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# Isospin-$1/2$ $D\pi$ scattering and the lightest $D_0^*$ resonance from lattice QCD

Based on [arXiv:2102.04973]


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for the Hadron Spectrum Collaboration


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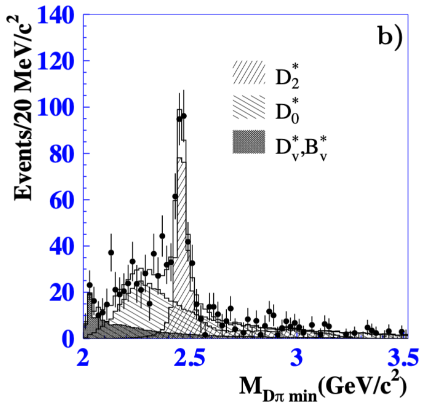
# Introduction: $D_0^*$ - the experimental puzzle

- $D_0^*$ lightest scalar charm-light resonance
- First observed by Belle and FOCUS in 2004: broad enhancement at 2300 - 2400 MeV
- Quark model construction: $q\bar{q}$ in relative $P$-wave
- Measured mass in agreement with predictions by quark model but has a large width




BELLE Collaboration [arXiv:hep-ex/0307021]

# Comparison: $D_{s0}^*$

- Charm-strange state $D_{s0}^*$ the same from view of quark model
- Mass difference w.r.t. $D_0^*$ due to different light-quark masses
  $\rightarrow$ predicted above $D_0^*$
- However: observed as narrow peak below $DK$ threshold - well below predictions by quark model; perhaps below $D_0^*$
  $\rightarrow$ What is going on?
- Proximity in mass of these two states in experiment and differing widths require better theoretical understanding!

# Enter: Lattice QCD

- Lattice QCD $\rightarrow$ first principles approach to understand QCD dynamics
- On Lattice: $D_0^*$ as part of $D\pi \rightarrow D\pi$ scattering
- Existent lattice studies:
  - $D_0^*$ in $D\pi \rightarrow D\pi$ at $m_\pi = 391$ MeV[1]
  - $D_{s0}^*$ in $DK \rightarrow DK$ at both $m_\pi = 391$ MeV and $m_\pi = 239$ MeV[2]
- Goal: complete the picture; better understand the quark-mass dependence

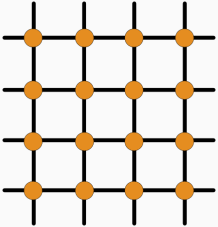
[1] G. Moir et al. [arXiv:1607.07093]

[2] G. K. C. Cheung et al. [arXiv:2008.06432]

# Calculation details

# Calculation details

- Lattice spacing: $a_s = 0.11$ fm, $a_t^{-1} = 6.079$ GeV
- $(L/a_s)^3 \times (T/a_t) = 32^3 \times 256 \to$ spatial volume: $(3.6 \text{ fm})^2$
- Anisotropic lattice ($a_t$ finer than $a_s$): $\xi \equiv a_s/a_t \approx 3.5$
- Scale set via comparison of $\Omega$ baryon masses $\to m_\pi = 239$ MeV
- $N_f = 2 + 1$ dynamical quark flavours
- 484 configurations



# Lattice → Amplitudes

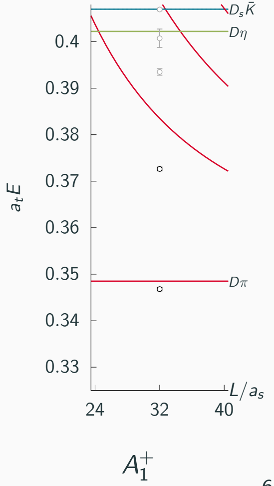
- Basis of interpolating operators (quark bilinears and meson-meson) with $C = 1$, $I = 1/2$ projected to irreducible representations (*irreps*) of the lattice
- Contractions make use of distillation framework[3] with 256 vectors
- Principal correlators computed using GEV method: $C_{ij}(t) v_j^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t, t_0) C_{ij}(t_0) v_j^{(\mathfrak{n})}$
- Correlator fits (sum of exponentials) → Finite volume spectrum
- Infinite volume amplitudes obtained from fit of spectrum to solutions of Lüscher quantisation condition: $\det\left[1 + i\boldsymbol{\rho}(s) \cdot \boldsymbol{t}(s) \cdot (1 + i\boldsymbol{\mathcal{M}}(s, L))\right] = 0$

