Decays of Exotic Quarkonia

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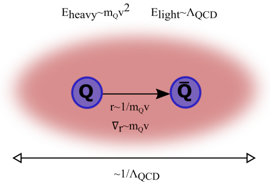
Motivation

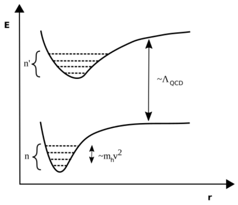
- The nature of many **exotic quarkonium** states is still not settled.
- Not many J^{PC} are easily accessible experimentally:
  - $\Rightarrow$ It is difficult to use spectrum predictions to validate different approaches.
  - $\Rightarrow$ Cannot test heavy quark spin symmetry multiplets in different approaches.
- On the other hand information on decay channels is always available.
- Many exotic states **discovered in channels with standard quarkonium** and light-quark mesons.

Our Aim

- Study exotic quarkonium transitions in **nonrelativistic EFT**.
- Interpret exotic states in the **Born-Oppenheimer picture**.
- We use the **multipole expansion**, thus we restrict ourselves to the bottomonium sector.

Exotic quarkonium in Born-Oppenheimer EFT



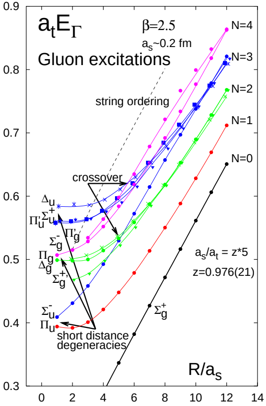


- Exotic quarkonium are formed by a **heavy quark-antiquark pair** and some **light degrees of freedom**.
- The natural starting point to study exotic quarkonium is **NRQCD in the static limit**.
- In this limit the spectrum is formed by the **static energies** which depend on:
  - The heavy quark-antiquark distance $\boldsymbol{r}$.
  - Light-quark and gluons quantum numbers: spin, parity, flavor...
  - Representation of $D_{\infty h}$ (cylindrical symmetry).
- Static energies are nonpertubative and should be computed on the lattice.
- Exotic quarkonium are the quark-antiquark **bound states on these static energies**. Berwein, Brambilla, JTC, Vairo Phys.Rev.D92 (2015); Oncala, Soto, Phys.Rev.D96 (2017); Brambilla, Krein, JTC, Vairo Phys.Rev.D97 (2018); Soto, JTC, Phys.Rev.D 102 (2020)

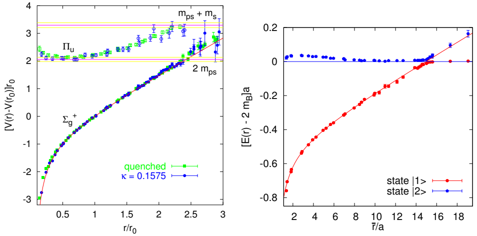
Static energies on the lattice

- **Quenched** lattice NRQCD.

Juge, Kuti, Morningstar Phys.Rev.Lett.90 (2003)



SESAM/TCL Col. Phys.Rev.D62 (2000), Phys.Rev.D71 (2005)



- The states over the quenched static energies are quarkonium hybrids.
- No significant difference between quenched and unquenched results for the ground and first excited state.
- However with dynamic light quarks new states appear such as thresholds.

Exotic quarkonium as Hybrids states

- **Hybrid quarkonium** should be a good approximation for $I = 0$ **exotic quarkonium** excluding molecular threshold states.
- Neutral exotic bottomonium states, all 1^{--}:
  - $\Upsilon(10753)$ mass within 40 MeV of ground state hybrid in Berwein, Brambilla, JTC, Vairo Phys.Rev.D92 (2015).
  - $\Upsilon(10860)$ lays very close to B mesons pair threshold, likely a molecular state.
  - $\Upsilon(11020)$ mass within 20 MeV of first excited hybrid in Berwein, Brambilla, JTC, Vairo Phys.Rev.D92 (2015).
- We are going to compute transitions of $\Upsilon(10753)$ and $\Upsilon(11020)$ into standard quarkonium and light-quark mesons.

