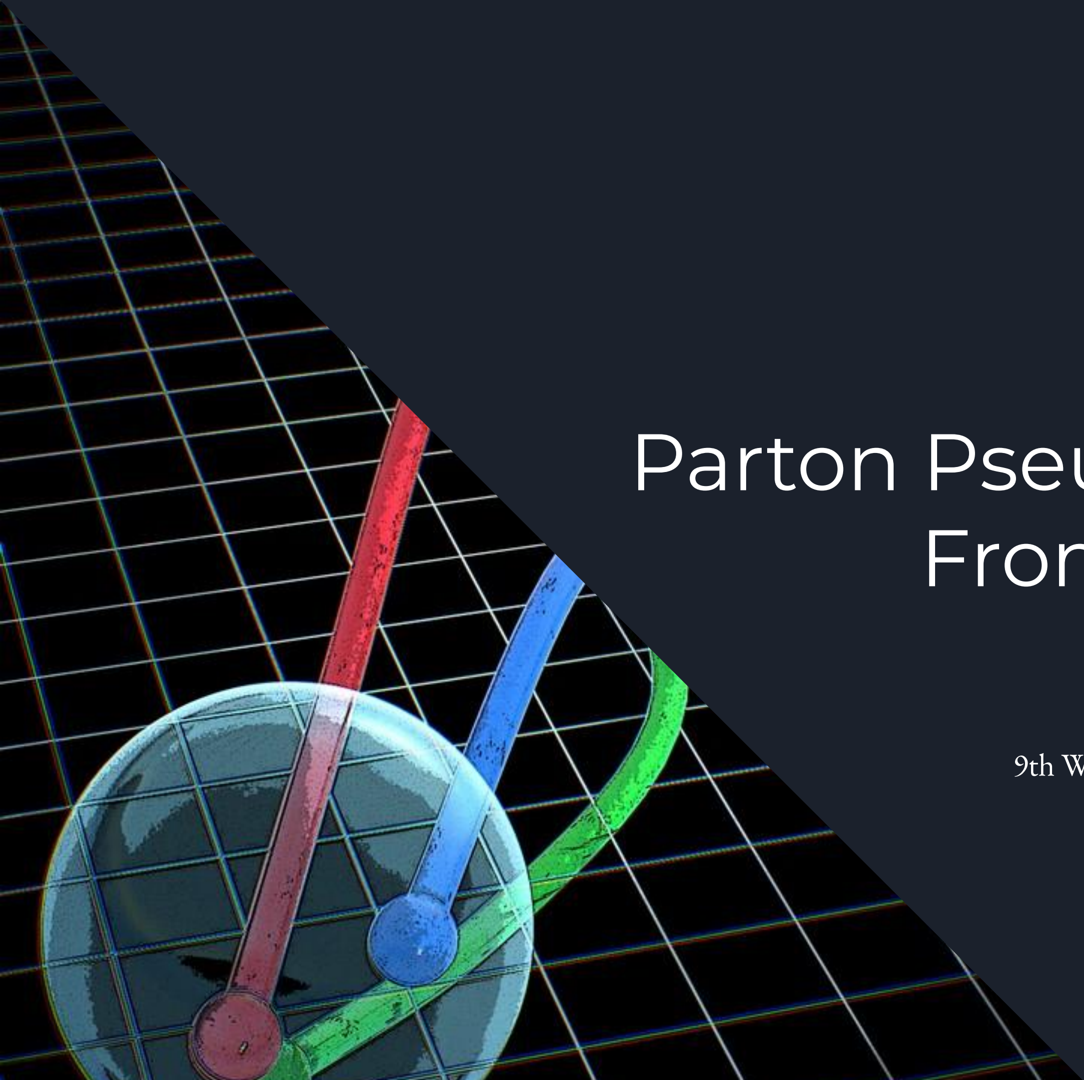


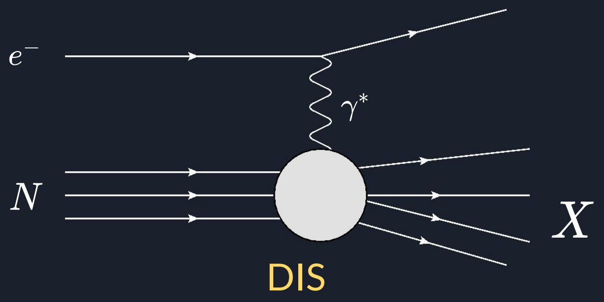
Parton Pseudo-Distributions From Lattice QCD via Distillation

9th Workshop of the APS Topical Group on Hadronic Physics
04/13/2021

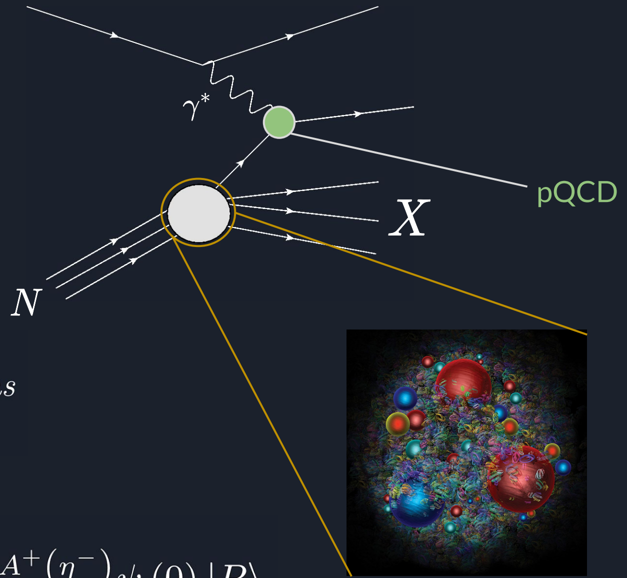
Colin Egerer

On Behalf of the
HadStruc Collaboration

Parton Distribution Functions (PDFs)



Parton Model



$$\sigma^{\mathrm{DIS}}\left(x, Q^2, \sqrt{s}\right) = \sum_{a=q,\bar{q},g} C_a\left(x, \frac{Q^2}{\mu^2}, \sqrt{s}\right) \otimes f_a\left(x, \mu^2\right) + power\ corrections$$

PDFs defined through light-like matrix elements

$$f_{a/h}\left(x, \mu\right) = \int \frac{\mathrm{d}\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P| \overline{\psi}\left(\xi^-, \mathbf{0_T}\right) \gamma^+ e^{-ig\int_0^{\xi^-} \mathrm{d}\eta^- A^+(\eta^-)} \psi\left(0\right) |P\rangle$$

PDF measurements - complement to hadron tomography efforts

- input to cross section predictions (e.g. LHC)
- affect precision measurements of SM parameters
- focus of upcoming facilities (EIC)

J. Gao et al., Phys. Rept. 742 (2018) 1-121

Lattice Gauge Theory (QCD)

Numerically solve QCD using Monte Carlo methods

- quantitatively study strong-coupled regimes

QCD action given as input $S_{\mathrm{QCD}}\left[\psi,\overline{\psi},G_\mu\right]$

- discretization (momentum cutoffs)
- path integral in Euclidean spacetime

$$\langle\hat{\mathcal{O}}\rangle_{\mathrm{E}} = \mathcal{Z}^{-1}\int \underbrace{\boxed{\mathcal{D}\psi\mathcal{D}\overline{\psi}}}_{\prod_{n\in\Lambda}\prod_{f,\alpha,c} \mathrm{d}\psi_{\mathrm{f}}\,(\mathrm{n})_\alpha^{\mathrm{c}}\,\mathrm{d}\overline{\psi}_{\mathrm{f}}\,(\mathrm{n})_\alpha^{\mathrm{c}}}\underbrace{\boxed{\mathcal{D}U}}_{\prod_{n\in\Lambda}\prod_{\mu=1}^{4}\mathrm{dU}_\mu\,(\mathrm{n})}\hat{\mathcal{O}}\left[\psi,\overline{\psi},U\right]e^{-S_{\mathrm{QCD}}^{\mathrm{E}}\left[\psi,\overline{\psi},U\right]}$$

Compute observables non-perturbatively

- i.e. correlation functions
(averaged over gluon configurations)

$$C_{2\mathrm{pt}}\left(\vec{p},t\right) = \langle 0|\,h\left(\vec{p},t\right)h^\dagger\left(0\right)|0\rangle$$

$$C_{3\mathrm{pt}}\left(\vec{p},\vec{q};t,\tau\right) = \langle 0|\,h\left(\vec{p},t\right)\mathcal{O}\left(\vec{q},\tau\right)h^\dagger\left(0\right)|0\rangle$$

- systematically improvable results

Roadmap

(Semi-) Inclusive Processes - PDFs

Cover image -- PC: CalLat / Bart van Lith.

Lattice QCD

Coordinate-space Factorizable Matrix Elements

Pseudo-Distributions in the forward limit (pseudo-PDFs)

Numerical Study

Distillation (at High-Momentum)

Selected Results

Unpolarized Valence/Plus Quark Distributions

Off-Forward Pseudo-Distributions

Generalized Pseudo-Distributions

3D Hadronic Structure Prospects

HadStruc Collaboration

Robert Edwards, Christos Kallidonis, Nikhil Karthik, Jianwei Qiu, David Richards, Eloy Romero, Frank Winter [1]

Balint Joó[2]

Carl Carlson, CE, Tanjib Khan, Christopher Monahan, Kostas Orginos, Raza Sufian[3]

Wayne Morris, Anatoly Radyushkin [4]

Joe Karpie [5]

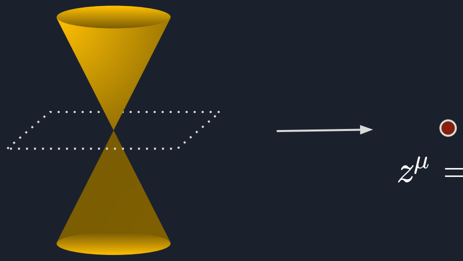
Savvas Zafeiropoulos [6]

Yan-Qing Ma [7]

Jefferson Lab [1], Oak Ridge [2], William and Mary [3], Old Dominion University [4], Columbia University [5], Aix Marseille University [6], Peking University [7]

PDFs From Lattice QCD

Collapse of light-cone in Euclidean space-time



$$f_{a/h}(x,\mu) = \int \frac{\mathrm{d}\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P | \overline{\psi}\left(\xi^-, \mathbf{0_T}\right) \gamma^+ e^{-ig\int_0^{\xi^-} \mathrm{d}\eta^- A^+(\eta^-)} \psi(0) | P \rangle$$

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OPE - Mellin moments of PDF
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$$\langle P | \overline{\psi}_f \Gamma_{\{\mu_1} D_{\mu_2} \cdots D_{\mu_n\}} \psi_f | P \rangle = \boxed{A^{(n)}_{\Gamma,f}} P_{\mu_1} \cdots P_{\mu_n}$$

