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decays of an exotic 1^{-+} hybrid meson in QCD

Jozef Dudek

hadspec
hadron spectrum collaboration
hadspec.org

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Jefferson Lab
Thomas Jefferson National Accelerator Facility

hybrid mesons

one hypothesis to go beyond the $q\bar{q}$ picture of mesons

– add an **excitation of the gluonic field** $q\bar{q}G$

– can give rise to J^{PC} not allowed for $q\bar{q}$

e.g. $0^{+-}, 1^{-+}, 2^{+-} \ldots$

long history of study within QCD-motivated models

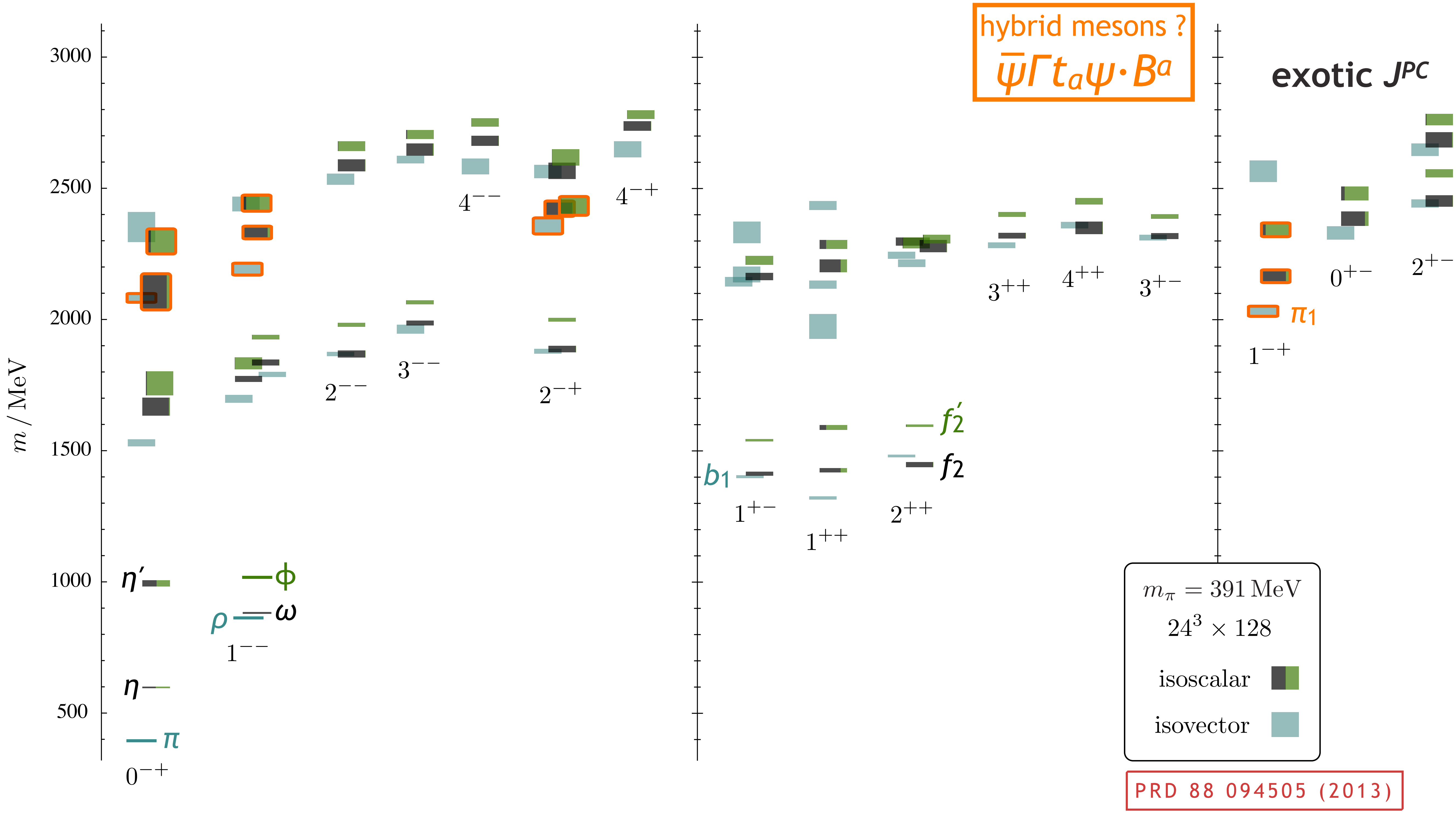
– constituent gluon
– bag model
– flux-tube model

⋮

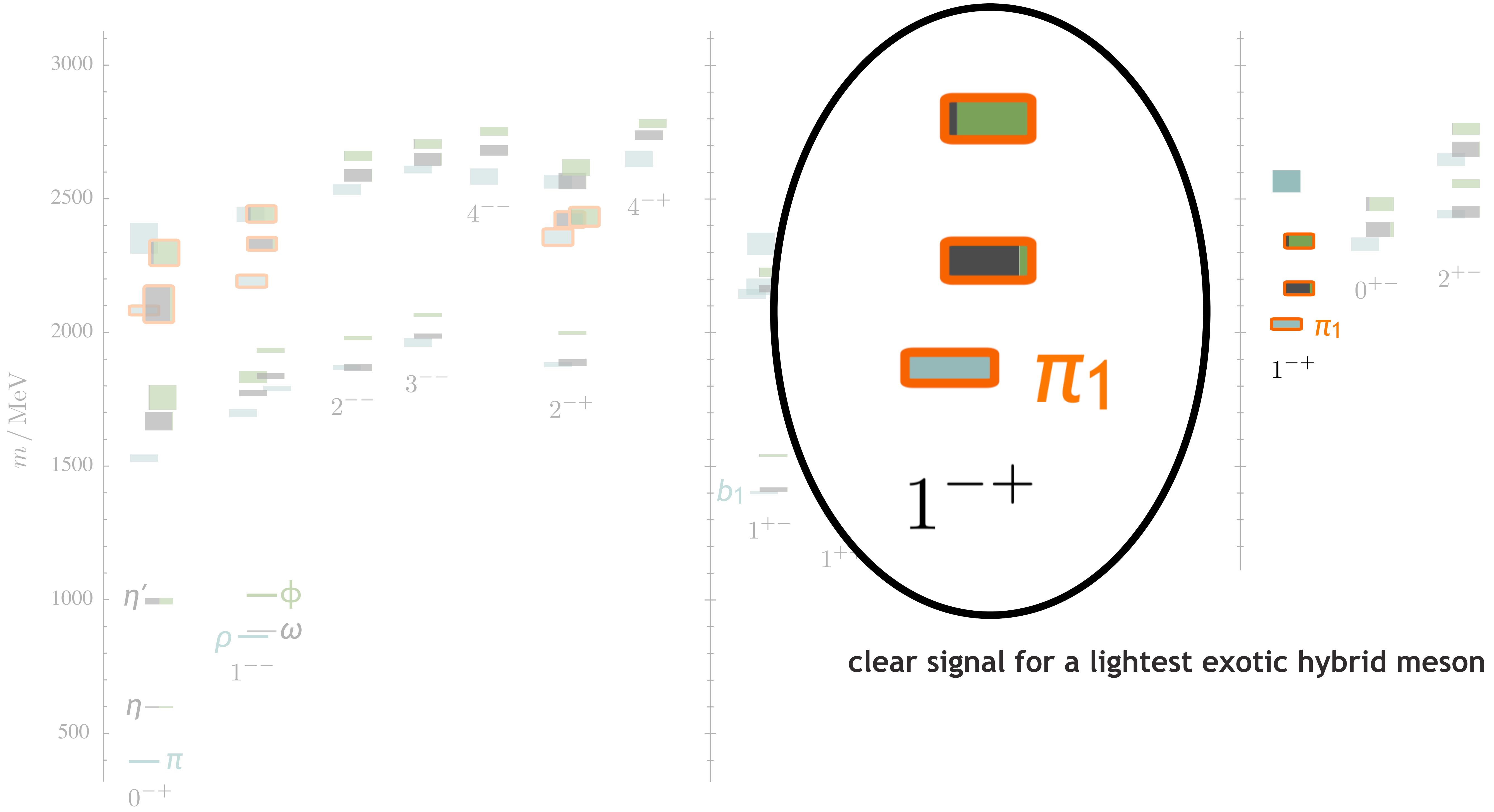
all have exotic J^{PC} mesons,
but spectra differ

more recently studied in (incomplete) **lattice QCD calculations** …

(incomplete) lattice QCD spectrum of mesons



(incomplete) lattice QCD spectrum of mesons



clear signal for a lightest exotic hybrid meson

experimental situation

a recent **JPAC** analysis of **COMPASS** data on $\pi p \to \pi\eta\, p$, $\pi p \to \pi\eta'\, p$

Determination of the Pole Position of the Lightest Hybrid Meson Candidate

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(Joint Physics Analysis Center)

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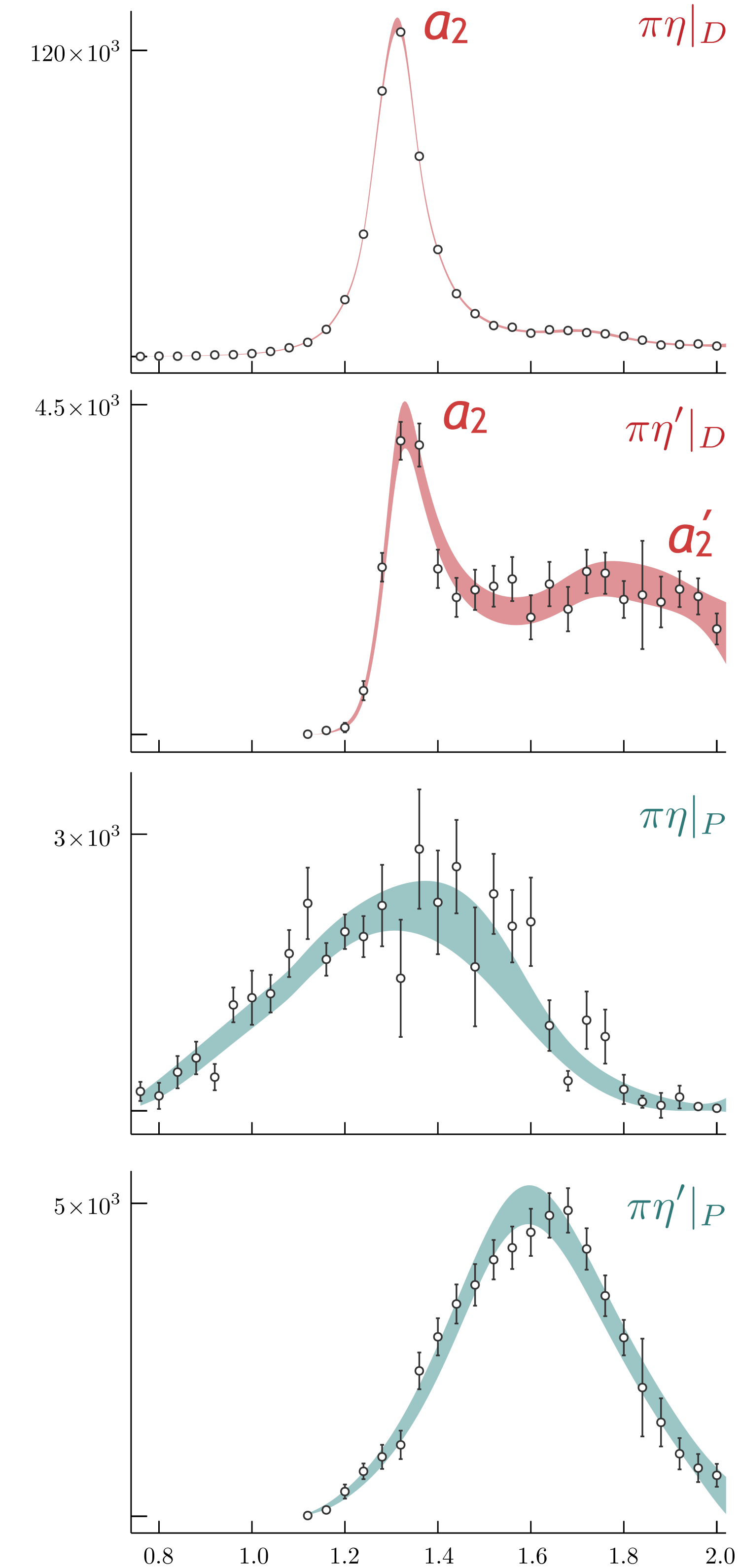
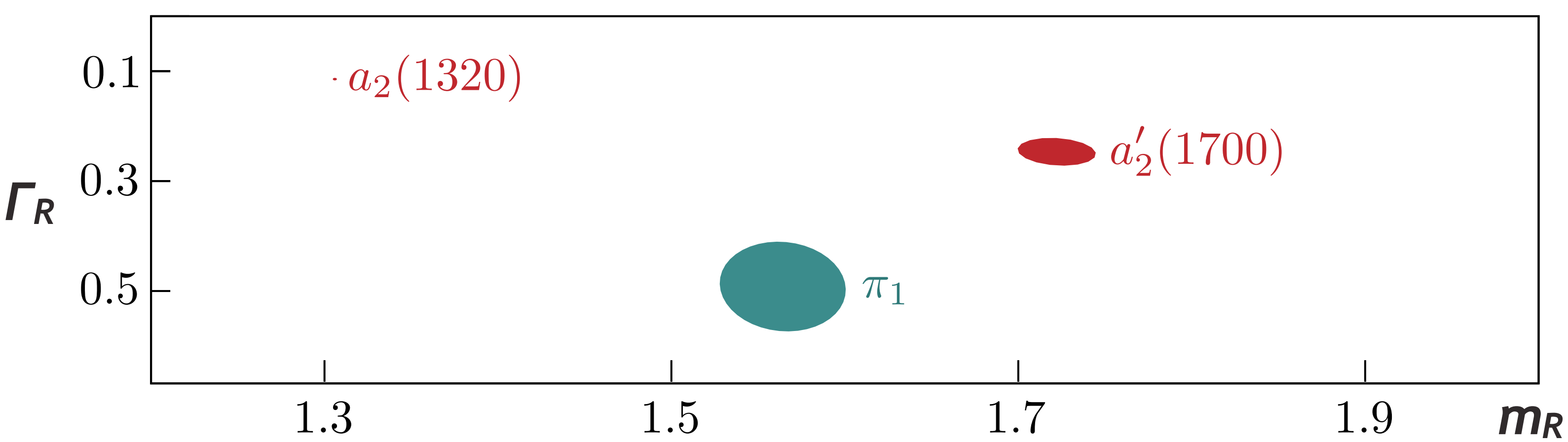


1−+

pole singularity of a π_1 resonance



m_R=1564(89) MeV, Γ_R=492(115) MeV

a rather broad resonance

a resonance in QCD ?

how would an unstable resonance appear in lattice QCD ?

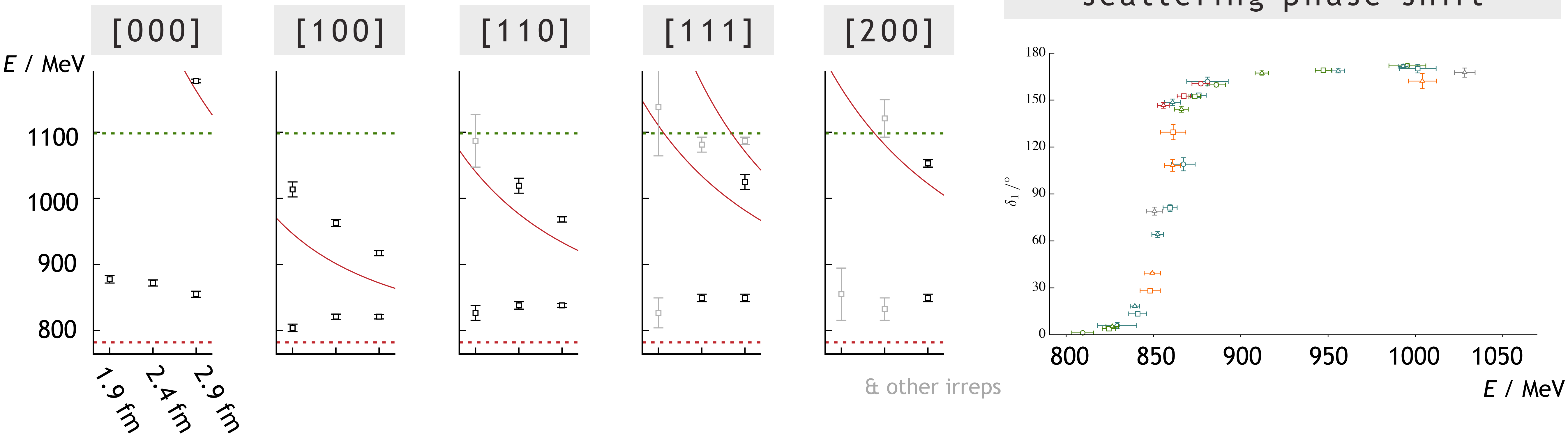
the lattice has a **finite-volume** ⇒ spectrum is **discrete**

but the mapping **discrete-spectrum** ⟷ **scattering matrix** is known

ππ I=1 $J^P=1^-$

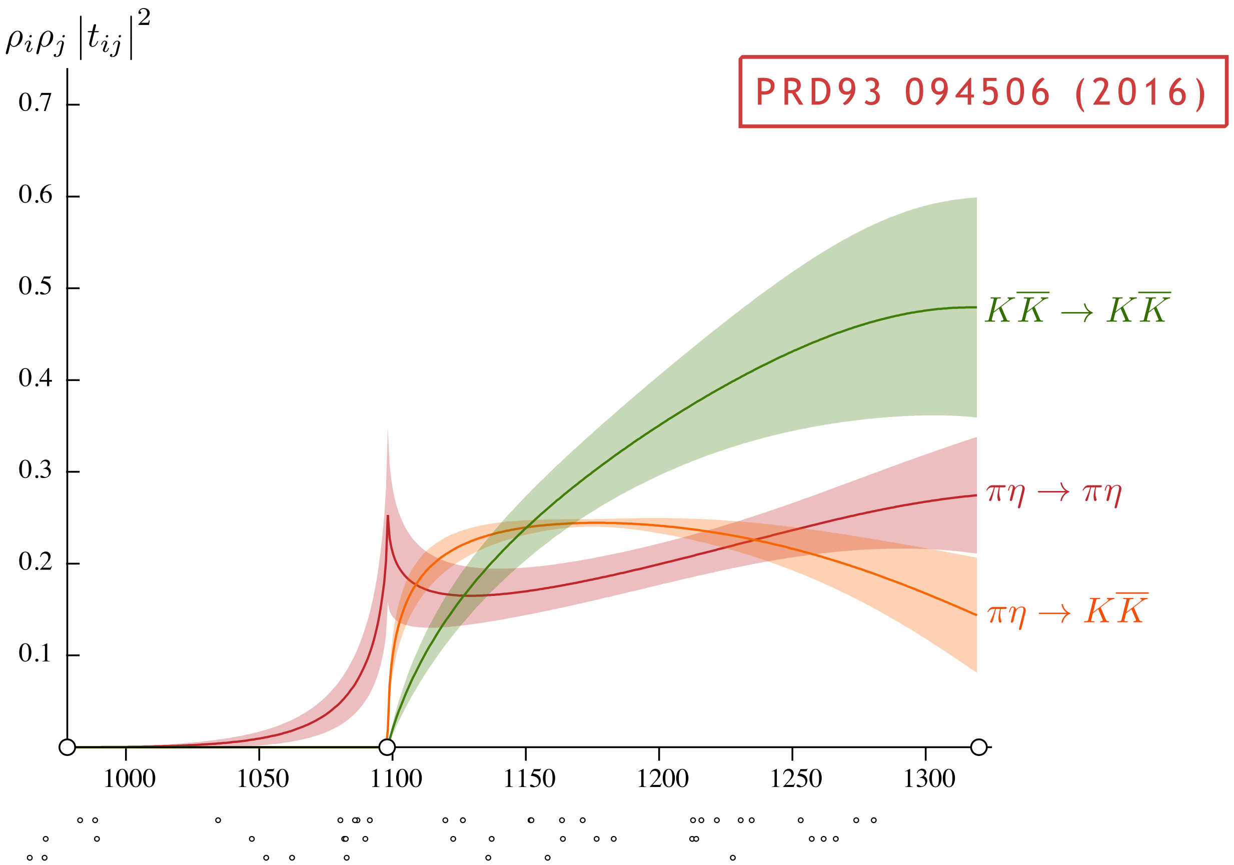
PRD87 034505 (2013)

