Radiative Transitions in Charmonium from Lattice QCD

9th Workshop of the APS Topical Group on Hadronic Physics
14th April 2021

James Delaney
Based on Work in Preparation by: C O'Hara, C. E. Thomas, S. Ryan

UNIVERSITY OF CAMBRIDGE

Radiative Transitions in Charmonium from Lattice QCD

$\chi_{c1}(3872), Y(4260), Z_c^{\pm}(3900)$

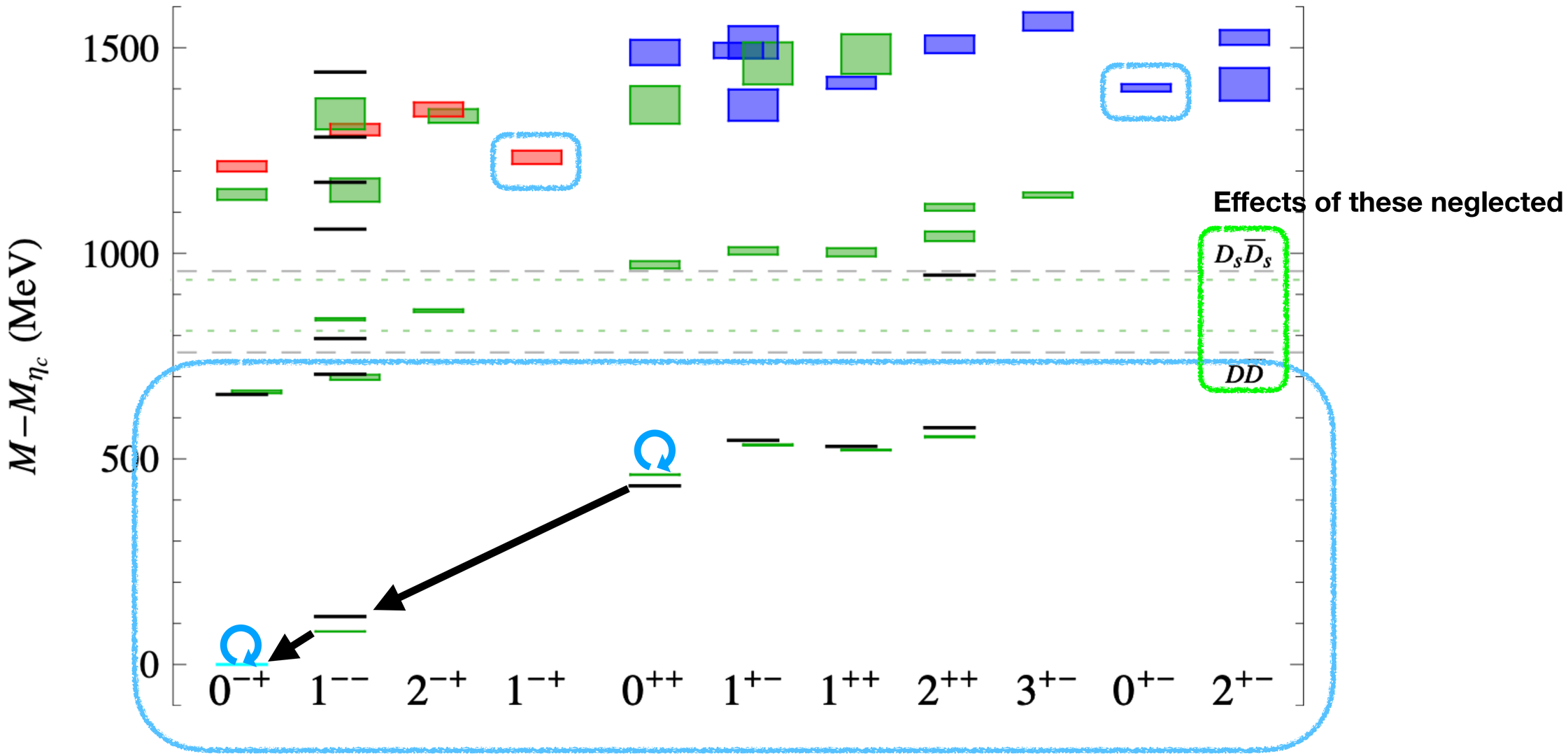
Meson structure, partial widths

First principles tool for investigating

Calculation Details

- Work performed on a single lattice
- $N_f = 2 + 1, \quad m_\pi = 391\,\mathrm{MeV}, \quad a_t \sim 0.12\,\mathrm{fm}, \quad \xi \sim 3.5, \quad m_\pi L \sim 4.8$
- Finite volume, momentum quantised $p \sim \dfrac{2\pi}{L}(n, m, l)$
- Couple only to c quark, can access $A \to A\gamma$ form factors
- Compute using $\mathcal{O}(a)$ improved current
- Correlators computed in the distillation framework