Standard and hybrid quarkonium in pNRQCD

- Potential NRQCD (pNRQCD) is and EFT which incorporates the heavy quark mass and multipole expansions. Pineda, Soto Nucl.Phys.B Proc.Suppl. 64 (1998); Brambilla, Pineda, Soto, Vairo Nucl.Phys.B 566 (2000)
- S heavy quark pair singlet field, O heavy quark pair octet field.
- The standard quarkonium static states are just

$$|\mathbf{R},\mathbf{r};\Sigma_g^+\rangle = S^\dagger\left(\boldsymbol{R},\boldsymbol{r}\right)|0\rangle\,.$$

- The static potential corresponds to the Σ_g^+ static energy

$$V^{(0)}_{\Sigma_g^+}(r) = \lim_{t\to\infty}\frac{i}{t}\ln\langle\mathbf{R},\mathbf{r};\Sigma_g^+;t/2|\mathbf{R},\mathbf{r};\Sigma_g^+;-t/2\rangle = E^{(0)}_{\Sigma_g^+}(r)$$

- A general standard quarkonium state

$$|S_m\rangle = \int d^3\boldsymbol{r}d^3\boldsymbol{R}\,\phi^{(m)}(\boldsymbol{R},\boldsymbol{r})|\boldsymbol{R},\boldsymbol{r};\Sigma_g^+\rangle\,,$$

- The wave function $\phi^{(m)}$ is obtained solving the Shrödinger eq. with $V^{(0)}_{\Sigma_g^+}(r)$.

Standard and hybrid quarkonium in pNRQCD

- The Hybrid static states contain a 1^{+-}, color octet, gluonic operator $\boldsymbol{G}_B^a \sim Z_B^{-1/2}\boldsymbol{B}^a + \ldots$:

$$|\mathbf{R},\mathbf{r};\lambda\rangle = \hat{\boldsymbol{r}}_\lambda \cdot \boldsymbol{G}_B^a(\boldsymbol{R}) O^{a\,\dagger}(\boldsymbol{R},\boldsymbol{r})\,|0\rangle$$

- $\hat{\boldsymbol{r}}_\lambda$ projection vectors $O(3) \to D_{\infty h}$.
- The static potentials

$$V_\lambda^{(0)}(r) = \lim_{t\to\infty} \frac{i}{t} \ln\langle \mathbf{R},\mathbf{r};\lambda;t/2|\mathbf{R},\mathbf{r};\lambda;-t/2\rangle = E_{|\lambda|}^{(0)}(r)$$

- $E_0^{(0)}(r) = E_{\Sigma_u^-}^{(0)}(r)$ and $E_{|\pm 1|}^{(0)}(r) = E_{\Pi_u}^{(0)}(r)$ taken from the lattice determination.
- A general hybrid state

$$|H_n\rangle = \int d^3\boldsymbol{r}\,d^3\boldsymbol{R} \sum_\lambda \psi_\lambda^{(n)}(\boldsymbol{R},\boldsymbol{r})|\boldsymbol{R},\boldsymbol{r};\lambda\rangle\,.$$

- The wave functions $\psi_\lambda^{(n)}$ are obtained from the coupled Shrödinger eqs. with $V_{\Sigma_u^-}^{(0)}(r)$ and $V_{\Pi_u}^{(0)}(r)$ Berwein, Brambilla, JTC, Vairo Phys.Rev.D92 (2015).
- Due to the mixing the hybrid wave functions are eigenstates of $(\boldsymbol{L}_{\bar{Q}Q} + \boldsymbol{S}_1)^2$ with eigenvalue $\ell(\ell+1)$.

LO Transitions

- Transitions from the **LO singlet-octet operator**:

$$\langle S_m \mathcal{O}_\pi | g \mathrm{Tr}\left[\mathrm{S}^\dagger \boldsymbol{r} \cdot \boldsymbol{E}\, \mathrm{O} \right] | H_n \rangle \sim Z_B^{-1/2} \underbrace{\langle \mathcal{O}_\pi | g^2 \boldsymbol{E} \cdot \boldsymbol{B} | 0 \rangle}_{\text{l.q. meson production}} \underbrace{\langle \phi^{(m)} | \boldsymbol{r} \cdot \hat{\boldsymbol{r}}_\lambda | \psi_\lambda^{(n)} \rangle}_{\text{Heavy quark matrix element}}$$