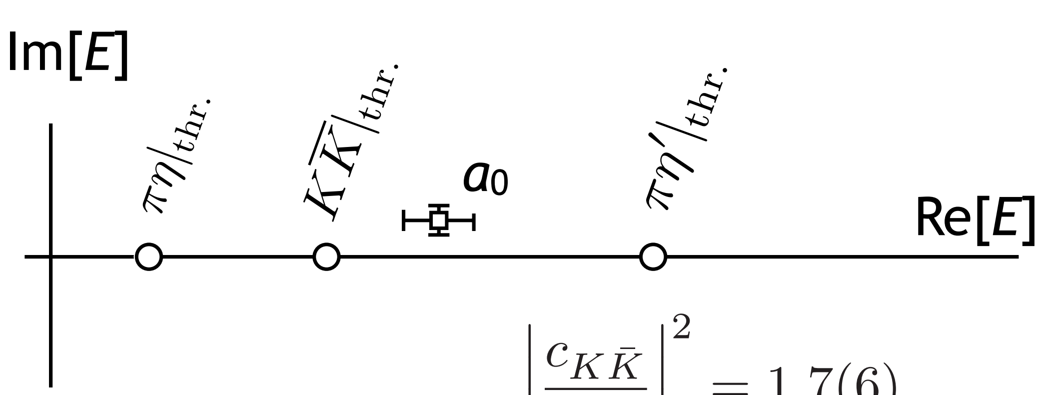
m_π ~ 391 MeV



coupled-channel resonances

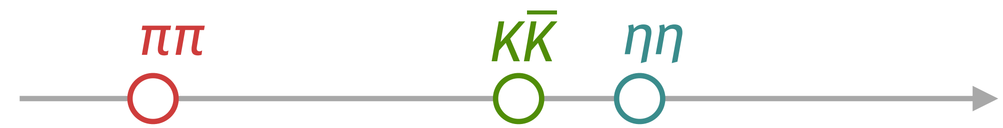
isospin=1 πη, $K\bar{K}$

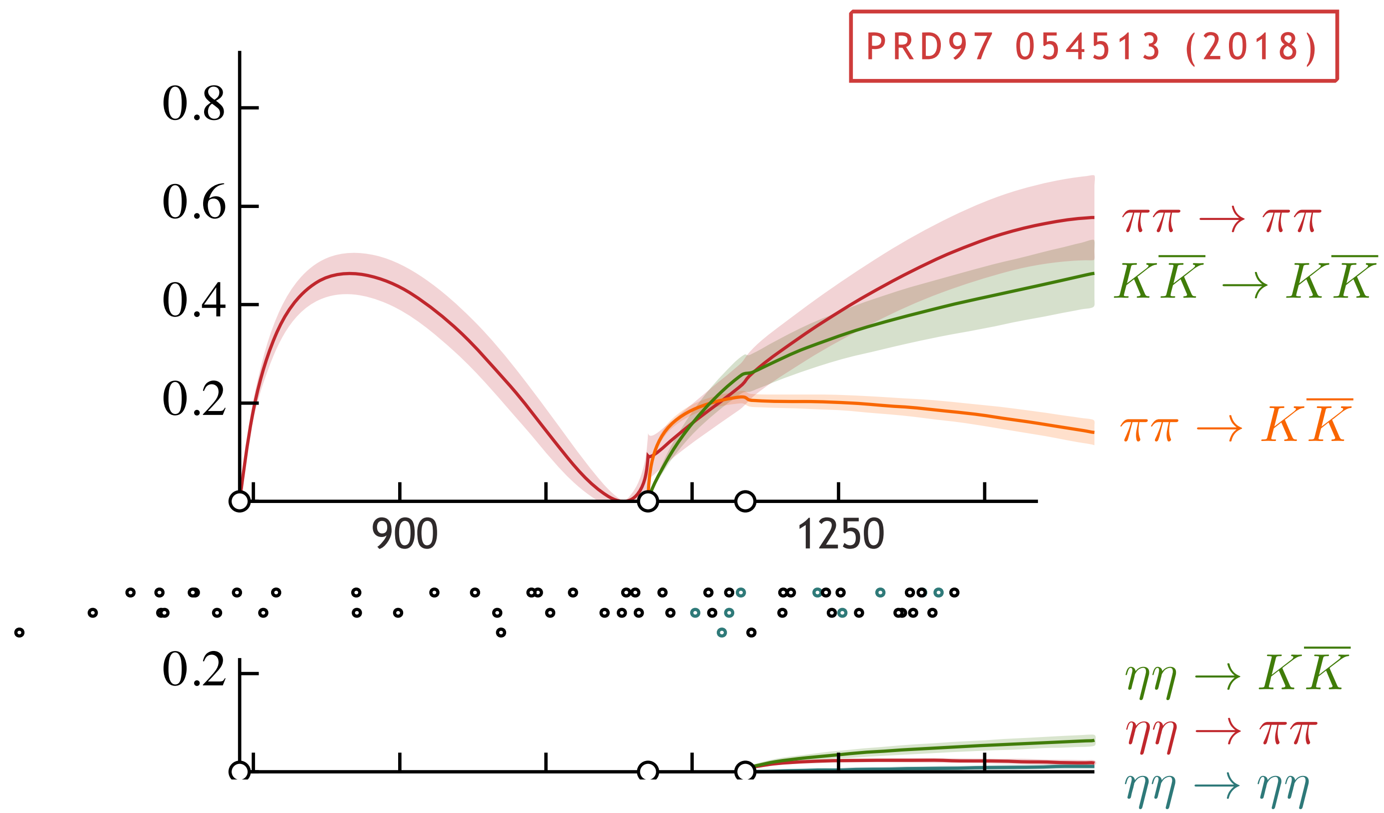


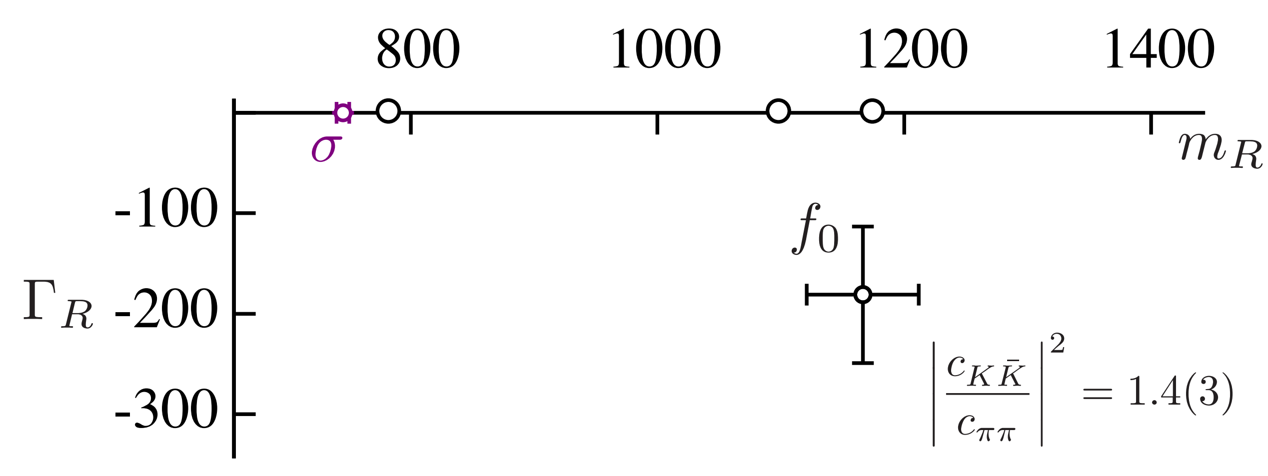


$$\left|\frac{c_{K\bar{K}}}{c_{\pi\eta}}\right|^2 = 1.7(6)$$

isospin=0 ππ, $K\bar{K}$, ηη



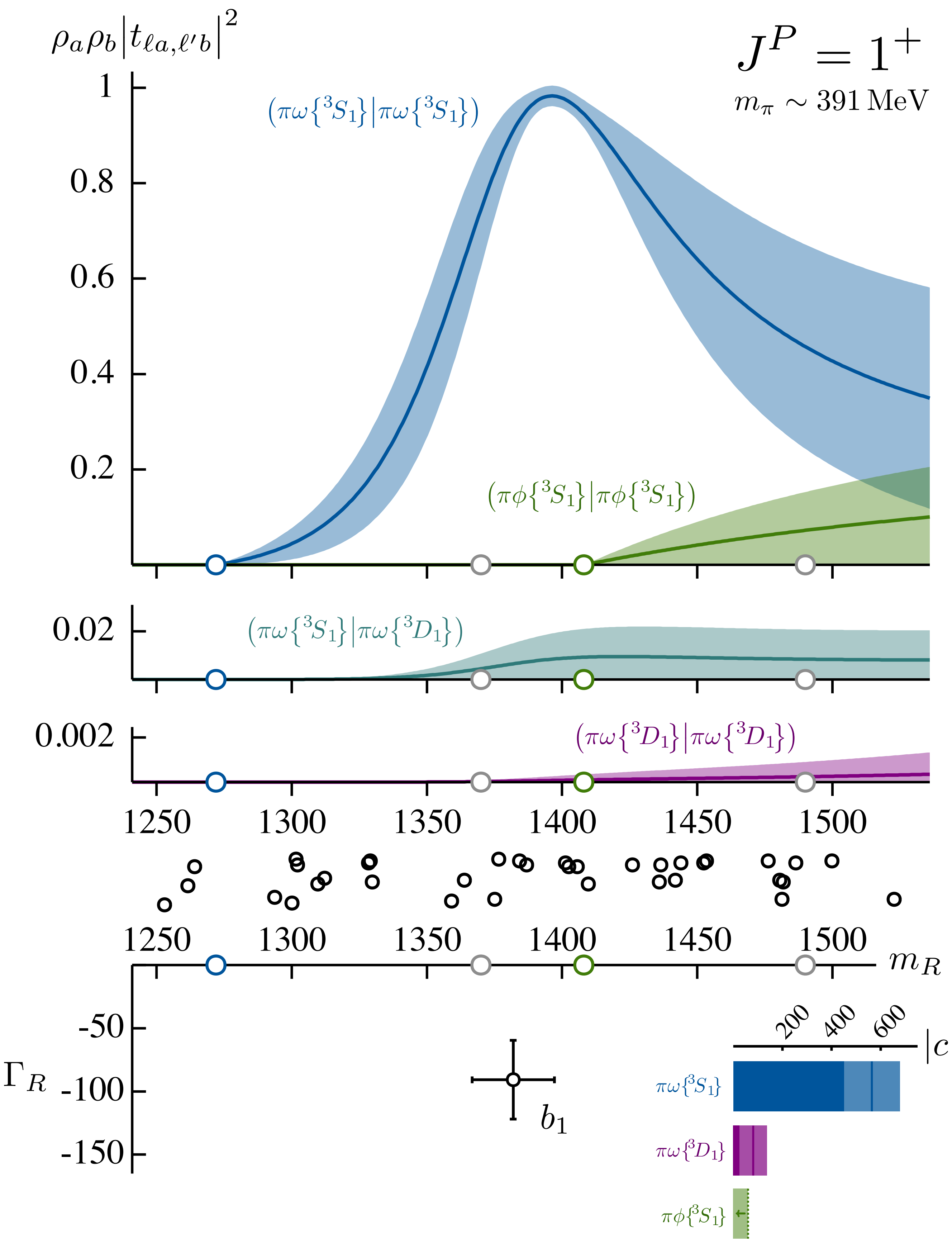




$$\left|\frac{c_{K\bar{K}}}{c_{\pi\pi}}\right|^2 = 1.4(3)$$

resonances decaying to hadrons with spin – $b_1 \to \omega\pi$

PRD100 054506 (2019)



ω is stable at m_π~391 MeV

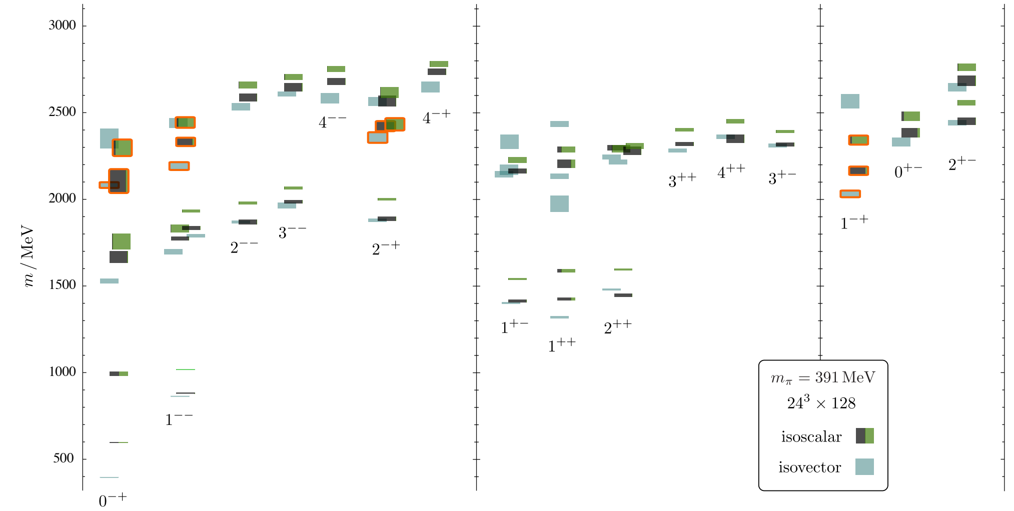
several successful calculations with m_π~391 MeV

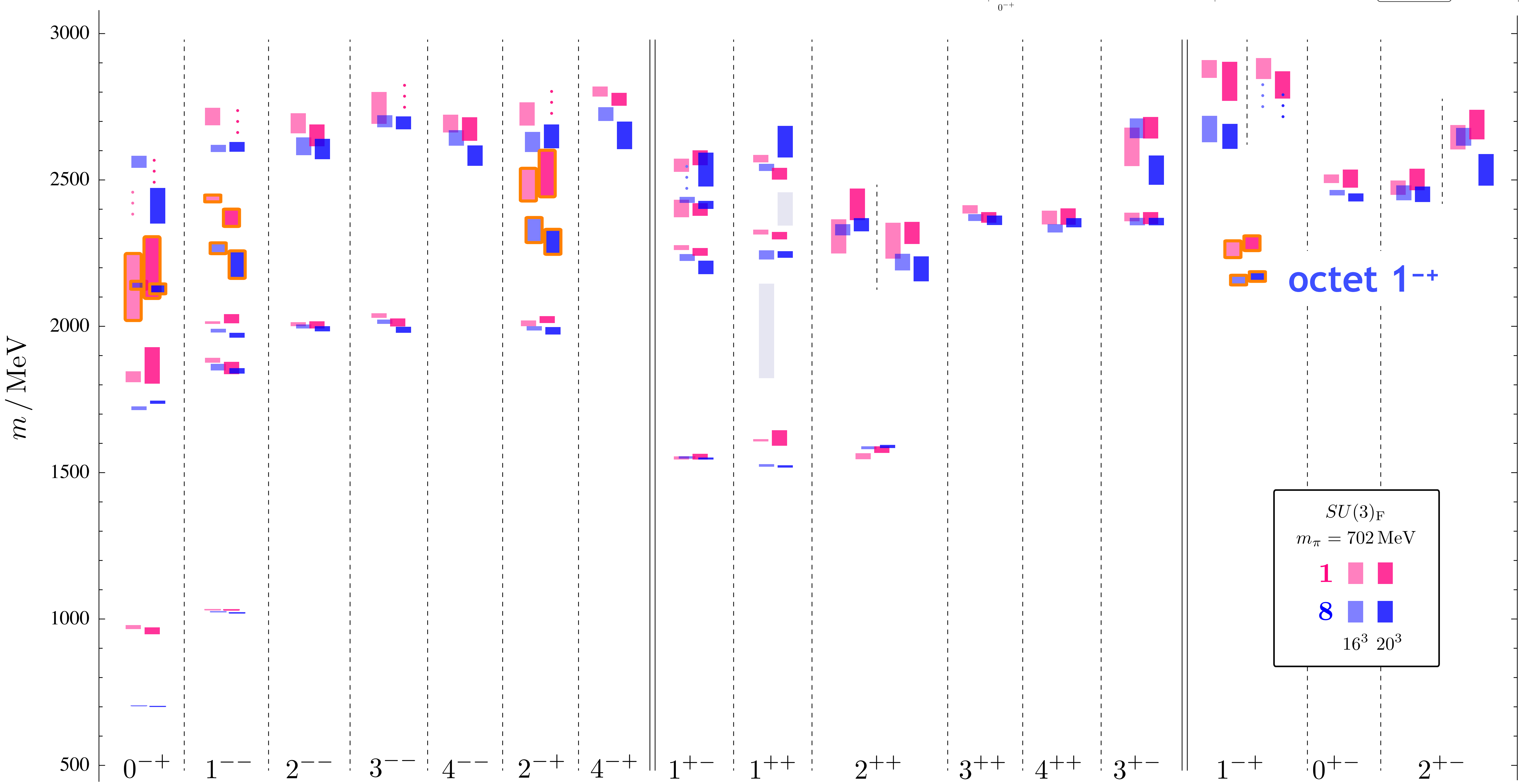
but a π_1 resonance potentially has a very large set of decay modes …

$m_u=m_d=m_s$ SU(3)$_F$ point

increase the light quark mass to the strange quark mass …

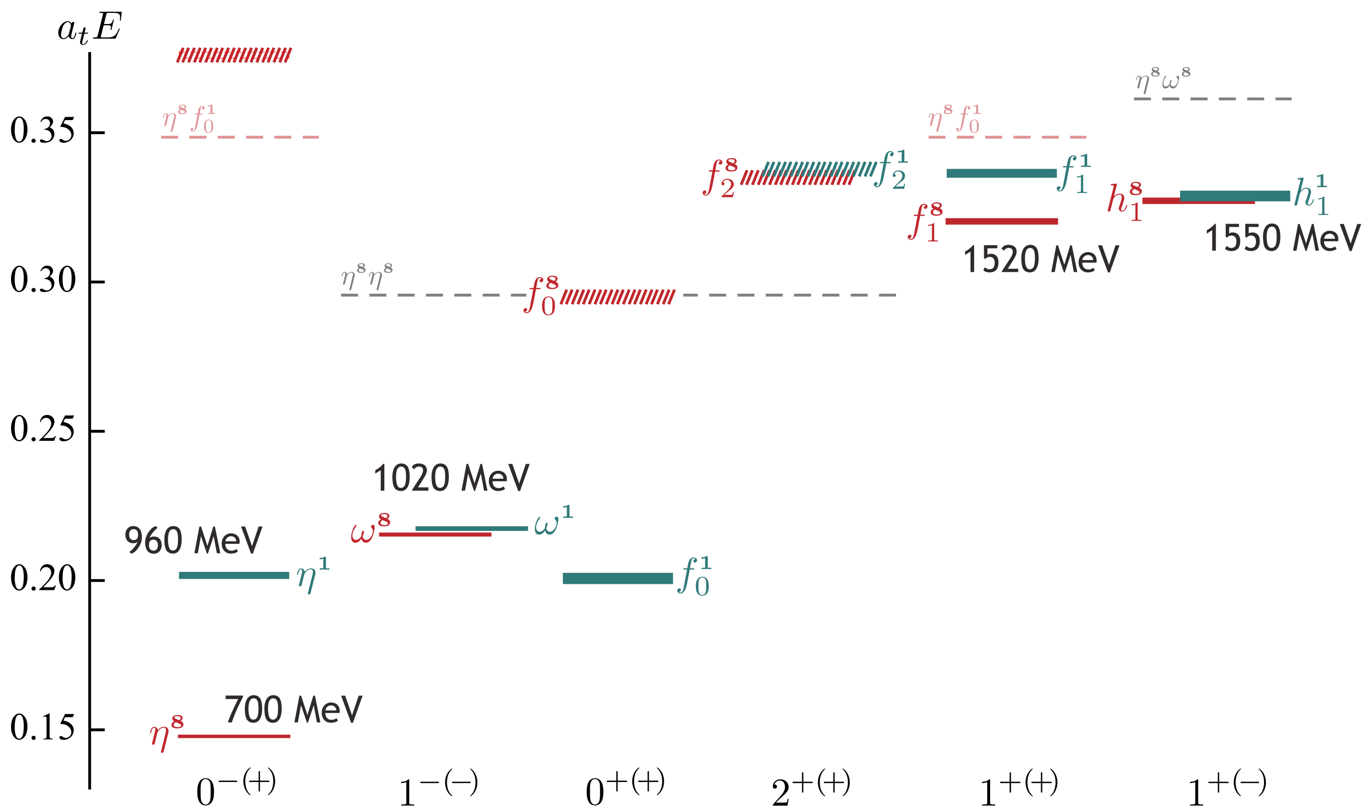
(incomplete) **lattice spectrum calculation** PRD 88 094505 (2013)