Spectroscopy Summary (1204.5425)



Correlators

- We want form factors, these can be accessed through correlators

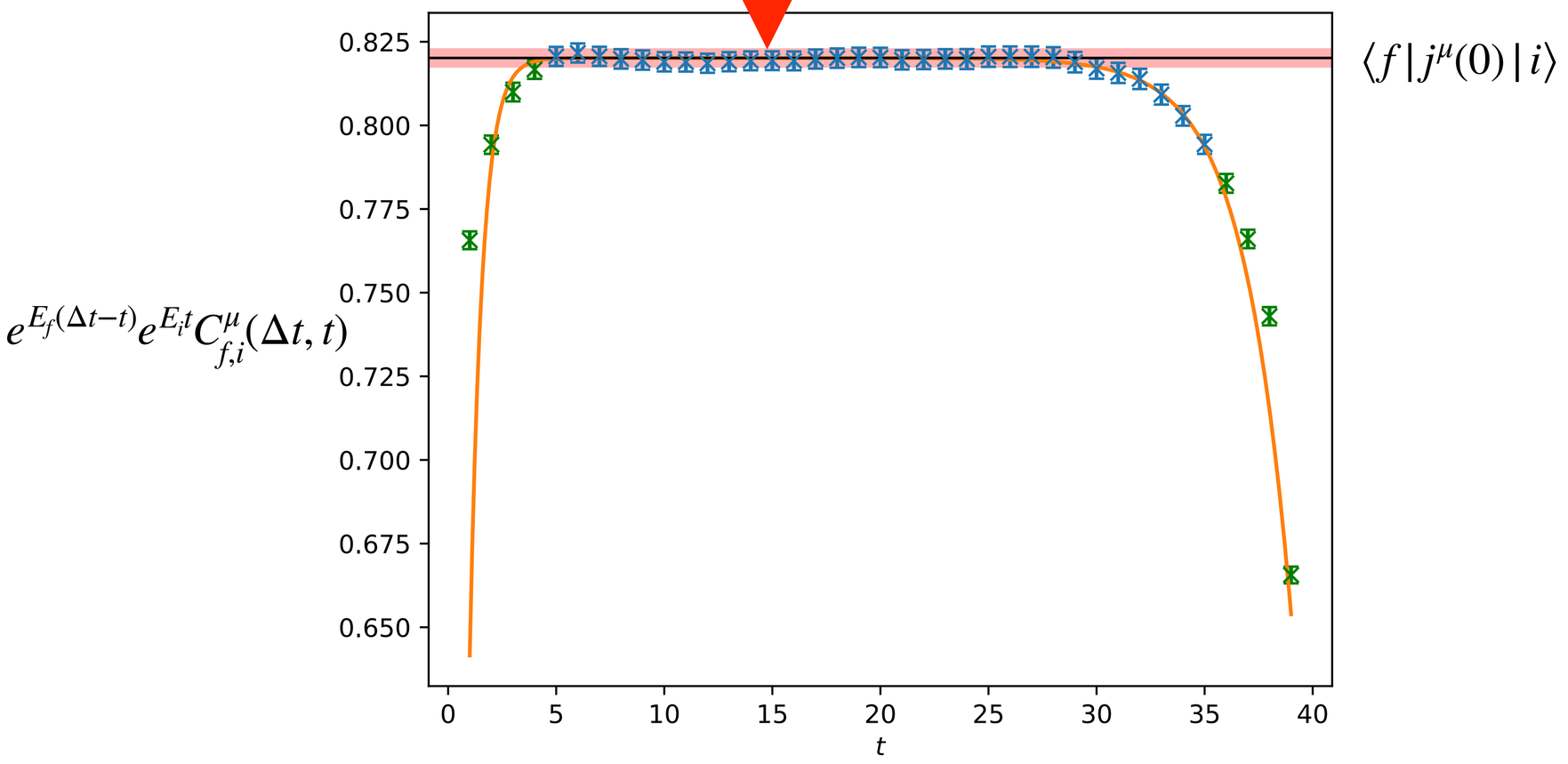
$$\langle 0|\Omega_f(\Delta t)j^\mu(t)\Omega_i^\dagger(0)|0\rangle = \sum_{n_i,n_f}\frac{1}{2E_{n_i}}\frac{1}{2E_{n_f}}e^{-E_{n_f}(\Delta t-t)}e^{-E_{n_i}t}\langle 0|\Omega_f(0)|n_f\rangle\langle n_f|j^\mu(0)|n_i\rangle\langle n_i|\Omega_i^\dagger(0)|0\rangle$$