[3] Hadron Spectrum collaboration [arXiv:0905.2160]

# Results

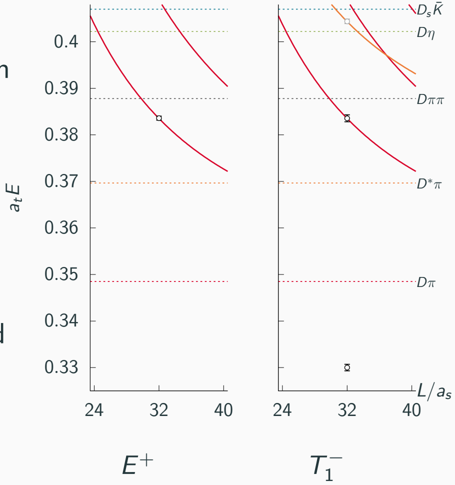
# Spectra at rest

- Irreps are labelled $[\vec{d}]\Lambda^{(P)}$ - with parity $P$ and lattice momentum $\vec{P} = 2\pi\vec{d}/L$
- At rest: neat separation of lowest partial waves
  - $A_1^+$: $S$-wave
  - $T_1^-$: $P$-wave
  - $E^+$: $D$-wave
- $A_1^+$: additional level around $a_t E_{\text{cm}} = 0.37$; levels above and below shifted up and down respectively $\rightarrow$ suggestive of non-trivial interactions



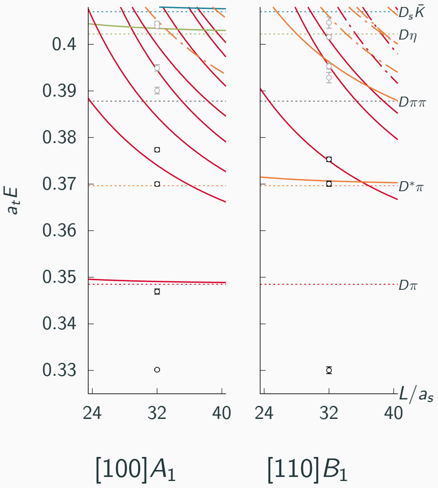
# Spectra at rest

- $T_1^-$: level far below threshold; little interaction above threshold
- $E^+$: level sits right on non-interacting energy $\rightarrow$ negligible $D$-wave interaction (we showed that the $D\pi$ $D$-wave phase shift is consistent with zero)
- Higher partial waves will be ignored (threshold suppression $\propto k^{2l}$)



# Spectra at non-zero momentum

- Moving-frame $\rightarrow$ rotational symmetry further broken $\rightarrow$ further mixing of partial waves
- $A_1$ irreps have contributions from $S$- and $P$-wave
- $[110]B_1/B_2$ and $[100]E_2$ irreps have a contribution from $D^*\pi$ $S$-wave



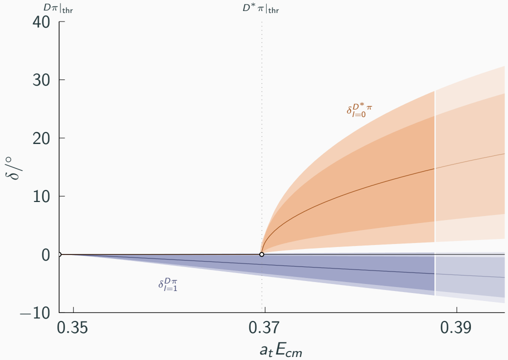
# $t$-matrix Parametrisations

- Parametric form of $\boldsymbol{t}$-matrix undetermined by Lüscher condition for multiple partial waves
- Unitarity and anlyticity provide constraints
- Using a single parametrisation could introduce bias
- We use a range of different parametrisations:
    - $K$-matrix: $(t^{(\ell)})^{-1}(s) = \frac{1}{(2k)^\ell} K^{-1}(s) \frac{1}{(2k)^\ell} + I(s)$
    - Effective range
    - Breit Wigner
    - Unitarized chiral amplitude[4]