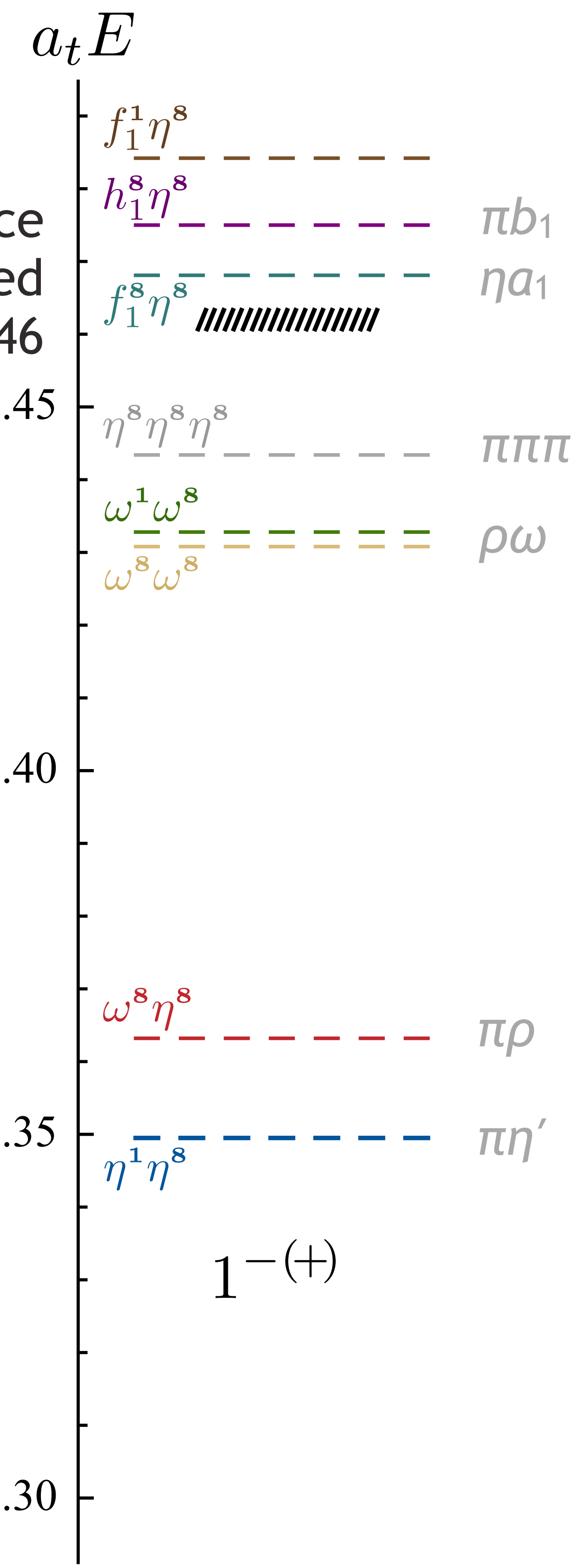
$m_u=m_d=m_s$ SU(3)$_F$ point

several stable mesons:



please forgive the obscure lattice units, will convert at the end ...

resonance expected near 0.46



$\eta^8 \rightsquigarrow \pi, K, \eta$

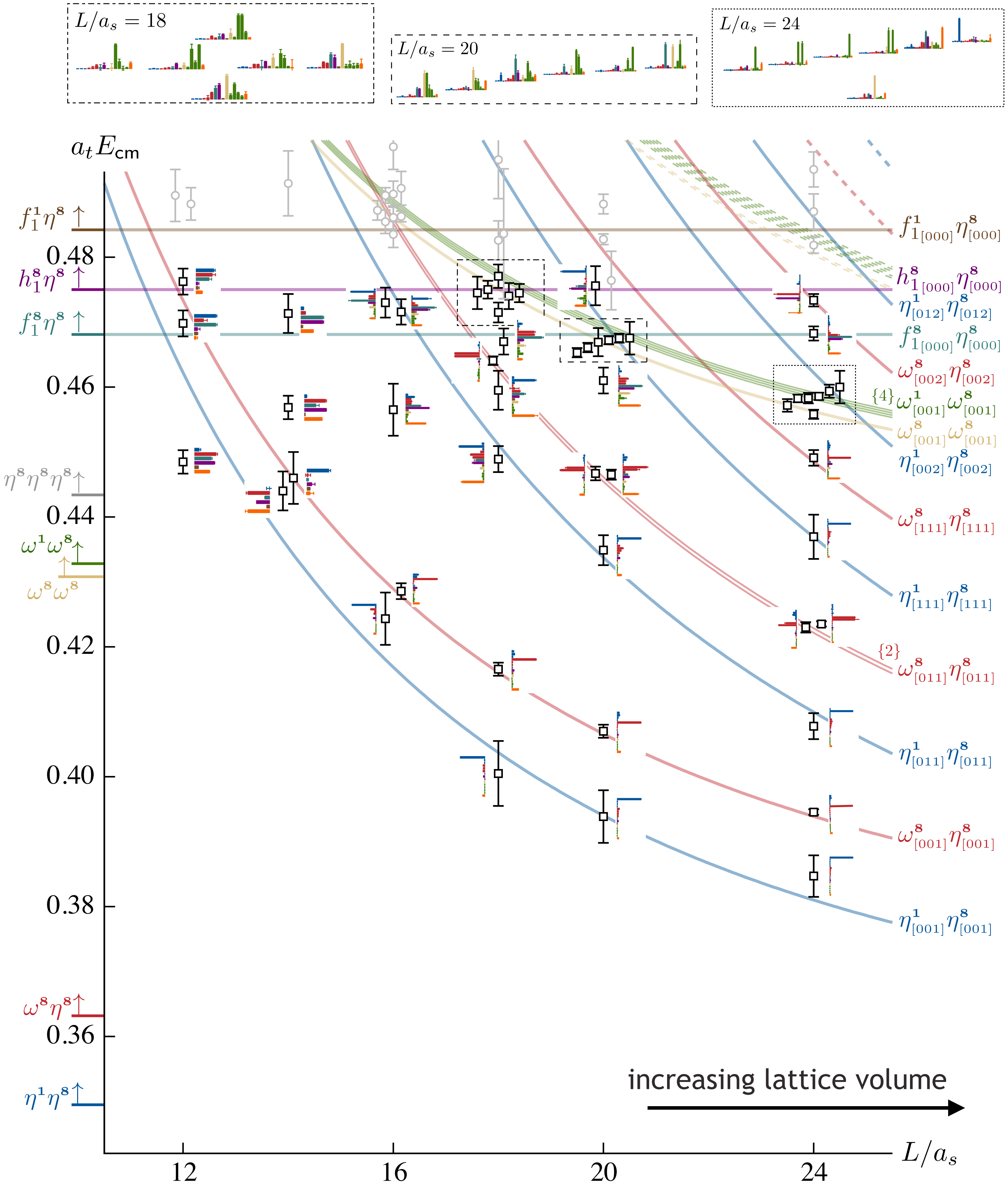
$\eta^1 \rightsquigarrow \eta'$

$\omega^8, \omega^1 \rightsquigarrow \rho, K^*, (\omega, \varphi)$

$h_1{}^8, h_1{}^1 \rightsquigarrow b_1, K_1, (h_1, h_1')$

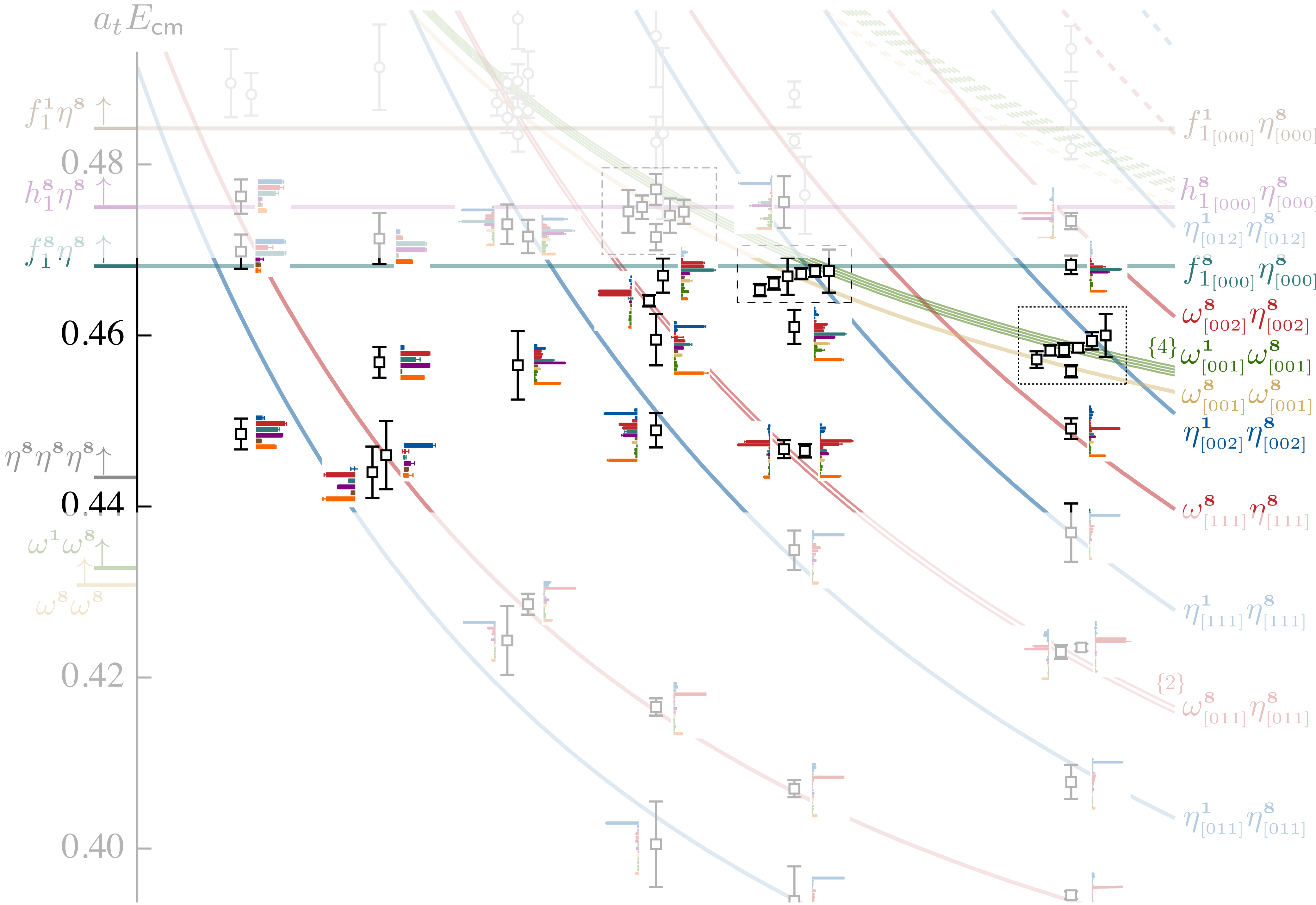
$f_1{}^8, f_1{}^1 \rightsquigarrow a_1, K_1, (f_1, f_1')$

lattice QCD spectrum computed in 6 volumes



53 energy levels to constrain 'eight' channel scattering

LQCD spectrum computed in 6 volumes



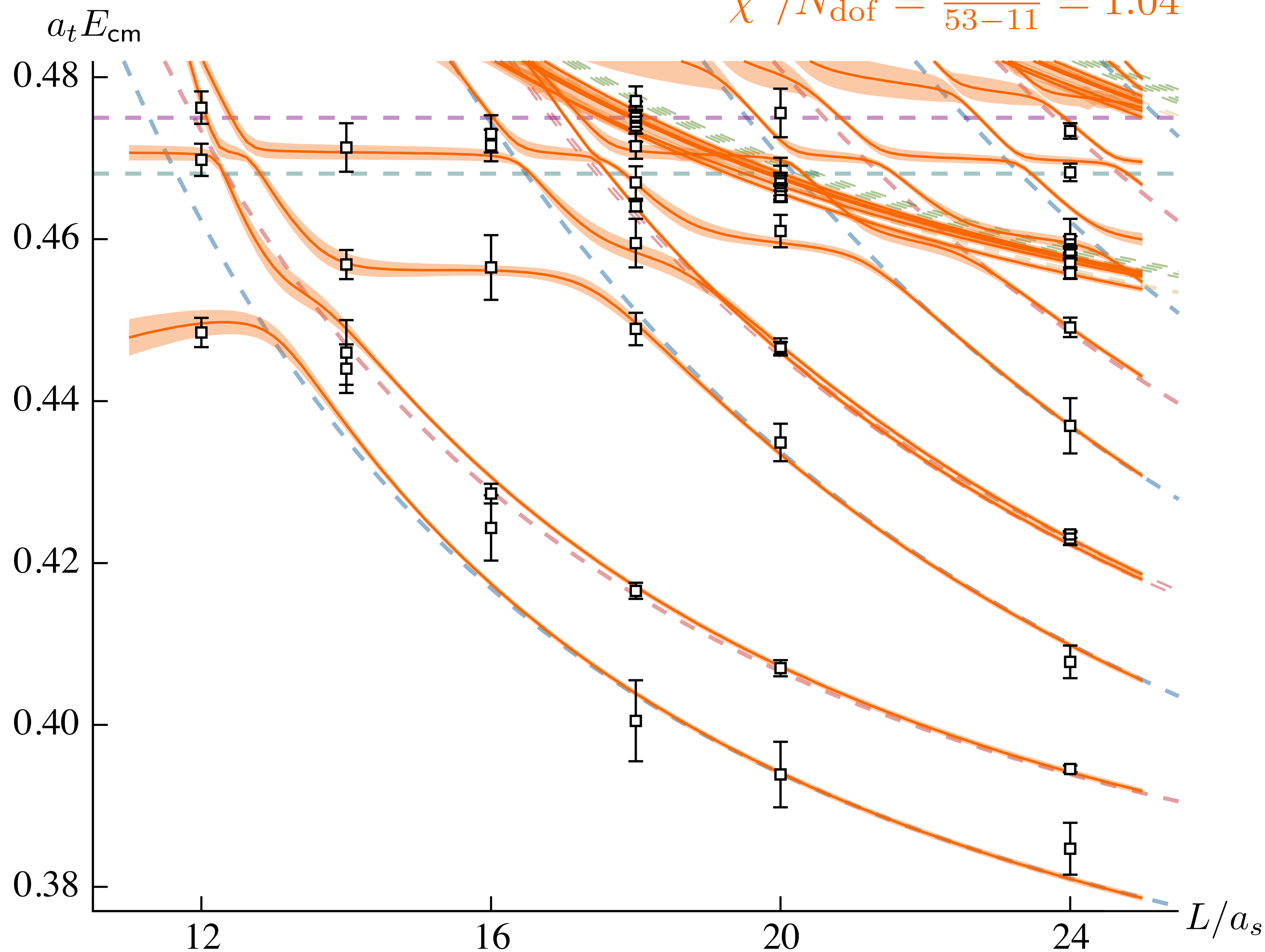
states have overlap with

$\bar{\psi}\Gamma t_a \psi \cdot B^a$

an ‘eight’ channel scattering amplitude

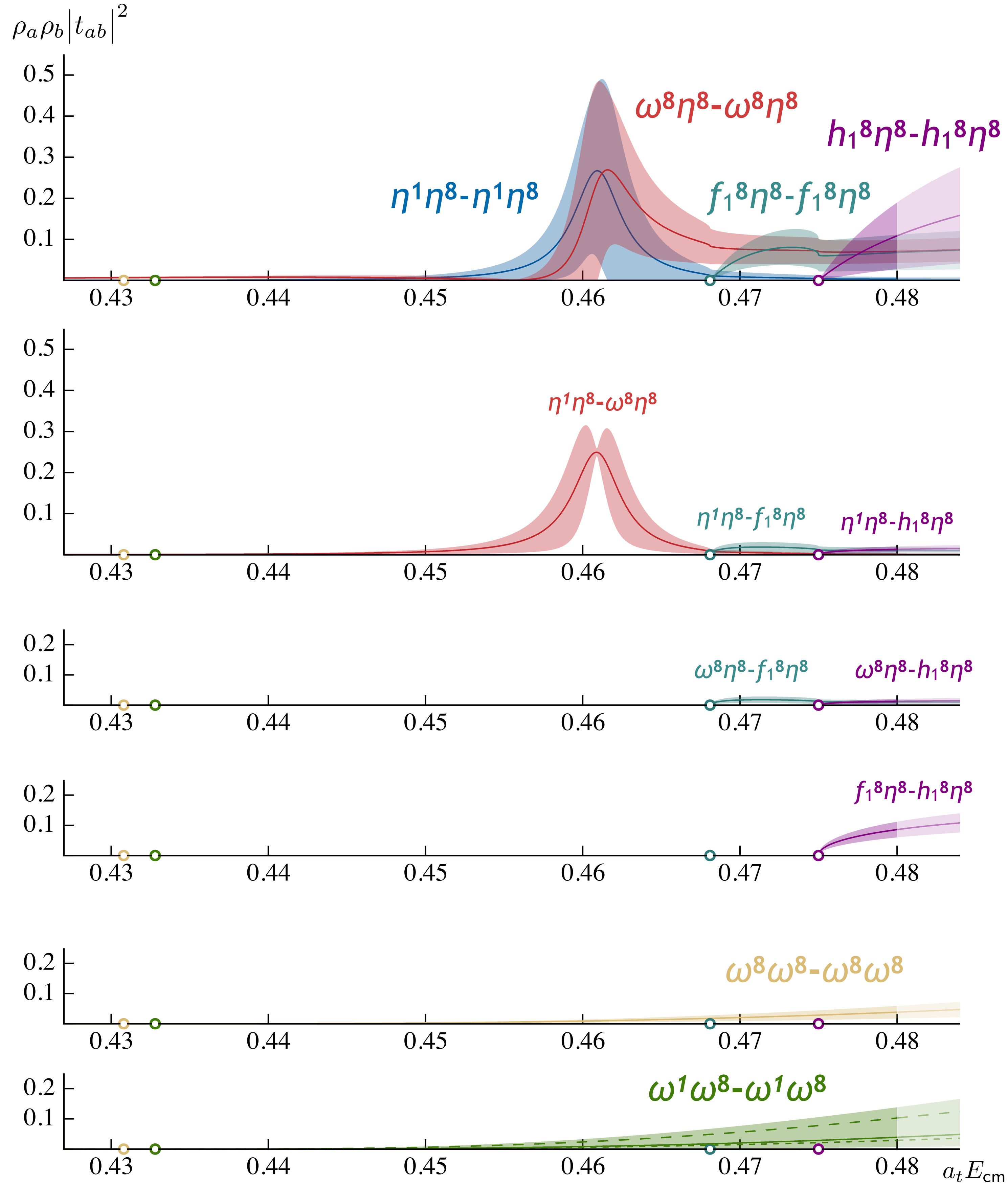
describe scattering by a unitarity-preserving *K*-matrix featuring a pole (11 free parameters)