Active development of methods to access light-cone structure of hadrons

- ➢ Hadronic Tensor — K.F. Liu et al., PRL 72, 1790 (1994) & Phys. Rev. D59/62
- ➢ Virtual Compton Amplitude [& OPE] — A.J. Chambers et al., Phys. Rev. Lett. 118 — "OPE without OPE" - G. Martinelli
- ➢ Auxiliary Quark Methods — U. Aglietti et al., Phys. Lett. B441; W. Detmold & C.J.D. Lin, Phys. Rev. D73; V. Braun & D. Mueller, Eur. Phys. J. C55
- ➢ Quasi-Distributions [PDFs/DAs/GPDs] — X. Ji, Phys. Rev. Lett. 110, 262002 (2013)
- ➢ Lattice "Cross Sections" — Y. Q. Ma & J. W. Qiu; Phys. Rev. D 98, no. 7, 074021 (2018) & Phys. Rev. Lett. 120, no. 2, 022003 (2018)
- ➢ Pseudo-Distributions [PDFs & GPDs] — A. V. Radyushkin, Phys. Rev. D 96, 034025 (2017); A. Radyushkin, Phys. Rev. D100, 116011 (2019); A. Radyushkin, Phys. Rev. D98, 014019 (2018)

Coordinate-space Factorizable Matrix Elements

- ❖ This talk: Lorentz-invariant amplitudes of coordinate-space matrix elements

$$\underbrace{\mathcal{T}_i\left(\nu, z^2\right)}_{\text{Pseudo-structure Functions (or Ioffe-time Pseudo-distributions)}} = \int_{-1}^{1} \mathrm{d}x \sum_j \underbrace{K_j^i\left(x\nu, z^2\mu^2\right)}_{\text{Hard Coefficients}} \underbrace{f_j\left(x, \mu^2\right)}_{\text{PDFs}} + \mathcal{O}\left(z^2\Lambda_{\mathrm{QCD}}^2\right)$$

$$\nu \equiv p \cdot z$$

B. L. Ioffe, Phys. Lett. 30B, 123 (1969)

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A spacelike matrix element [pseudo- & quasi-distributions]

$$M^\alpha(z,p) = \langle h(p)|\,\overline{\psi}(z)\,\gamma^\alpha W(z,0;A)\,\psi(0)\,|h(p)\rangle$$

$p \sim \sqrt{s}$ $\qquad$ $z^2 \sim \frac{1}{Q^2}$

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Pseudo-structure Functions (or Ioffe-time Pseudo-distributions)

Hard Coefficients

PDFs

A spacelike matrix element [pseudo- & quasi-distributions]

Ioffe-Time Pseudo-Distributions (pseudo-ITDs)

$$M^\alpha\left(z,p\right) = \langle h(p)|\,\overline{\psi}\left(z\right)\gamma^\alpha W\left(z,0;A\right)\psi\left(0\right)|h(p)\rangle = 2p^\alpha \boxed{\mathcal{M}_{\mathrm{p}}\left(\nu, z^2\right)} + z^\alpha \mathcal{M}_{\mathrm{z}}\left(\nu, z^2\right)$$

Leading-twist

$$\begin{cases} p^\mu = \left(p^0, 0, 0, p_3\right) \\ z^\mu = \left(0, 0, 0, z_3\right) \end{cases}$$

$p \sim \sqrt{s}$ $\qquad$ $z^2 \sim \frac{1}{Q^2}$

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Leading-twist $\left\{ \begin{array}{l} p^\mu = \left(p^0, 0, 0, p_3\right) \\ z^\mu = \left(0, 0, 0, z_3\right) \end{array} \right.$

$p \sim \sqrt{s}$ $\qquad z^2 \sim \frac{1}{Q^2}$

➢ Pseudo-PDFs
- ○ Fourier transform for constant z_3^2
- ○ Lorentz-invariant generalizations of PDFs to spacelike intervals
- ○ Lack probabilistic interpretation

A. Radyushkin, Phys.Rev.D 96 (2017) 3, 034025

$$\mathcal{P}\left(x, z_3^2\right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-ix\nu} \mathcal{M}_{\mathrm{p}}\left(\nu, z_3^2\right)$$

(neglecting divergences) converges to PDF in $z_3^2 \to 0$

More on Pseudo-ITDs

A. V. Radyushkin, Phys. Rev. D96, 034025 (2017)
A. Radyushkin, Phys. Rev. D98, 014019 (2018)

Pseudo-ITD shares log. divergence with PDF

- ➢ spacelike Wilson line acquires *power divergence*
- ➢ all-order link related divergence

$$Z_{\text{link}}\left(z_3, a\right) \simeq e^{-A|z_3|/a}$$

- independent of Ioffe-time
- multiplicatively renormalizable

T. Ishikawa, et al., Phys. Rev. D96 (2017) 094019

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Reduced Distribution (or “reduced pITD”)

K. Orginos, et al., Phys. Rev. D96, 094503 (2017)

$$\mathfrak{M}\left(\nu, z_3^2\right) \equiv \frac{\mathcal{M}_p\left(\nu, z_3^2\right)}{\mathcal{M}_p\left(0, z_3^2\right)}$$

$$= \underbrace{\left(\frac{\mathcal{M}_p\left(\nu, z_3^2\right)}{\mathcal{M}_p\left(\nu, 0\right)|_{z_3=0}}\right) \times \left(\frac{\mathcal{M}_p\left(0, 0\right)|_{p=0, z_3=0}}{\mathcal{M}_p\left(0, z_3^2\right)|_{p=0}}\right)}_{\text{RGI!}}$$

Ioffe-time indep. + unity in zero separation limit
Same PDF

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Matching relationship (1-loop) between pseudo-ITD and light-cone Ioffe-time Distribution (ITD)

$$\mathfrak{M}\left(\nu, z^2\right) = \mathcal{Q}\left(\nu, \mu^2\right) + \frac{\alpha_s C_F}{2\pi}\int_0^1 {\rm d}u \left[\ln\left(z^2\mu^2\frac{e^{2\gamma_E+1}}{4}\right) B\left(u\right) + L\left(u\right)\right]\mathcal{Q}\left(u\nu, \mu^2\right) + \sum_{k=1}^{\infty}\mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$$

$$\mathcal{Q}\left(\nu, \mu^2\right) = \int_{-1}^{1} dx\ e^{i\nu x} q\left(x, \mu^2\right)$$

Must ensure perturbative regime $|\vec{z}| \ll \Lambda_{\rm QCD}^{-1}$
minimize h.t. effects

Lattice Implementation

ID	a (fm)	m_π (MeV)	$L^3 \times N_t$	N_{cfg}	N_{srcs}	N_{vec}
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	349	4	64

JLab/WM/LANL
2+1 Flavor
Isotropic Lattice

Spatial smearing - increase interpolator overlap with ground-state

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- Distillation: Low-rank approximation of a gauge-covariant smearing kernel
  M. Peardon et al., Phys. Rev. D80, 054506 (2009)

$$J_{\sigma,n_\sigma} = e^{\sigma\nabla^2} = \sum_\lambda e^{-\sigma\lambda}\,|\lambda\rangle\langle\lambda|$$

$$\Box\,(\vec{x},\vec{y};t)_{ab} = \sum_{k=1}^{N} \xi_a^{(k)}\,(\vec{x},t)\,\xi_b^{(k)\dagger}\,(\vec{y},t)$$

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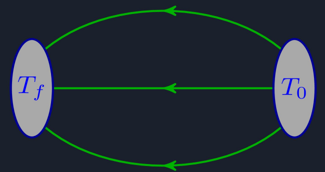

"Perambulators"

$$\tau^{kl}_{\alpha\beta}(t_f, t_0) = \xi^{(k)\dagger}(t_f)\, M^{-1}_{\alpha\beta}(t_f, t_0)\, \xi^{(l)}(t_0)$$

"Elementals"

$$\Phi^{(i,j,k)}_{\mu\nu\sigma}(t) = \epsilon^{abc}\left(\mathcal{D}_1 \xi^{(i)}\right)^a \left(\mathcal{D}_2 \xi^{(j)}\right)^b \left(\mathcal{D}_3 \xi^{(k)}\right)^c (t)\, S_{\mu\nu\sigma}$$

Irrep. projection

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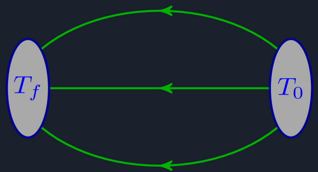
Wick contract "distilled" (smeared) fields

$$C_{mn}(t) = \sum_{\vec{x}, \vec{y}} \langle 0 | \mathcal{O}_m(t, \vec{x})\, \mathcal{O}^\dagger_n(0, \vec{y}) | 0 \rangle$$

$$\equiv \mathrm{Tr}\left[\Phi_m(t) \otimes \tau(t,0)\, \tau(t,0)\, \tau(t,0) \otimes \Phi_n(0)\right]$$

Lattice Implementation



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$$\square\left(\vec{x},\vec{y};t\right)_{ab} = \sum_{k=1}^N \xi^{(k)}_a\left(\vec{x},t\right)\xi^{(k)\dagger}_b\left(\vec{y},t\right)$$

Why Distillation ?

- *factorization of correlation functions*
- *explicit momentum projections - all times*
- *excited-state control* [1]
- *reusability*
- *admits efficient implementation of variational method*

Key to control of excited-states

R. Briceno et al., Phys.Rev.D 97 (2018) 5, 054513
J. Dudek et. al., Phys.Rev.D 88 (2013) 9, 094505
J. Dudek et al., Phys.Rev.D 87 (2013) 3, 034505
L. Liu, et. al., JHEP 07, (2012) 126
J. Dudek, et. al., Phys. Rev.D83, 111502 (2011)
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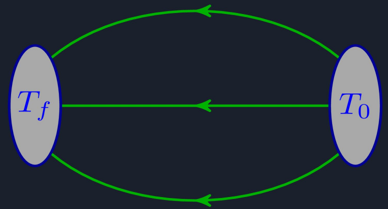
$$\equiv \mathrm{Tr}\left[\Phi_m\left(t\right)\otimes\tau\left(t,0\right)\tau\left(t,0\right)\tau\left(t,0\right)\otimes\Phi_n\left(0\right)\right]$$

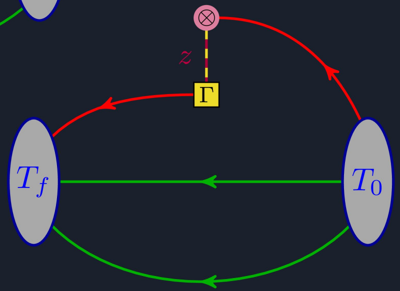
[1] C.E., D. Richards, F. Winter, Phys. Rev. D 99 (2019) 3, 034506
C.E. et al., Phys. Rev. D103, 034502 (2021)

Numerical Workload

Correlation functions needed:



$$C_2\left(p_z,T\right)=\langle\mathcal{N}\left(-p_z,T\right)\overline{\mathcal{N}}\left(p_z,0\right)\rangle=\sum_n\left|\mathcal{A}_n\right|^2e^{-E_nT}$$