[4] Z.-H. Guo et al. [arXiv:1811.05585]

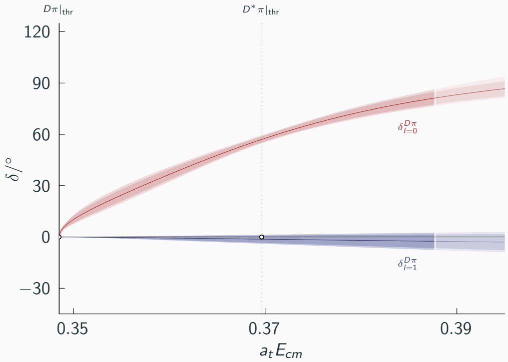
# $D\pi$ P-wave and $D^*\pi$ S-wave

- Determined from spectrum fits in $[000]T_1^-$, $[100]E_2$, $[110]B_1$ and $[110]B_2$
- Deeply bound level in all irreps $\rightarrow$ $J^P = 1^-$ $D^*$ bound state
- $D^*\pi$ S-wave $\rightarrow$ contribution in moving frames
- Parametrisation: $K$-matrix with 2 channels with a pole term in $D\pi$ $P$-wave
- Phase shift indicates very weak effect of $P$-wave above threshold



# $D\pi$ S- and P-wave

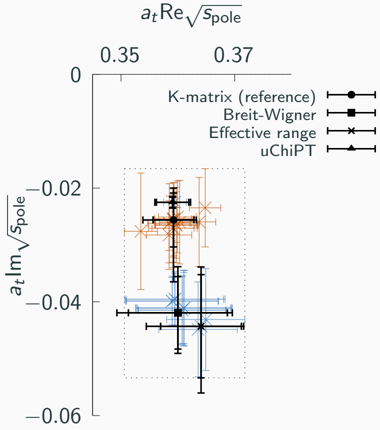
- Fit of energy levels below $D\pi\pi$ threshold in $A_1^+$, $T_1^-$ & moving-frame $A_1$ irreps
- Excluding irreps that have $D^*\pi$ contribution
- Deeply bound level in all irreps with $P$-wave contribution; "extra" level in irreps with $S$-wave contribution
- Parametrisation: $K$-matrix for 2 partial waves, both containing a pole term



# Poles

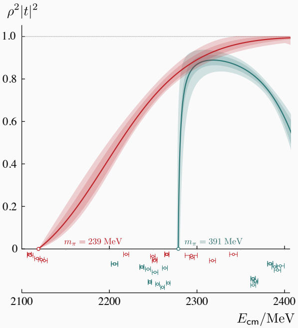
- Cluster of poles from 30 different parametrisations; all above threshold
  → Resonance
- Amplitudes similar at real energies but differ in complex plane; pole common feature
- Scatter of poles: single parametrisation might underestimate uncertainties
- Mass and coupling considering all parametrisations:

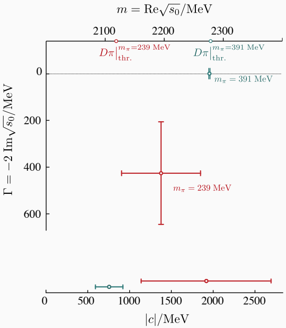
$$\sqrt{s_0}/\mathrm{MeV} = (2196 \pm 64) - \tfrac{i}{2}(425 \pm 224)$$
$$c/\mathrm{MeV} = (1916 \pm 776) \exp i\pi(-0.59 \pm 0.41)$$