$\chi^2/N_{\rm dof} = \frac{43.6}{53-11} = 1.04$

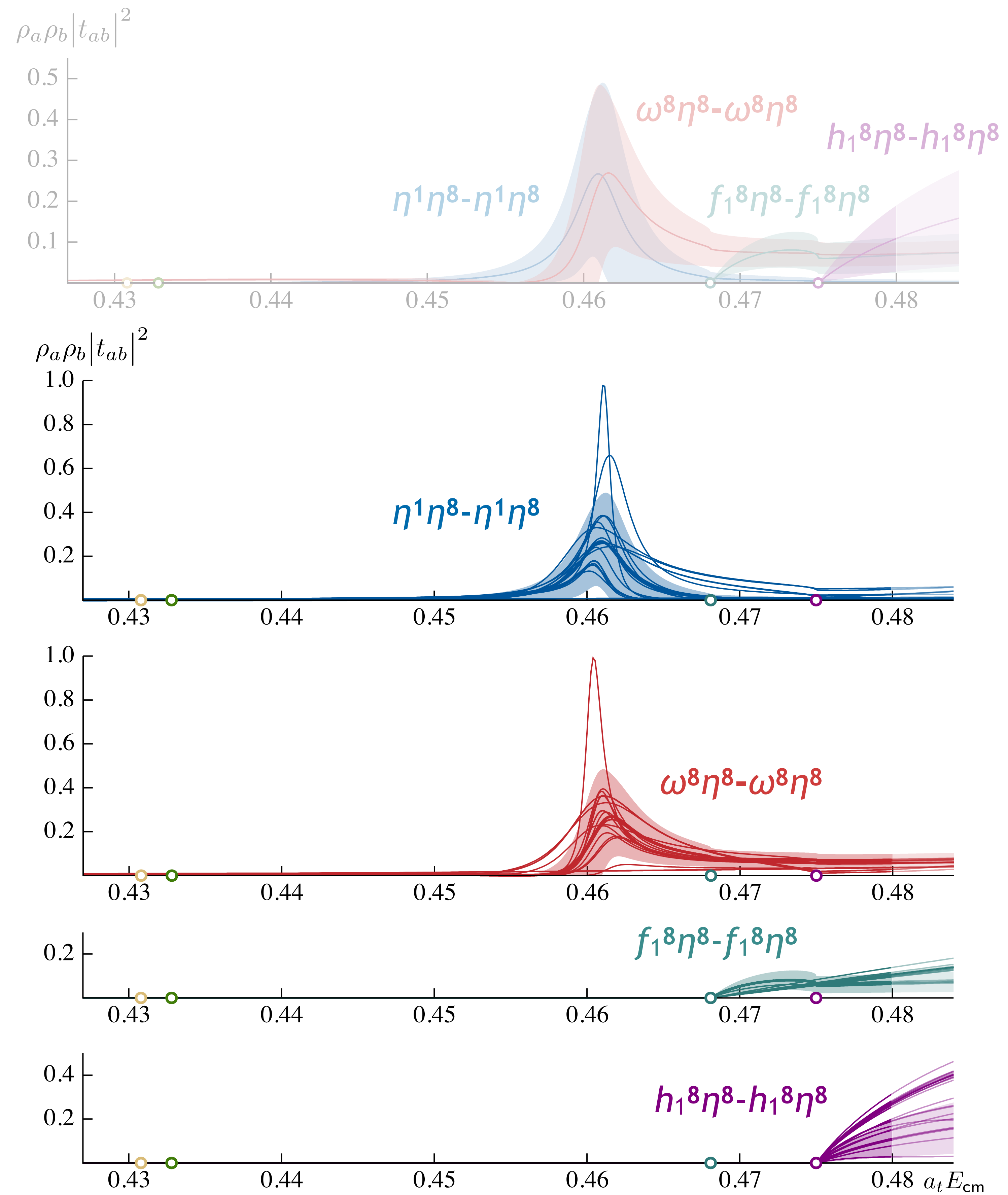


a good description of the spectrum …

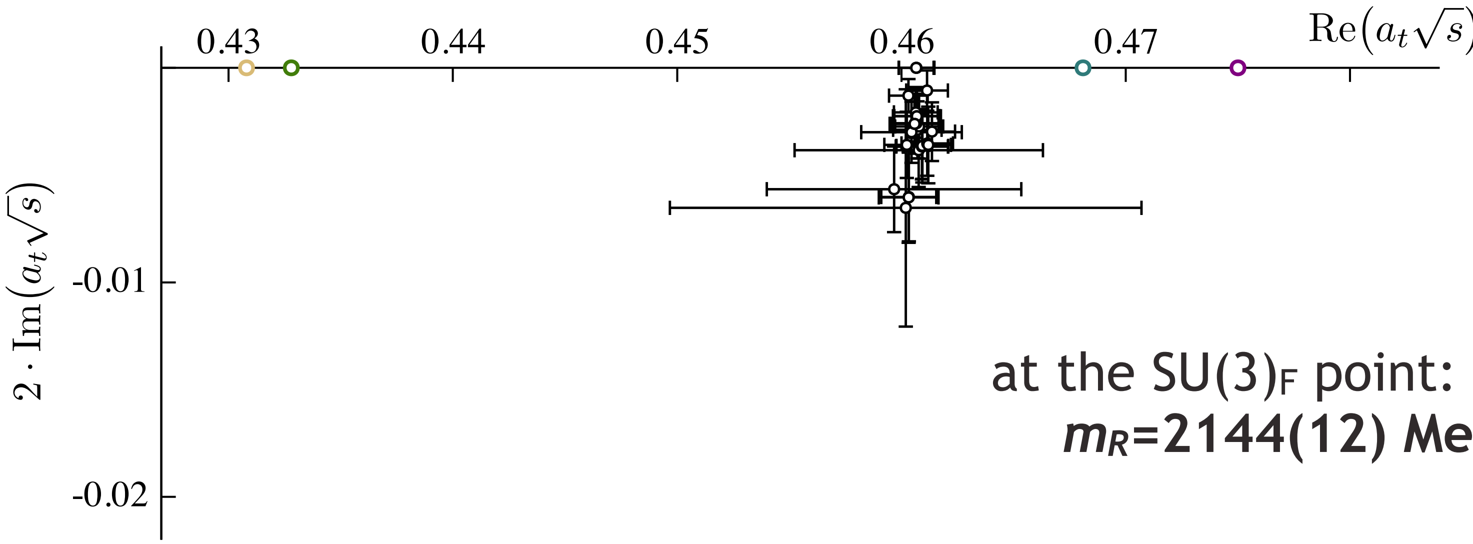
an ‘eight’ channel scattering amplitude



‘eight’ channel scattering amplitudes - varying parameterization



octet 1^{-+} resonance pole & couplings

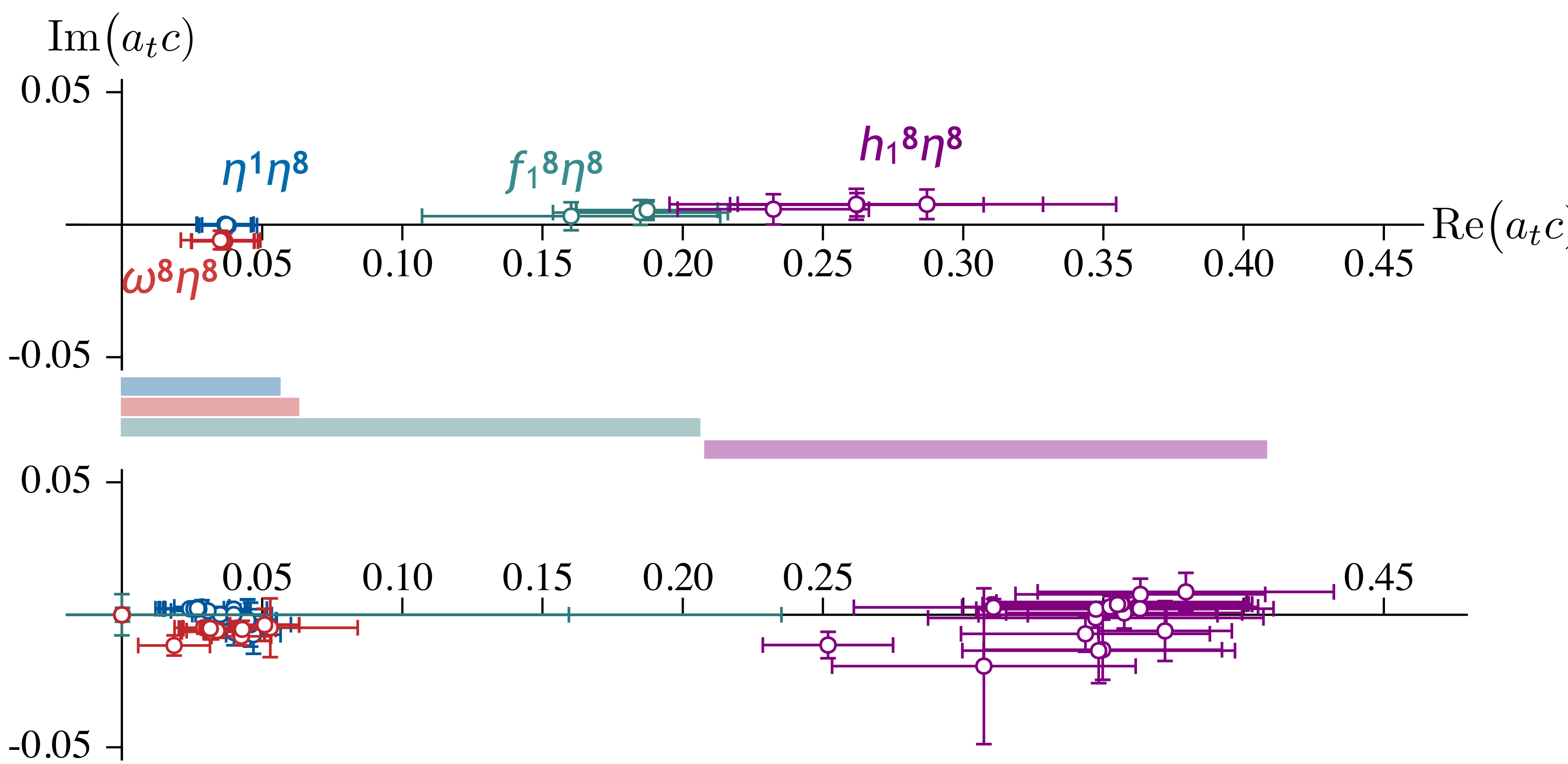


$$t_{ab}(s) \sim \frac{c_a c_b}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\tfrac{1}{2}\Gamma_R$$

at the $SU(3)_F$ point:

***m_R*=2144(12) MeV, *Γ_R*=21(21) MeV** (a narrow resonance)



resonance below $h_1{}^8\eta^8$ threshold, but with a large coupling

crude extrapolation to physical point

core assumption: couplings scale only with the relevant barrier factor $\boldsymbol{k^\ell}$

use PDG masses & COMPASS/JPAC π_1 mass

generates for a π_1 at 1564 MeV:

Γ_{TOT} ~ **140-600 MeV**

$\Gamma(\pi\eta) \lesssim$ 1 MeV

$\Gamma(\pi\eta') \lesssim$ 20 MeV

$\Gamma(\pi\rho) \lesssim$ 12 MeV

$\boldsymbol{\Gamma(\pi b_1)}$ ~ **140-530 MeV**

JPAC/COMPASS candidate:

Γ_{TOT} ~ **492(115) MeV**

Kopf et al analysis:

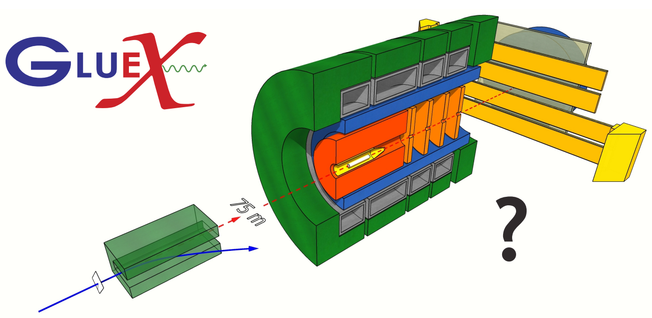
Γ_{TOT} ~ **388(10) MeV**

$\Gamma(\pi\eta') \,/\, \Gamma(\pi\eta)$ ~ 6.5(1)

if correct, suggests prior observations in $\pi\eta$, $\pi\eta'$, $\pi\rho$
are in heavily suppressed decay channels

$\pi b_1 \rightarrow \pi\pi\omega \rightarrow \pi\pi\pi\pi\pi$

GLUEX



first ever calculation of an **exotic hybrid meson** as a **resonance** in QCD

simplified scattering system using exact $SU(3)_F$ and m_π~700 MeV

flavor octet 1^{-+} state appears as a narrow resonance

crude extrapolation to physical kinematics
suggests a **potentially broad resonance**

what about other exotic J^{PC} ?

can we build a phenomenology of hybrid decays starting from QCD ?

challenge of **reducing quark mass** really the challenge of **including three-meson decays**

progress in this direction, see Max Hansen's talk …

extrapolation

$$|c|^{\text{phys}} = \left| \frac{k^{\text{phys}}(m_R^{\text{phys}})}{k(m_R)} \right|^{\ell} |c|.$$

$$\Gamma(R \to i) = \frac{|c_i^{\text{phys}}|^2}{m_R^{\text{phys}}} \cdot \rho_i(m_R^{\text{phys}}).$$

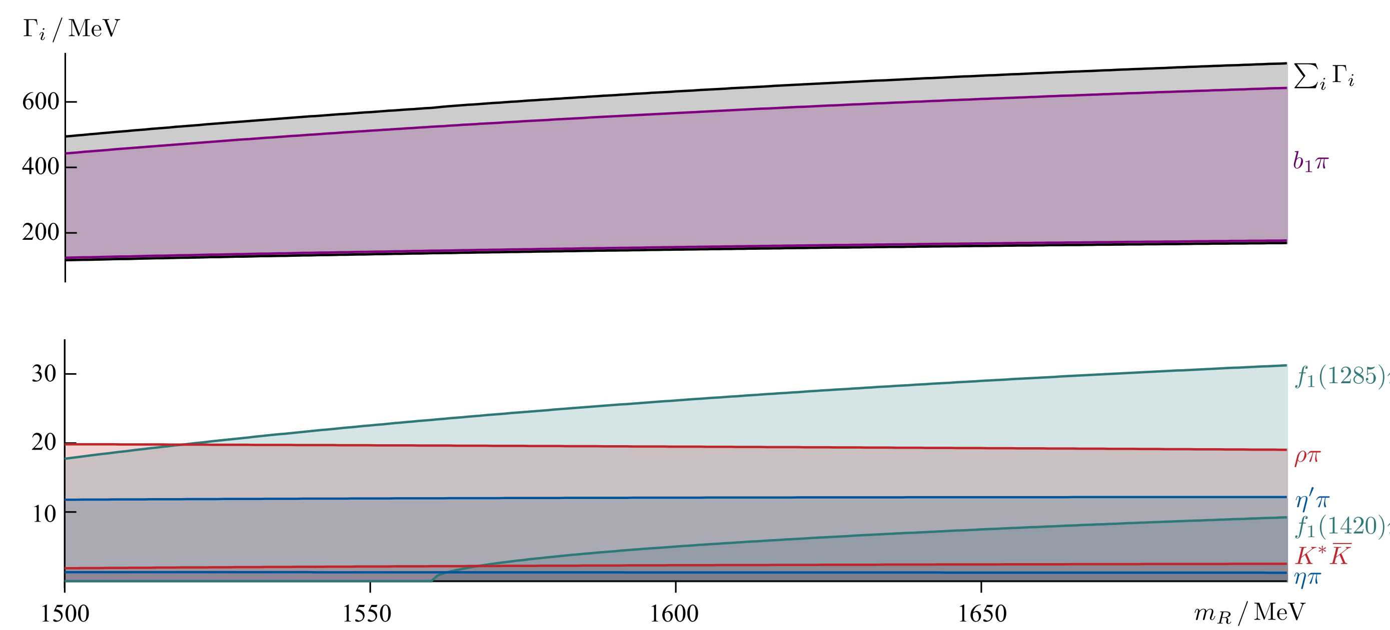
example ‘success’ – f_2, f_2' calculated at m_π~400 MeV

	Scaled	PDG
$\lvert c(f_2 \to \pi\pi)\rvert$	488(28)	453^{+9}_{-4},
$\lvert c(f_2 \to K\bar{K})\rvert$	139(27)	132(7),
$\lvert c(f_2' \to \pi\pi)\rvert$	103(32)	33(4),
$\lvert c(f_2' \to K\bar{K})\rvert$	321(50)	389(12),

$$\frac{1}{\sqrt{3}}(\pi^+\rho^0 - \pi^0\rho^+) + \frac{1}{\sqrt{6}}(K^+\bar{K}^{*0} - \bar{K}^0 K^{*+}),$$

$$-\sqrt{\frac{3}{10}}(K_{1A}^+\bar{K}^0 + \bar{K}_{1A}^0 K^+) + \frac{1}{\sqrt{5}}(a_1^+\eta_8 + (f_1)_8\pi^+),$$

$$\frac{1}{\sqrt{6}}(K_{1B}^+\bar{K}^0 - \bar{K}_{1B}^0 K^+) + \frac{1}{\sqrt{3}}(b_1^+\pi^0 - b_1^0\pi^+),$$

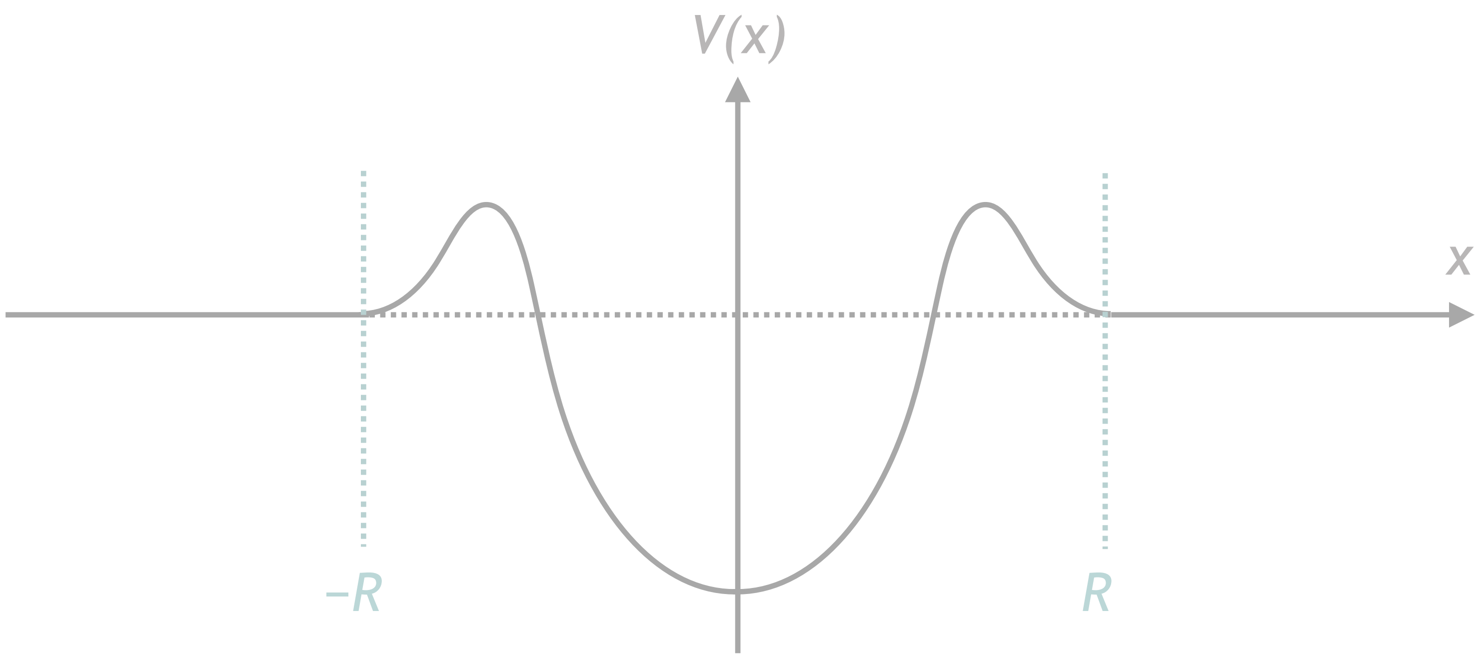


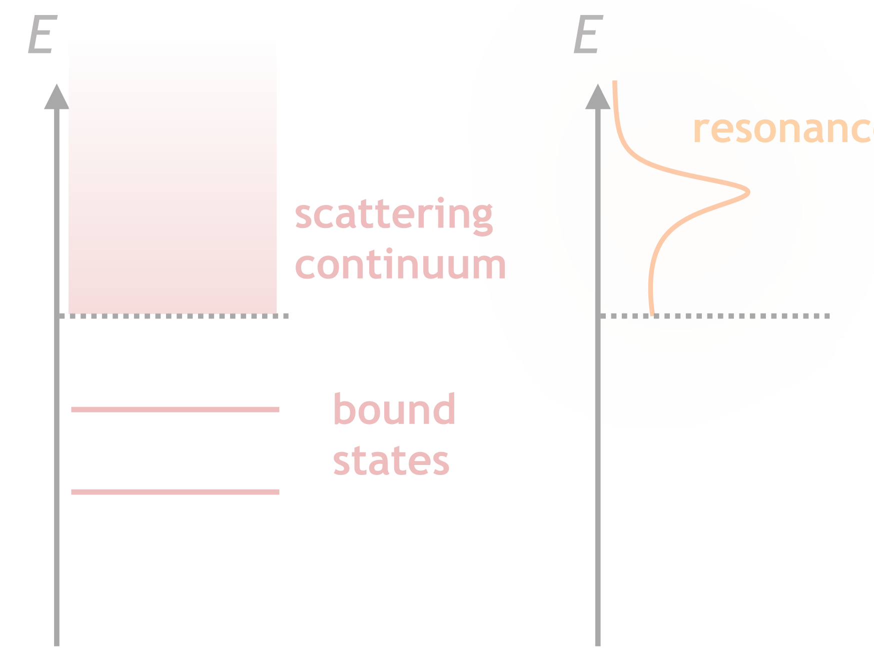
illustrative K-matrix form

$$
\boldsymbol{K}_{\not{V}\not{V}}(s) = \frac{\boldsymbol{g}\boldsymbol{g}^T}{m^2 - s} + \begin{bmatrix} \gamma_{\eta^1\eta^8\{^1P_1\}} & 0 & 0 & 0 \\ 0 & \gamma_{\omega^8\eta^8\{^3P_1\}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
$$