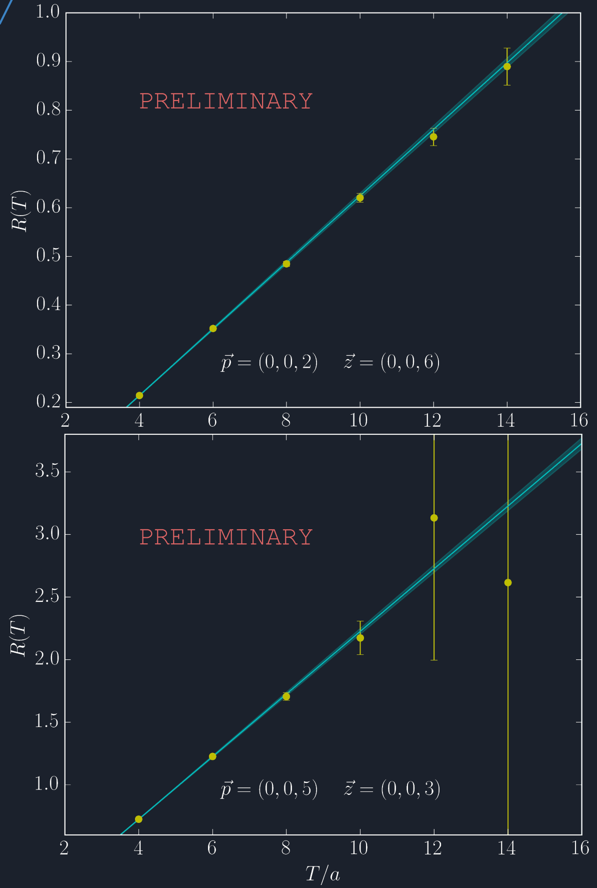
$$C_3\left(p_z,\vec{z};T,t_{\text{ins}}\right)=\sum_{\vec{z}}\langle\mathcal{N}\left(-p_z,T\right)\overline{\psi}\left(\vec{z},t_{\text{ins}}\right)\gamma^{\alpha}_{u-d}W\left(0,\vec{z};A\right)\psi\left(0,t_{\text{ins}}\right)\overline{\mathcal{N}}\left(p_z,0\right)\rangle$$
$$=\sum_{n,n'}\langle\mathcal{N}|n'\rangle\langle n|\overline{\mathcal{N}}\rangle\langle n'|\hat{\mathcal{O}}_{\gamma_4}|n\rangle e^{-E_{n'}(T-t_{\text{ins}})}e^{-E_nT}$$

➢ Distillation induces expensive generalized perambulator (“genprop”)

$$\tilde{\tau}^{ij}_{\text{pITD}}\left(t_f,t_0;\tau,[\vec{z}];\vec{q}\right)=\sum_{\vec{z}}e^{i\vec{q}\cdot\vec{z}}\xi^{(i)\dagger}\left(t_f\right)D^{-1}\left(t_f;\vec{z}+\vec{z}_0,\tau\right)\Gamma W\left(\vec{z}+\vec{z}_0,\vec{z}_0;\tau\right)D^{-1}\left(\vec{z}_0,\tau;t_0\right)\xi^{(j)}\left(t_0\right)$$

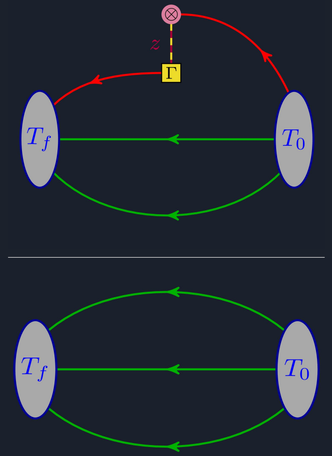
t_{sep}/a	$p_z\ \left(\times\frac{2\pi}{L}\right)$	z/a
$4,6,\cdots,14$ $0.38,\cdots 1.32$ fm	$0,\pm1,\cdots,\pm6$ $0,0.411,\cdots,2.47$ GeV	$0,\pm1,\cdots,\pm12,\cdots$ $0,0.094,\cdots,1.13$ fm

Selected Matrix Element Isolation



Matrix element from ratio of correlators

$$R(p_z, z_3; T) = \sum_{t_{\text{ins}}/a=1}^{T-1} \frac{C_3(p_z, z_3; T, t_{\text{ins}})}{C_2(p_z, T)} =$$



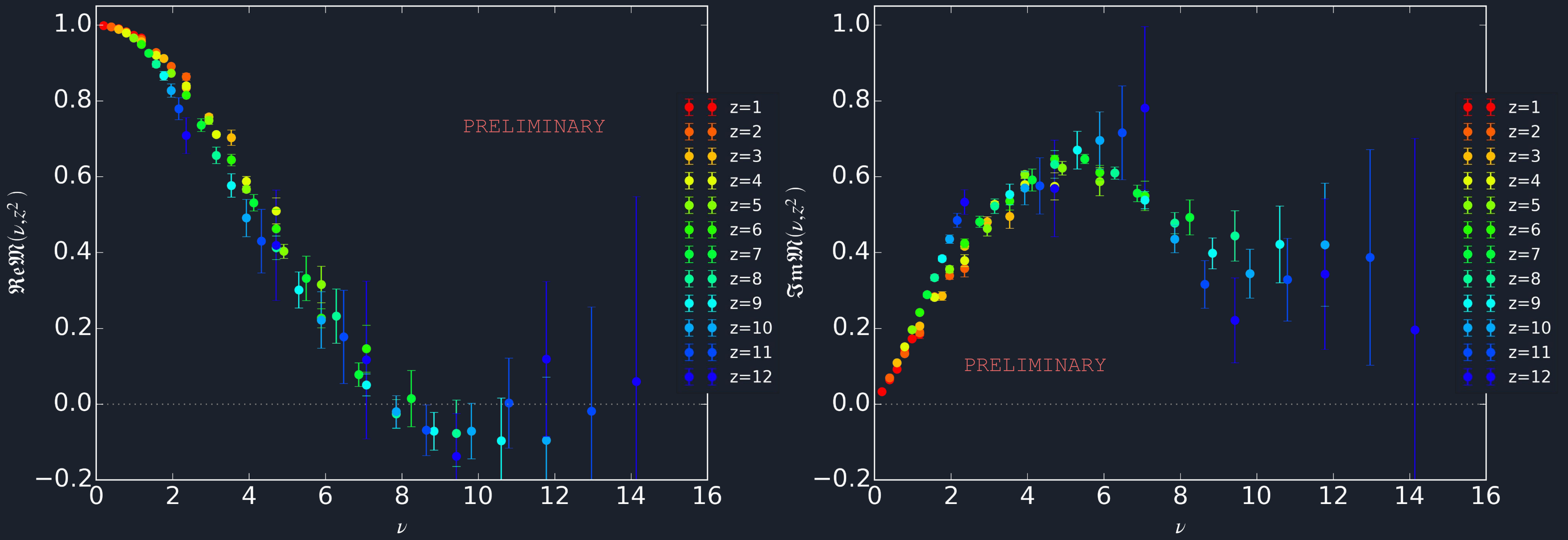
L. Maiani et al., Nucl. Phys. B 293 (1987)
C. Bouchard et al., Phys. Rev. D 96, no. 1, 014504 (2017)

$$R(p_z, z_3; T) = C + \mathcal{M}T + \mathcal{O}\left(e^{-\Delta E_{10} T}\right)$$

"Summation Method"

- greater suppression of excited-states

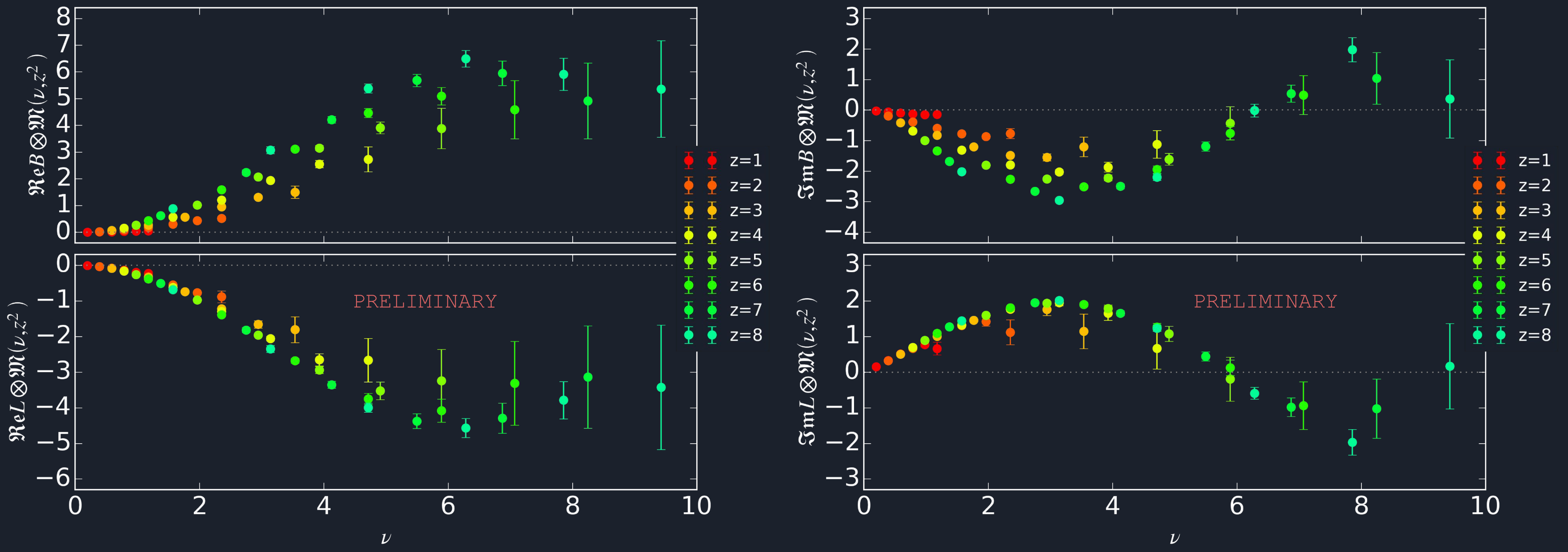
Unpolarized Ioffe-time Pseudo-Distributions



- ➢ Evolution/matching - need smooth interpolation of $\mathfrak{M}\left(\nu, z^2\right)$
  - ○ polynomial to each z_3 independently

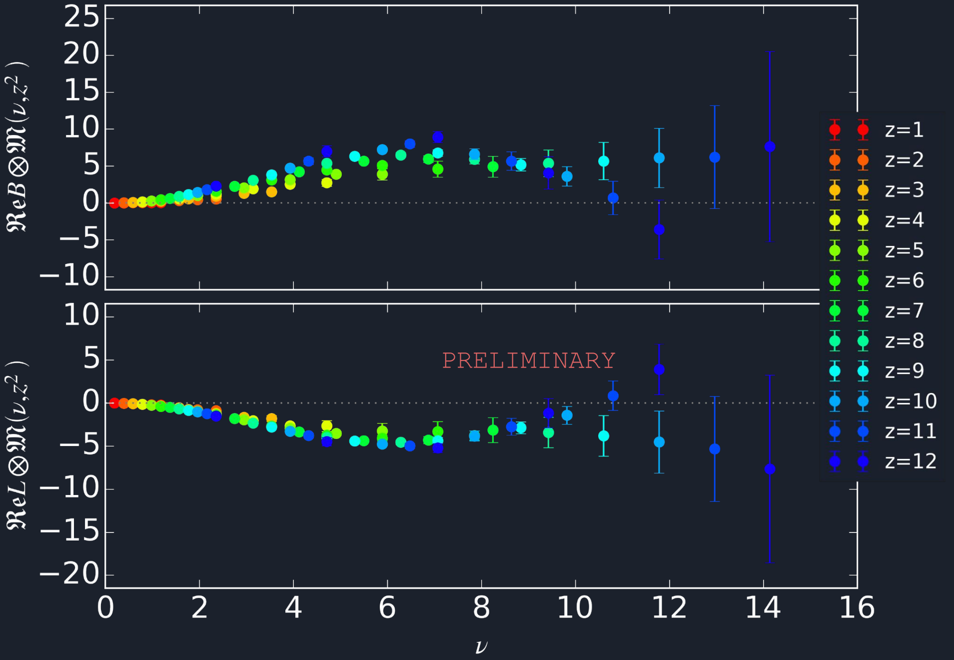
$$\mathfrak{M}\left(\nu, z_3^2\right) = 1 + \sum_{n=1}^{3}\left(c_{2n}\nu^{2n} + ic_{2n-1}\nu^{2n-1}\right)$$