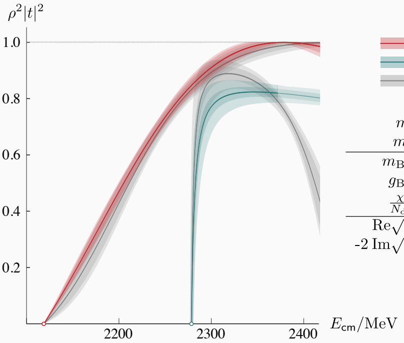
# The big picture: Comparison with other calculations

- Earlier study of $D\pi \to D\pi$ at $m_\pi = 391$ MeV: shallow bound-state ($\approx 2 \pm 1$ MeV below threshold)
- At 239 MeV: pole migrates into complex plane ($\approx 77 \pm 64$ MeV above threshold)
- Mass below reported experimental value (despite heavier-than-physical light quarks)
- Strong coupling of poles to $D\pi$ channel in both cases



# Study: parametrising $D\pi$ $S$-wave at different masses




| | | |
|---|---|---|
| $m_\pi$/MeV | 239 | 391 |
| $m_D$/MeV | 1880 | 1887 |
| $m_{\text{BW}}$/MeV | 2380(36) | 2206(32) |
| $g_{\text{BW}}$ | 5.39(56) | 7.62(75) |
| $\frac{\chi^2}{N_{\text{dof}}}$ | $\frac{14.6}{20-4}$ | $\frac{36.0}{29-5}$ |
| $\text{Re}\sqrt{s_0}$/MeV | 2189(72) | 2275(1) |
| -2 $\text{Im}\sqrt{s_0}$/MeV | 510(97) | - |
| $\lvert c\rvert$/MeV | 2391(411) | 826(133) |

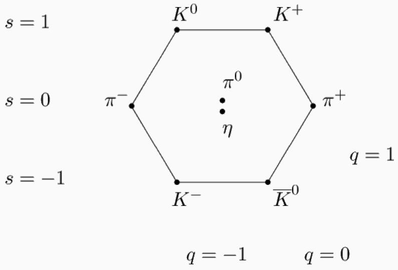
- Comparison: $K$-matrix and Breit-Wigner
- Real parts of the poles are comatible between both parametrisations
- Breit-Wigner mass parameter incompatible with pole location

# SU(3) flavour symmetry

- When $m_u = m_d = m_s$ $\pi$ and $K$ are rows of the same SU(3) octet
  $\rightarrow D\pi$ and $DK$ scattering related by SU(3) flavour symmetry

$$\bar{\mathbf{3}} \otimes \mathbf{8} \rightarrow \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$$

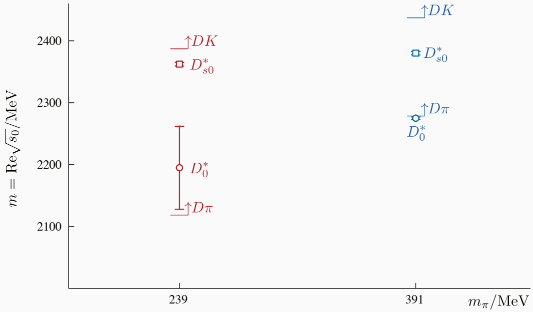
- Symmetry is less broken at heavier light-quark masses
- We expect the number of poles to stay the same as function of quark mass




https://en.wikipedia.org/wiki/Eightfold_way_(physics) - Creative Commons

# $D\pi$ and $D_s\bar{K}$ at different light-quark masses

- Locations of poles match expectation from SU(3) symmetry
- $D_0^*$ shallow bound state at $m_\pi = 391$ MeV - becomes a resonance at $m_\pi = 239$ MeV
- Pole mass decreases with pion mass $\rightarrow$ extrapolation to physical pion mass would suggest $D_0^*$ well below $D_{s0}^*$
- $D_{s0}^*$ bound at both masses