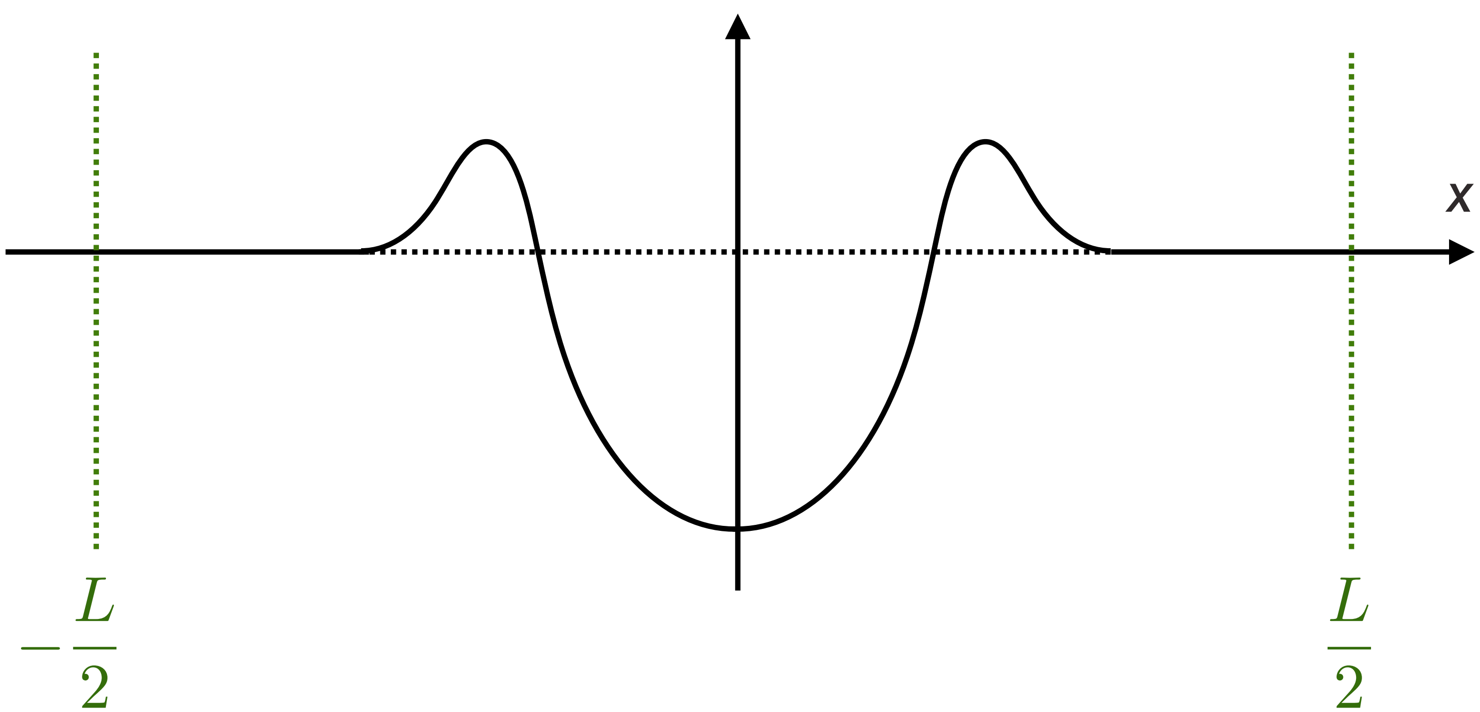
$$
\boldsymbol{g} = \left(g_{\eta^1\eta^8\{^1P_1\}}, g_{\omega^8\eta^8\{^3P_1\}}, g_{f_1^8\eta^8\{^3S_1\}}, g_{h_1^8\eta^8\{^3S_1\}}\right),
$$

$$
\boldsymbol{K}_{VV}(s) = \begin{bmatrix} \gamma_{\omega^8\omega^8\{^3P_1\}} & 0 & 0 & 0 \\ 0 & \gamma_{\omega^1\omega^8\{^1P_1\}} & 0 & 0 \\ 0 & 0 & \gamma_{\omega^1\omega^8\{^3P_1\}} & 0 \\ 0 & 0 & 0 & \gamma_{\omega^1\omega^8\{^5P_1\}} \end{bmatrix}.
$$

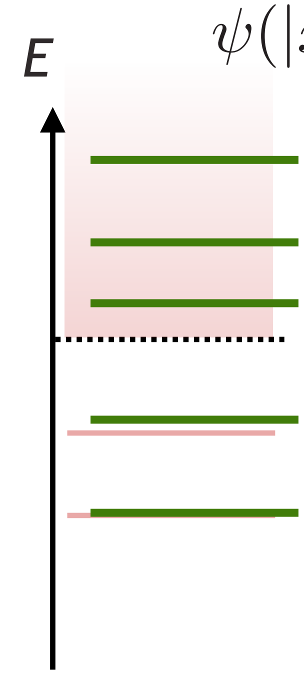




but in a periodic volume …



$$\psi(|x|>R) \sim \cos\left(p|x| + \delta(p)\right)$$



applying the boundary conditions

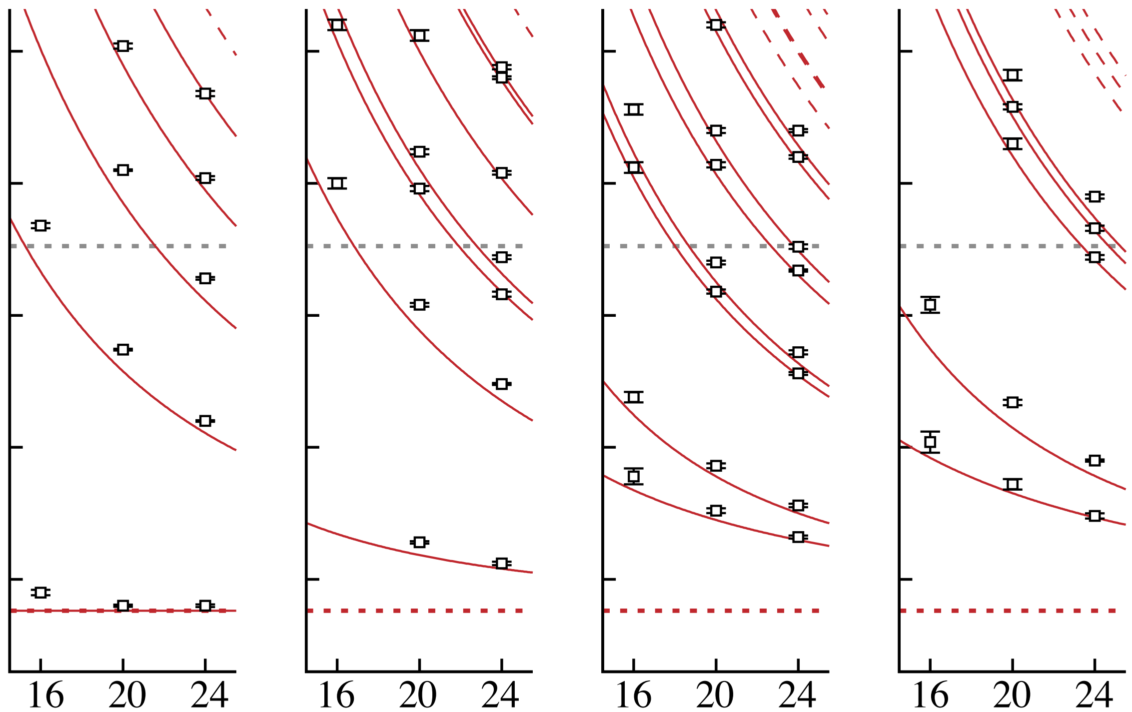
$$p = \frac{2\pi}{L} n - \frac{2}{L}\delta(p)$$

solved by **discrete** $p_n(L)$

non-resonant elastic scattering — $\pi^+\pi^+$ (isospin=2)

m_π ~ 391 MeV

[000] [100] [110] [111]

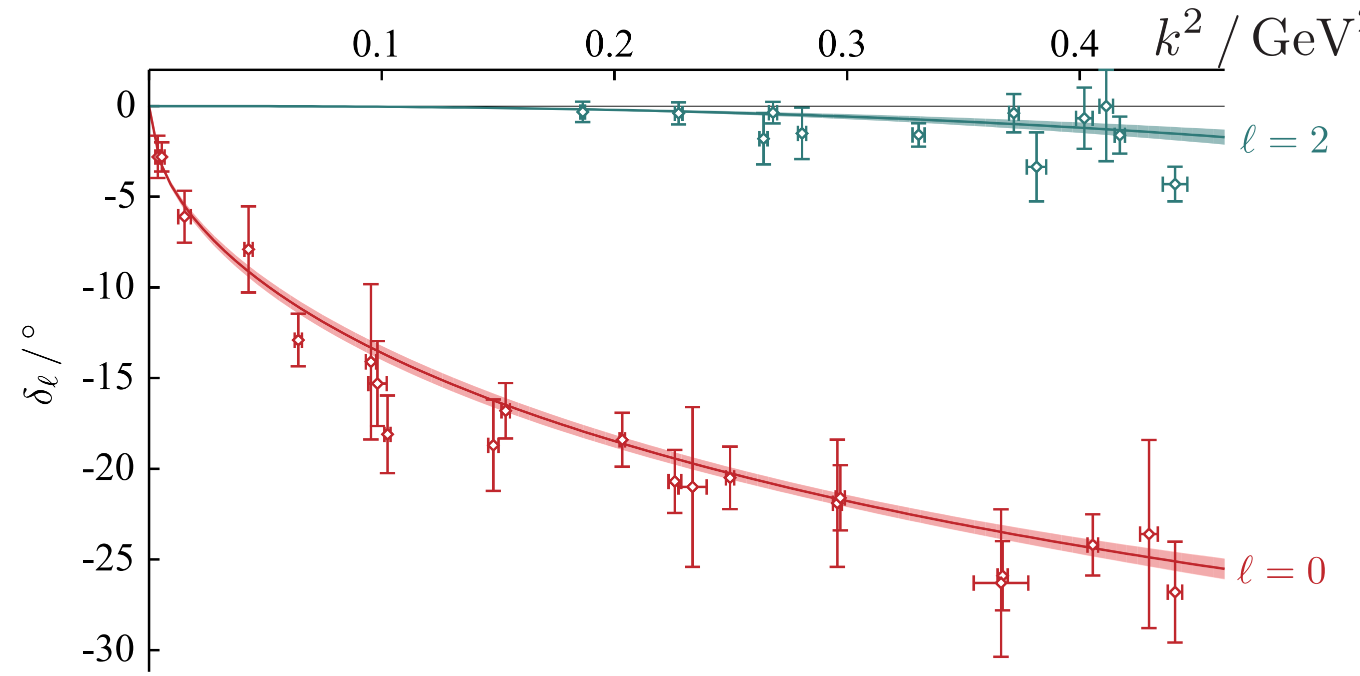


1.9 fm 2.4 fm 2.9 fm

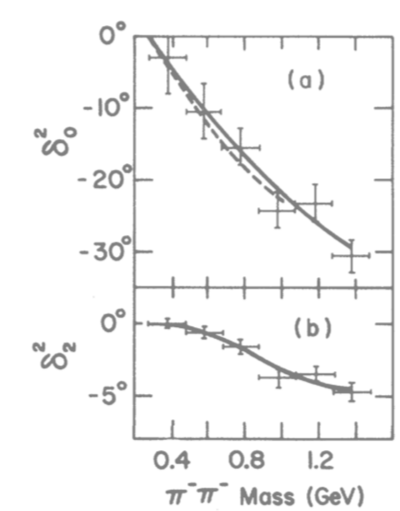
$m_\pi L$ 4 5 6

increasing total momentum

scattering phase-shifts



repulsive scattering, no resonances

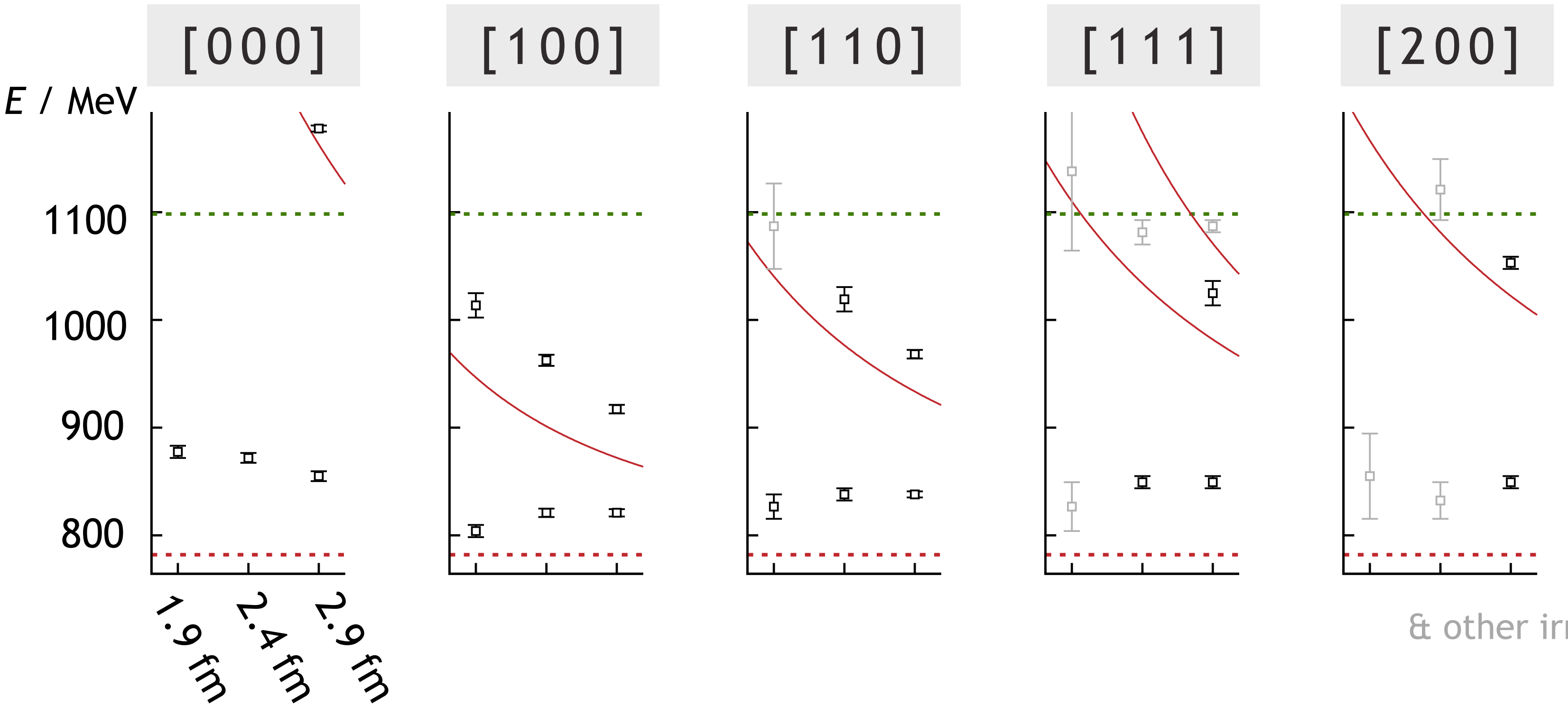


Cohen 1972

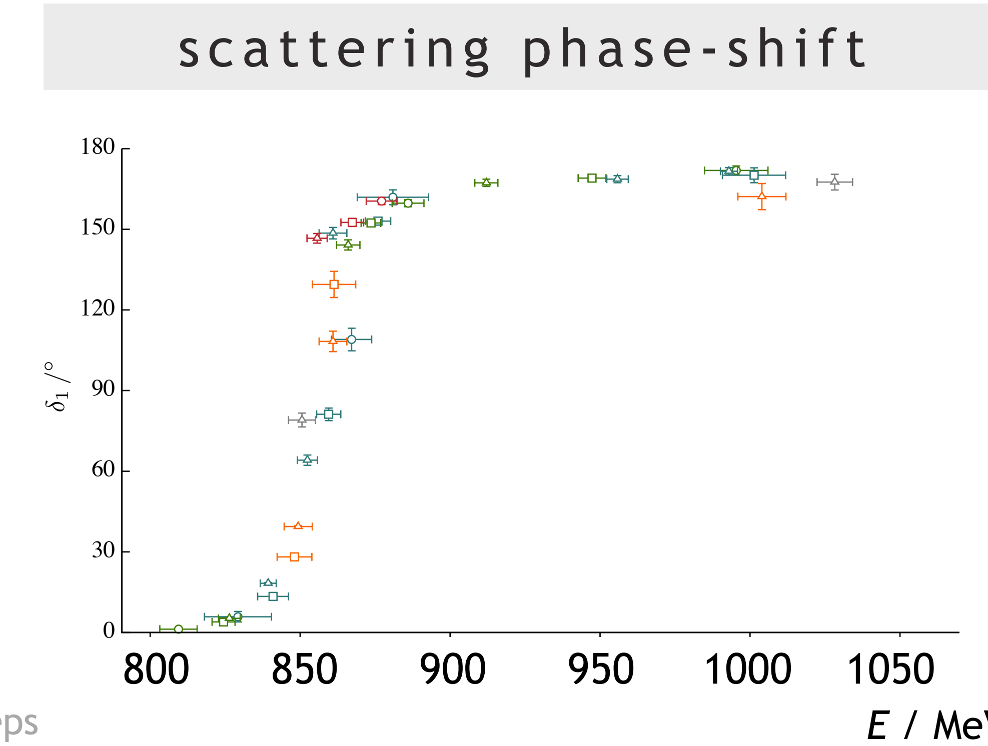
an elastic resonance — the ρ in $\pi\pi$ (isospin=1)