$$\Omega_{n_i}^\dagger|0\rangle = 2E_{n_i}|n_i\rangle + \sum_{j\neq i}\varepsilon_j|n_j\rangle$$

$$\langle n_i|j^\mu|n_f\rangle = \sum_j K_j^\mu[\lambda_i,\lambda_f,p_i,p_f]\,F_j(Q^2)$$

Extracting Matrix Elements

$$e^{E_f(\Delta t - t)}\, e^{E_i t}\, C^{\mu}_{fi}(\Delta t, t) = \langle f \,|\, j^{\mu}(0) \,|\, i \rangle + \varepsilon_f\, e^{-\delta E_f (\Delta t - t)} + \varepsilon_i\, e^{-\delta E_i t} + \mathcal{O}(\varepsilon^2)$$

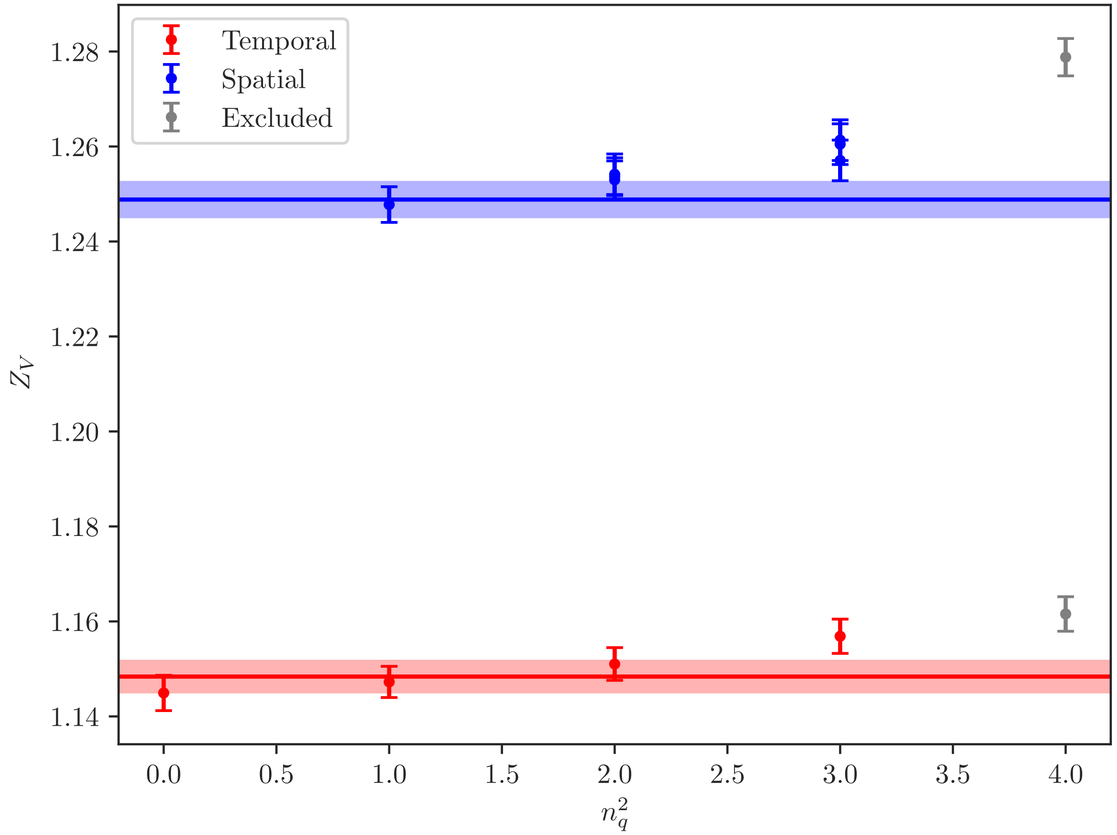


Z_V Determination

$$Z_V = \frac{F^{cont.}_{\eta_c}(0)}{F^{lat.}_{\eta_c}(0)} = \frac{1}{F^{lat.}_{\eta_c}(0)}$$

