

Gluon pseudo-distributions at short distances: Forward case

Wayne Morris

Ian Balitsky, Anatoly Radyushkin

as part of the

HadStruc Collaboration

based on

arXiv:1910.13963v2

April 13, 2021



- Lattice calculations of parton distribution functions (PDFs) are now a subject of considerable interest and efforts
- PDFs not directly calculable on the lattice, $z^2 = 0$ doesn't work in Euclidean space
- X. Ji's ground-breaking proposal to consider equal-time versions of nonlocal operators: quasi-PDFs [Ji, 2013]. Taking $z = (0, 0, 0, z_3)$:

$$\tilde{q}(x, \mu^2, P_3) = \int \frac{dz}{4\pi} e^{-ixzP_3} \langle P | \bar{\psi}(z) \gamma^3 \exp\left(-ig \int_0^z dz' A^z(z')\right) \psi(0) | P \rangle \quad (1)$$

- PDFs are obtained from the large-momentum $P_3 \rightarrow \infty$ limit of quasi-PDFs
- A. Radyushkin introduced a coordinate-space oriented approach [Radyushkin, 2017]

$$\langle p | \phi(z) \phi(0) | p \rangle = \mathcal{M}(p_3 z_3, z_3^2) = \int_{-1}^1 dx e^{ixp_3 z_3} \mathcal{P}(x, z_3^2), \quad \mathcal{P}(x, 0) = f(x) \quad (2)$$

- PDFs are obtained from $z_3 \rightarrow 0$ limit of pseudo-PDFs
- offe-time distribution (ITD) $\mathcal{M}(\nu, z_3^2)$, with $\nu = p_3 z_3$

- At small z_3 , $1/z_3$ is analogous to the renormalization parameter μ of scale-dependent PDFs $f(x, \mu^2)$ of the standard OPE approach
- But z_3^2 dependence comes from evolution logarithms: $\log(z_3^2 \mu_{IR}^2)$ and UV logarithms: $\log(z_3^2 \mu_{UV}^2)$
- Since UV divergences have no ν dependence at leading log, and if $\mathcal{M}(\nu, z_3^2)$ is multiplicatively renormalizable, can define reduced ITD:

$$\mathfrak{M}(\nu, z_3^3) = \frac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)} \quad (3)$$

- This leads to the evolution equation:

$$\frac{d}{d \log z_3^2} \mathfrak{M}(\nu, z_3^3) = -\frac{\alpha_s}{2\pi} C \int_0^1 du B(u) \mathfrak{M}(u\nu, z_3^3) \quad (4)$$

- Taking $z_3 \rightarrow 0$ to extract light-cone PDF is singular, and one needs to use matching relations to go from Euclidean lattice data to PDFs

- The gluon distribution calculation is complicated by gauge-invariance
- Effective to use external field method along with the Schwinger representation for the propagator [Balitsky, Braun, 1988]
- External field method involves separating fields into a fluctuating quantum field with virtualities between μ_2^2 and μ_1^2 and a slowly varying “classical” field with virtualities below μ_1^2 ($A_\mu = A_\mu^q + A_\mu^{cl}$ and $\psi = \psi_q + \psi_{cl}$)

$$\mathcal{L} = -\frac{1}{4g^2} \left(G_{\mu\nu}^{cl,a} + D_\mu A_\nu^{a,q} - D_\nu A_\mu^{a,q} + f^{abc} A_\mu^{b,q} A_\nu^{c,q} \right)^2 + (\bar{\psi}_q + \bar{\psi}_{cl}) (i\not{D} + A_\mu^{a,q} \gamma^\mu t^a) (\psi_q + \psi_{cl}) + \mathcal{L}_{GF} + \mathcal{L}_g \quad (5)$$

- Coupling is absorbed into A_μ , $gA_\mu \rightarrow A_\mu$, and $D_\mu = \partial_\mu - iA_\mu^{cl}$