PRD87 034505 (2013)

m_π ~ 391 MeV

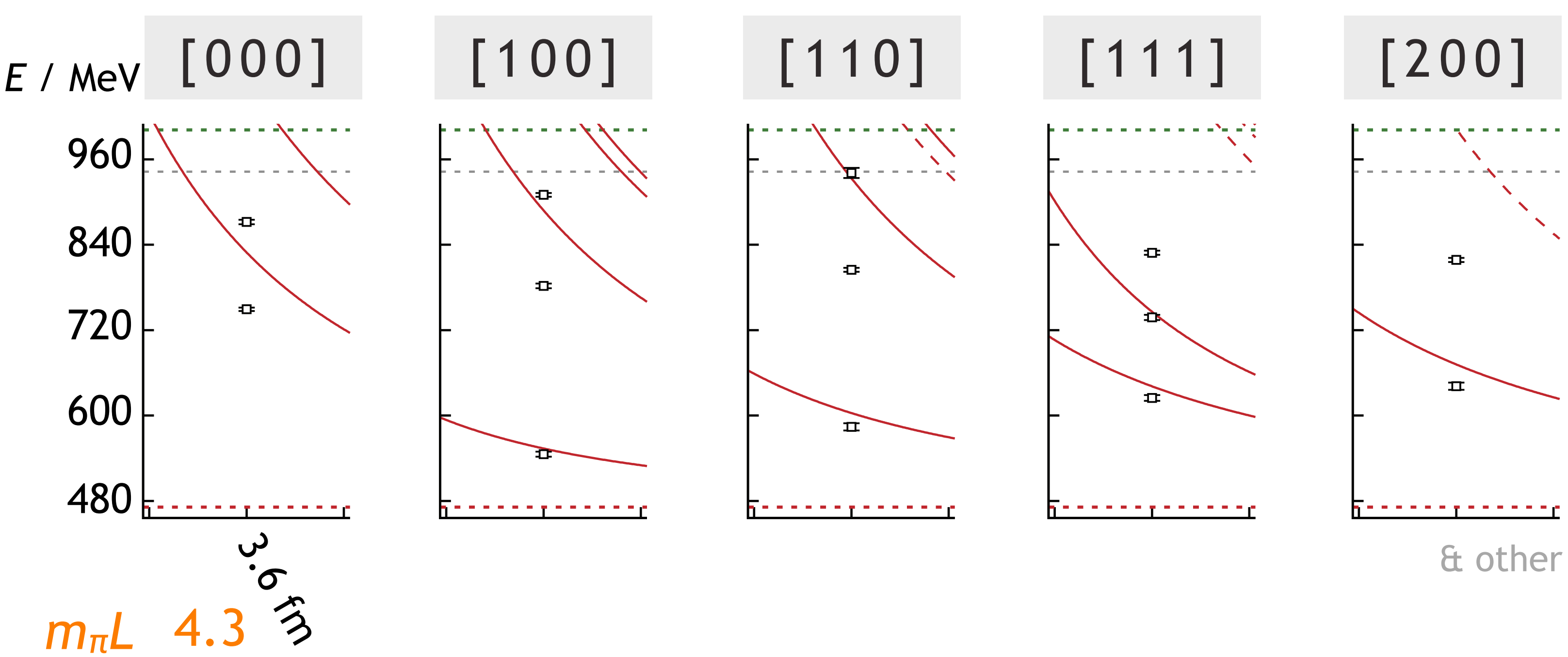


& other irreps

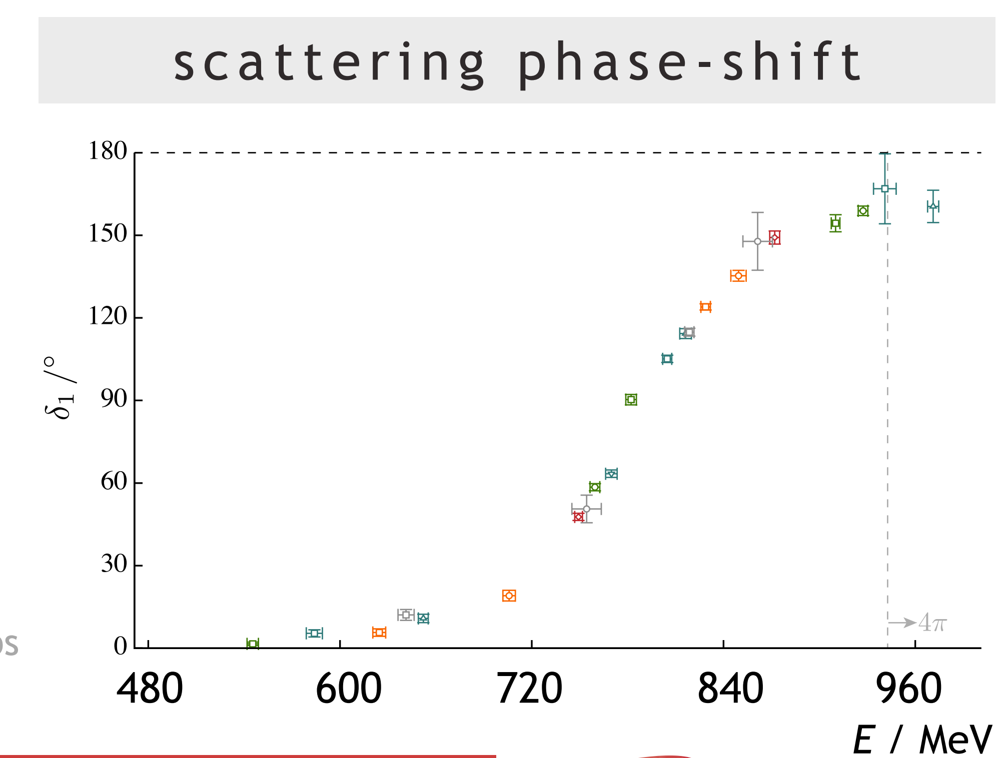


PRD92 094502 (2015)

m_π ~ 236 MeV



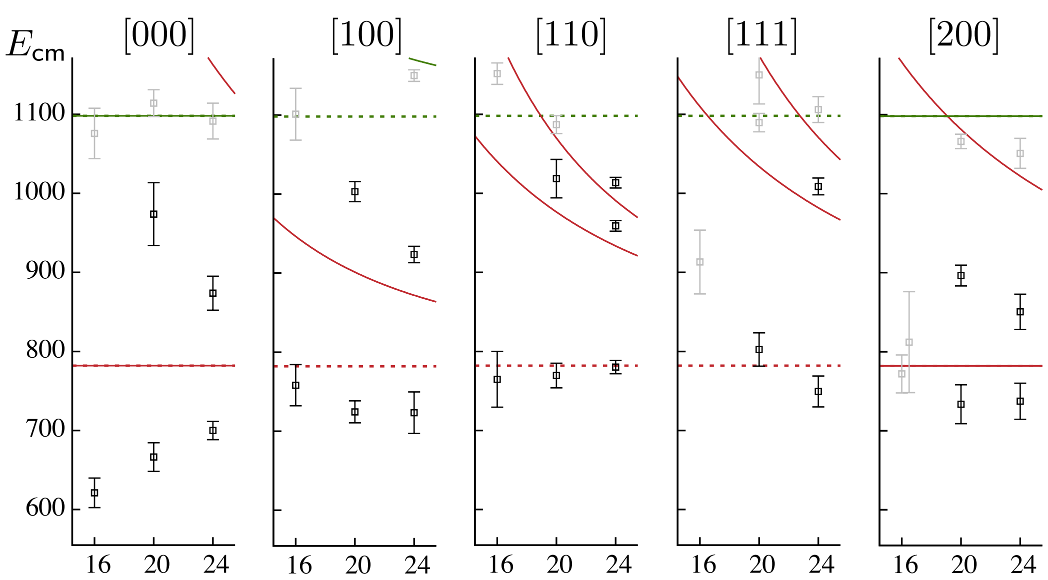
& other irreps



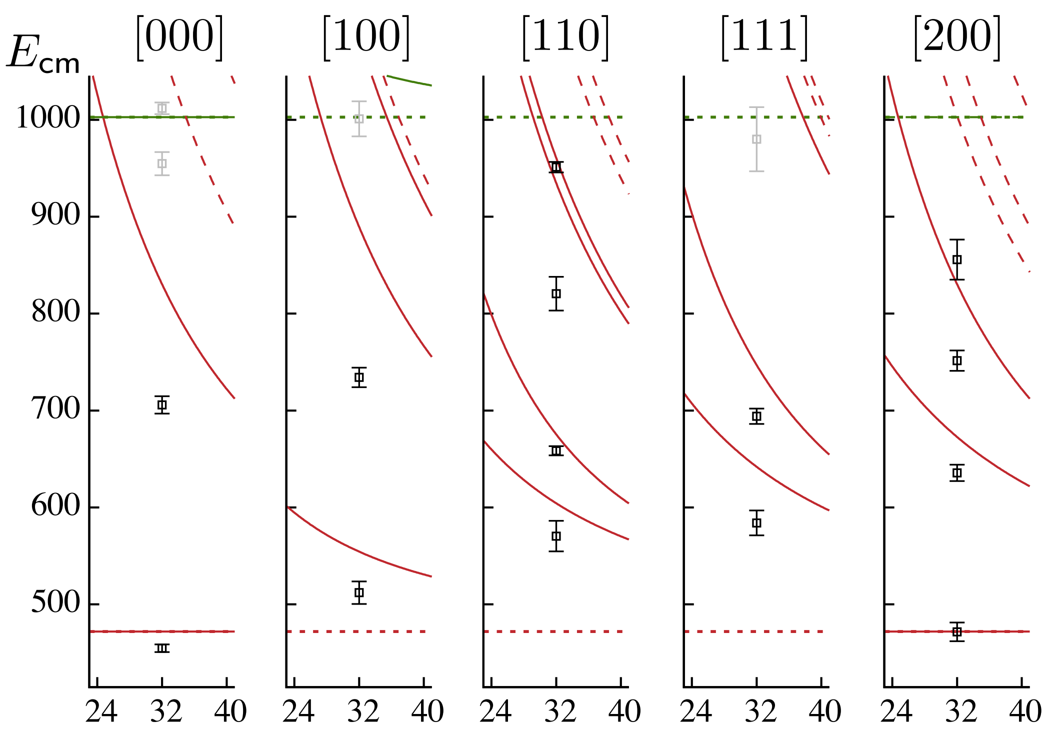
$m_\pi L$ 4.3

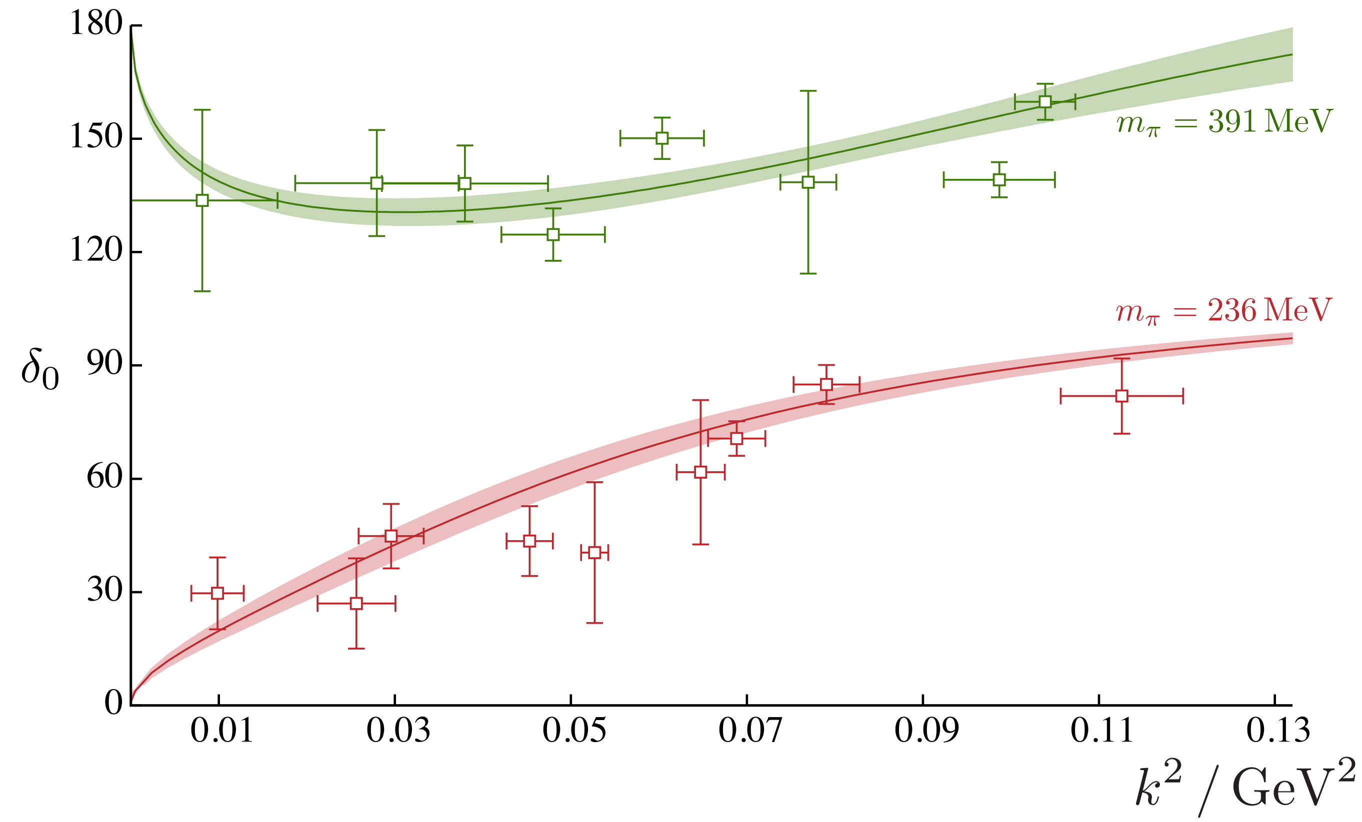
ππ isospin=0

m_π~391 MeV



m_π~236 MeV





heavier quark mass — **a bound-state**

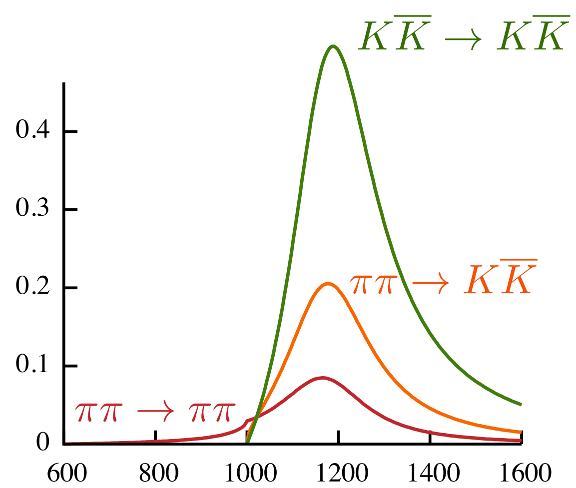
lighter quark mass — attraction, maybe a **broad resonance** ?

c.f. the experimental σ resonance …

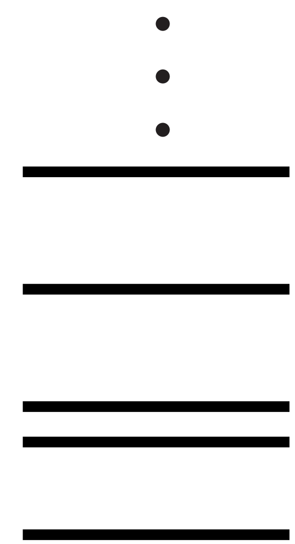
in the case of **coupled-channel scattering** it’s more challenging …

e.g. some energy region where $\pi\pi$, $K\bar{K}$ accessible

$$\mathbf{S}(E) = \begin{bmatrix} S_{\pi\pi,\pi\pi}(E) & S_{\pi\pi,K\bar{K}}(E) \\ S_{\pi\pi,K\bar{K}}(E) & S_{K\bar{K},K\bar{K}}(E) \end{bmatrix} \xrightarrow{L\times L\times L} E_n(L)$$



you want to know this



lattice QCD gives you this

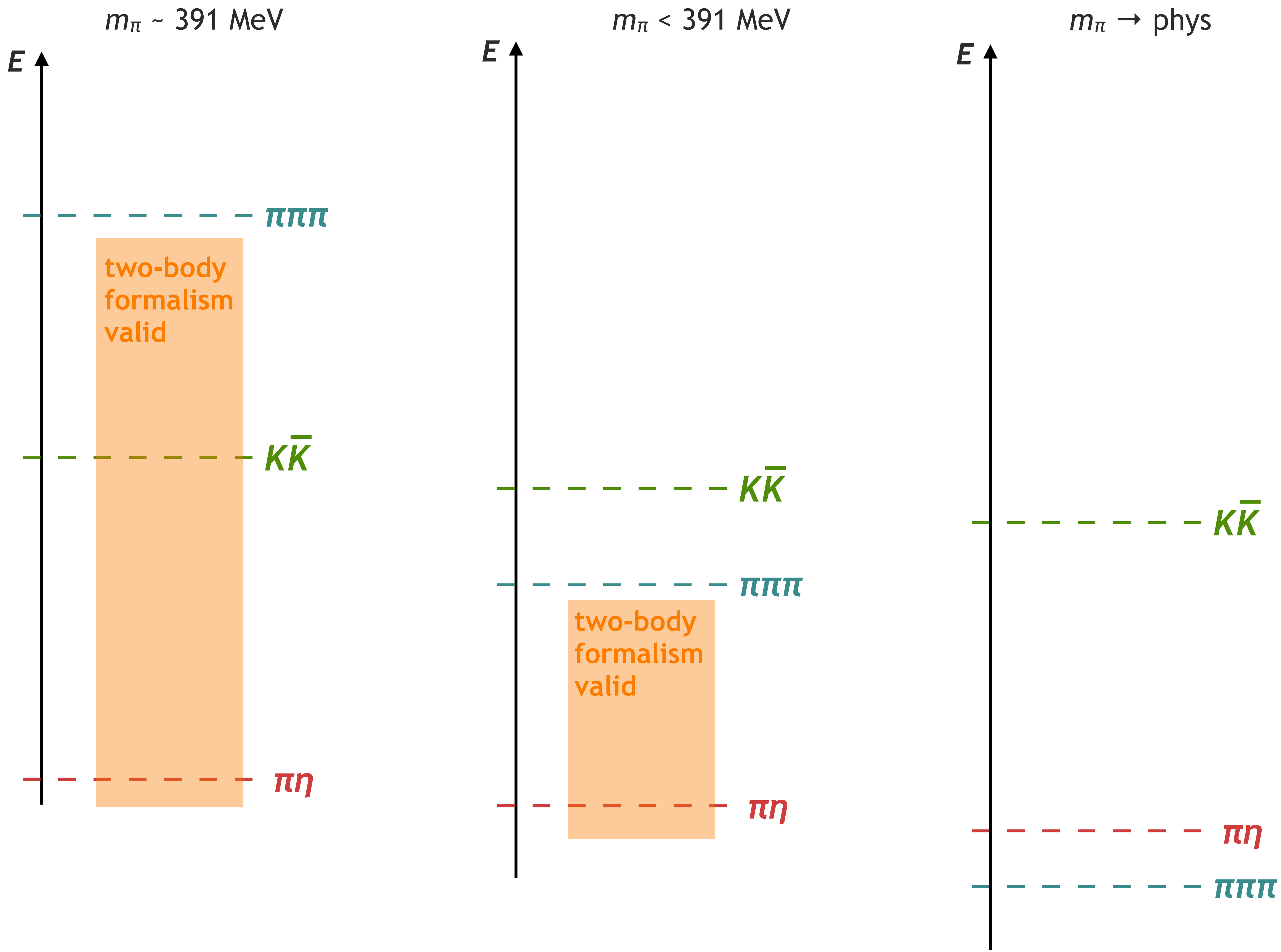
?