- Z_B can be related to the gluon condensate.
- Heavy quark matrix element:
  - Selection rules for final quarkonium states: $\Delta s = 0$, $\ell = l$.
  - $\Upsilon(10753)$ and $\Upsilon(11020)$ decay into $h_b(m^1P_1)$ and light quark mesons.
- Light-quark meson production
  - Allowed final light-quark states: 0^{-+}, $I = 0$ such as π^0, η, η', η-like resonances or odd numbers of pseudoscalar mesons.
  - Matrix elements for production of π^0, η, η' can be determined from $U(1)_A$ anomaly and a mixing scheme.
  - We use the Feldmann-Kroll-Stech scheme Feldmann, Kroll, Stech, Phys.Rev.D58 (1998); Kroll, Mod. Phys. Lett. A20 (2005).

LO Transitions

- Transition width predictions

$$\Gamma_{\Upsilon(10753)\to h_b(1P)\pi^0} = 2.57(\pm1.03)_{\mathrm{m.e.}}(\pm0.14)_{Z_B}(\pm0.16)_{\omega_{\pi^0}}\ \mathrm{keV}$$
$$\Gamma_{\Upsilon(10753)\to h_b(1P)\eta} = 2.29(\pm0.92)_{\mathrm{m.e.}}(\pm0.13)_{Z_B}(\pm0.08)_{\omega_{\eta}}\ \mathrm{MeV}$$
$$\Gamma_{\Upsilon(10753)\to h_b(2P)\pi^0} = 0.168(\pm0.067)_{\mathrm{m.e.}}(\pm0.009)_{Z_B}(\pm0.010)_{\omega_{\pi^0}}\ \mathrm{keV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(1P)\pi^0} = 2.04(\pm0.82)_{\mathrm{m.e.}}(\pm0.11)_{Z_B}(\pm0.13)_{\omega_{\pi^0}}\ \mathrm{keV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(1P)\eta} = 2.04(\pm0.81)_{\mathrm{m.e.}}(\pm0.11)_{Z_B}(\pm0.07)_{\omega_{\eta}}\ \mathrm{MeV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(1P)\eta'} = 9.23(\pm3.69)_{\mathrm{m.e.}}(\pm0.51)_{Z_B}(\pm0.39)_{\omega_{\eta'}}\ \mathrm{MeV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(2P)\pi^0} = 0.104(\pm0.042)_{\mathrm{m.e.}}(\pm0.006)_{Z_B}(\pm0.006)_{\omega_{\pi^0}}\ \mathrm{keV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(2P)\eta} = 81.8(\pm32.7)_{\mathrm{m.e.}}(\pm4.6)_{Z_B}(\pm2.7)_{\omega_{\eta}}\ \mathrm{keV}$$

- Uncertainties labeled by the origin m.e.=multipole expansion, ω=Production matrix element.

NLO Transitions

- Transitions from the **NLO singlet-octet operator**

$$\langle S_m \mathcal{O}_{\pi\pi} | \frac{g c_F}{m_Q} \mathrm{Tr}\left[\mathrm{S}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2) \cdot \boldsymbol{B}\, \mathrm{O} \right] | H_n \rangle \sim \frac{c_F}{m_Q} Z_B^{-1/2} \underbrace{\langle \mathcal{O}_{\pi\pi} | \boldsymbol{B}^2 | 0 \rangle}_{\text{l.q. meson production}} \underbrace{\langle \phi^{(m)} | (\boldsymbol{S}_1 - \boldsymbol{S}_2) \cdot \hat{\boldsymbol{r}}_\lambda | \psi_\lambda^{(n)} \rangle}_{\text{Heavy quark matrix element}}$$

- Heavy quark matrix element:
  - Selection rules: $\Delta s = 1$, $l = \ell \pm 1$.
  - $\Upsilon(10753)$ and $\Upsilon(11020)$ decay into $\Upsilon(m^3S_1)$ or $\Upsilon(m^3D_1)$ and light quark mesons.
- Light-quark meson production
  - Allowed final light-quark states: 0^{++} and $I = 0$ such as $\pi^+\pi^-$, K^+K^-, pairs of π^0 or η as well as f_0 resonances.
  - We use a dispersive representation for the matrix elements for the production $\pi^+\pi^-$, K^+K^-. Donoghue, Gasser, Leutwyler, Nucl.Phys.B343 (1990); Moussallam, Eur.Phys.J.C 14 (2000); Celis, Cirigliano, Passemar, Phys.Rev.D89 (2014)