$$Z_V^t = 1.148(3)$$

$$Z_V^s = 1.249(4)$$



η_c Form Factor

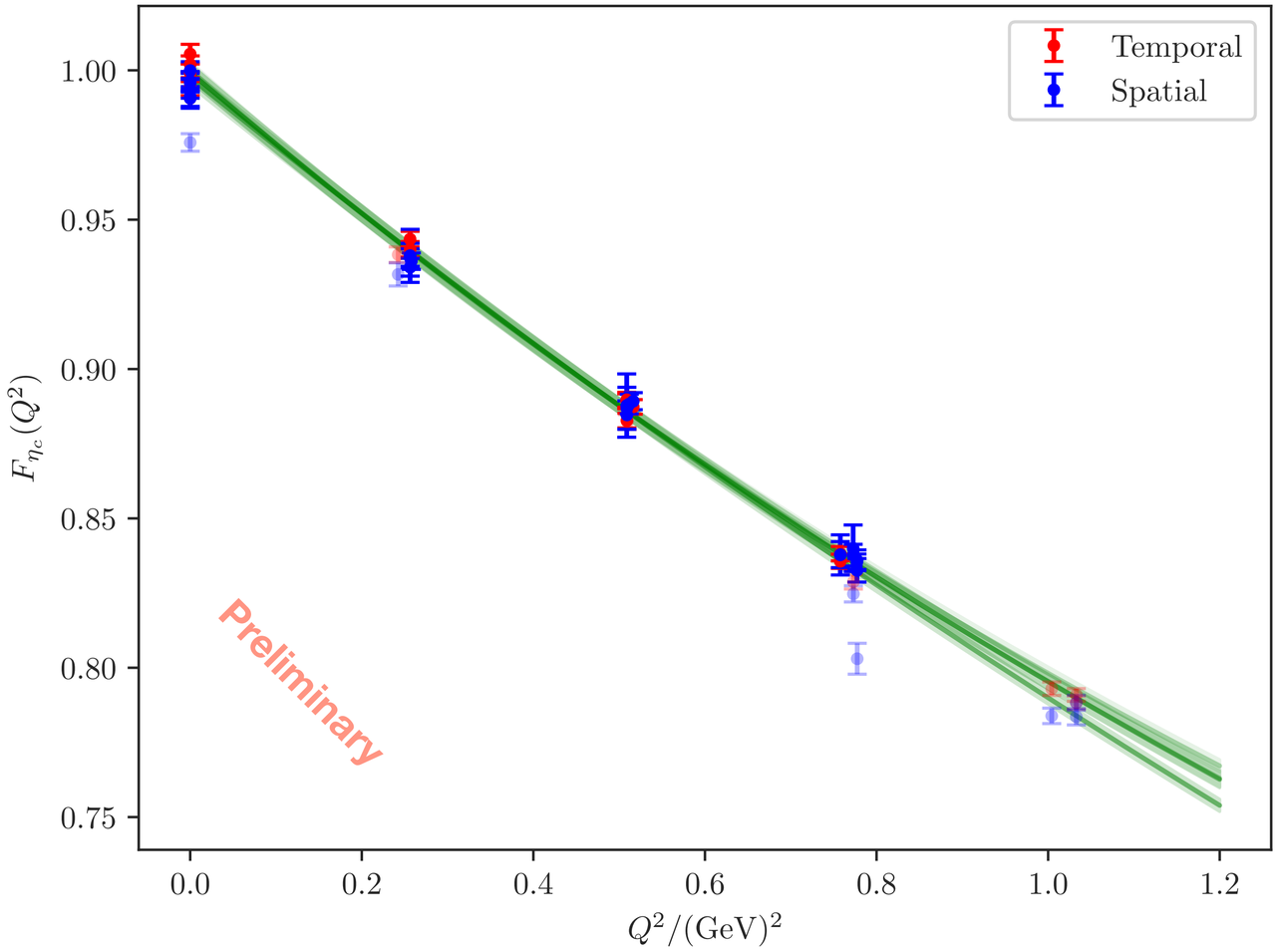
$$\langle \eta_c(p') | j^\mu | \eta_c(p) \rangle = (p + p')^\mu F(Q^2)$$

$$\frac{\langle \eta_c(p') | j^\mu | \eta_c(p) \rangle}{(p + p')^\mu} = F(Q^2)$$

$$\langle r^2 \rangle^{\frac{1}{2}} \propto \frac{\mathrm{d}}{\mathrm{d}Q^2} F(Q^2) \Big|_{Q^2=0}$$