an approach:

parameterize the energy dependence of the scattering matrix

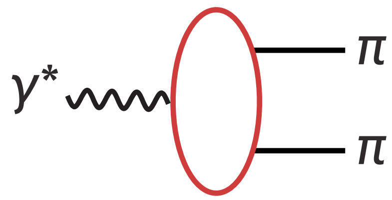
an important observation
— not like experiment
— can’t study ‘channel-by-channel’
— all channels contribute, have to solve the ‘whole problem’

physical pion masses = low-lying multipion channels



coupling resonances to currents

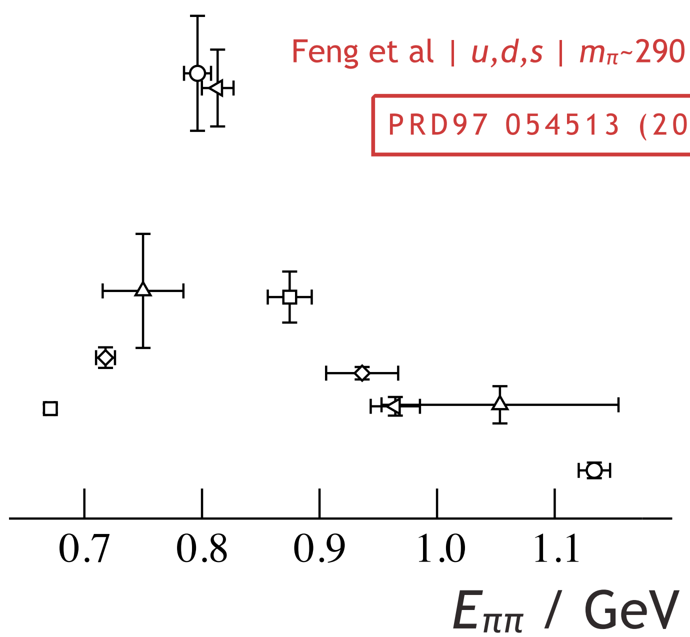
$\gamma^* \to \pi\pi$



$F_\pi(E_{\pi\pi})$

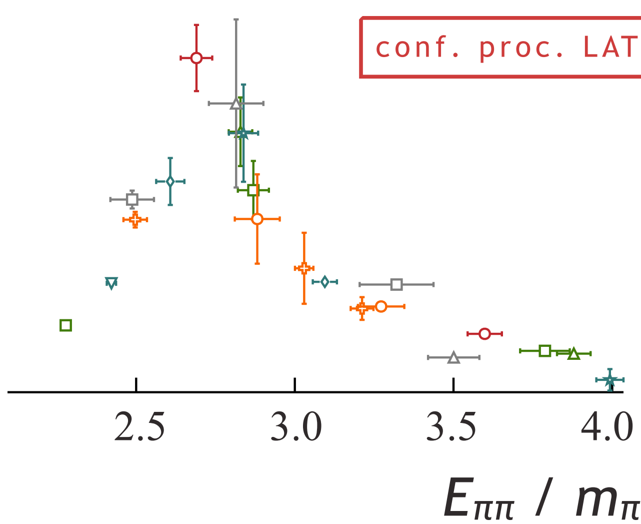
Feng et al | *u,d,s* | m_π~290 MeV

PRD97 054513 (2018)

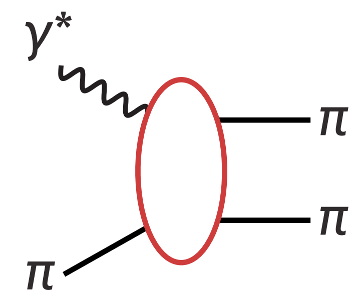


Bulava et al | *u,d,s* | m_π~280 MeV

conf. proc. LAT '15



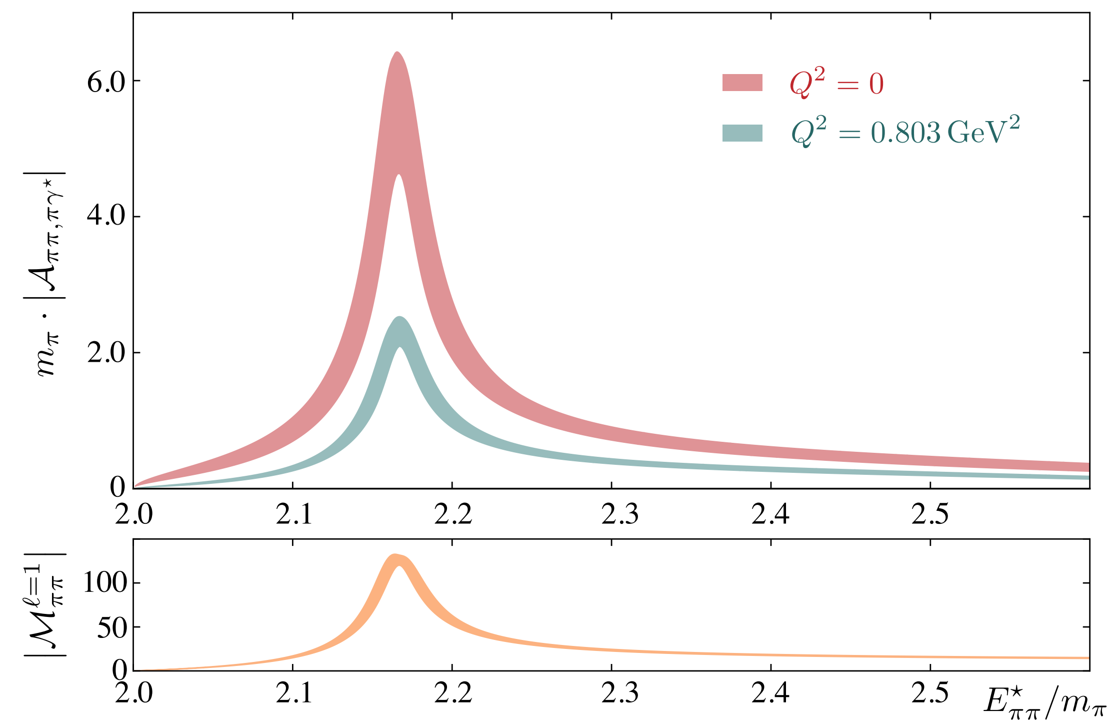
$\gamma^* \pi \to \pi\pi$

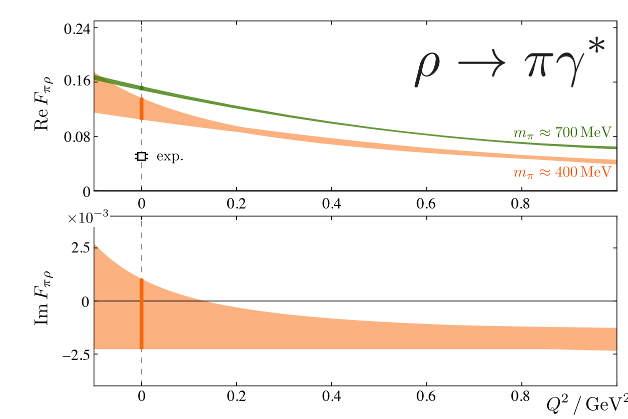


$\mathcal{A}(E_{\pi\pi}, Q^2)$

Briceno et al | *u,d,s* | m_π~391 MeV

PRL115 242001 (2015)





what do you calculate

calculate correlation functions

e.g. $\langle 0 | \mathcal{O}_i(t) \mathcal{O}_j(0) | 0 \rangle$ where the operators are constructed from quark and gluon fields and have the quantum numbers of the hadronic system you want to study

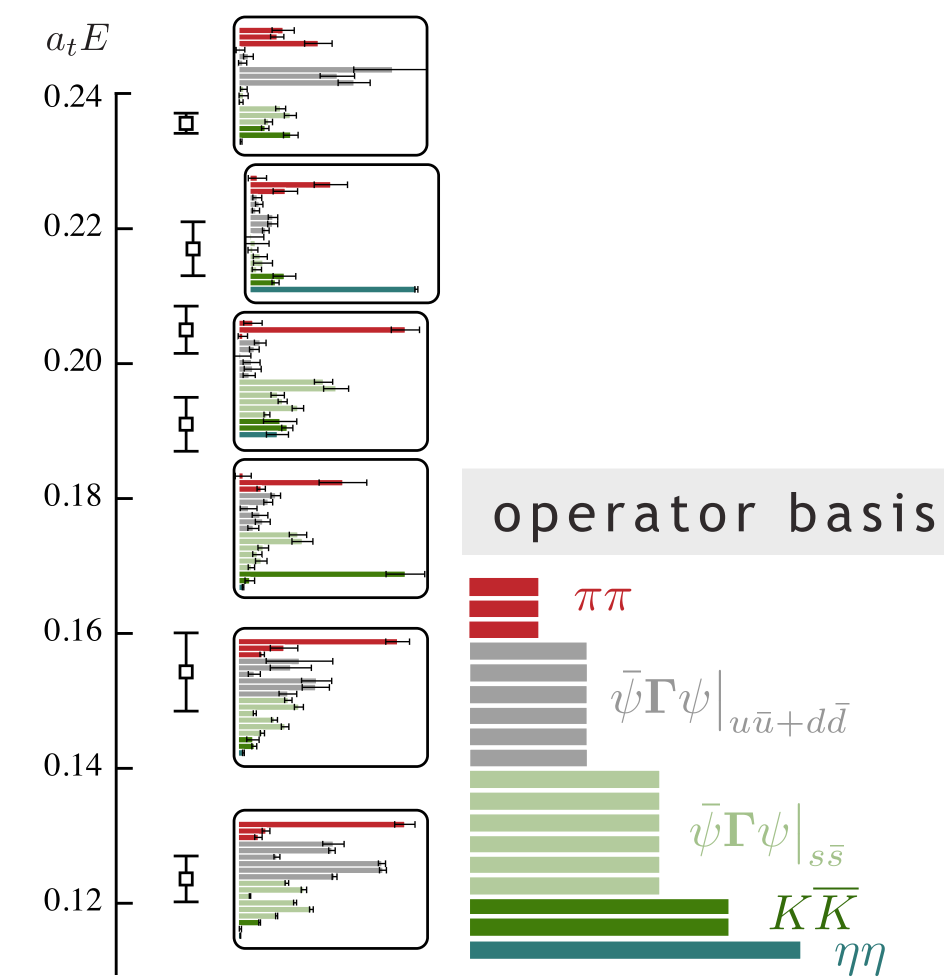
$$\langle 0 | \mathcal{O}_i(t) \mathcal{O}_j(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle \langle n | \mathcal{O}_j | 0 \rangle \, e^{-E_n t}$$

a superposition of the (finite-volume) eigenstates of QCD

powerful approach:

- use a large basis of operators*
- form a matrix of correlation functions
- diagonalize this matrix

e.g. [000] A_1^+ 24^3



* could give a whole interesting talk on the construction of these operators

operator basis — I=0 $\pi\pi$, $K\bar{K}$, $\eta\eta$

operator basis: ‘single-meson’ + ‘meson-meson’

$$\bar{\psi}\boldsymbol{\Gamma}\psi$$

(& if you like,
tetraquark & …)

$$\sum_{\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2} C(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p})\, M_1(\mathbf{p}_1)\, M_2(\mathbf{p}_2)$$

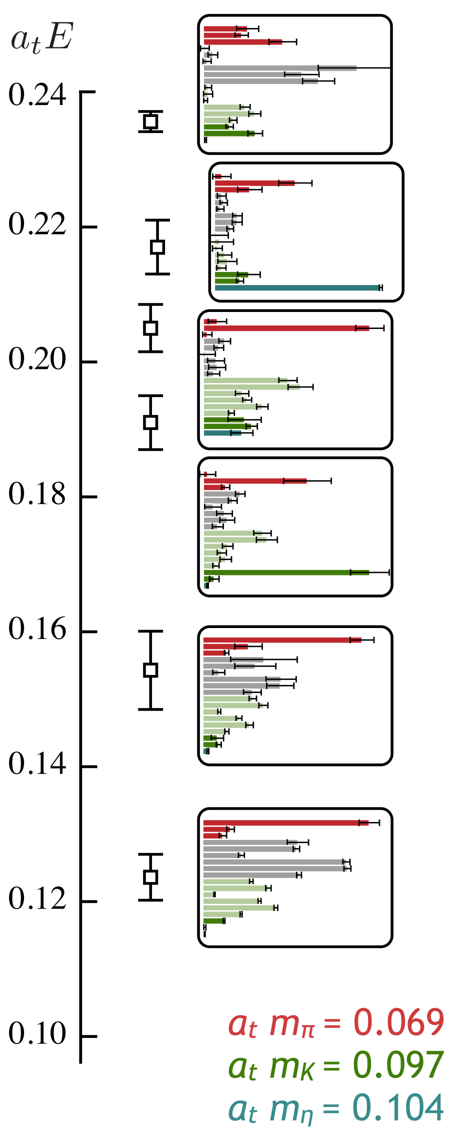
$$\mathbf{p} = \frac{2\pi}{L}[n_x, n_y, n_z]$$

maximum momentum guided by non-interacting energies

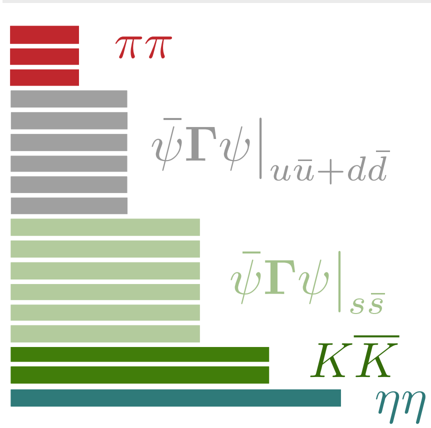
$$\sqrt{m_1^2+\mathbf{p}_1^2}+\sqrt{m_2^2+\mathbf{p}_2^2}$$

solutions of the det equation when **t** = **0**

[000] A_1^+ 24^3



operator basis



operator basis

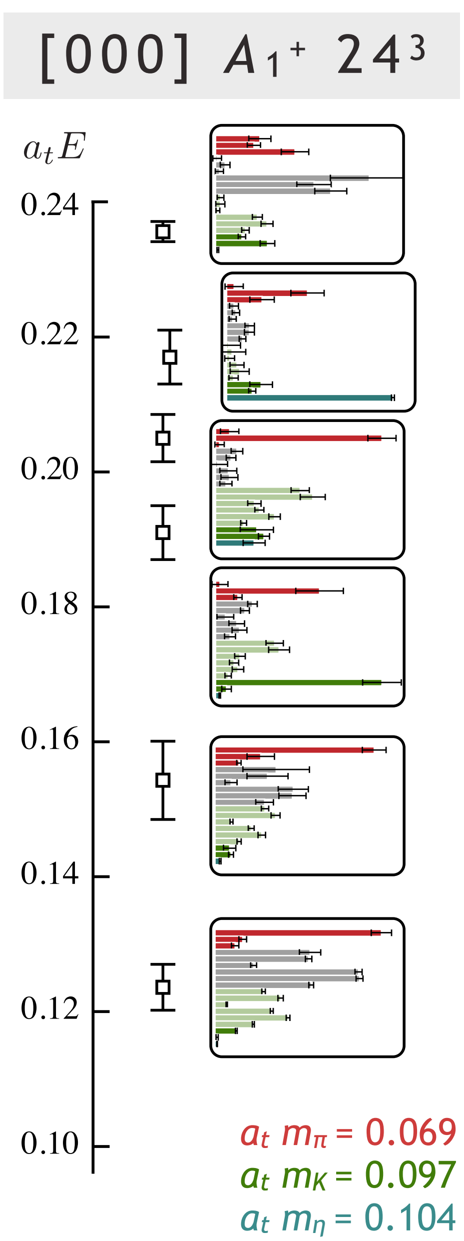
operator basis: ‘single-meson’ + ‘meson-meson’

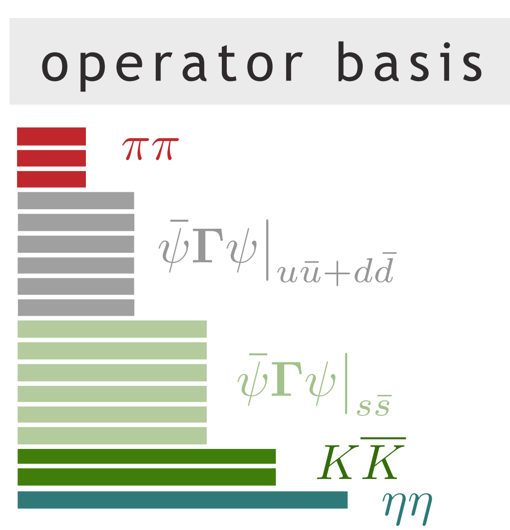
$$\bar{\psi}\mathbf{\Gamma}\psi$$

$$\sum_{\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2} C(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p})\, M_1(\mathbf{p}_1)\, M_2(\mathbf{p}_2)$$

$$\sum_{\mathbf{x}} e^{i\mathbf{p}_1\cdot\mathbf{x}}\,\bar{\psi}_{\mathbf{x}}\mathbf{\Gamma}\psi_{\mathbf{x}}\;\sum_{\mathbf{y}} e^{i\mathbf{p}_2\cdot\mathbf{y}}\,\bar{\psi}_{\mathbf{y}}\mathbf{\Gamma}'\psi_{\mathbf{y}}$$

sampling the whole lattice volume





prefer to use

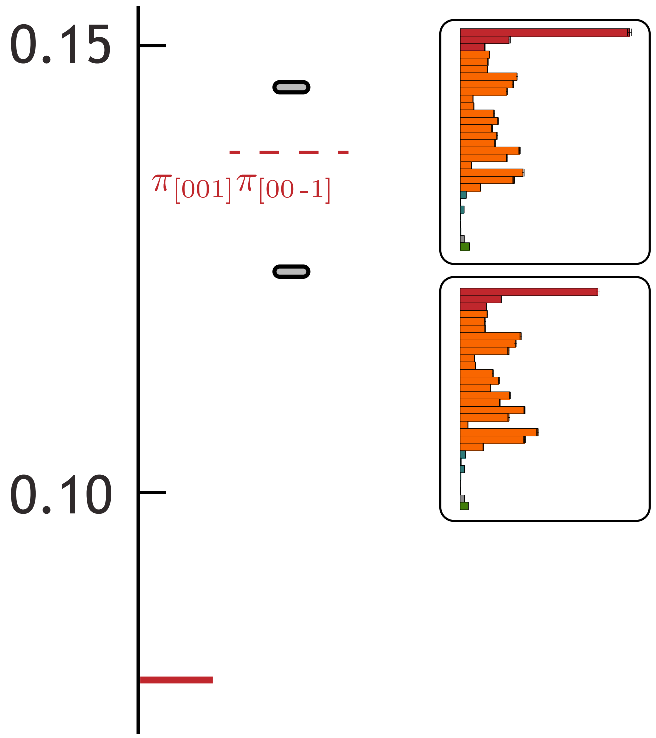
optimized single-meson operators …

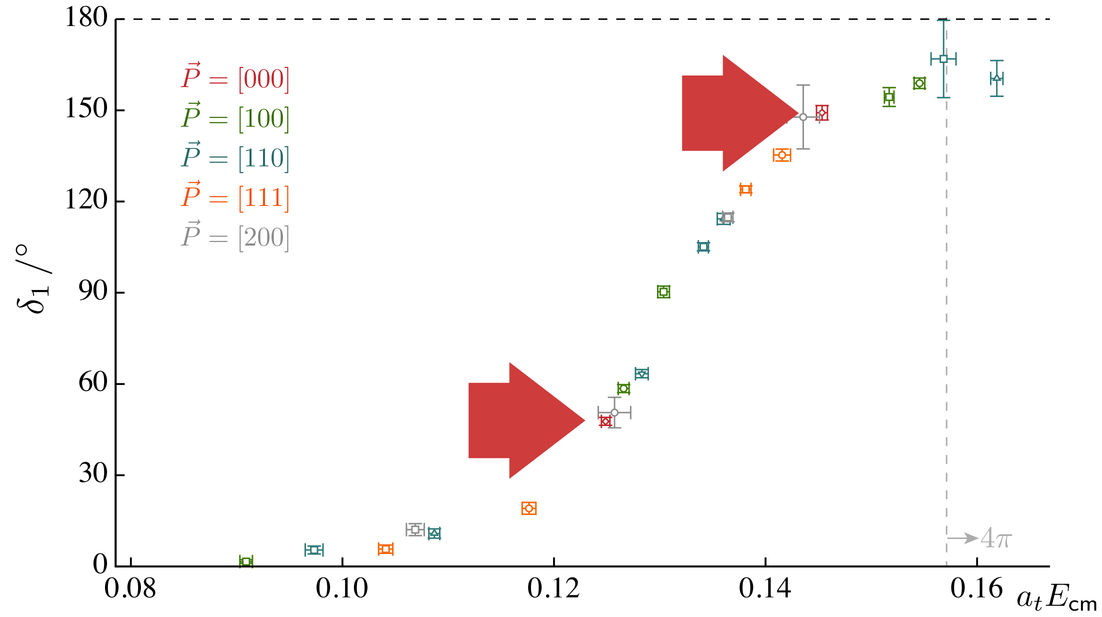
‘dropping’ ops in $\rho \to \pi\pi$

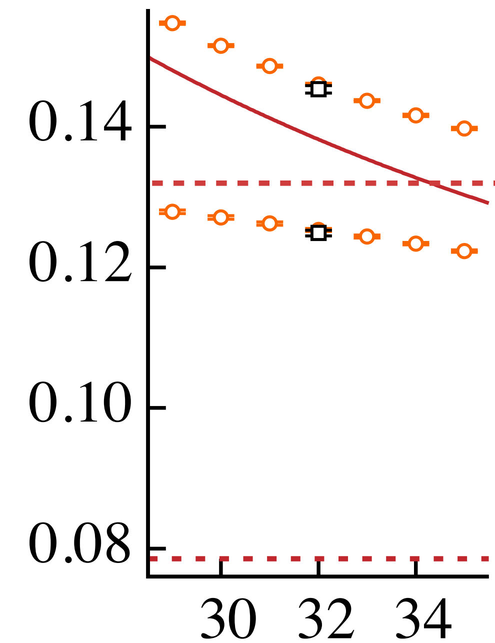
$m_\pi = 0.039$
$m_K = 0.083$
$L \sim 3.8$ fm

what's happening here ?

