decays of an exotic $1^{-+}$ hybrid meson in QCD
hybrid mesons

one hypothesis to go beyond the $q\bar{q}$ picture of mesons

- add an excitation of the gluonic field $q\bar{q}G$
- can give rise to $J^{PC}$ not allowed for $q\bar{q}$
  e.g. $0^{+-}, 1^{-+}, 2^{+-} \ldots$

long history of study within QCD-motivated models

- constituent gluon
- bag model
- flux-tube model

all have exotic $J^{PC}$ mesons, but spectra differ

more recently studied in (incomplete) lattice QCD calculations ...
(incomplete) lattice QCD spectrum of mesons

hybrid mesons?

exotic $J^{PC}$

$m_\pi = 391 \text{ MeV}$

$24^3 \times 128$

iso scalar

iso vector

$\bar{\psi} \Gamma t_\alpha \psi \cdot B^\alpha$

$\psi \Gamma t_\alpha \psi \cdot B^\alpha$

PRD 88 094505 (2013)
(incomplete) lattice QCD spectrum of mesons

clear signal for a lightest exotic hybrid meson
A recent JPAC analysis of COMPASS data on $\pi p \rightarrow \pi \eta \ p$, $\pi p \rightarrow \pi \eta' \ p$

Determination of the Pole Position of the Lightest Hybrid Meson Candidate


(Joint Physics Analysis Center)

pole singularity of a $\pi_1$ resonance

$m_R=1564(89) \text{ MeV}, \Gamma_R=492(115) \text{ MeV}$

a rather broad resonance
decays of an exotic $\pi_1$ hybrid | 13 Apr 2021 | APS GHP 2021

hence how would an unstable resonance appear in lattice QCD?

the lattice has a finite-volume $\Rightarrow$ spectrum is discrete

but the mapping discrete-spectrum $\leftrightarrow$ scattering matrix is known

$\pi\pi J^{P}=1^{-}$

$\pi\pi l=1$ $m_\pi \sim 391$ MeV

& other irreps

scattering phase-shift

$E$ / MeV

$0 \rightarrow 180$

$0 \rightarrow 90$

$0 \rightarrow 30$
coupled-channel resonances

**isospin=1** \(\pi\eta, K\bar{K}\)

- \(\pi\eta\)
- \(K\bar{K}\)
- \(\pi\eta'\)

\[
|c_{K\bar{K}}/c_{\pi\eta}|^2 = 1.7(6)
\]

**isospin=0** \(\pi\pi, K\bar{K}, \eta\eta\)

- \(\pi\pi\)
- \(K\bar{K}\)
- \(\eta\eta\)

\[
|c_{K\bar{K}}/c_{\pi\pi}|^2 = 1.4(3)
\]
resonances decaying to hadrons with spin $- b_1 \rightarrow \omega \pi$

$\rho_a \rho_b |t_{\ell a, \ell'b}|^2$

$J^P = 1^+$

$m_\pi \sim 391$ MeV

$\omega$ is stable at $m_\pi \sim 391$ MeV

but a $\pi_1$ resonance potentially has a very large set of decay modes ...

several successful calculations with $m_\pi \sim 391$ MeV
$m_u = m_d = m_s \text{ SU}(3)_F \text{ point}$

increase the light quark mass to the strange quark mass ...

(incomplete) lattice spectrum calculation  PRD 88 094505 (2013)
Several stable mesons:

\[ \eta^8 \sim \pi, K, \eta \]
\[ \eta^1 \sim \eta' \]
\[ \omega^8, \omega^1 \sim \rho, K^*, (\omega, \varphi) \]
\[ h_1^8, h_1^1 \sim b_1, K_1, (h_1, h_1') \]
\[ f_1^8, f_1^1 \sim a_1, K_1, (f_1, f_1') \]

Decays of an exotic \( \pi_1 \) hybrid | 13 Apr 2021 | APS GHP 2021
lattice QCD spectrum computed in 6 volumes

53 energy levels to constrain ‘eight’ channel scattering
LQCD spectrum computed in 6 volumes

states have overlap with $\bar{\psi} \Gamma a \psi \cdot B^a$
describe scattering by a unitarity-preserving $K$-matrix featuring a pole (11 free parameters)

$$\frac{\chi^2}{N_{\text{dof}}} = \frac{43.6}{53-11} = 1.04$$

a good description of the spectrum ...
an ‘eight’ channel scattering amplitude
‘eight’ channel scattering amplitudes - varying parameterization
octet $1^{--}$ resonance pole & couplings

at the SU(3)$_F$ point:
$m_R = 2144(12)$ MeV, $\Gamma_R = 21(21)$ MeV (a narrow resonance)