Averaging over a range of fit forms and data partitions

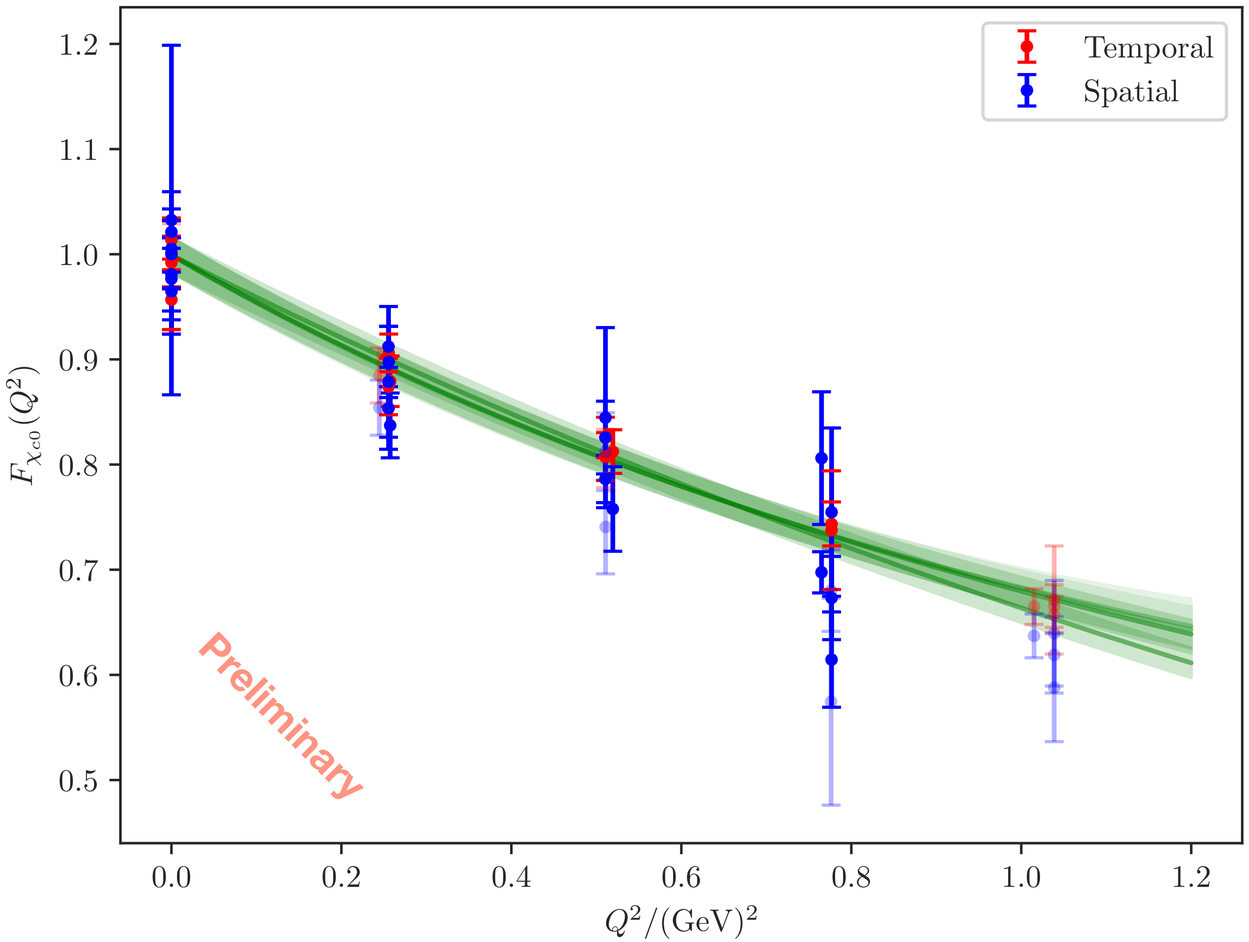
$$\langle r^2 \rangle^{\frac{1}{2}} = 0.232\,\mathrm{fm} - 0.246\,\mathrm{fm}$$



