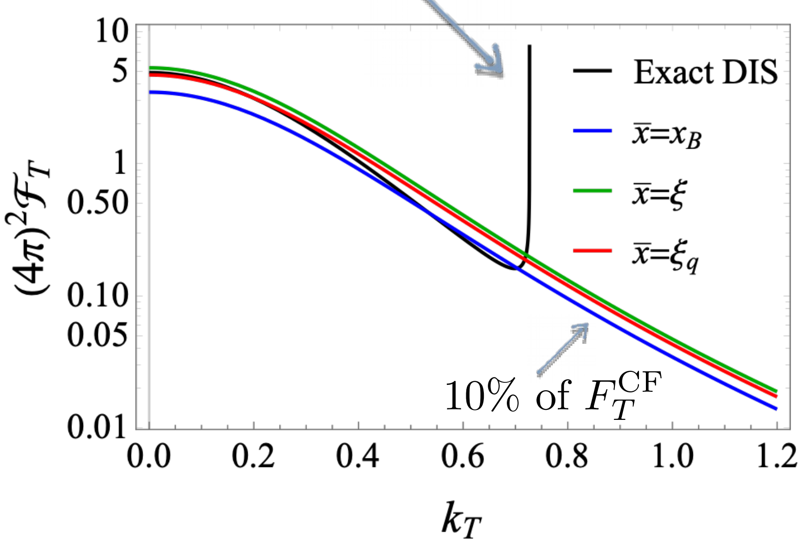


Test #2: testing the CF limits

- **Breaking of factorization at large x_B**
  - Even with best *collinear* kinematic approx. ($\bar{x} = \xi_q$) F_T^{CF} overestimates the full F_T^{CF} at $x \gtrsim 0.6$
  - x_B threshold grows with Q^2



$$k_{T,\max}^2 = \frac{(W^2-(m_\phi+m_q)^2)(W^2-(m_\phi-m_q)^2)}{4W^2}$$



- Breaking due to violation of transverse momentum conservation

  See also Moffat et al (2017)
  - Happens when ~10% of F_T^{CF} beyond $k_{T,\max}$
- **Irreducible in CF**
  - need TMD formalism to overcome

Conclusions and Outlook

Conclusions

- Collinear Factorization can be extended to large x_B
  - **Carefully distinguish**
    - Dynamical twist **expansion** (systematically improvable)
    - Kinematic **approximations** (localized to external legs, minimized)
  - **Obtain more freedom**, adaptability to subasymptotic regimes

- **Testing in a diquark spectator model**
  - Best collinear kinematic approx for "light" quarks:

    $\rightarrow \bar{x} = \xi\left(1 + m_q^2/Q^2\right)$
  - kT effects not negligible:
    - At medium/large xB

      → controllable with HT power corrections
    - At largest xB, irreducible breaking of CF