$t_{ab}(s) \sim \frac{c_ac_b}{s_0 - s}$

$\sqrt{s_0} = m_R - i\frac{1}{2}\Gamma_R$

resonance below $h_1^8\eta^8$ threshold, but with a large coupling
core assumption: couplings scale only with the relevant barrier factor $k^\ell$

use PDG masses & COMPASS/JPAC $\pi_1$ mass

generates for a $\pi_1$ at 1564 MeV:

- $\Gamma_{TOT} \sim 140$-$600$ MeV
- $\Gamma(\pi\eta) \approx 1$ MeV
- $\Gamma(\pi\eta') \approx 20$ MeV
- $\Gamma(\pi\rho) \approx 12$ MeV
- $\Gamma(\pi b_1) \sim 140$-$530$ MeV

JPAC/COMPASS candidate:

- $\Gamma_{TOT} \sim 492(115)$ MeV

Kopf et al analysis:

- $\Gamma_{TOT} \sim 388(10)$ MeV
- $\Gamma(\pi\eta') / \Gamma(\pi\eta) \sim 6.5(1)$

if correct, suggests prior observations in $\pi\eta$, $\pi\eta'$, $\pi\rho$
are in heavily suppressed decay channels

\[ \pi b_1 \rightarrow \pi\pi\omega \rightarrow \pi\pi\pi\pi\pi \]
first ever calculation of an **exotic hybrid meson** as a **resonance** in QCD

simplified scattering system using exact SU(3)$_F$ and $m_{\pi} \sim 700$ MeV

flavor octet $1^{-+}$ state appears as a narrow resonance

crude extrapolation to physical kinematics suggests a **potentially broad resonance**

what about other exotic $J^{PC}$?

can we build a phenomenology of hybrid decays starting from QCD?

challenge of **reducing quark mass** really the challenge of **including three-meson decays**

progress in this direction, see Max Hansen’s talk ...
$$|c|^{\text{phys}} = \left| \frac{k^{\text{phys}}(m_R^{\text{phys}})}{k(m_R)} \right| |c|.$$  

$$\Gamma(R \to i) = \frac{|c_i^{\text{phys}}|^2}{m_R^{\text{phys}}} \cdot \rho_i(m_R^{\text{phys}}).$$

### Example 'success' — $f_2,f_2'$ calculated at $m_\pi \sim 400$ MeV

<table>
<thead>
<tr>
<th></th>
<th>Scaled</th>
<th>PDG</th>
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<tbody>
<tr>
<td>$</td>
<td>c(f_2 \to \pi\pi)</td>
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<tr>
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<td>c(f_2 \to K\bar{K})</td>
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<td>$</td>
<td>c(f_2' \to \pi\pi)</td>
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<tr>
<td>$</td>
<td>c(f_2' \to K\bar{K})</td>
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$$\frac{1}{\sqrt{3}}(\pi^+\rho^0 - \pi^0\rho^+) + \frac{1}{\sqrt{6}}(K^+\bar{K}^*0 - \bar{K}^0K^{*+}),$$  

$$-\sqrt{\frac{3}{10}}(K_{1A}^+\bar{K}^0 + \bar{K}_{1A}^0K^+) + \frac{1}{\sqrt{5}}(a_1^+\eta_8 + (f_1)_{8\pi}^+),$$  

$$\frac{1}{\sqrt{6}}(K_{1B}^+\bar{K}^0 - \bar{K}_{1B}^0K^+) + \frac{1}{\sqrt{3}}(b_1^+\pi^0 - b_1^0\pi^+),$$

![Graph](image-url)
illustiative K-matrix form

\[
K_{VV}(s) = \frac{gg^T}{m^2 - s} + \begin{bmatrix}
\gamma_{\eta^1\eta^8\{1P_1}\}} & 0 & 0 & 0 \\
0 & \gamma_{\omega^8\eta^8\{3P_1}\}} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

\[
g = (g_{\eta^1\eta^8\{1P_1}\}}, g_{\omega^8\eta^8\{3P_1}\}}, g_{f^1\eta^8\{3S_1}\}}, g_{h^1\eta^8\{3S_1}\}}),
\]

\[
K_{VV}(s) = \begin{bmatrix}
\gamma_{\omega^8\omega^8\{3P_1}\}} & 0 & 0 & 0 \\
0 & \gamma_{\omega^1\omega^8\{1P_1}\}} & 0 & 0 \\
0 & 0 & \gamma_{\omega^1\omega^8\{3P_1}\}} & 0 \\
0 & 0 & 0 & \gamma_{\omega^1\omega^8\{5P_1}\}} \\
\end{bmatrix}.
\]
resonances in a finite volume?

but in a periodic volume ...