χ_{c0} Form Factor

$$\langle \chi_{c0}(p') | j^\mu | \chi_{c0}(p) \rangle = (p + p')^\mu F_{\chi_{c0}}(Q^2)$$

$$\langle r^2 \rangle^{\frac{1}{2}} = 0.29\,\text{fm} - 0.37\,\text{fm}$$



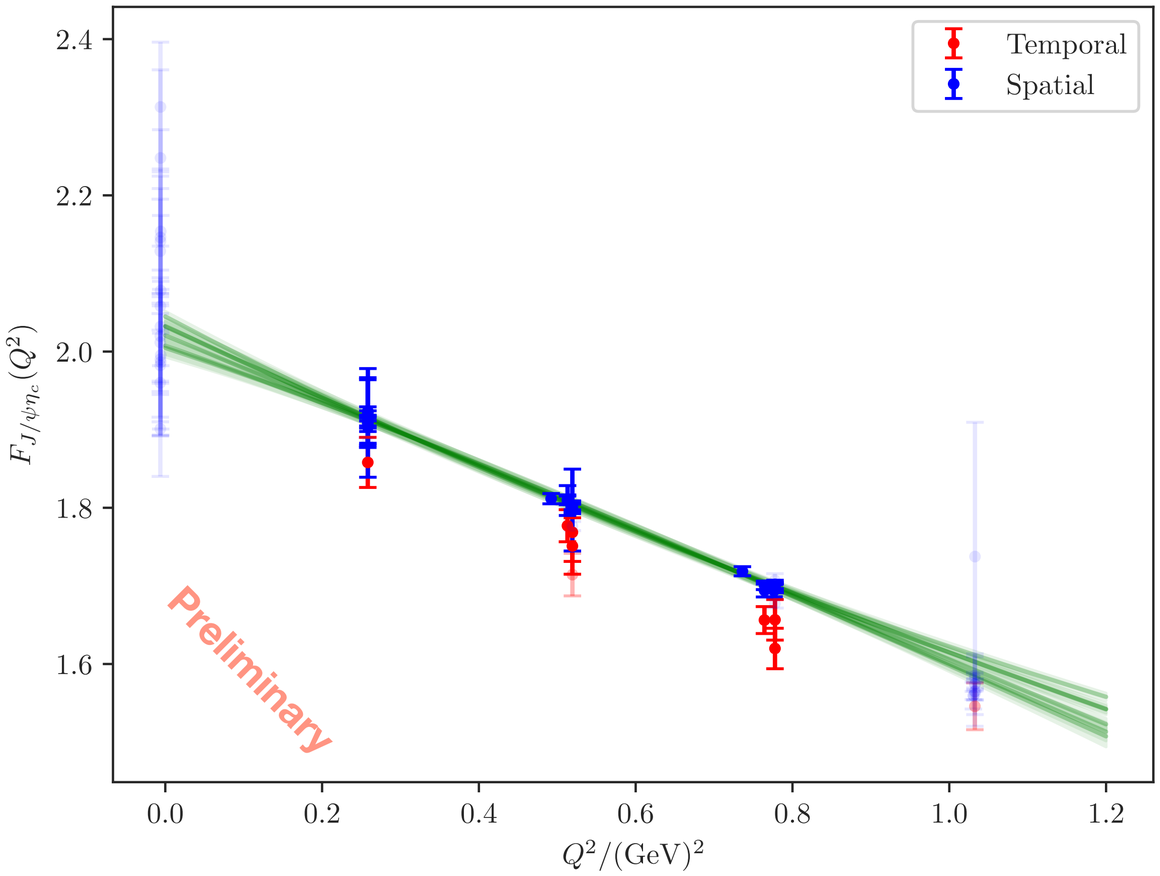
$J/\psi \to \eta_c \gamma$

$$\langle \eta_c(\vec{p}\,')\,|\,j^\mu(0)\,|\,J/\psi(\lambda,\vec{p})\rangle = \epsilon^{\mu\nu\rho\sigma} p'_\nu p_\rho \epsilon_\sigma(\lambda,\vec{p})\,\frac{2}{m_{J/\psi}+m_{\eta_c}} F_{J/\psi\eta_c}(Q^2)$$

$$\Gamma(J/\psi \to \eta_c\gamma) = \frac{64\alpha}{27}\frac{|\mathbf{q}|^3}{(m_{J/\psi}+m_{\eta_c})^2}\,|F(0)|^2$$