focus on the lowest two states





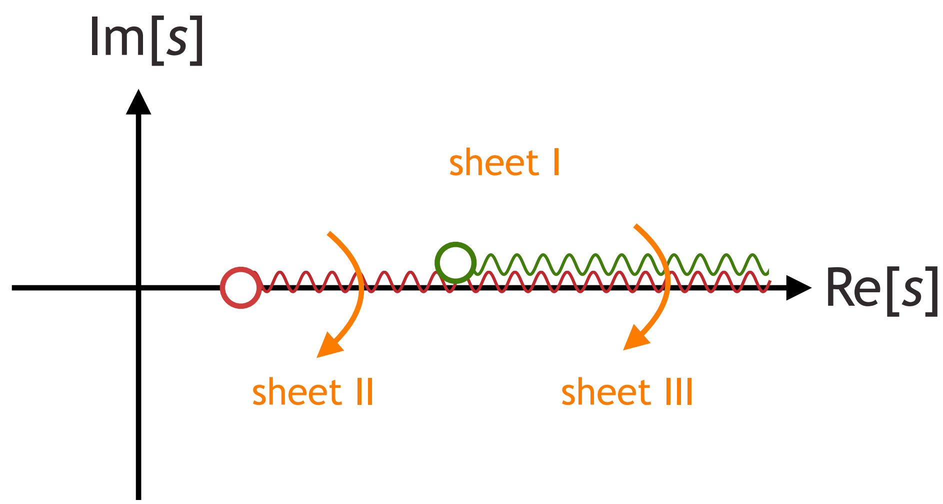


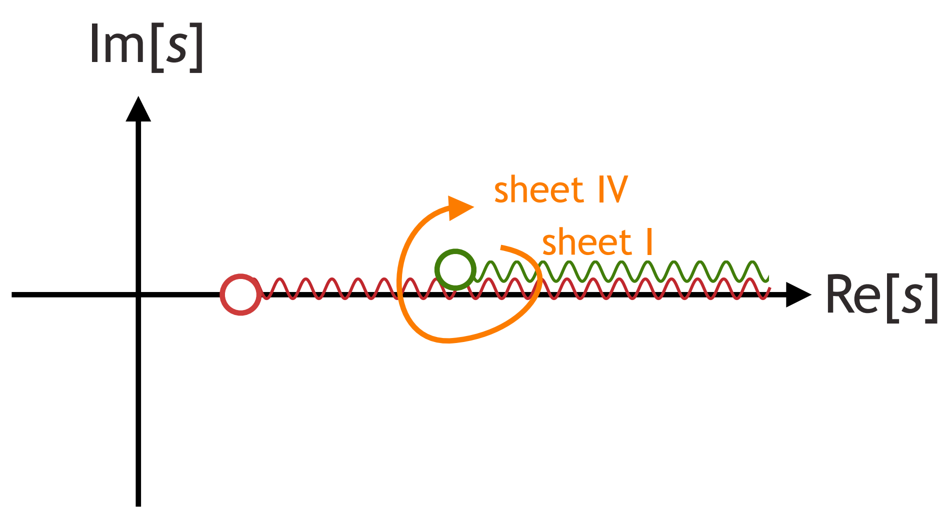
an avoided level crossing

coupled-channel Riemann sheet structure

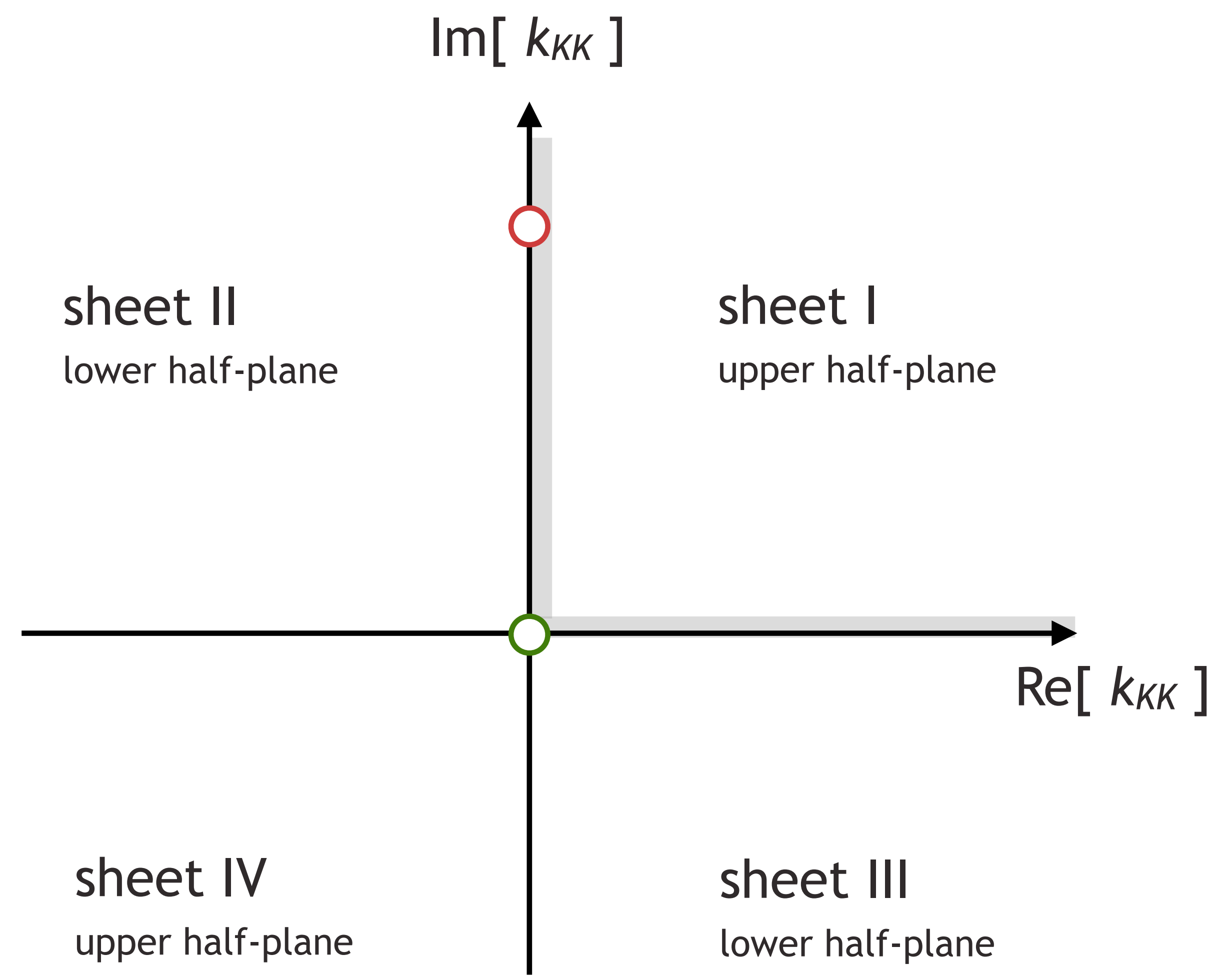
for each new channel, each sheet splits in two ⇒ 2^N sheets for N channels

e.g. two channels

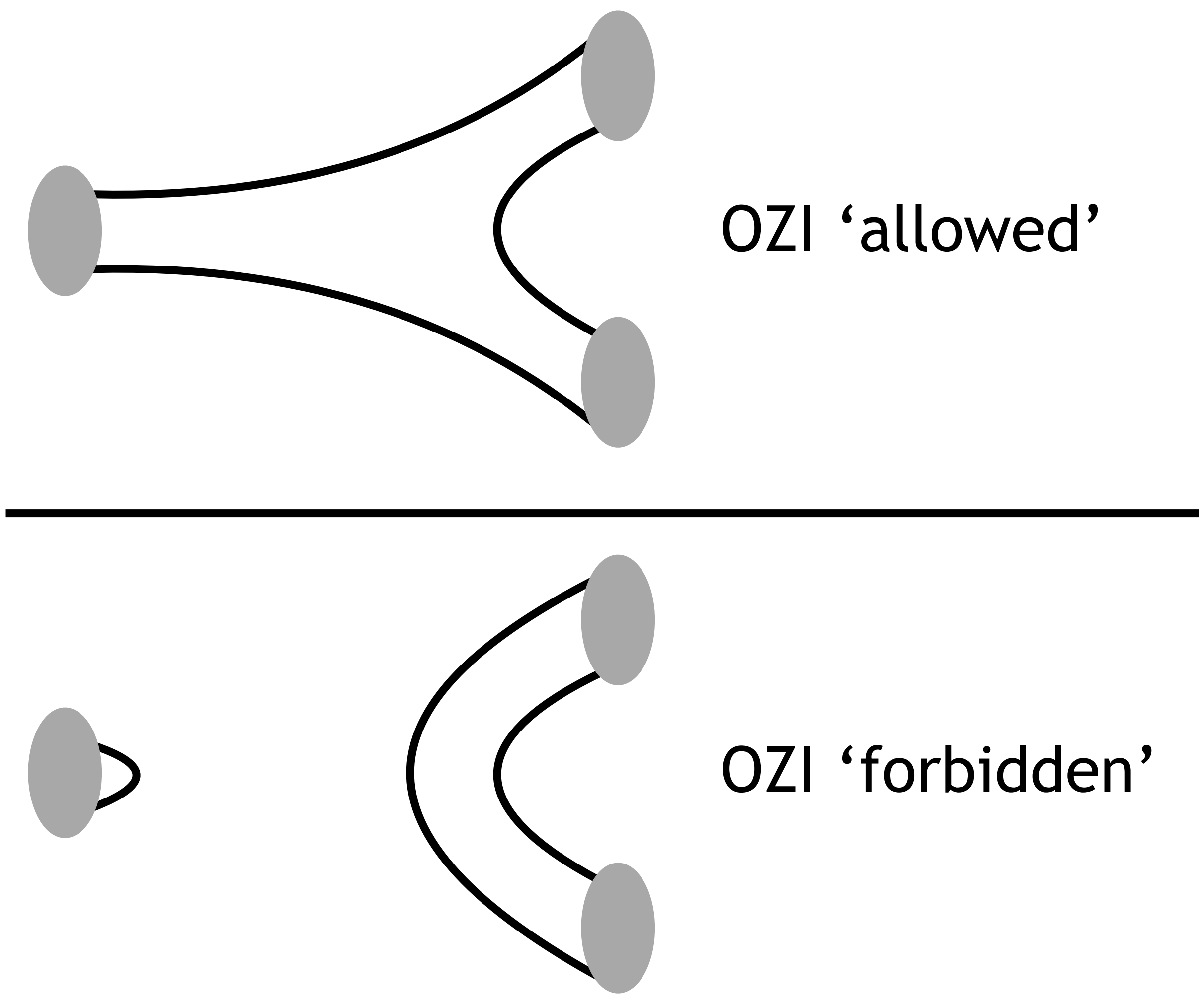


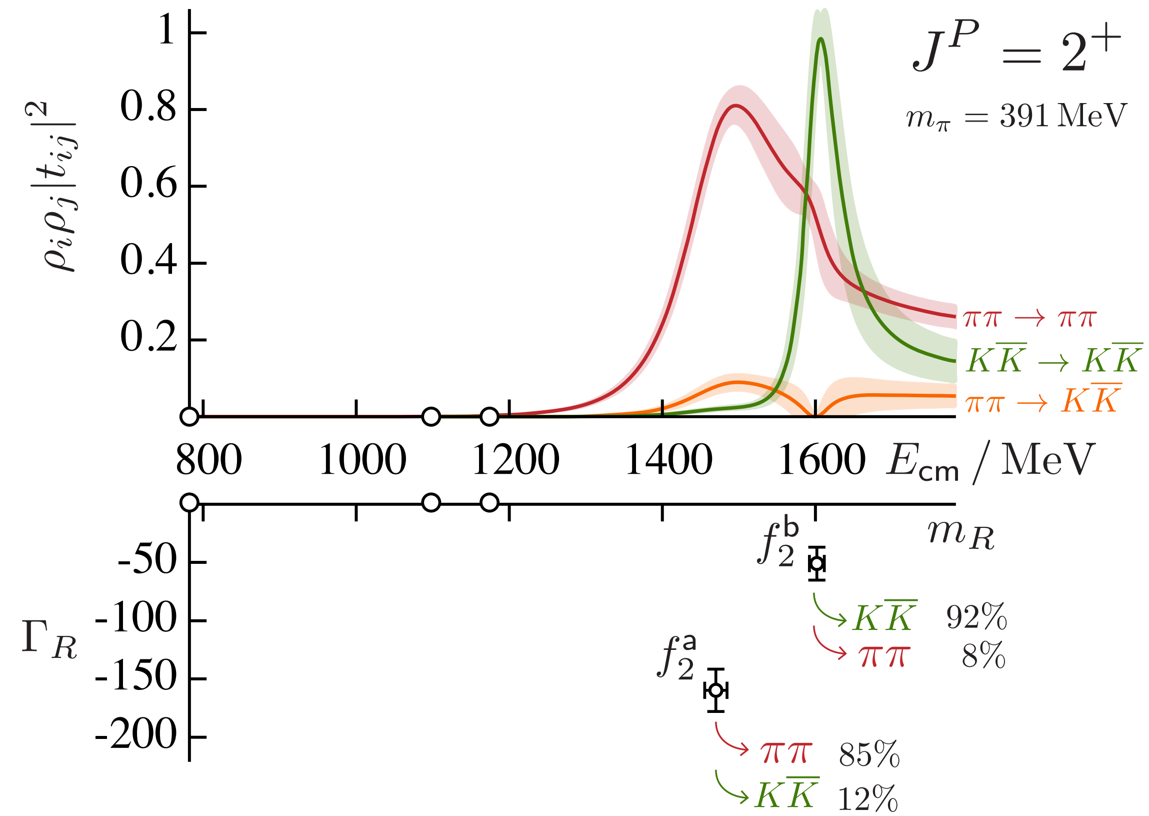


	$\mathrm{Im}[\,k_{\pi\pi}\,]$	$\mathrm{Im}[\,k_{KK}\,]$
sheet I	+	+
sheet II	−	+
sheet III	−	−
sheet IV	+	−



f_2 resonances — decay couplings & OZI





$$f_2^{\mathsf{a}} \sim u\bar{u} + d\bar{d} \qquad f_2^{\mathsf{b}} \sim s\bar{s}$$

couplings from pole residue

	$\frac{a_t \lvert c_{\pi\pi} \rvert}{(a_t k_{\pi\pi})^2}$	$\frac{a_t \lvert c_{K\bar{K}} \rvert}{(a_t k_{K\bar{K}})^2}$
f_2^{a}	7.1(4)	4.8(9)
f_2^{b}	1.0(3)	5.5(8)

zero in 'OZI' limit
— requires $s\bar{s}$ annihilation

Luescher finite-volume functions

$$0 = \det\Big[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\mathbf{t}(E)\cdot\big(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\big)\Big]$$

$$\overline{\mathcal{M}}_{\ell J m, \ell' J' m'} = \sum_{m_\ell, m'_\ell, m_S} \langle \ell m_\ell; 1 m_S | J m\rangle \langle \ell' m'_\ell; 1 m_S | J' m'\rangle$$

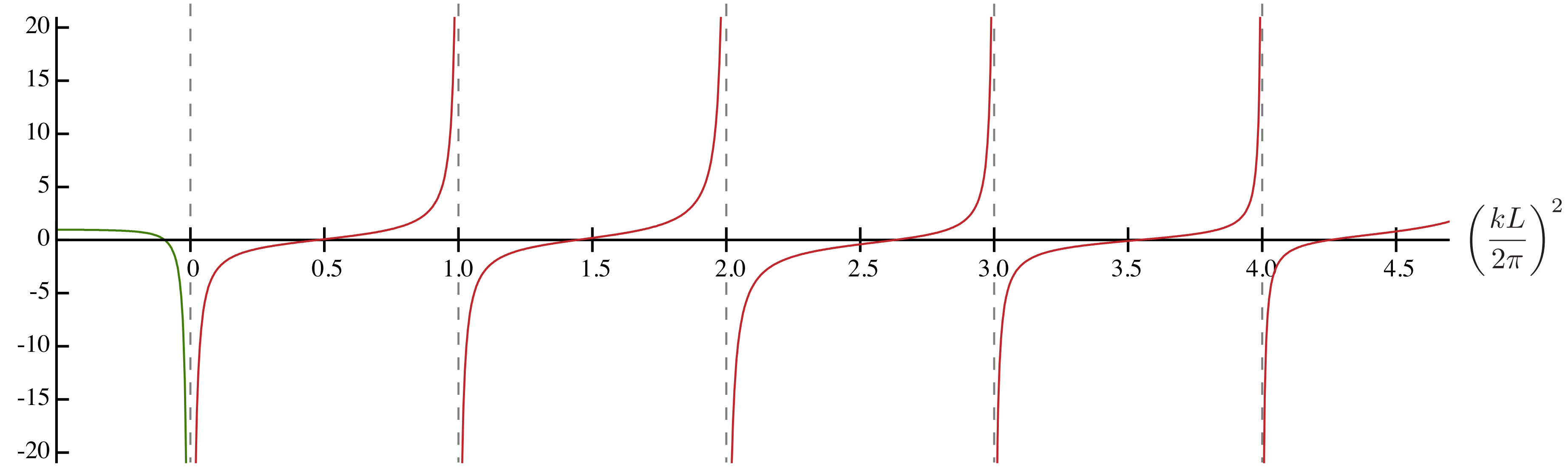
$$\times \sum_{\bar\ell, \bar m_\ell} \frac{(4\pi)^{3/2}}{k_{\mathsf{cm}}^{\bar\ell+1}}\, c^{\vec n}_{\bar\ell, \bar m_\ell}(k^2_{\mathsf{cm}}; L) \int d\Omega\, Y^*_{\ell m_\ell} Y^*_{\bar\ell \bar m_\ell} Y_{\ell' m'_\ell}$$

"spinless" Luescher functions

to respect the lattice symmetries,
need to subduce into irreducible representations

$$\overline{\mathcal{M}}^{\vec n, \Lambda}_{\ell J n, \ell' J' n'}\, \delta_{\Lambda,\Lambda'}\delta_{\mu,\mu'} = \sum_{\substack{m,\lambda \\ m',\lambda'}} \mathcal{S}^{J\lambda\,*}_{\Lambda\mu n}\, D^{(J)*}_{m\lambda}(R)\; \overline{\mathcal{M}}^{\vec n}_{\ell J m, \ell' J' m'}\; \mathcal{S}^{J'\lambda'}_{\Lambda'\mu' n'}\, D^{(J')}_{m'\lambda'}(R)$$

e.g. $\mathcal{M}_0$



zeroes of the determinant

e.g. a two-channel Flatté form — [000] A_1^+ irrep in L=2.4 fm box

m_π = 300 MeV
m_K = 500 MeV

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\mathbf{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right]$$