\[ \psi(|x| > R) \sim \cos (p|x| + \delta(p)) \]

applying the boundary conditions

\[ p = \frac{2\pi}{L} n - \frac{2}{L} \delta(p) \]

solved by discrete \( p_n(L) \)
non-resonant elastic scattering — $\pi^+\pi^+$ (isospin=2)

$m_\pi \sim 391$ MeV

$\ell = 2$

repulsive scattering, no resonances

$\ell = 0$

scattering phase-shifts

$\delta\ell/\pi$

$\ell \sim k^2 / \text{GeV}^2$

increasing total momentum

$1.9$ fm $2.4$ fm $2.9$ fm

$16$ $20$ $24$

$m_\pi L$ $4$ $5$ $6$

decays of an exotic $\pi_1$ hybrid | 13 Apr 2021 | APS GHP 2021
an elastic resonance — the $\rho$ in $\pi\pi$ (isospin=1)

$m_\pi \sim 391$ MeV

$m_\pi \sim 236$ MeV

$E / \text{MeV}$

$E / \text{MeV}$

$m_\pi L \ 4.3$

William & Mary
decays of an exotic π₁ hybrid | 13 Apr 2021 | APS GHP 2021

ππ isospin=0

\[ m_\pi \sim 391 \text{ MeV} \]

\[ E_{cm} \]

[000] [100] [110] [111] [200]

\[ m_\pi \sim 236 \text{ MeV} \]

\[ E_{cm} \]

[000] [100] [110] [111] [200]

heavier quark mass — a bound-state
lighter quark mass — attraction, maybe a broad resonance?

c.f. the experimental σ resonance ...

\[ k^2 / \text{GeV}^2 \]

\[ \delta_0 \]
in the case of **coupled-channel scattering** it’s more challenging ...

e.g. some energy region where $\pi\pi$, $\bar{K}K$ accessible

$$S(E) = \begin{bmatrix} S_{\pi\pi,\pi\pi}(E') & S_{\pi\pi,\bar{K}K}(E') \\ S_{\pi\pi,\bar{K}K}(E) & S_{\bar{K}K,\bar{K}K}(E) \end{bmatrix} \rightarrow E_n(L)$$

[a](#), [b](#), [c](#), [d](#), [e](#), [f](#)

you want to know this

an approach:

- parameterize the energy dependence of the scattering matrix

an important observation
- not like experiment
- can’t study ‘channel-by-channel’
- all channels contribute, have to solve the ‘whole problem’

lattice QCD gives you this
physical pion masses = low-lying multipion channels

\[ m_\pi \sim 391 \text{ MeV} \]

\[ m_\pi < 391 \text{ MeV} \]

\[ m_\pi \rightarrow \text{phys} \]
coupling resonances to currents

$\gamma^* \rightarrow \pi \pi$

$F_{\pi}(E_{\pi\pi})$

Feng et al. | u,d,s | $m_\pi$=290 MeV

PRD97 054513 (2018)

$\gamma^* \pi \rightarrow \pi \pi$

$A(E_{\pi\pi}, Q^2)$

Briceno et al. | u,d,s | $m_\pi$=391 MeV

PRL115 242001 (2015)

$E_{\pi\pi} / \text{GeV}$

Bulava et al. | u,d,s | $m_\pi$=280 MeV

conf. proc. LAT '15

$E_{\pi\pi} / m_\pi$

$|A|_{\gamma^*, \pi\pi}$

$E^*_{\pi\pi} / m_\pi$

$\rho \rightarrow \pi \gamma^*$

$\text{Re} F_{\pi\gamma}$

$\text{Im} F_{\pi\gamma}$

Jefferson Lab

decays of an exotic $\pi_1$ hybrid | 13 Apr 2021 | APS GHP 2021
calculate correlation functions

e.g. \( \langle 0 \vert O_i(t)O_j(0) \vert 0 \rangle \) where the operators are constructed from quark and gluon fields and have the quantum numbers of the hadronic system you want to study

\[
\langle 0 \vert O_i(t)O_j(0) \vert 0 \rangle = \sum_n \langle 0 \vert O_i \vert n \rangle \langle n \vert O_j \vert 0 \rangle e^{-E_n t}
\]

a superposition of the (finite-volume) eigenstates of QCD

powerful approach:

– use a large basis of operators*
– form a matrix of correlation functions
– diagonalize this matrix

* could give a whole interesting talk on the construction of these operators
operator basis $- l=0 \pi\pi, K\bar{K}, \eta\eta$

operator basis: ‘single-meson’

$\bar{\psi}\Gamma\psi$

( & if you like, tetraquark & … )

[000] $A_{1+}^+$ 24$^3$

\[
\begin{align*}
[000] & A_{1+}^+ \quad 24^3 \\
\hline
\text{a}_t E & \\
0.24 & \hline
0.22 & \\
0.20 & \\
0.18 & \\
0.16 & \\
0.14 & \\
0.12 & \\
0.10 & \hline
\end{align*}
\]