# Conclusion

# Conclusion and Outlook

- Found a $D_0^*$ *resonance* pole at
  - mass $m = (2194 \pm 64)$ MeV - $(77 \pm 64)$ MeV above $D\pi$ threshold
  - width $\Gamma = (425 \pm 224)$ MeV

  from first principles (no external inputs after fixing quark masses)
- Considered a range of parametrisations (major contribution to uncertainty)
- Pole strongly coupled to $D\pi$ channel; coupling compatible with $D_{s0}^* \to DK$ (broken $SU(3)$ flavour symmetry)
- Result indicates slight decrease in pole mass with decreasing pion mass
- Value significantly lower than currently reported experimental one
  $\to$ puzzling $D_0^*$ heavier than $D_{s0}^*$ not reproduced by Lattice

# Questions?

# Backup

# Correlators on the lattice

- Compute matrix of (euclidean) correlators:

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle \,,$$

- $\mathcal{O}_i(t)$ have quantum numbers of $I = 1/2\ D\pi$
- Find "optimal" interpolators by solving *Generalised Eigenvalue* (GEV) problem

$$C_{ij}(t)v_j^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t,t_0)C_{ij}(t_0)v_j^{(\mathfrak{n})},$$

- Fit Principal correlators (eigenvalues):

$$\lambda_{\mathfrak{n}}(t,t_0) = (1-A_{\mathfrak{n}})e^{-E_{\mathfrak{n}}(t-t_0)} + A_{\mathfrak{n}}e^{-E'_{\mathfrak{n}}(t-t_0)}.$$

# Operator Table ($S$-wave)

| $A_1^+[000]$ | $A_1[100]$ | $A_1[110]$ | $A_1[111]$ | $A_1[200]$ |
|---|---|---|---|---|
| $D_{[000]}\ \pi_{[000]}$ | $D_{[000]}\ \pi_{[100]}$ | $D_{[000]}\ \pi_{[110]}$ | $D_{[000]}\ \pi_{[111]}$ | $D_{[100]}\ \pi_{[100]}$ |
| $D_{[100]}\ \pi_{[100]}$ | $D_{[100]}\ \pi_{[000]}$ | $D_{[100]}\ \pi_{[100]}$ | $D_{[100]}\ \pi_{[110]}$ | $D_{[110]}\ \pi_{[110]}$ |
| $D_{[110]}\ \pi_{[110]}$ | $D_{[100]}\ \pi_{[110]}$ | $D_{[110]}\ \pi_{[000]}$ | $D_{[110]}\ \pi_{[100]}$ | $D_{[200]}\ \pi_{[000]}$ |
| $D_{[111]}\ \pi_{[111]}$ | $D_{[100]}\ \pi_{[200]}$ | $D_{[110]}\ \pi_{[110]}$ | $D_{[111]}\ \pi_{[000]}$ | $D_{[210]}\ \pi_{[100]}$ |
| $D_{[000]}\ \eta_{[000]}$ | $D_{[110]}\ \pi_{[100]}$ | $D_{[111]}\ \pi_{[100]}$ | $D_{[211]}\ \pi_{[100]}$ | $D_{[200]}\ \eta_{[000]}$ |
| $D_{[100]}\ \eta_{[100]}$ | $D_{[110]}\ \pi_{[111]}$ | $D_{[210]}\ \pi_{[100]}$ | $D^*{}_{[110]}\ \pi_{[100]}$ | |
| $D_{s\,[000]}\ \bar{K}_{[000]}$ | $D_{[111]}\ \pi_{[110]}$ | $D^*{}_{[100]}\ \pi_{[100]}$ | $D_{[111]}\ \eta_{[000]}$ | |
| | $D_{[200]}\ \pi_{[100]}$ | $D^*{}_{[111]}\ \pi_{[100]}$ | $D_{s\,[111]}\ \bar{K}_{[000]}$ | |
| | $D_{[210]}\ \pi_{[110]}$ | $D_{[110]}\ \eta_{[000]}$ | | |
| | $D_{[000]}\ \eta_{[100]}$ | $D_{s\,[110]}\ \bar{K}_{[000]}$ | | |
| | $D_{[100]}\ \eta_{[000]}$ | | | |
| | $D_{s\,[000]}\ \bar{K}_{[100]}$ | | | |
| | $D_{s\,[100]}\ \bar{K}_{[000]}$ | | | |
| $8 \times \bar{\psi}\boldsymbol{\Gamma}\psi$ | $18 \times \bar{\psi}\boldsymbol{\Gamma}\psi$ | $18 \times \bar{\psi}\boldsymbol{\Gamma}\psi$ | $9 \times \bar{\psi}\boldsymbol{\Gamma}\psi$ | $16 \times \bar{\psi}\boldsymbol{\Gamma}\psi$ |