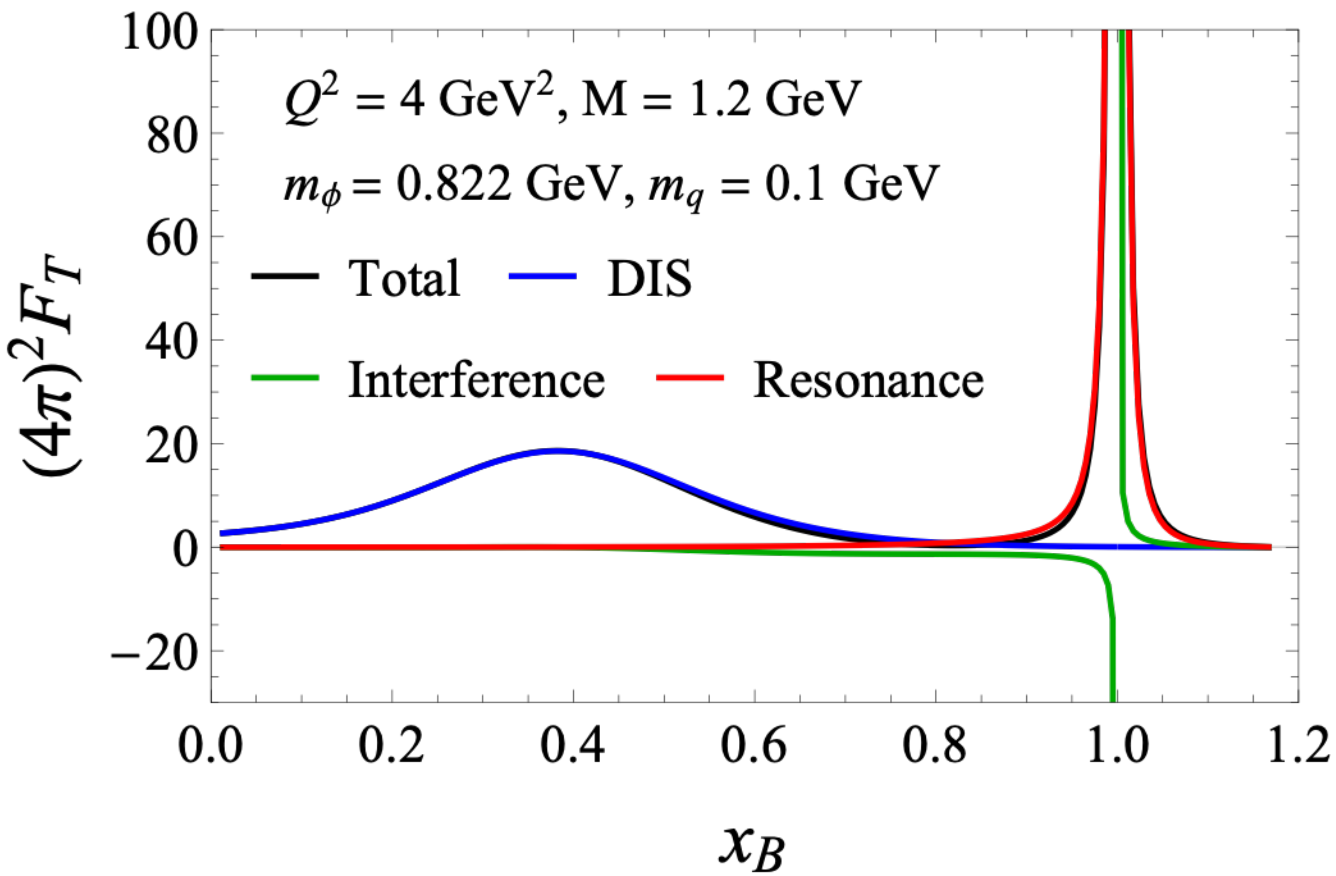
      → need TMD formalism for further progress

Outlook

- **Fit full F_T pseudo-data**
  - Up to what x are fitted PDF ~ full PDFs ?
  - Can fitted $1/Q^2$ correction:
    - control INT and isolate DIS ?
    - simulate k_T corrections at medium-large x_B ?
- **Push analysis to NLO**
- **Extend to heavy quarks**
  - Twist expansion around heavy fermions
  - Kinematics with $v^2=m_q^2$
- **Include hadronization effects**
  - simplified Lund model
- **Mass corrections to TMD fctorization!**