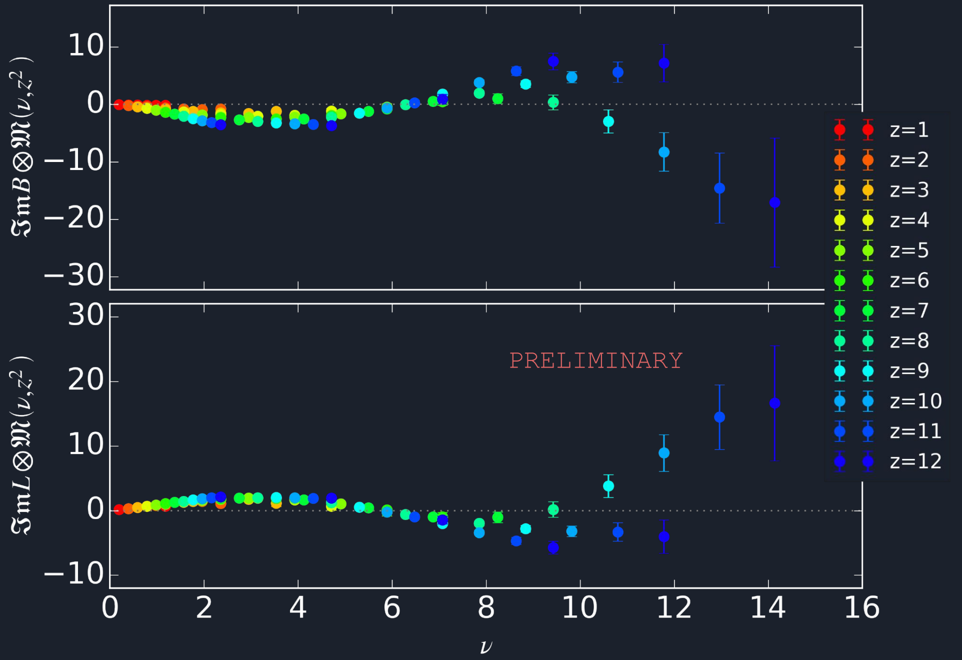
Effect of Evolution/Matching



$$B\left(u\right)=\left[\frac{1+u^2}{1-u}\right]_+ \qquad L\left(u\right)=\left[4\frac{\ln\left(1-u\right)}{1-u}-2\left(1-u\right)\right]_+$$

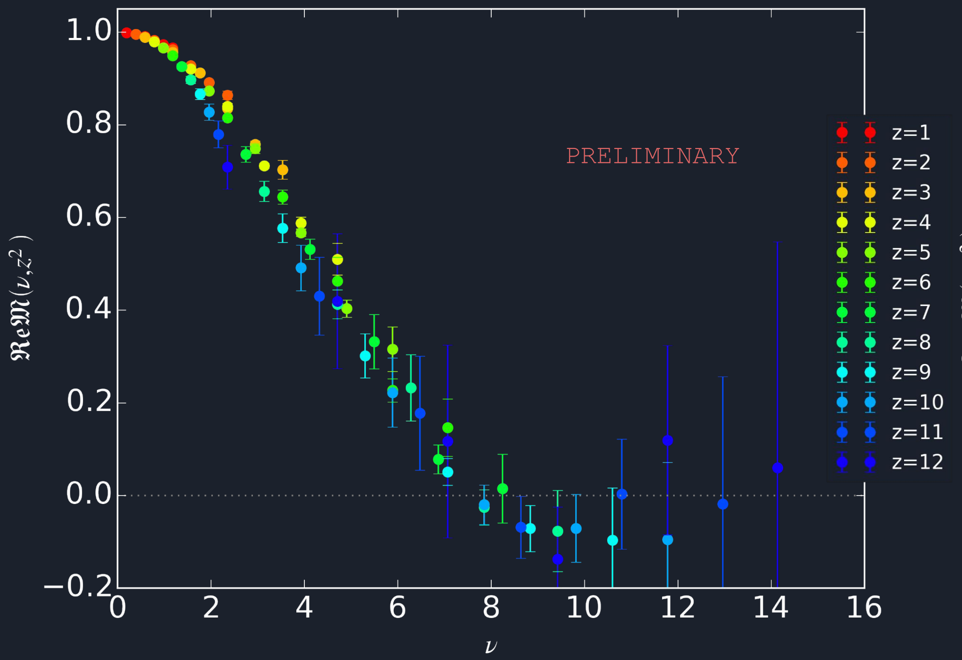
Effect of Evolution/Matching

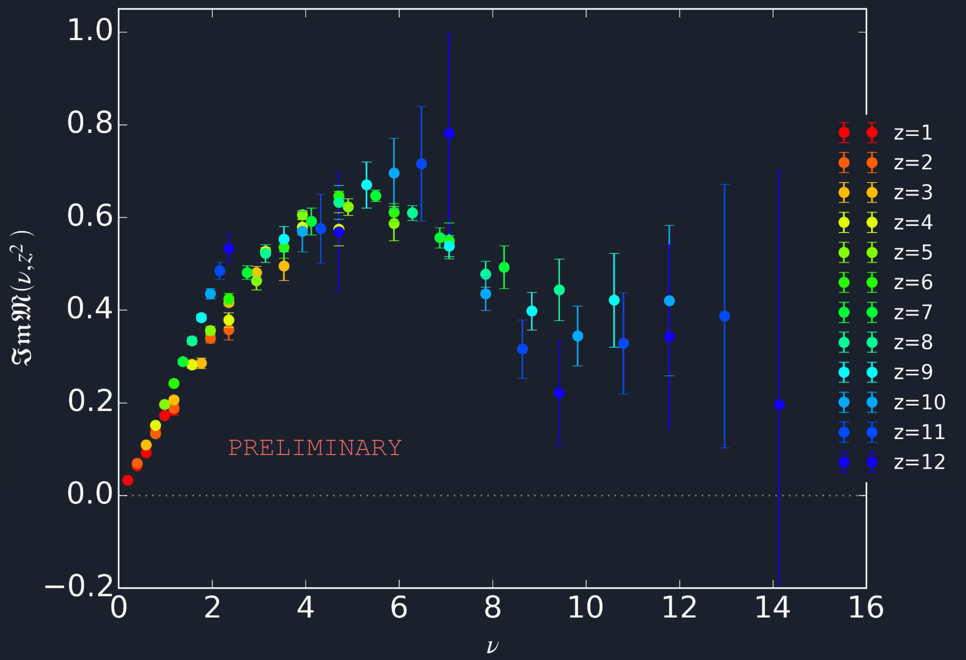




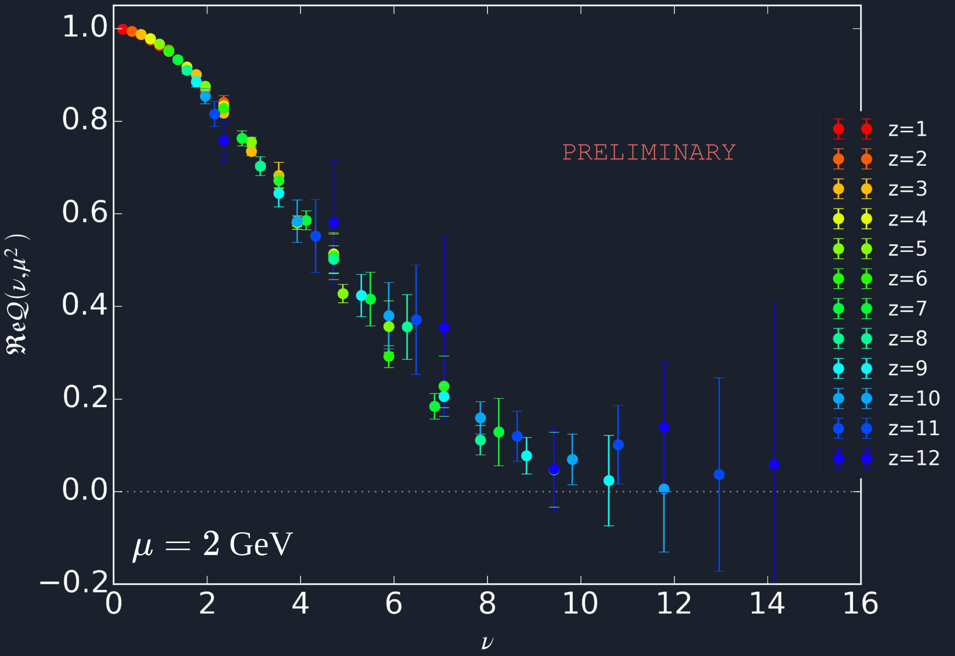
$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+ \qquad L(u) = \left[4\frac{\ln(1-u)}{1-u} - 2(1-u)\right]_+$$

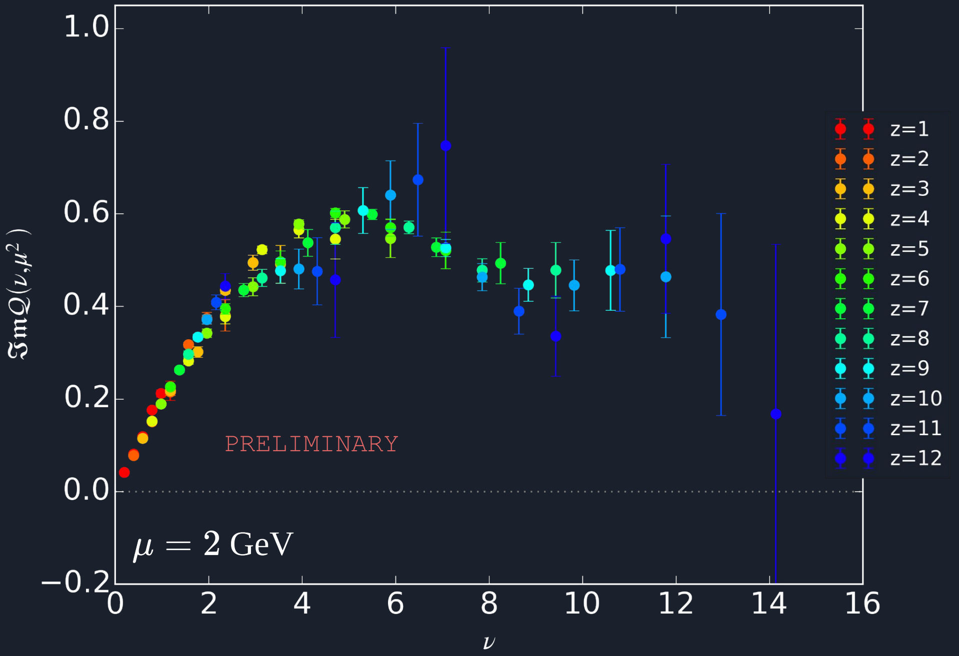
Unpolarized Ioffe-time Pseudo-Distributions