Operators used in the S-wave fits. Subscripts indicate momentum types. $\boldsymbol{\Gamma}$ represents some monomial of $\gamma$ matrices and derivatives.

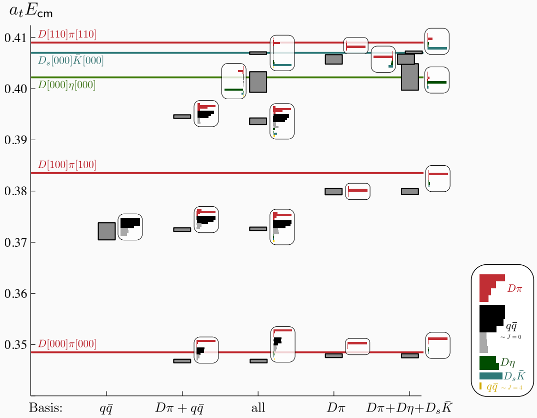
# Operator Table ($P$-wave)

| $T_1^-[000]$ | $E_2[100]$ | $B_1[110]$ | $B_2[110]$ |
|---|---|---|---|
| $D_{[100]}\ \pi_{[100]}$ | $D_{[100]}\ \pi_{[110]}$ | $D_{[100]}\ \pi_{[100]}$ | $D_{[100]}\ \pi_{[111]}$ |
| $D_{[110]}\ \pi_{[110]}$ | $D_{[110]}\ \pi_{[100]}$ | $D_{[110]}\ \pi_{[110]}$ | $D_{[110]}\ \pi_{[110]}$ |
| $D^*{}_{[100]}\ \pi_{[100]}$ | $D^*{}_{[000]}\ \pi_{[100]}$ | $D_{[210]}\ \pi_{[100]}$ | $D_{[111]}\ \pi_{[100]}$ |
| | $D^*{}_{[100]}\ \pi_{[000]}$ | $D^*{}_{[100]}\ \pi_{[100]}$ | $D^*{}_{[000]}\ \pi_{[110]}$ |
| | | $D^*{}_{[110]}\ \pi_{[000]}$ | $D^*{}_{[100]}\ \pi_{[100]}\ \{2\}$ |
| | | | $D^*{}_{[110]}\ \pi_{[000]}$ |
| | | | $D^*{}_{[111]}\ \pi_{[100]}$ |
| $6 \times \bar{\psi}\mathbf{\Gamma}\psi$ | $18 \times \bar{\psi}\mathbf{\Gamma}\psi$ | $18 \times \bar{\psi}\mathbf{\Gamma}\psi$ | $20 \times \bar{\psi}\mathbf{\Gamma}\psi$ |

Operators used in the P-wave fits. Subscripts indicate momentum types. $\mathbf{\Gamma}$ represents some monomial of $\gamma$ matrices and derivatives. The number in curly parentheses indicates the number of operators of this momentum combination.

# Operator basis variations

- Varying the basis affects the spectrum
- $I = 1/2$ allows both meson-meson and $q\bar{q}$-like operator constructions
- Interpolating the complete spectrum requires both types of operator
- Other meson-meson operators do not play a significant role below coupled-channel threshold




Spectrum in $A_1^+$

# How are operators constructed?

- Two types of interpolating operator:
  - quark bilinears: $\bar{\psi}\Gamma D ... \psi$
  - meson-meson like operators: $\sum_{\vec{p_1}+\vec{p_2}=\vec{P}} \mathcal{C}(\vec{p_1},\vec{p_2})\Omega^{\dagger}_{M_1}(\vec{p_1})\Omega^{\dagger}_{M_2}(\vec{p_2})$
- Rotational symmetry broken $\Rightarrow$ eigenstates labelled by irreducible representations of $O_h$ or $LG(\vec{P})$ (*irreps*)
- Continuum spins *subduce* into one or more finite volume irreps; operators are projected into irreps
- Correlators are computed using distillation with 256 vectors