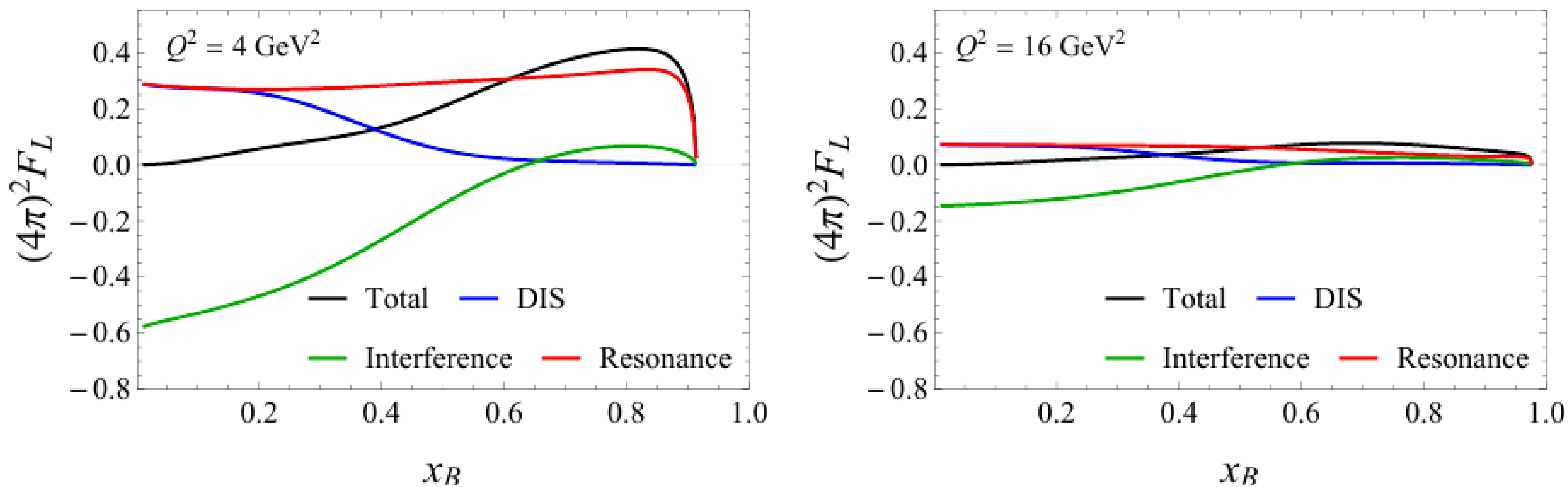
Backup

Proton “resonance”



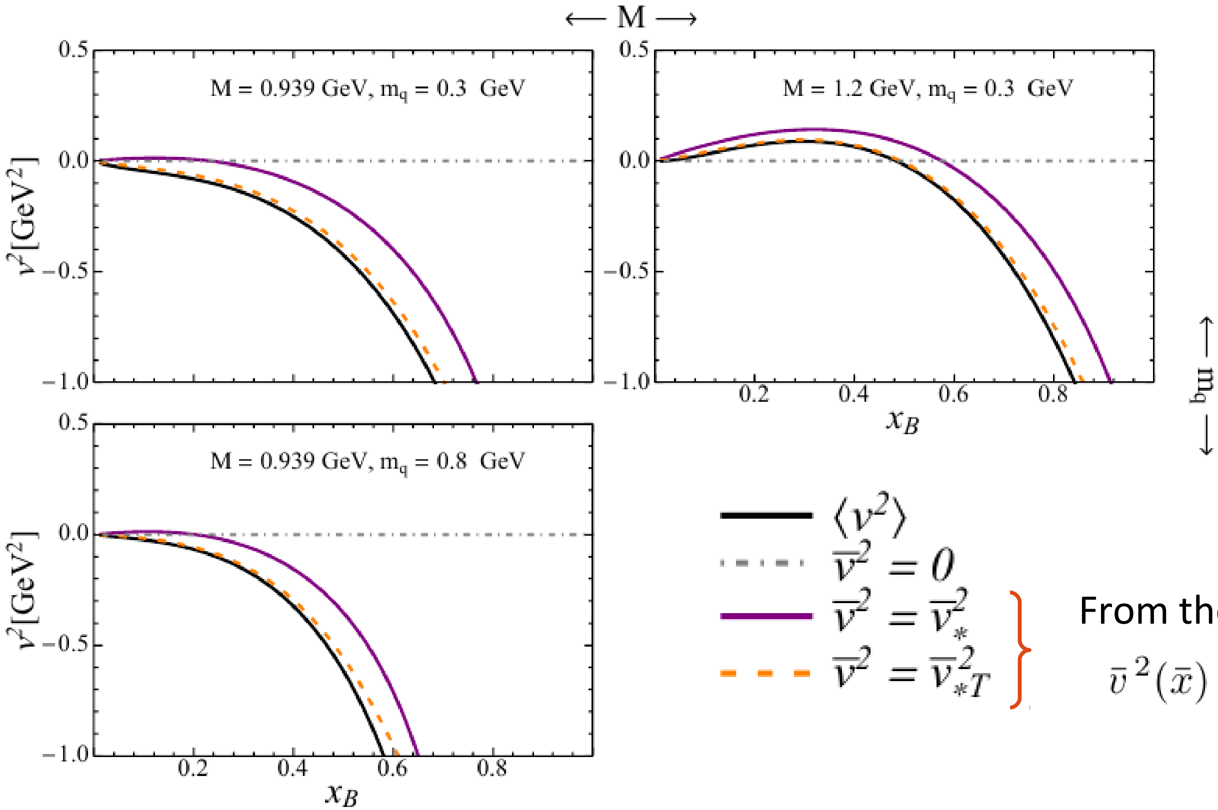
Longitudinal structure function decompositions

- At small x → 0, gauge invariance imposes
  - FL → 0 exactly
  - RES ~ INT ~ DIS : breakdown of factorization
  - non-DIS contrib. partly be simualted by “HT” x ~ s/Q2 term (since y →1 b/c of phase space)