Dispersive representation

- The two pion or kaon production matrix elements can be decomposed into S **and** D **wave pieces**:

$$\langle P^+P^-|\frac{\beta_0\alpha_s}{2\pi}\boldsymbol{B}^2|0\rangle = F_P^{(0)}(s) + \left[(u-t)^2 - \frac{4}{3}m_n^2\sigma_P^2(s)\rho_P^2(s)\right] F_P^{(2)}(s)\,, \quad P=\pi, K$$

- Only the cuts in the right-hand side of the complex s-plane corresponding to **two-pion and two-kaon rescattering** need to be considered.
- From **Watson's Theorem** the **imaginary part** of $F^{(l)}$ is known

$$\mathrm{Im}\left[n_P F_P^{(l)}(s)\right] = \sum_{P'=\pi,K} (T_l^{0*}(s))_{PP'}\sigma_{P'}(s) n_{P'} F_{P'}^{(l)}(s)\theta(s-4m_{P'}^2)$$

- Two-channel **Muskhelishvili-Omnès problem**: to find a functional form of the form factors with this imaginary part, is analytic in the complex s-plane, except on the cuts, and is real in the real s axis below the cuts. Muskhelishvili, *Singular integral equations*; Omnes, Nuovo Cim.8 (1958)

Dispersive representation

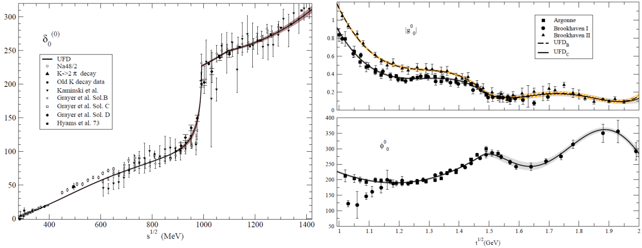
- A general solution is

$$n_P F_P^{(I)}(s) = \Omega_{PP'}^{(I)}(s) Q_{P'}^{(I)}(s)$$

- $\boldsymbol{Q}^{(I)}(s) = (Q_1^{(I)},\ Q_2^{(I)})$ are subtraction polynomials.
- Ω-matrix satisfies the singular integral equations

$$\boldsymbol{\Omega}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s} \left(\boldsymbol{T}_l^0(s') \right)^* \boldsymbol{\Sigma}(s') \boldsymbol{\Omega}(s')$$

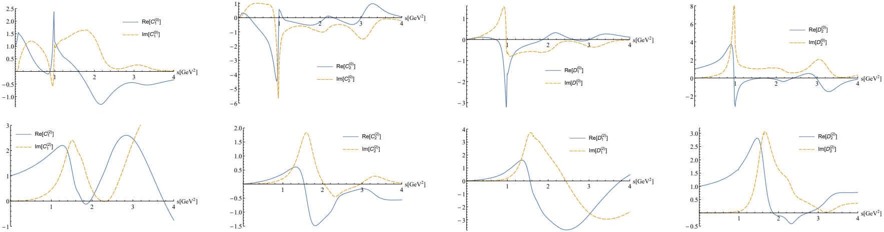
- Input needed: $\pi\pi \to \pi\pi$ and $\pi\pi \to K\bar{K}$ T-matrix for each partial wave.



- Parametrizations (and figure) from Garcia-Martin et al Phys.Rev.D83 (2011); Pelaez, Rodas Eur.Phys.J. C78 (2018)

Dispersive representation

- (Numerical) $\Omega^{(I)}$ solutions Moussallam, Eur.Phys.J.C 14 (2000); Descotes-Genon Ph.D. thesis (2000)

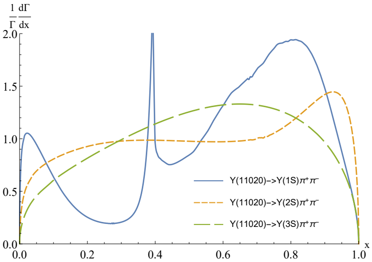


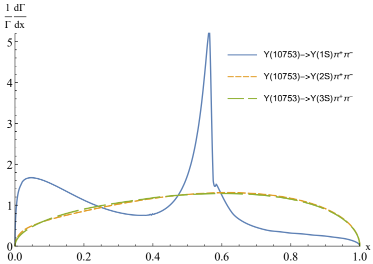
- The subtraction polynomials are obtained matching to a chiral representation of the form factors.
- Two Chiral low-energy constants determined using the Scale anomaly, Feynman-Hellmann theorem. Voloshin, Zakharov, Phys.Rev.Lett.45 (1980); Novikov, Shifman, Z.Phys.C8 (1981); Chivukula et al, Annals Phys. 192 (1989).