Dependent on hyperfine splitting

Study	$\lvert F(0)\rvert$	N_f
this work	1.80 - 2.05	2+1
Donald 2012	1.90(7)(11)	2+1
Becirevic 2012	1.92(3)(2)	2
Chen 2011	2.01(2)	2
Dudek 2006	1.85(4)	0
Dudek 2009	1.89(3)	0
Gui 2019	1.933(41)	0
PDG	1.57(18)	



Matrix Elements to Form Factors

- In general a single matrix element overlaps onto multiple form factors
- To solve for form factors we create redundancy at a given Q^2 then invert using linear regression

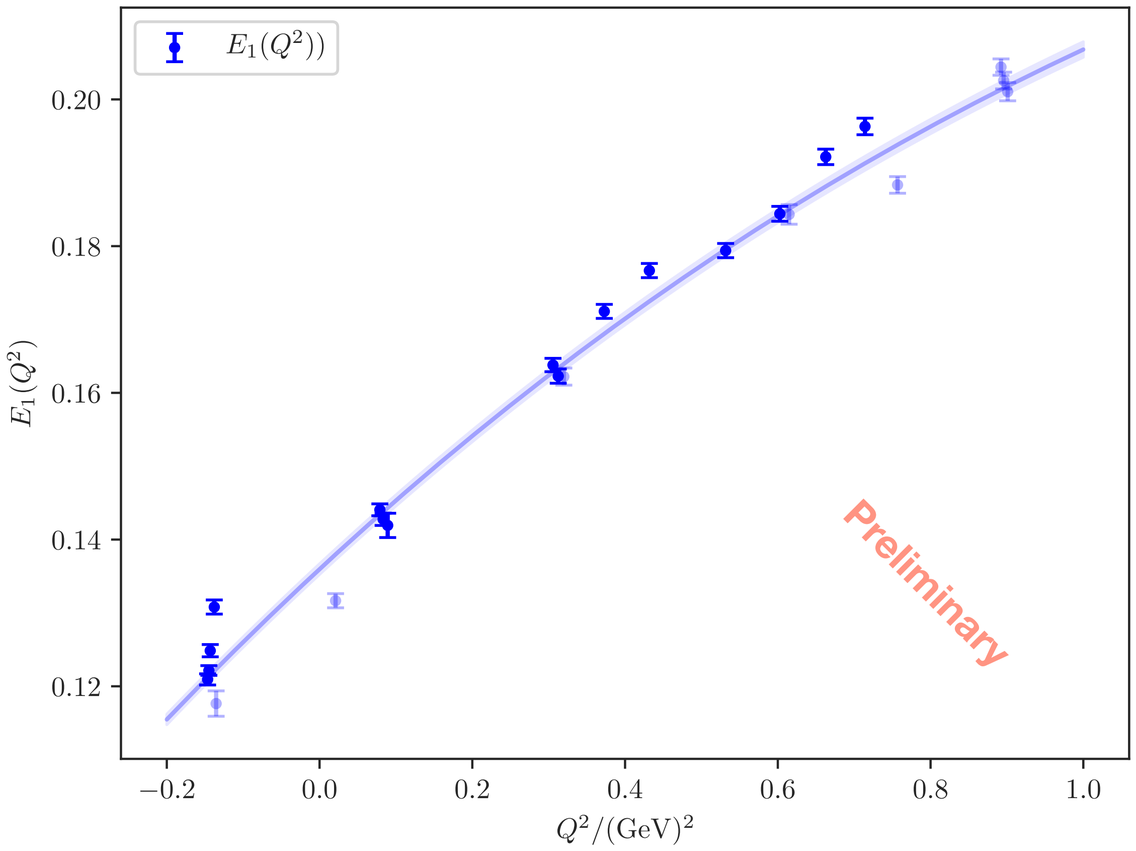
$$\Gamma_\alpha = K_{i\alpha} F_i(Q^2) \qquad\qquad F_i = (K^\dagger K)_{ij} K^\dagger_{j\alpha} \Gamma_\alpha$$

$\chi_{c0} \to J/\psi\gamma$ E_1 Form Factor

$$\langle \chi_{c0}(\vec{p}) | j^\mu(0) | J/\psi(\vec{p}\,', \lambda) \rangle = E_1(Q^2) K_1^\mu(\vec{p}, \vec{p}\,', \epsilon(\lambda, \vec{p}\,')) + \frac{C_1(Q^2)}{\sqrt{Q^2}} K_2^\mu(\vec{p}, \vec{p}\,', \epsilon(\lambda, \vec{p}\,'))$$

$$\Gamma(\chi_{c0} \to J/\psi\gamma) = \frac{16\alpha}{9} \frac{|\mathbf{q}|}{m_{\chi_{c0}}^2} |E_1(0)|^2$$