Initial state light cone virtuality - 1

- **Average light cone virtuality:**
  - Quark is on the light cone at small x
  - Farther and farther away as $x_B \longrightarrow 1$

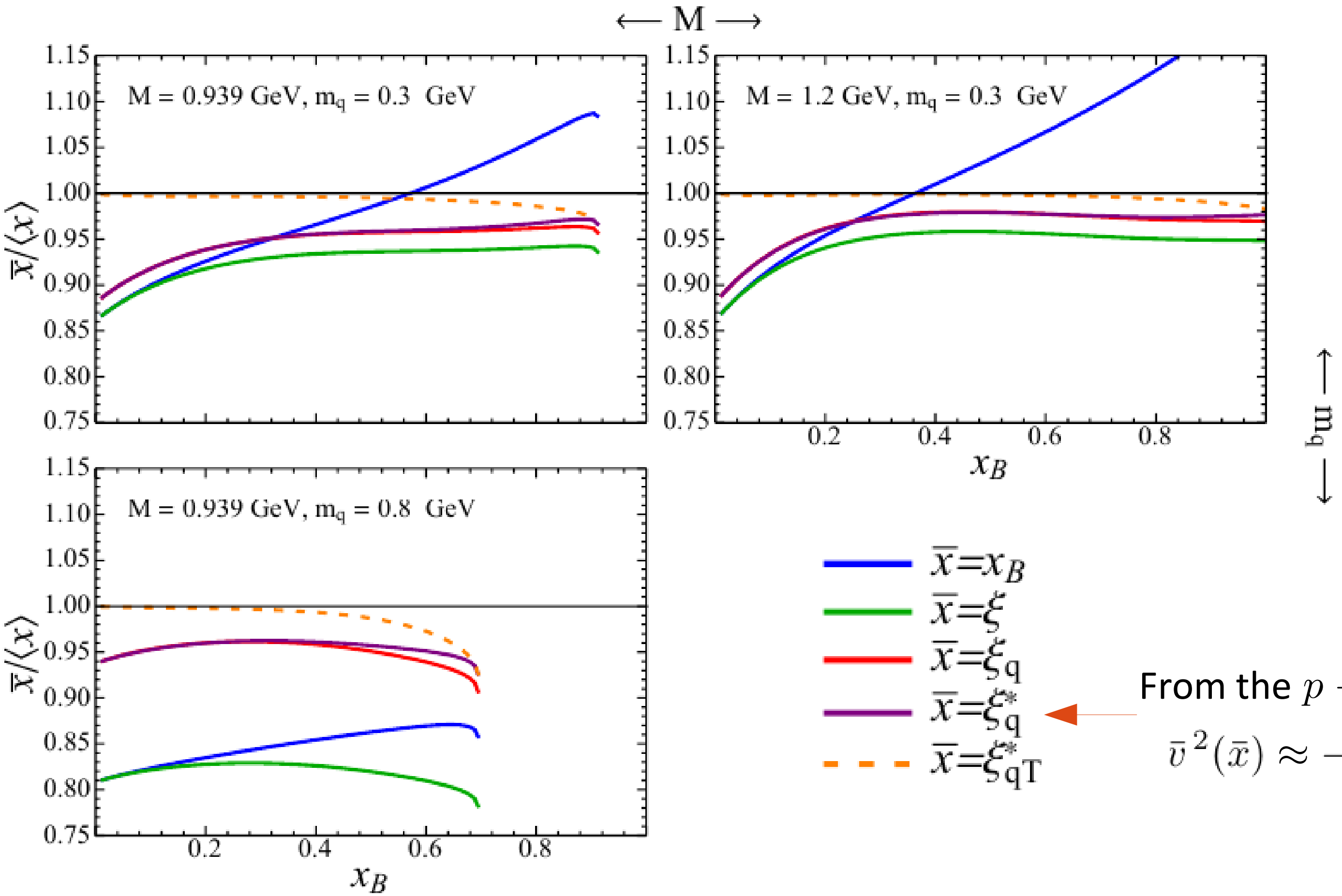


From the $p \to q + \phi$ vertex' kinematics:

$$\bar{v}^{\,2}(\bar{x}) \approx -\frac{\bar{x}}{1-\bar{x}}\left(m_\phi^2 + (\bar{x}-1)M^2\right)$$

Initial state light cone virtuality - 2

- ...but only gives very minor corrections to collinear kinematics
  - Because it only contributes starting at $O(1/Q^4)$



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