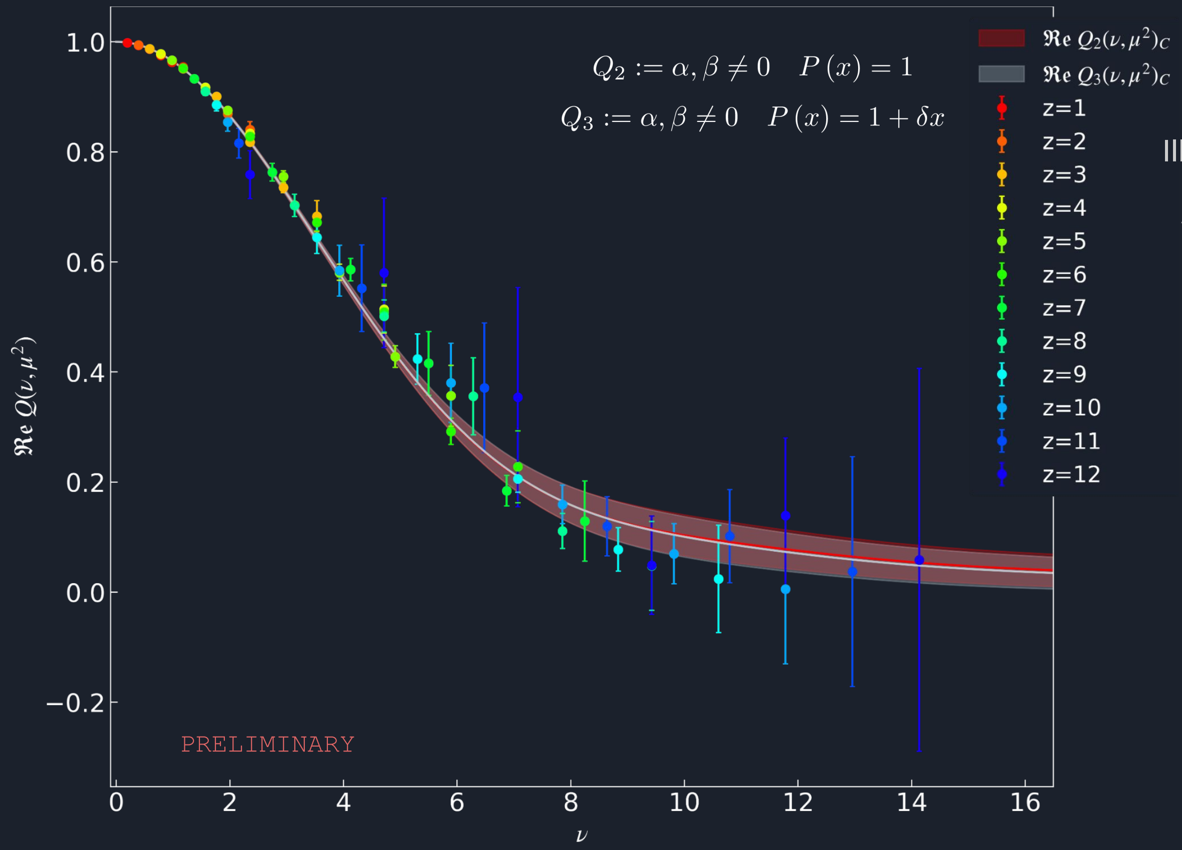
Unpolarized Ioffe-time Distributions





c.f. Phys.Rev.D 103 (2021) 3, 034510; Phys.Rev.Lett. 125 (2020) 23, 232003
JHEP 12 (2019) 081; Phys.Rev.D 96 (2017) 9, 094503

PDFs from Ioffe-time Distribution Fits



$$\mathfrak{Re}\, Q\left(\nu,\mu^2\right) = \int_0^1 dx \cos\left(\nu x\right) q_{\rm v}\left(x,\mu^2\right)$$

Ill-posed ITD - PDF relation [How to Proceed?]

A) Supply extra physically motivated information

B) Parametric fits (model bias - i.e. functional forms & at what stage)

$$q_{\rm v}\left(x\right) = N_{\rm v} x^{\alpha}\left(1-x\right)^{\beta} P\left(x\right)$$

C) Smooth function to connect nominal behavior

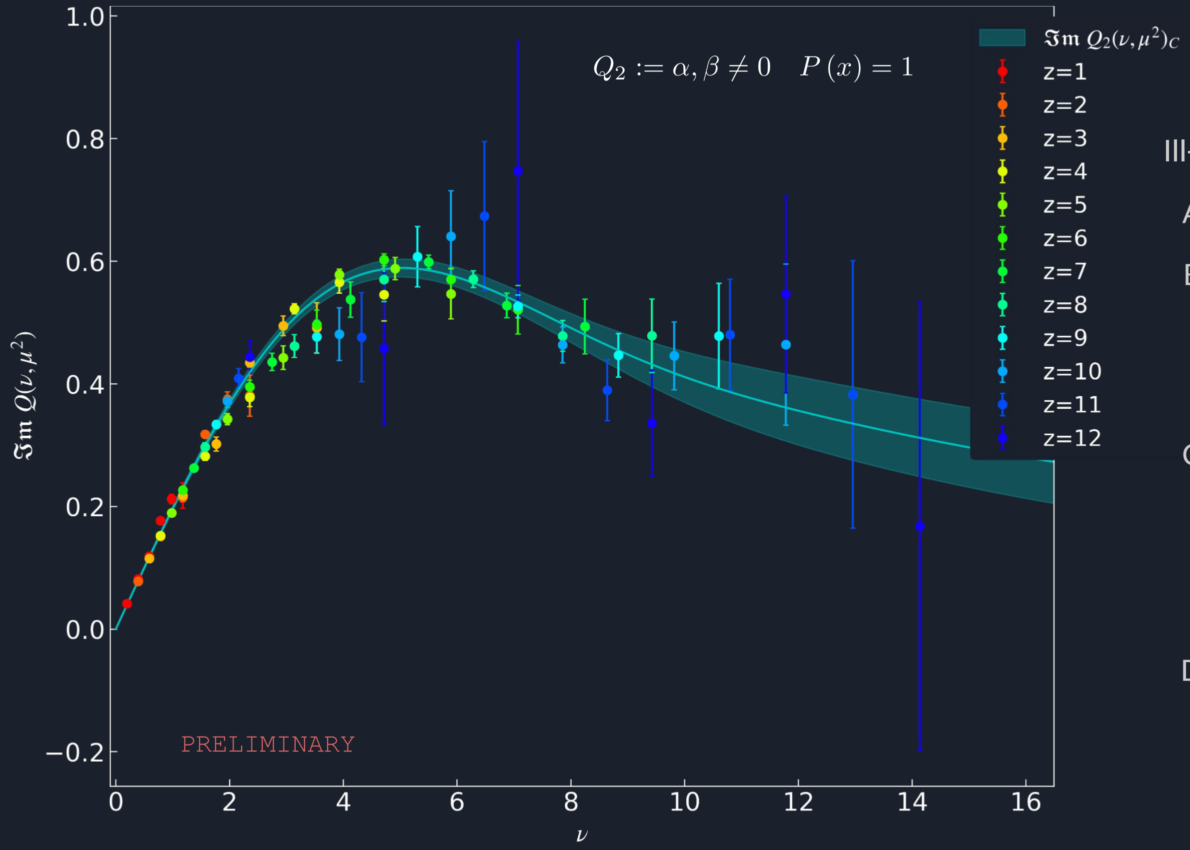
$$P\left(x\right) = 1 + \sum_k \lambda_k x^{(k+1)/2}$$

$$N_{\rm v} = B\left(\alpha+1,\beta+1\right) + \sum_k \lambda_k B\left(\alpha+1+\frac{k+1}{2},\beta+1\right)$$

D) Least-squares fit to matched ITD

$$\chi^2 = \sum_{\nu_{min}}^{\nu_{max}} \frac{\left[Q\left(\nu,\mu^2\right) - Q_{fit}\left(\nu,\mu^2\right)\right]^2}{\sigma_Q^2}$$

PDFs from Ioffe-time Distribution Fits



$$\mathfrak{Im}\, Q\left(\nu, \mu^2\right) = \int_0^1 dx \sin\left(\nu x\right) q_+\left(x, \mu^2\right)$$

Ill-posed ITD - PDF relation [How to Proceed?]

A) Supply extra physically motivated information

B) Parametric fits (model bias - i.e. functional forms & at what stage)

$$q_+\left(x\right) = N_+ x^{\alpha_+}\left(1 - x\right)^{\beta_+} P\left(x\right)$$

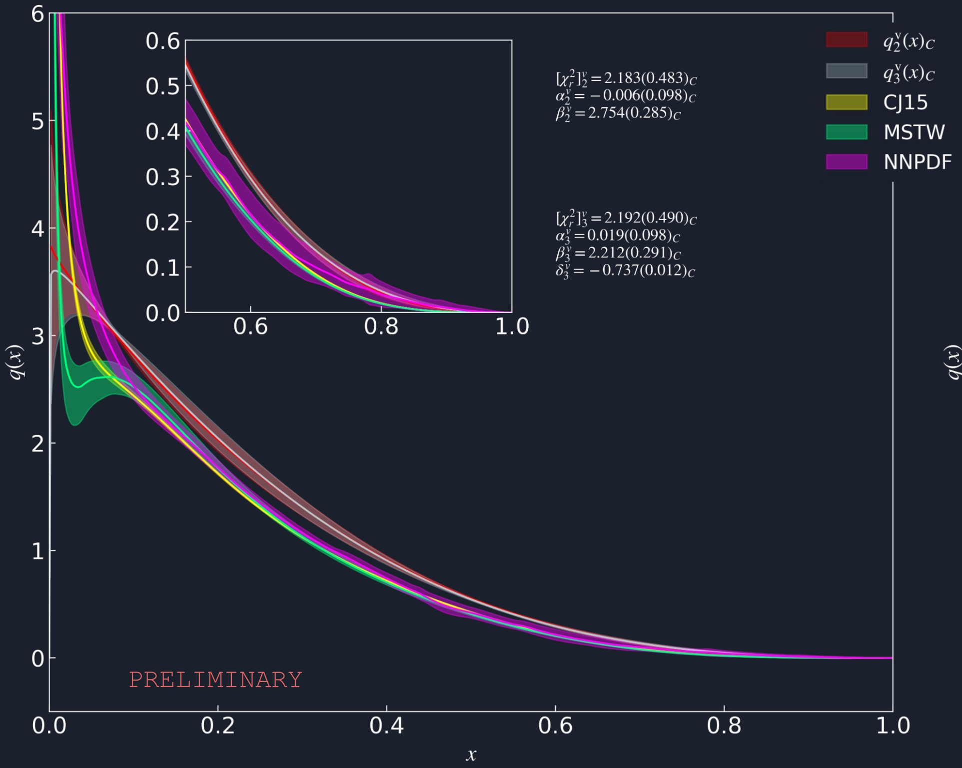
C) Smooth function to connect nominal behavior