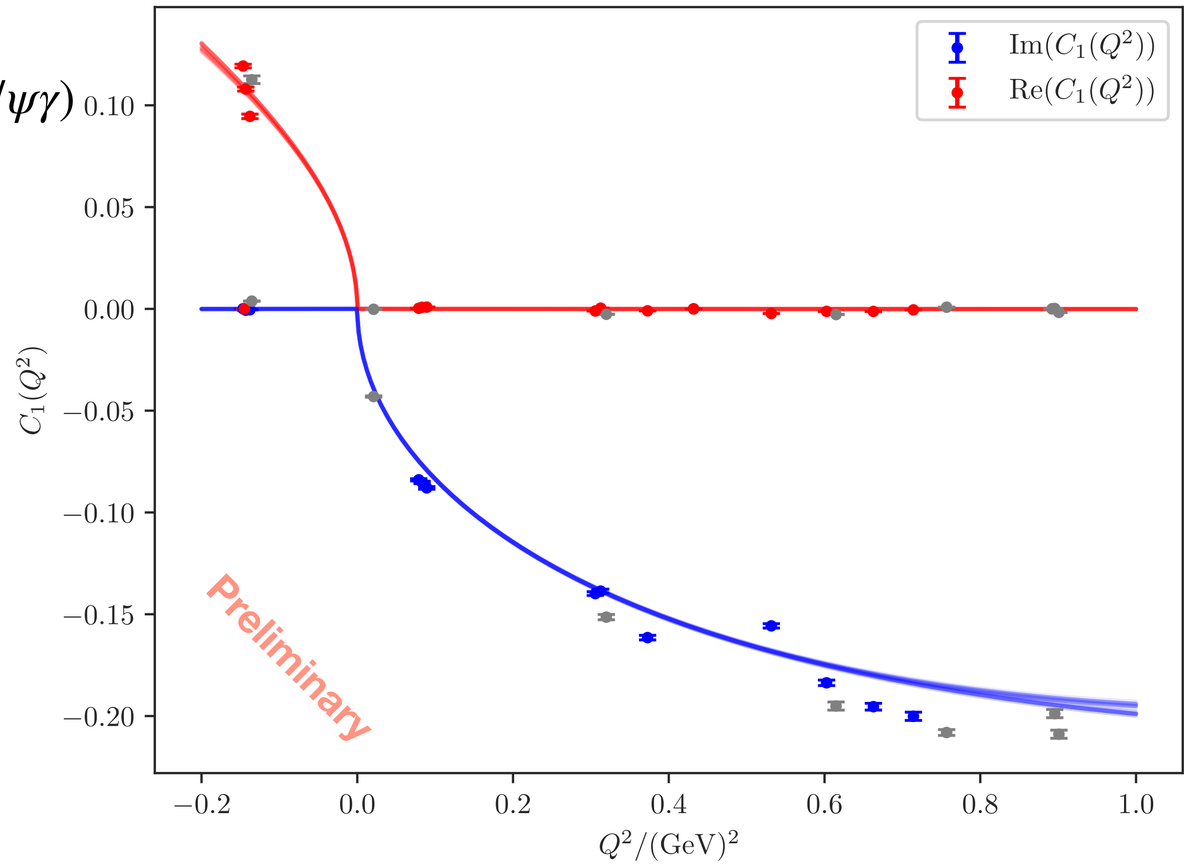
$$\Gamma(\chi_{c0} \to J/\psi\gamma) = 195 - 235\,\text{keV}$$



$\chi_{c0} \to J/\psi\gamma$ C_1 Form Factor

$$\langle \chi_{c0}(\vec{p}) \,|\, j^\mu(0) \,|\, J/\psi(\vec{p}\,', \lambda) \rangle = E_1(Q^2) K_1^\mu(\vec{p}, \vec{p}\,', \epsilon(\lambda, \vec{p}\,')) + \frac{C_1(Q^2)}{\sqrt{Q^2}} K_2^\mu(\vec{p}, \vec{p}\,', \epsilon(\lambda, \vec{p}\,'))$$

As $C_1(Q^2 = 0) = 0$ this does not contribute to $\Gamma(\chi_{c0} \to J/\psi\gamma)$



Summary

- Demonstrated technology for low lying charmonia, showing aspects needed for exotics:
  - Excited states, multiple form factors