## Subduction Table

| $\vec{P}$ | Irrep $\Lambda$ | $J^P$ $(\vec{P}=\vec{0})$ $\lvert\lambda\rvert^{(\tilde{\eta})}$ $(\vec{P}\neq\vec{0})$ | $D\pi$ $J^P_{[N]}$ | $D^*\pi$ $J^P_{[N]}$ |
|---|---|---|---|---|
| [000] | $A_1^+$ | $0^+$, $4^+$ | $0^+$, ... | ... |
| | $T_1^-$ | $1^-$, $3^-$ | $1^-$, ... | ... |
| | $E^+$ | $2^+$, $4^+$ | $2^+$, ... | ... |
| $[n00]$ | $A_1$ | $0^{(+)}$, 4 | $0^+$, $1^-$, $2^+$, ... | ... |
| | $E_2$ | 1, 3 | $1^-$, $2^+$, ... | $1^+$, ... |
| $[nn0]$ | $A_1$ | $0^{(+)}$, 2, 4 | $0^+$, $1^-$, $2^+_{[2]}$, ... | ... |
| | $B_2$, $B_2$ | 1, 3 | $1^-$, $2^+$, ... | $1^+$, ... |
| $[nnn]$ | $A_1$ | $0^{(+)}$, 3 | $0^+$, $1^-$, $2^+$, ... | ... |

Lowest $D\pi$ and $D^*\pi$ continuum $J^P$ and helicity $\lambda$ subductions by irrep

# Masses and thresholds

| | $a_t m$ |
|---|---|
| $\pi$ | 0.03928(18) |
| $K$ | 0.08344(7) |
| $\eta$ | 0.09299(56) |
| $D$ | 0.30923(11) |
| $D_s$ | 0.32356(12) |
| $D^*$ | 0.33058(24) |

| | $a_t E_{\mathrm{threshold}}$ |
|---|---|
| $D\pi$ | 0.34851(21) |
| $D\pi\pi$ | 0.38779(27) |
| $D\eta$ | 0.40222(57) |
| $D_s\bar{K}$ | 0.40700(14) |
| $D^*\pi\pi$ | 0.40914(35) |

Left: A summary of the stable hadron masses relevant for this calculation. Right: kinematic thresholds relevant for $I = 1/2$ $D\pi$ scattering.

# From the spectrum to scattering amplitudes

- Need a mapping between finite-volume spectrum and infinite volume scattering amplitudes $\to$ Lüscher quantisation condition

$$\det\left[1 + i\boldsymbol{\rho}(s) \cdot \boldsymbol{t}(s) \cdot (1 + i\boldsymbol{\mathcal{M}}(s, L))\right] = 0$$

  - $\rho(s) = 2k(s)/\sqrt{s}$ with $k(s)$ the COM-momentum function
  - $\boldsymbol{t}(s)$ = infinite volume t-matrix
  - $\boldsymbol{\mathcal{M}}(s, L))$ encodes finite-volume effects (dense in partial waves)
- Procedure
  - solve equation (25) for a given parametrisation of $\boldsymbol{t}(s)$ to obtain a spectrum
  - vary the parameters in $\boldsymbol{t}(s)$ in a $\chi^2$-minimisation to best match the spectrum obtained from the lattice

# Combined $D\pi$ $S+P$-wave and $D^*\pi$ $S$-wave

- Sanity check: Fit of all relevant partial waves below three-body threshold
- Fit of energy levels below $D\pi\pi$ threshold in all irreps we computed
- Parametrisation: $K$-matrix with 2 channels / 3 partial waves
- Pole term in $D\pi$ $S$- and $P$-wave
- Constant in $D^*\pi$ $S$-wave
- Results compatible with fit excluding $D^*\pi$