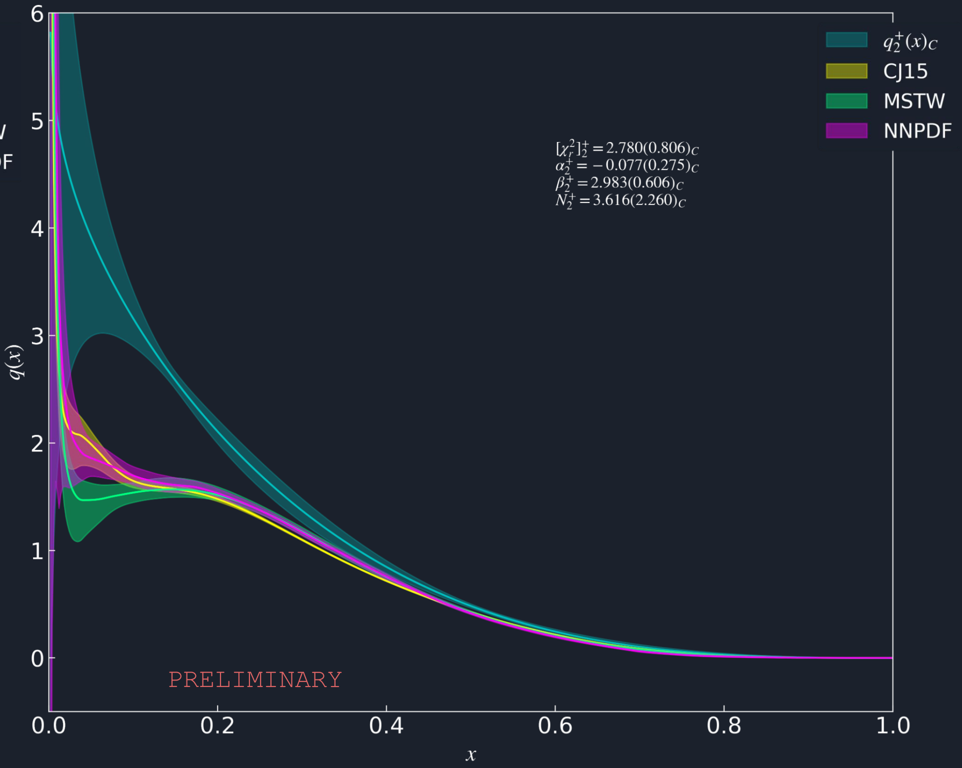
$$P\left(x\right) = 1 + \sum_k \lambda_k x^{(k+1)/2}$$

D) Least-squares fit to matched ITD

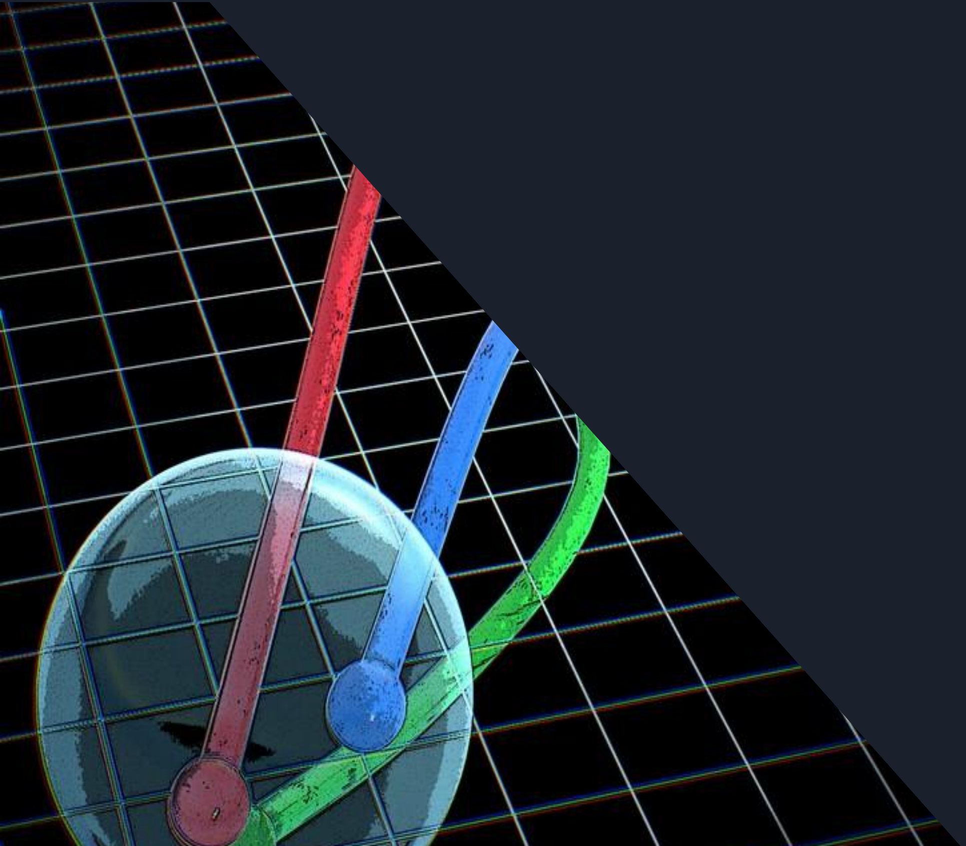
$$\chi^2 = \sum_{\nu_{min}}^{\nu_{max}} \frac{\left[Q\left(\nu, \mu^2\right) - Q_{fit}\left(\nu, \mu^2\right)\right]^2}{\sigma_Q^2}$$

PDFs and Phenomenological Comparison



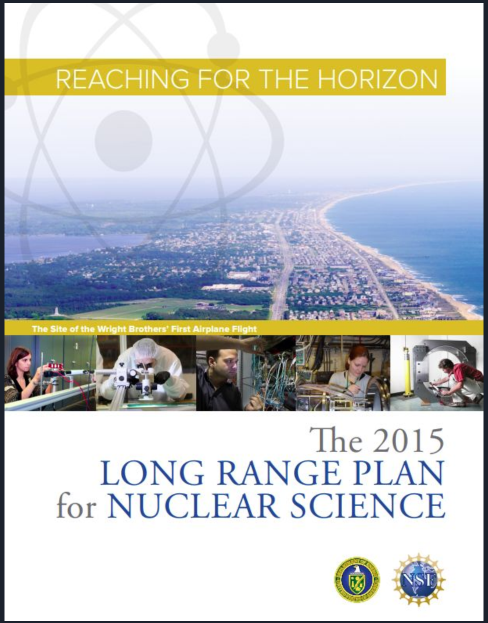


Pseudo-Distributions in the Off-Forward Case



Towards a Three-Dimensional Image of Hadrons from Lattice QCD

The Nuclear Science Long-Range Plan



DOE Office of Science & NSF Directorate of Mathematical and Physical Sciences charge to Nuclear Science Advisory Committee (NSAC)

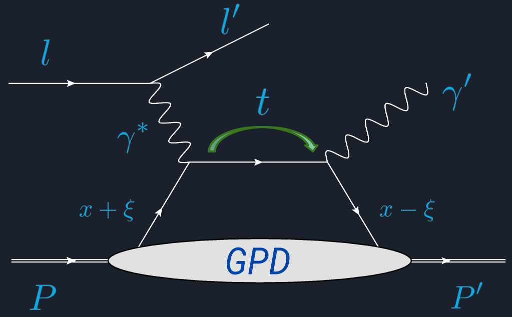
"To meet challenges and realize full scientific potential of current/future experiments, we require new investments in theoretical/computational nuclear physics."

"... a multidimensional description of nucleon structure is emerging that is providing profound new insights"

"[GPDs] will transform the current picture of hadronic structure"

Challenge to extract GPDs from experimental data

- ➢ variety of exclusive channels/observables needed for complete GPD extraction (e.g. DVCS/DVMP)
- ➢ DVCS observables & Compton Form Factors



$$CFF \sim \int_{-1}^{1} \mathrm{d}x \frac{GPD\,(x,\xi,t)}{x-\xi+i\epsilon} + \cdots$$

Generalized Parton Distributions and Double Ioffe-time Pseudo-Distributions

A. Radyushkin, Phys. Rev. D100, 116011 (2019)
A. Radyushkin, arXiv: Int.J.Mod.Phys.A 35 (2020) 05, 2030002

Double Ioffe-Time Pseudo-Distributions

An off-forward spacelike matrix element of interest

$$\mathrm{M}^{\alpha}\left(p_2, p_1, z\right) \equiv \langle h(p_2)|\,\overline{\psi}\left(0\right)\boxed{\frac{\tau^3}{2}}\Gamma^{\alpha} W\left(0, z; A\right)\psi\left(z\right)|h\left(p_1\right)\rangle \;=\; \frac{\left(p_2+p_1\right)^{\alpha}}{2} M\left(\nu_2, \nu_1, t; z^2\right) + z^{\alpha} N\left(\nu_2, \nu_1, t; z^2\right)$$

Isovector projection - numerically cheaper

Generalized Ioffe-time Pseudo-Distribution (pGITD)

- ioffe-times define skewness/momentum transfer

$$\xi = \frac{(p_1 z) - (p_2 z)}{(p_1 z) + (p_2 z)} \qquad \nu_i \equiv -\left(p_i \cdot z\right) \qquad t = \left(p_1 - p_2\right)^2$$

$$M\left(\nu_2, \nu_1, t; z^2\right) \mapsto \mathcal{M}\left(\nu, \xi, t; z_3^2\right)$$

Manage power divergence

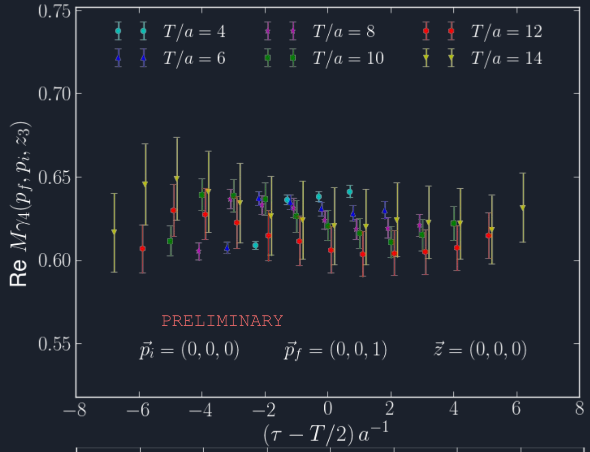
- factorization relationship to Generalized Ioffe-time Distribution