- Use background field gauge: $D^\mu A_\mu^q = 0$ for quantum fields, and Fock-Schwinger gauge: $x^\mu A_\mu^{cl}(x) = 0$ for “classical” fields
- Fock-Schwinger gauge leads to the important relation:

$$A_\nu^{cl}(x) = \int_0^1 dv v x^\mu G_{\mu\nu}^{cl}(vx) \quad (6)$$

- Schwinger representation for the propagator

$$\frac{i}{P^2 + i\epsilon} = \int_0^\infty ds \exp[is(P^2 + i\epsilon)] \quad (7)$$

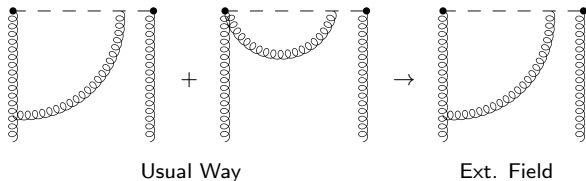
- Gluon propagator in terms of external gluon fields (omitting ϵ term for convenience):

$$g^{-2} i A_\mu^a(x) A_\nu^b(0) = \langle x | \left(\frac{1}{P^2 g_{\mu\nu} + 2iG_{\mu\nu}} \right)^{ab} | 0 \rangle = -i \int_0^\infty ds \langle x | e^{is(P^2 g_{\mu\nu} + 2iG_{\mu\nu})} | 0 \rangle \quad (8)$$

- Expanding and integrating over s , and momentum:

$$\begin{aligned} & g^{-2} \langle x | \frac{1}{P^2 g_{\alpha\beta} + 2iG_{\alpha\beta}} | 0 \rangle \\ &= -i g_{\alpha\beta} \frac{\Gamma(d/2 - 1)}{4\pi^2 (-x^2)^{d/2-1}} + \frac{\Gamma(d/2 - 2)}{16\pi^2 (-x^2)^{d/2-2}} \int_0^1 du \{ 2G_{\alpha\beta}(ux) - \bar{u}u D_\sigma G^{\sigma\rho}(ux) x_\rho g_{\alpha\beta} \\ &\quad - 2i g_{\alpha\beta} \int_0^u dv \bar{u}v x^\lambda G_{\lambda\xi}(ux) x^\rho G_\rho^\xi(vx) \} - \frac{i\Gamma(d/2 - 3)}{16\pi^2 (-x^2)^{d/2-3}} \\ &\quad \times \int_0^1 du \int_0^u dv \left[G_{\alpha\xi}(ux) G_\beta^\xi(vx) - \frac{1}{2} g_{\alpha\beta} \bar{u}v G_{\eta\xi}(ux) G^{\eta\xi}(vx) - \frac{1}{2} i \bar{u} D^2 G_{\alpha\beta}(ux) \right] \\ &\quad + \mathcal{O}(\text{twist } 3) \end{aligned} \quad (9)$$

External field method of calculation has fewer diagrams:



Linear divergences are 'hidden' inside the vertex diagram:

$$\mathcal{O}_{\mu\alpha;\nu\beta}^V(x)$$

$$\begin{aligned} \rightarrow & \frac{g^2 N_c \Gamma(d/2 - 1)}{4\pi^2 (-x^2)^{d/2-1}} \int_0^1 du \int_0^{\bar{u}} dv \left\{ \delta(u) \left(\frac{v^{3-d} - v}{d-2} \right) G_{\mu\alpha}(\bar{u}x) (x_\beta G_{x\nu}(vx) - x_\nu G_{x\beta}(vx)) \right. \\ & \left. + \delta(v) \left(\frac{u^{3-d} - u}{d-2} \right) (x_\alpha G_{x\mu}(\bar{u}x) - x_\mu G_{x\alpha}(\bar{u}x)) G_{\nu\beta}(vx) \right\} \\ & + \frac{N_c \Gamma(d/2 - 2)}{8\pi^2 (-x^2)^{d/2-2}} \int_0^1 du \int_0^{\bar{u}} dv \left\{ \delta(u) \left[\frac{v^{3-d} - 1}{d-3} \right]_+ + \delta(v) \left[\frac{u^{3-d} - 1}{d-3} \right]_+ \right\} G_{\mu\alpha}(\bar{u}x) G_{\nu\beta}(vx) \end{aligned} \quad (10)$$