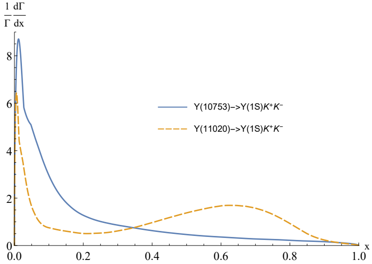
$$\langle P^+P^-|\frac{\beta_0\alpha_s}{2\pi}\boldsymbol{B}^2|0\rangle = -\left[\left(1-\frac{3\kappa}{4}\right)s+\left(1+\frac{3}{2}\kappa\right)m_P^2+\frac{3\kappa}{2}\left((m_n-m_m)^2-\left(\frac{u-t}{2m_n}\right)^2\right)\right]$$

- κ obtained from $\psi(2S)\to J/\psi\,\pi^+\pi^-$ and $\Upsilon(2S)\to\Upsilon(1S)\,\pi^+\pi^-$ Pineda, JTC, Phys.Rev.D100 (2019).

NLO Transitions differential widths







NLO Transitions widths

- Renormalization group improved expression for $c_F(1\ \mathrm{GeV}) = 0.879$ and $m_b(1\ \mathrm{GeV}) = 4.885$ GeV in RS' scheme Peset, Pineda, Segovia JHEP 09 (2018).

$$\Gamma_{\Upsilon(10753)\to\Upsilon(1S)\pi^+\pi^-} = 43.4(\pm 17.3)_{\mathrm{m.e.}}(\pm 2.4)_{Z_B}(\pm 8.6)_{\alpha_s}(^{+0.5}_{-0.0})_\kappa\ \mathrm{keV}$$

$$\Gamma_{\Upsilon(10753)\to\Upsilon(2S)\pi^+\pi^-} = 2.75(\pm 1.10)_{\mathrm{m.e.}}(\pm 0.15)_{Z_B}(\pm 0.55)_{\alpha_s}(^{+0.13}_{-0.12})_\kappa\ \mathrm{keV}$$

$$\Gamma_{\Upsilon(10753)\to\Upsilon(3S)\pi^+\pi^-} = 0.98(\pm 0.39)_{\mathrm{m.e.}}(\pm 0.05)_{Z_B}(\pm 0.19)_{\alpha_s}(\pm 0.03)_\kappa\ \mathrm{eV}$$

$$\Gamma_{\Upsilon(11020)\to\Upsilon(1S)\pi^+\pi^-} = 99.1(\pm 39.6)_{\mathrm{m.e.}}(\pm 5.5)_{Z_B}(\pm 19.7)_{\alpha_s}(^{+26.3}_{-21.8})_\kappa\ \mathrm{keV}$$

$$\Gamma_{\Upsilon(11020)\to\Upsilon(2S)\pi^+\pi^-} = 3.96(\pm 1.58)_{\mathrm{m.e.}}(\pm 0.22)_{Z_B}(\pm 0.70)_{\alpha_s}(^{-0.16}_{+0.17})_\kappa\ \mathrm{keV}$$

$$\Gamma_{\Upsilon(11020)\to\Upsilon(3S)\pi^+\pi^-} = 1.33(\pm 0.53)_{\mathrm{m.e.}}(\pm 0.07)_{Z_B}(\pm 0.27)_{\alpha_s}(\pm 0.02)_\kappa\ \mathrm{keV}$$

$$\Gamma_{\Upsilon(10753)\to\Upsilon(1S)K^+K^-} = 3.98(\pm 1.59)_{\mathrm{m.e.}}(\pm 0.22)_{Z_B}(\pm 0.79)_{\alpha_s}(^{-0.50}_{+0.67})_\kappa\ \mathrm{keV}$$

$$\Gamma_{\Upsilon(11020)\to\Upsilon(1S)K^+K^-} = 5.93(\pm 2.37)_{\mathrm{m.e.}}(\pm 0.33)_{Z_B}(\pm 1.18)_{\alpha_s}(^{+1.75}_{-1.18})_\kappa\ \mathrm{keV}$$