$\begin{align*}
\text{a}_t m_\pi & = 0.069 \\
\text{a}_t m_K & = 0.097 \\
\text{a}_t m_\eta & = 0.104
\end{align*}$

‘meson-meson’

\[
\sum_{\hat{p}_1, \hat{p}_2} C(p_1, p_2; p) M_1(p_1) M_2(p_2)
\]

$p = \frac{2\pi}{L} [n_x, n_y, n_z]$

maximum momentum guided by non-interacting energies

\[
\sqrt{m_1^2 + p_1^2} + \sqrt{m_2^2 + p_2^2}
\]

solutions of the det equation when $t = 0$
operator basis: 

\[ \bar{\psi} \Gamma \psi \quad + \quad \sum_{\hat{p}_1, \hat{p}_2} C(p_1, p_2; p) \left( M_1(p_1) M_2(p_2) \right) \]

\[ \sum_x e^{i p_1 \cdot x} \bar{\psi}_x \Gamma \psi_x \quad \sum_y e^{i p_2 \cdot y} \bar{\psi}_y \Gamma' \psi_y \]

prefer to use optimized single-meson operators ...

sampling the whole lattice volume
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$m_\pi = 0.039$, $m_K = 0.083$, $L \sim 3.8$ fm

'dropping' ops in $\rho \rightarrow \pi \pi$
what’s happening here?

focus on the lowest two states

an avoided level crossing
for each new channel, each sheet splits in two $\Rightarrow 2^N$ sheets for $N$ channels

e.g. two channels

\begin{align*}
\text{Im}[s] & \quad \text{Re}[s] \\
\text{Im}[s] & \quad \text{Re}[s]
\end{align*}

\begin{align*}
\text{sheet I} & \quad + \quad + \\
\text{sheet II} & \quad - \quad + \\
\text{sheet III} & \quad - \quad - \\
\text{sheet IV} & \quad + \quad -
\end{align*}
$f_2$ resonances – decay couplings & OZI

OZI ‘allowed’

OZI ‘forbidden’

$J^P = 2^+$
$m_w = 391$ MeV

$f_{2}^a \sim u\bar{u} + d\bar{d}$
$f_{2}^b \sim s\bar{s}$

couplings from pole residue

|       | $a_t |c_{\pi\pi}|/(a_t k_{\pi\pi})^2$ | $a_t |c_{K\bar{K}}|/(a_t k_{K\bar{K}})^2$ |
|-------|--------------------------------|--------------------------------|
| $f_{2}^a$ | 7.1(4)                           | 4.8(9)                          |
| $f_{2}^b$ | 1.0(3)                           | 5.5(8)                          |

zero in ‘OZI’ limit
– requires $s\bar{s}$ annihilation
Luescher finite-volume functions

\[ 0 = \det \left[ 1 + i \rho(E) \cdot t(E) \cdot (1 + i \mathcal{M}(E, L)) \right] \]

\[ \overline{\mathcal{M}}_{\ell J m, \ell' J' m'} = \sum_{m_\ell, m'_\ell, m_S} \langle \ell m_\ell; 1 m_S | J m \rangle \langle \ell' m'_\ell; 1 m_S | J' m' \rangle \]

\[ \times \sum_{\ell, m_\ell} \frac{(4\pi)^{3/2}}{k_{\text{cm}}^{\ell+1}} c_{\ell, m_\ell}^2 (k_{\text{cm}}^2; L) \int d\Omega \ Y_{\ell m_\ell}^* Y_{\ell' m'_\ell}^* Y_{\ell' m'_\ell} \]

to respect the lattice symmetries, need to subduce into irreducible representations

\[ \overline{\mathcal{M}}_{\tilde{n}, \Lambda}^{\tilde{n}, \Lambda} = \sum_{m, \lambda} \delta_{\Lambda, \Lambda'} \delta_{\mu, \mu'} \]

\[ \equiv \sum_{m, \lambda} S_{\Lambda \mu n}^{J \lambda} D_{m \lambda}^{(J^*)}(R) \overline{\mathcal{M}}_{\ell J m, \ell' J' m'}^{\tilde{n}} S_{\Lambda' \mu' n'}^{J' \lambda'} D_{m' \lambda'}^{(J')}(R) \]

e.g. \[ \mathcal{M}_0 \]

\[ \left( \frac{k L}{2\pi} \right)^2 \]
zeroes of the determinant

e.g. a two-channel Flatté form — [000] $A_1^+$ irrep in $L=2.4$ fm box

$det \left[ 1 + i \rho(E) \cdot t(E) \cdot (1 + i \mathcal{M}(E, L)) \right]$