Nucleon spin-averaged matrix elements with non-contracted indices

$$M_{\mu\alpha;\nu\beta}(z, p) \equiv \langle p | G_{\mu\alpha}(z) [z, 0] G_{\nu\beta}(0) | p \rangle \quad (11)$$

with straight-line gauge link in the adjoint representation

$$[x, y] \equiv \mathcal{P} \exp \left[ig \int_0^1 du (x - y)^\mu A_\mu (ux + (1 - u)y) \right] \quad (12)$$

with Lorentz decomposition

$$\begin{aligned} M_{\mu\alpha;\nu\beta}(z, p) &= (g_{\mu\nu} p_\alpha p_\beta - g_{\mu\beta} p_\alpha p_\nu - g_{\alpha\nu} p_\mu p_\beta + g_{\alpha\beta} p_\mu p_\nu) \mathcal{M}_{pp}(\nu, z^2) \\ &+ (g_{\mu\nu} z_\alpha z_\beta - g_{\mu\beta} z_\alpha z_\nu - g_{\alpha\nu} z_\mu z_\beta + g_{\alpha\beta} z_\mu z_\nu) \mathcal{M}_{zz}(\nu, z^2) \\ &+ (g_{\mu\nu} z_\alpha p_\beta - g_{\mu\beta} z_\alpha p_\nu - g_{\alpha\nu} z_\mu p_\beta + g_{\alpha\beta} z_\mu p_\nu) \mathcal{M}_{zp}(\nu, z^2) \\ &+ (g_{\mu\nu} p_\alpha z_\beta - g_{\mu\beta} p_\alpha z_\nu - g_{\alpha\nu} p_\mu z_\beta + g_{\alpha\beta} p_\mu z_\nu) \mathcal{M}_{pz}(\nu, z^2) \\ &+ (p_\mu z_\alpha p_\nu z_\beta - p_\alpha z_\mu p_\nu z_\beta - p_\mu z_\alpha p_\beta z_\nu + p_\alpha z_\mu p_\beta z_\nu) \mathcal{M}_{ppzz}(\nu, z^2) \\ &+ (g_{\mu\nu} g_{\alpha\beta} - g_{\mu\beta} g_{\alpha\nu}) \mathcal{M}_g(\nu, z^2) \end{aligned} \quad (13)$$

The light-cone distribution is obtained from

$$g^{\alpha\beta} M_{+\alpha,\beta+}(z_-, p) = -2p_+^2 \mathcal{M}_{pp}(\nu, 0) \quad (14)$$

so

$$-\mathcal{M}_{pp}(\nu, 0) = \frac{1}{2} \int_{-1}^1 dx e^{-ix\nu} x f_g(x) \quad (15)$$

Taking other projections, there are three multiplicatively renormalizable [Zhang et. al, 2018] quantities

$$\langle p | G_{3i}(z) G_{i3}(0) | p \rangle = -2\mathcal{M}_g + 2p_3^2 \mathcal{M}_{pp} + 2z_3^2 \mathcal{M}_{zz} + 2z_3 p_3 (\mathcal{M}_{zp} + \mathcal{M}_{pz}) \quad (16)$$

$$\langle p | G_{0i}(z) G_{i0}(0) | p \rangle = 2\mathcal{M}_g + 2p_0^2 \mathcal{M}_{pp} \quad (17)$$

$$\langle p | G_{0i}(z) G_{i3}(0) + G_{3i}(z) G_{i0}(0) | p \rangle = 4p_0 p_3 \mathcal{M}_{pp} + 2p_0 z_3 (\mathcal{M}_{pz} + \mathcal{M}_{zp}) \quad (18)$$

There are higher twist contaminations, but can isolate \mathcal{M}_{pp} through:

$$M_{0i;i0} + M_{ji;ij} = 2p_0^2 \mathcal{M}_{pp} \quad (19)$$

where $M_{ji;ij} = -2\mathcal{M}_g$ shares the same anomalous dimension as $M_{0i;i0}$.