- Uncertainties labeled by the origin m.e.=multipole expansion, κ and α_s uncertainties on dispersive representation.
- Experimental values for with **ranges** Belle col. JHEP 10, 220 (2019)

$$\Gamma^{\mathrm{exp}}_{\Upsilon(11020)\to\Upsilon(1S)\pi^+\pi^-} = 85^{+33}_{-36}\ \mathrm{keV}$$

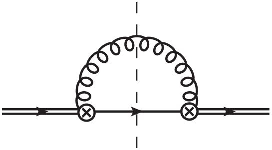
$$\Gamma^{\mathrm{exp}}_{\Upsilon(11020)\to\Upsilon(2S)\pi^+\pi^-} = 120^{+105}_{-107}\ \mathrm{keV}$$

$$\Gamma^{\mathrm{exp}}_{\Upsilon(11020)\to\Upsilon(3S)\pi^+\pi^-} = 61^{+37}_{-38}\ \mathrm{keV}$$

- **Promising agreement** for $\Upsilon(11020) \to \Upsilon(1S)\pi^+\pi^-$.

Semi-inclusive widths

- If the **energy gap** between a hybrid and a standard quarkonium state is **large** semi-inclusive transition widths can be computed. Oncala, Soto Phys.Rev.D96 (2017)

$$\Gamma_{\text{s.i.}} = \langle H_n | \mathrm{Im}[$$

$$]|H_n\rangle$$

$$\Gamma^{\mathrm{LO}}_{\Upsilon(11020)\to h_b(1P)} = 20(\pm 9)_{\alpha_s}\ \mathrm{MeV}$$
$$\Gamma^{\mathrm{NLO}}_{\Upsilon(10753)\to \Upsilon(1S)} = 9.7(\pm 3.8)_{\alpha_s}\ \mathrm{MeV}$$
$$\Gamma^{\mathrm{NLO}}_{\Upsilon(11020)\to \Upsilon(1S)} = 7.3(\pm 2.5)_{\alpha_s}\ \mathrm{MeV}$$
$$\Gamma^{\mathrm{NLO}}_{\Upsilon(11020)\to \Upsilon(2S)} = 1.1(\pm 0.5)_{\alpha_s}\ \mathrm{MeV}$$

- The sum of the three $\Upsilon(11020) \to h_b(1P)$ channels computed is compatible with the semi-inclusive width.
- The sum of the two pion/kaon channels is much smaller than the semi-inclusive widths $\Rightarrow$ other channels dominate.
- The **sum of semi-inclusive widths** for $\Gamma^{\mathrm{LO+NLO}}_{\Upsilon(11020)} = 28.4 \pm 9.4$ MeV is **compatible** with the **experimental** value of the **total width** $\Gamma^{\mathrm{exp}}_{\Upsilon(11020)} = 24^{+8}_{-6}$ MeV.

Summary

- We have computed the transition widths of $\Upsilon(10753)$ and $\Upsilon(11020)$ into standard quarkonium states and light quark mesons.
- We have assumed that these two states are the first two lowest laying 1^{--} hybrid bottomonium states.
- We have used an EFT framework incorporating the heavy quark mass and multipole expansions (pNRQCD).
- LO transitions:
  - $\Upsilon(10753)/\Upsilon(11020)$ to $h_b(mP)$ and 0^{-+} light quark mesons
  - π^0, η, η' production matrix elements obtained using $U(1)_A$ anomaly and Feldmann-Kroll-Stech mixing scheme.
- NLO transitions:
  - $\Upsilon(10753)/\Upsilon(11020)$ to $\Upsilon_b(mS)$ (or $\Upsilon_b(mD)$) and 0^{++} light quark mesons.
  - $\pi^+\pi^-$, K^+K^- production matrix elements obtained with a dispersive representation matched to χPT.
- Total width, mass and transition width to $\Upsilon(1S)\pi^+\pi^-$ are consistent with the interpretation of $\Upsilon(11020)$ as a hybrid in the Born-Oppenheimer picture.

Thank you for your attention