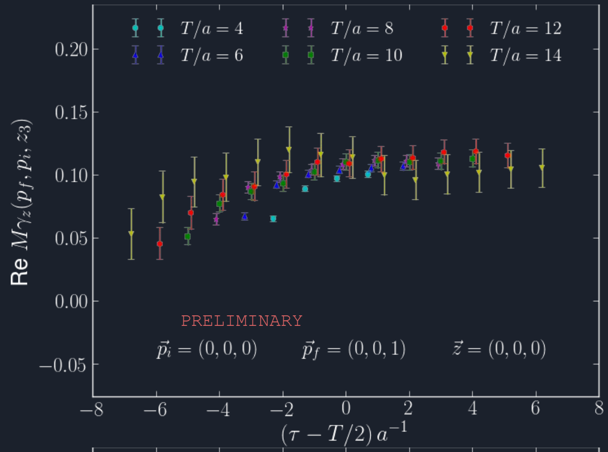
$$\widetilde{\mathcal{I}}\left(\nu, \xi, t, \mu^2\right) = \int_{-1}^{1} \mathrm{d}x\, e^{ix\nu} H\left(x, \xi, t; \mu^2\right)$$

$$\widetilde{\mathfrak{M}}\left(\nu, \xi, t, z_3^2\right) \equiv \frac{\widetilde{\mathcal{M}}\left(\nu, \xi, t, z_3^2\right)}{\widetilde{\mathcal{M}}\left(0, 0, 0, z_3^2\right)}$$

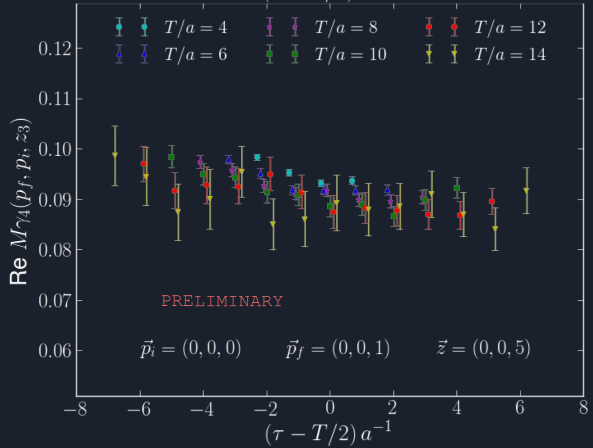
$$\widetilde{\mathfrak{M}}\left(\nu, \xi, t, z_3^2\right) = K\left(\xi\nu, z^2\mu^2; \alpha_s\right) \otimes \widetilde{\mathcal{I}}\left(\nu, \xi, t, \mu^2\right)$$

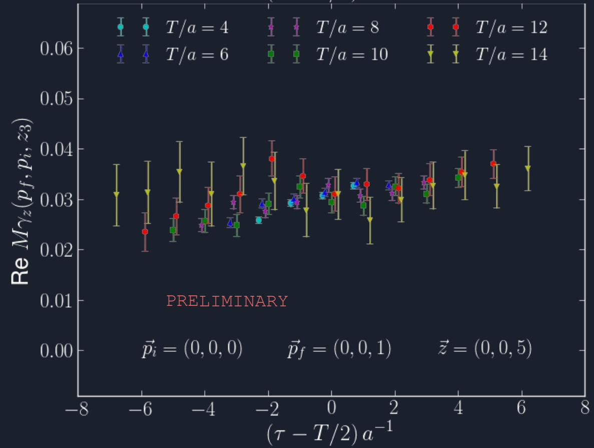
Selected Off-Forward Matrix Elements



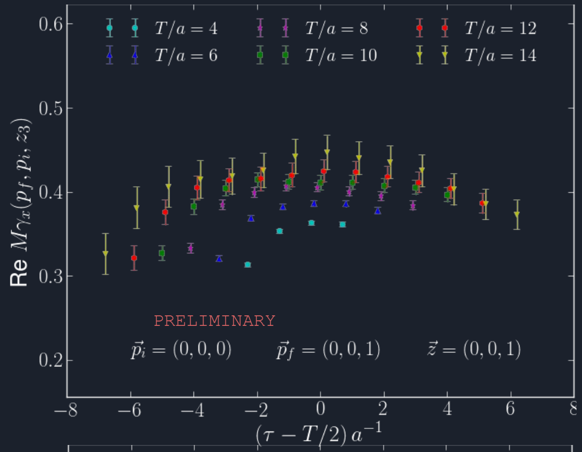


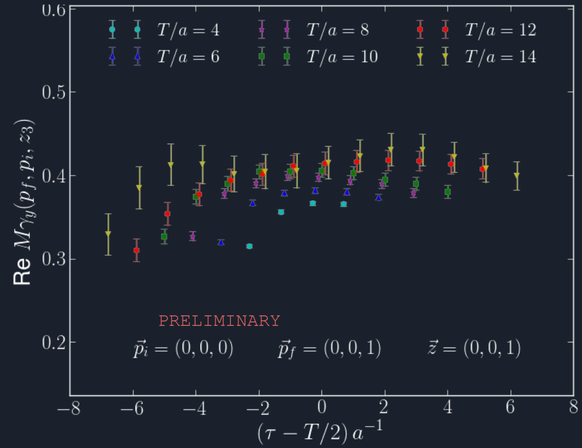
Matrix element extractions are underway



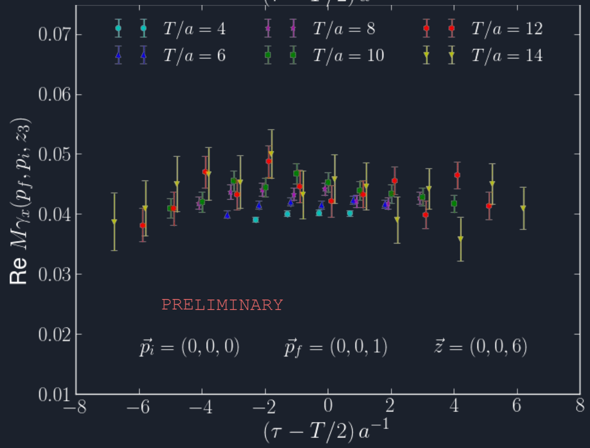


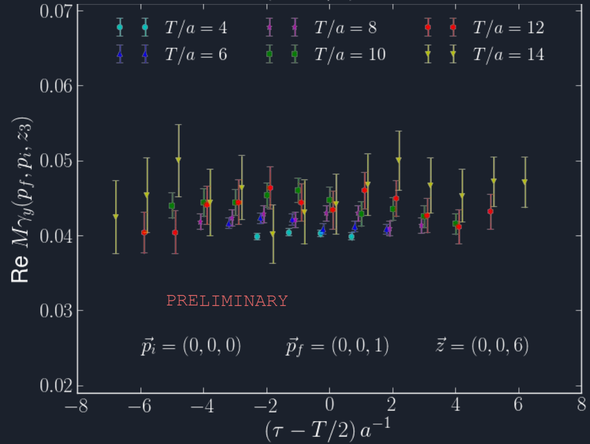
Selected Off-Forward Matrix Elements





Matrix element extractions are underway





Wrap Up & Outlook

Hadronic structure accessible from lattice calculable matrix elements

Pseudo-Distribution formalism & Distillation spatial smearing program - improved pseudo-ITD determination

- ✓ controllable effects from excited-states
- ✓ lattice systematics potentially visible

Nucleon valence/plus quark PDFs - a proving ground

- ✓ constrained by reach in Ioffe-time
- ✓ higher-momenta - reducing lattice spacing (underway)
- ✓ discrepancies with phenomenology observed - non-physical simulation parameters

Generalized Parton Distributions - a vast landscape wherein LQCD is starting to provide insight

- ✓ inverse problem more challenging to control
- ✓ parametrizations and polynomiality

X. D. Ji, Phys. Rev. Lett. 78, 610 (1997)
D. Müller et al., Fortsch. Phys. 42, 101 (1994)
A. Radyushkin, Phys. Rev. D 56, 5524 (1997)

Thank You!

Supplements

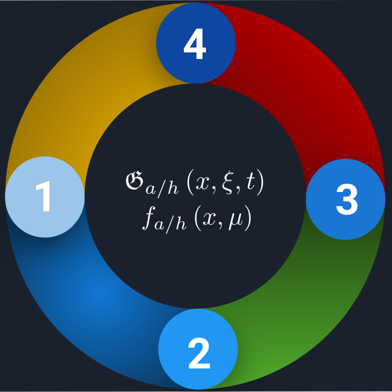
From Lattice Data to PDFs/GPDs

Matrix Element

- GLCS
- Pseudo-PDF/GPD

Renormalization

- Trivial factor [GLCS]
- Reduced distribution



Inverse Problem

A grossly ill-posed convolutional relationship connecting lattice data to desired structure function

A serious systematic that must be confronted

Analogous challenge faced by global fitting community!

Evolution/Matching

Coordinate-space factorization; perturbative matching kernels

How to Proceed?

A) Parametric fits (potential bias - i.e. functional forms & at what stage)

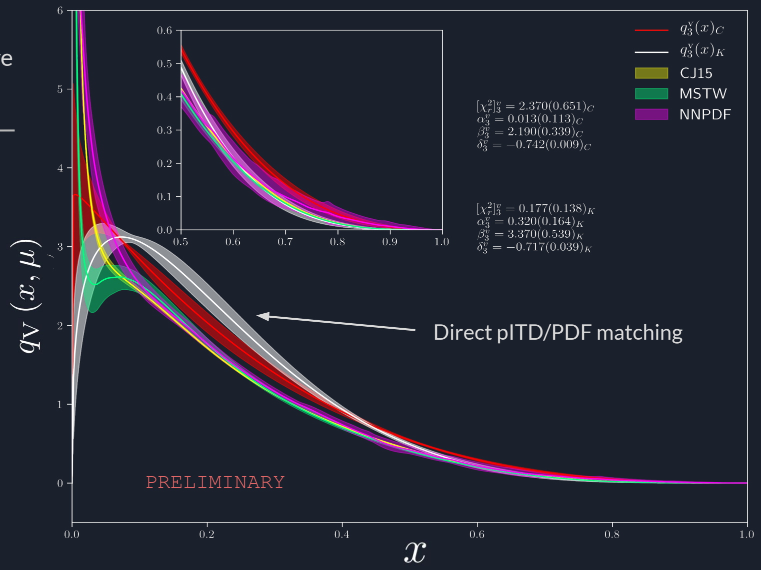
$$q_V(x,\mu) = \frac{x^\alpha (1-x)^\beta \left(1+\sum_k \lambda_k x^{(k+1)/2}\right)}{B(\alpha+1,\beta+1)+\sum_k \lambda_k B\left(\alpha+1+\frac{k+1}{2},\beta+1\right)}$$