Leading twist gluon calculation for $M_{0i;i0} + M_{ji;ij}$:

$$\begin{aligned}
 & \mathcal{M}_{pp}(\nu, z_3^2) \\
 & \rightarrow \frac{\alpha_s N_c}{2\pi} \left\{ \left(\frac{1}{\epsilon_{UV}} + \log(z_3^2 \mu_{UV}^2 e^{2\gamma}/4) + 2 \right) \delta(\bar{u}) \right. \\
 & \quad - \int_0^1 du \left[\frac{2}{3} (1 + 6u - 6u^2 - u^3) + \frac{4u + 4 \log(\bar{u})}{\bar{u}} \right]_+ \\
 & \quad \left. + 2 \left(\frac{1}{\epsilon_{IR}} - \log(z_3^2 \mu_{IR}^2 e^{2\gamma}/4) \right) \int_0^1 du \left[\frac{(1 - \bar{u}u)^2}{\bar{u}} \right]_+ \right\} \mathcal{M}_{pp}(u\nu, 0) \quad (20)
 \end{aligned}$$

Evolution kernel is then:

$$B_{gg}(u) = 2 \left[\frac{(1 - \bar{u}u)^2}{\bar{u}} \right]_+ \quad (21)$$

UV divergence cancels in the reduced Ioffe time distribution:

$$\mathfrak{M}(\nu, z_3^2) = \frac{\mathcal{M}_{pp}(\nu, z_3^2)}{\mathcal{M}_{pp}(0, z_3^2)} \quad (22)$$

$$\begin{aligned}
 & 2p_0^2 \mathcal{M}_{pp}(\nu, z_3^2) \\
 & \rightarrow \frac{g^2 C_F}{8\pi^2 z_3} \left(\frac{1}{\epsilon_{\text{IR}}} - \log(z_3^2 e^{\gamma_E}) \right) \frac{p^0}{p_3} \int_0^1 du (2\bar{u} + \delta(\bar{u})) \langle p | \mathcal{O}_q(uz_3) | p \rangle
 \end{aligned} \tag{23}$$

with singlet combination:

$$\mathcal{O}_q(z_3) = \frac{i}{2} \sum_f (\bar{\psi}_f(0) \gamma^0 \psi_f(z_3) - \bar{\psi}_f(z_3) \gamma^0 \psi_f(0)) \tag{24}$$

Evolution kernel: $B_{gq}(u) = 2\bar{u} + \delta(\bar{u})$

Related to ITD through parametrization of the matrix element and oddness in z_3 :

$$\frac{1}{z_3} \int_0^1 du B(u) \langle p | \mathcal{O}(uz_3) | p \rangle = p^0 p_3 \int_0^1 dw \mathcal{B}_{gq}(w) \mathcal{I}(w\nu) \tag{25}$$

where

$$\mathcal{B}_{gq}(w) = \int_w^1 du B_{gq}(u) \implies \mathcal{B}_{gq}(w) = 1 + (1-w)^2 \tag{26}$$

Relating reduced Ioffe-time pseudo-distribution to light-cone Ioffe time distribution

$$\begin{aligned}
 & \mathfrak{M}(\nu, z_3^2) \mathcal{I}_g(0, \mu^2) \\
 &= \mathcal{I}_g(\nu, \mu^2) - \frac{\alpha_s N_c}{2\pi} \int_0^1 du \mathcal{I}_g(u\nu, \mu^2) \left\{ \ln(z_3^2 \mu^2 e^{2\gamma_E}/4) B_{gg}(u) \right. \\
 & \quad \left. + 4 \left[\frac{u + \log(\bar{u})}{\bar{u}} \right]_+ + \frac{2}{3} [1 + 6u - 6u^2 - u^3]_+ \right\} \\
 & \quad - \frac{\alpha_s C_F}{2\pi} \ln(z_3^2 \mu^2 e^{2\gamma_E}/4) \int_0^1 dw [\mathcal{I}_S(w\nu, \mu^2) - \mathcal{I}_S(0, \mu^2)] \mathcal{B}_{gq}(w)
 \end{aligned} \tag{27}$$

Can be directly related to light-cone PDFs using:

$$\mathcal{I}_g(\nu, \mu^2) = \frac{1}{2} \int_{-1}^1 dx e^{ix\nu} x f_g(x, \mu^2), \quad \mathcal{I}_g(0, \mu^2) = \langle x \rangle_{\mu^2} \tag{28}$$

New kernel form:

$$\mathfrak{M}(\nu, z_3^2) = \int_0^1 dx \frac{x f_g(x, \mu^2)}{\langle x \rangle_{\mu^2}} R_{gg}(x\nu, z_3^2 \mu^2) + \int_0^1 dx \frac{x f_S(x, \mu^2)}{\langle x \rangle_{\mu^2}} R_{gq}(x\nu, z_3^2 \mu^2) \tag{29}$$

Need to independently calculate $\langle x \rangle_{\mu^2}$, and calculate or estimate singlet quark function $\mathcal{I}_S(w\nu, \mu^2)$.

Matching Relation

New kernels found by cosine transformation only, because sine part integrates to zero ($xf_g(x, \mu^2)$ is even):

$$R(y) = \int_0^1 du B(u) \cos(uy) \quad (30)$$

Gluon kernel given by:

$$R_{gg}(y, z_3^2 \mu^2) = \cos y - \frac{\alpha_s N_c}{2\pi} \left\{ \ln(z_3^2 \mu^2 e^{2\gamma_E}/4) \underset{\text{Evolution}}{R_B(y)} + \underset{\text{log}}{R_L(y)} + \underset{\text{Constant}}{R_C(y)} \right\} \quad (31)$$

Mixing kernel given by:

$$R_{gq}(y, z_3^2 \mu^2) = -\frac{\alpha_s N_c}{2\pi} \ln(z_3^2 \mu^2 e^{2\gamma_E}/4) R_B(y) \quad (32)$$










Calculation of R_L gives hypergeometric function ${}_3F_3(1, 1, 1; 2, 2, 2; -iy)$, while calculation of evolution and constant part involve $\text{Si}(y)$, $\text{Ci}(y)$, $\sin y$, $\cos y$, and inverse powers of y .

Important points:

- $R(y, z_3^2, \mu^2)$ kernels are given by explicit perturbatively calculable expressions
- Lattice data and LC PDFs directly relatable
- Taking some parametrization of $f_g(x, \mu^2)$ and $f_S(x, \mu^2)$ distributions, one can fit parameters and α_s from the lattice data for $\mathfrak{M}(\nu, z_3^2)$
- Essentially same procedure as that used in the “good lattice cross sections” approach [Ma, Qiu, 2018]

- Paper on gluon helicity pseudo-distribution coming soon
- Currently working on transverse quark pseudo-distribution
- Also working on gluon “condensate” calculation

Thank you

-  A. V. Radyushkin, *Quasi-PDFs, momentum distributions and pseudo-PDFs*, Phys.Rev. D96 no.3, 034025 (2017).
-  I. I. Balitsky, V. M. Braun, *Evolution Equations for QCD String Operators*, Nuclear Physics B311 541-584 (1988/89).
-  J. Zhang et. al, *Renormalization of gluon quasi-PDF in large momentum effective theory* arXiv:1808.10824v1 [hep-ph] (2018).
-  X. Ji, *Parton Physics on Euclidean Lattice*, Phys. Rev. Lett. 110, 262002 (2013).
-  Y.Q. Ma, J.W. Qiu, Phys. Rev. D 98 (7) (2018) 074021.
-  Y.Q. Ma, J.W. Qiu, Phys. Rev. Lett. 120 (2) (2018) 022003.
-  V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov *Calculations in External Fields in Quantum Chromodynamics. Technical Review*. Fortschr. Phys. 32 (1984) 11, 585-622
-  A. Radyushkin, PoS QCDEV 2017 (2017) 021.
-  K. Orginos, A. Radyushkin, J. Karpie, S. Zafeiropoulos, Phys. Rev. D 96 (9) (2017) 094503.