B) Advanced reconstructions

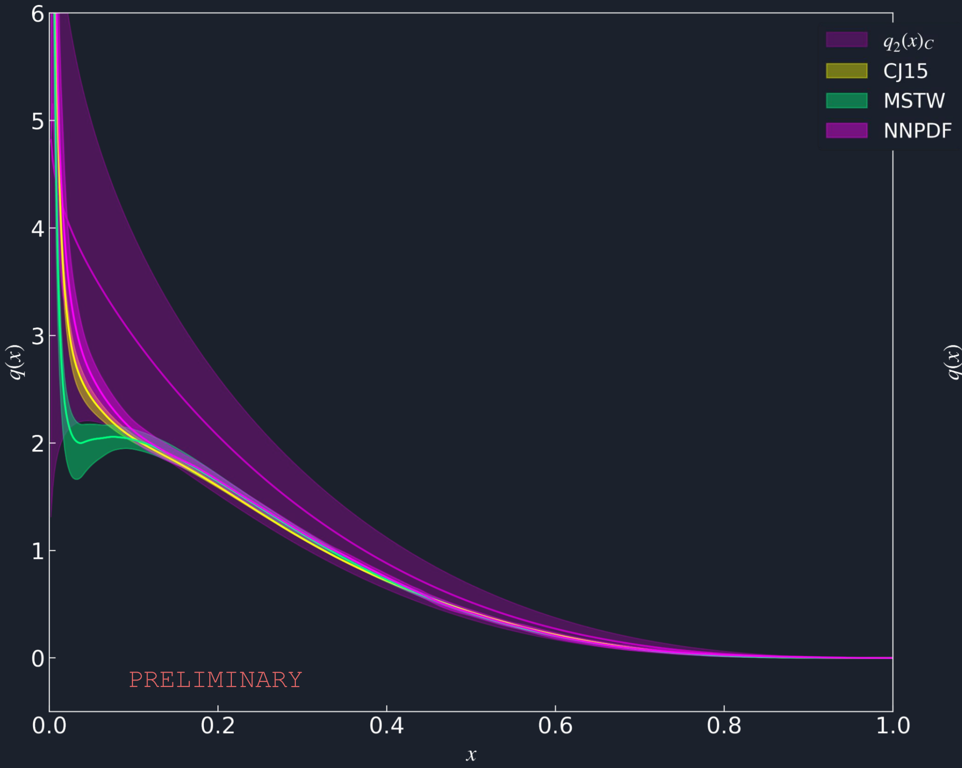
a) Bayesian reconstruction, Backus-Gilbert, Maximum Entropy, etc — J. Karpie, et al., JHEP 04, 057 (2019)

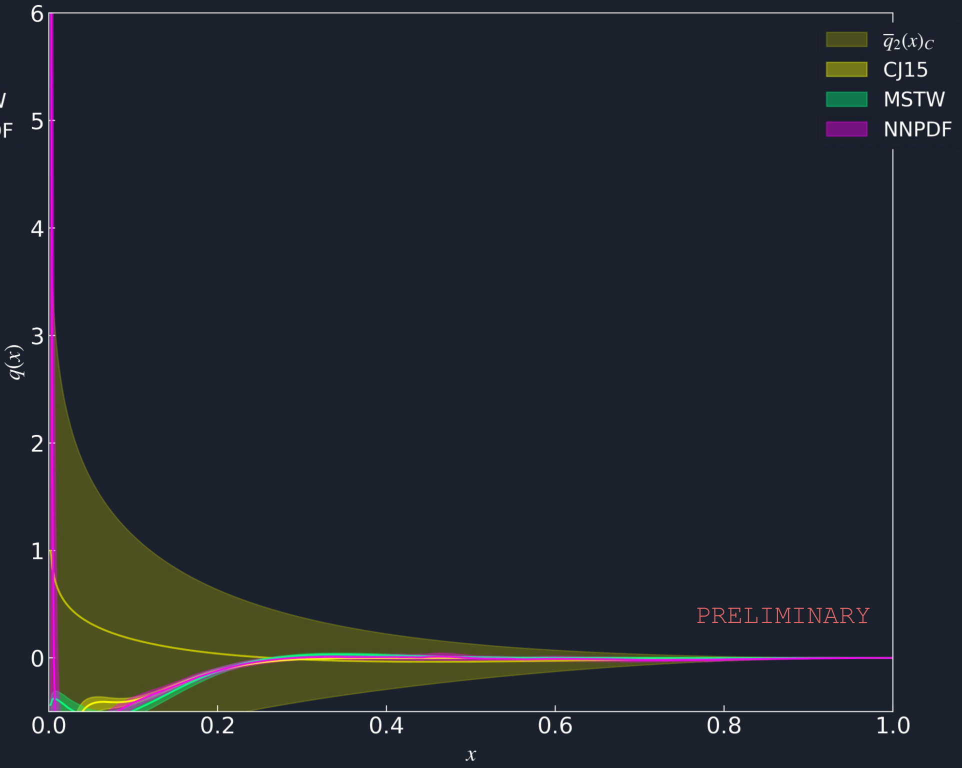
b) sensitivity of default model/pre-conditioning

C) Novel deep-learning methods — L. D. Debbio, et al., arXiv: 2010.03996 [hep-ph]; K. Cichy, L. D. Debbio, T. Giani, JHEP 10 (2019) 137



(Connected) Flavor Separated PDFs and Phenomenological Comparison





NLO Matching pseudo-GITD to Generalized Ioffe-time Distribution

Technicality: matching w.r.t. "symmetric" operator

$$\begin{aligned}\langle p_2|\,\overline{\psi}(0)\cdots\psi(z)\,|p_1\rangle &= e^{-i(p_1 z)/2+i(p_2 z)/2}\langle p_2|\,\overline{\psi}(-z/2)\cdots\psi(z/2)\,|p_1\rangle \\ &= e^{i\xi\nu}\langle p_2|\,\overline{\psi}(-z/2)\cdots\psi(z/2)\,|p_1\rangle \\ &= e^{i\xi\nu}\widetilde{\mathcal{M}}\end{aligned}$$

Compute in LQCD

Matching valid here

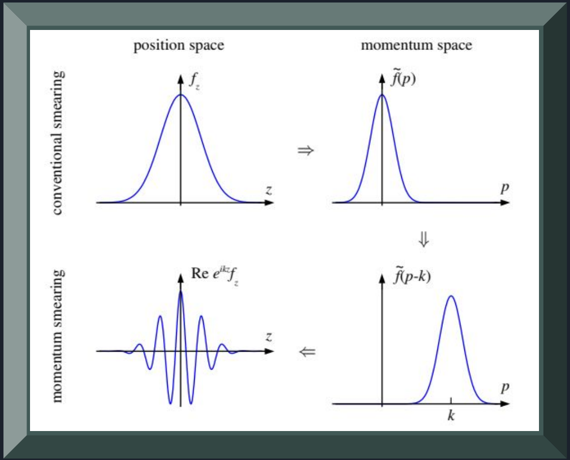
Scale dep. of pGITD

$$\widetilde{\mathcal{I}}\left(\nu,\xi,t,\mu^2\right)=\widetilde{\mathfrak{M}}\left(\nu,\xi,t,z_3^2\right)+\frac{\alpha_s C_F}{2\pi}\int_0^1 du\,\widetilde{\mathfrak{M}}\left(u\nu,\xi,t,z_3^2\right)$$
$$\times\left\{\boxed{\ln\left[\frac{e^{2\gamma_E+1}}{4}z_3^2\mu^2\right]\left(\left[\frac{2u}{1-u}\right]_+\cos\left(\bar{u}\xi\nu\right)+\frac{\sin\left(\bar{u}\xi\nu\right)}{\xi\nu}-\frac{\delta\left(\bar{u}\right)}{2}\right.}\right)$$
$$\left.+\boxed{4\left[\frac{\ln\left(1-u\right)}{1-u}\right]_+\cos\left(\bar{u}\xi\nu\right)-2\frac{\sin\left(\bar{u}\xi\nu\right)}{\xi\nu}+\delta\left(\bar{u}\right)}\right\}$$

Matching to $\overline{\mathrm{MS}}$

Boosted Distillation

- High-momentum needed to maintain perturbative regime
  - Increase boosted interpolator-state overlaps
  - *Momentum Smearing* G. S. Bali et al. Phys. Rev. D93, 094515 (2016)
- Momentum space overlaps increase for excited-states!
  - Dense spectrum (broken symmetries)



Distillation affords control over excited-states

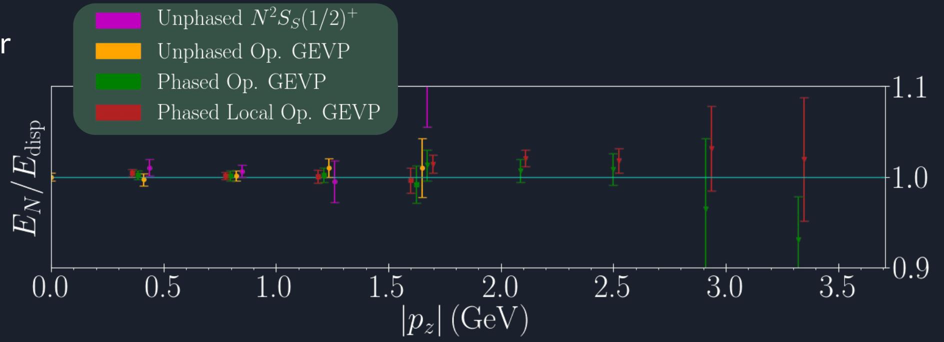
- "Phased" (momentum smeared) Distillation
  - spatially varying phases & pre-computed eigenvector basis

$$\tilde{\xi}_a^{(k)}(\vec{z},t) \equiv e^{i\vec{\zeta}\cdot\vec{z}}\xi_a^{(k)}(\vec{z},t)$$

$$\tilde{\xi}_{\pm}^{(k)}(\vec{z},t) \equiv e^{i\vec{\zeta}_{\pm}\cdot\vec{z}}\xi^{(k)}(\vec{z},t)$$

$$\vec{\zeta}_{\pm} = \pm 2\cdot\frac{2\pi}{L}\hat{z}$$

Restricted to allowed lattice momenta



C. Egerer et al., Phys. Rev. D103, 034502 (2021)

PDFs from ITDs & Pseudo-ITDs

$$\mathfrak{M}\left(\nu, z^2\right) = \int_0^1 \mathrm{d}u \boxed{\mathcal{C}\left(u, z^2\mu^2, \alpha_s\left(\mu\right)\right)} \mathcal{Q}\left(u\nu, \mu^2\right) + \sum_{k=1}^{\infty} \mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$$

T. Izubuchi, et al., Phys.Rev. D98 (2018) no.5, 056004
A. Radyushkin, Phys.Lett. B781 (2018) 433-442
A. Radyushkin, Phys. Rev. D 98 (2018) no.1, 014019
J.-H. Zhang, et al., Phys.Rev. D97 (2018) no.7, 074508

$$\mathcal{C}\left(u, z^2\mu^2, \alpha_s\left(\mu\right)\right) = \delta\left(1-u\right) - \frac{\alpha_s}{2\pi} C_F \left[\ln\left(\frac{e^{2\gamma_E+1} z^2\mu^2}{4}\right) B\left(u\right) + D\left(u\right)\right]$$

- Evolve/Match pseudo-ITD to ITD
  - parametrize PDF

$$q_{\mathrm{V}}\left(x, \mu\right) = \frac{x^{\alpha}\left(1-x\right)^{\beta}\left(1+\sum_k \lambda_k x^{(k+1)/2}\right)}{B\left(\alpha+1, \beta+1\right) + \sum_k \lambda_k B\left(\alpha+1+\frac{k+1}{2}, \beta+1\right)}$$

  - numerically fit cosine-transform to ITD
  - [method C]

- Cast ITD as cosine-transform of PDF
  - closed-form expression

A. Radyushkin, Phys.Rev.D 100 (2019) 11, 116011
T. Izubuchi et al., Phys.Rev.D 98 (2018) 5, 056004

  - directly fit PDF parameters to pseudo-ITD data

Caution: $\quad {}_3F_3\left(111; 222; -ix\nu\right)$

  - [method K]