Extending lattice PDF computations to new theoretical regimes

GHP 2021

Nikhil Karthik

JLab and WM

Based on papers accepted in PRD:

arXiv 2101.11632 by X. Gao, NK, S. Mukherjee, P. Petreczky, S. Syritsyn, Y. Zhao

arXiv 2101.02224 by NK

LaMET and SDF methodologies

Distill leading-twist terms from Euclidean matrix elements at large-hadron momentum and/or short-distances

X. Ji ’13
A. Radyushkin ’17
V. Braun, D. Muller ‘08

$$\mathcal{M}(P_z z, z^2) = \langle H(\vec{P}) | \overline{\psi}(0)\gamma_t \; W(0,z) \; \psi(z) | H(\vec{P}) \rangle$$

Perturbative matching

$$\mathcal{M}(P^+ z_-) = \langle H(\vec{P}) | \overline{\psi}(0)\gamma_+ \; W(0,z_-) \; \psi(z_-) | H(\vec{P}) \rangle$$

LaMET and SDF methodologies

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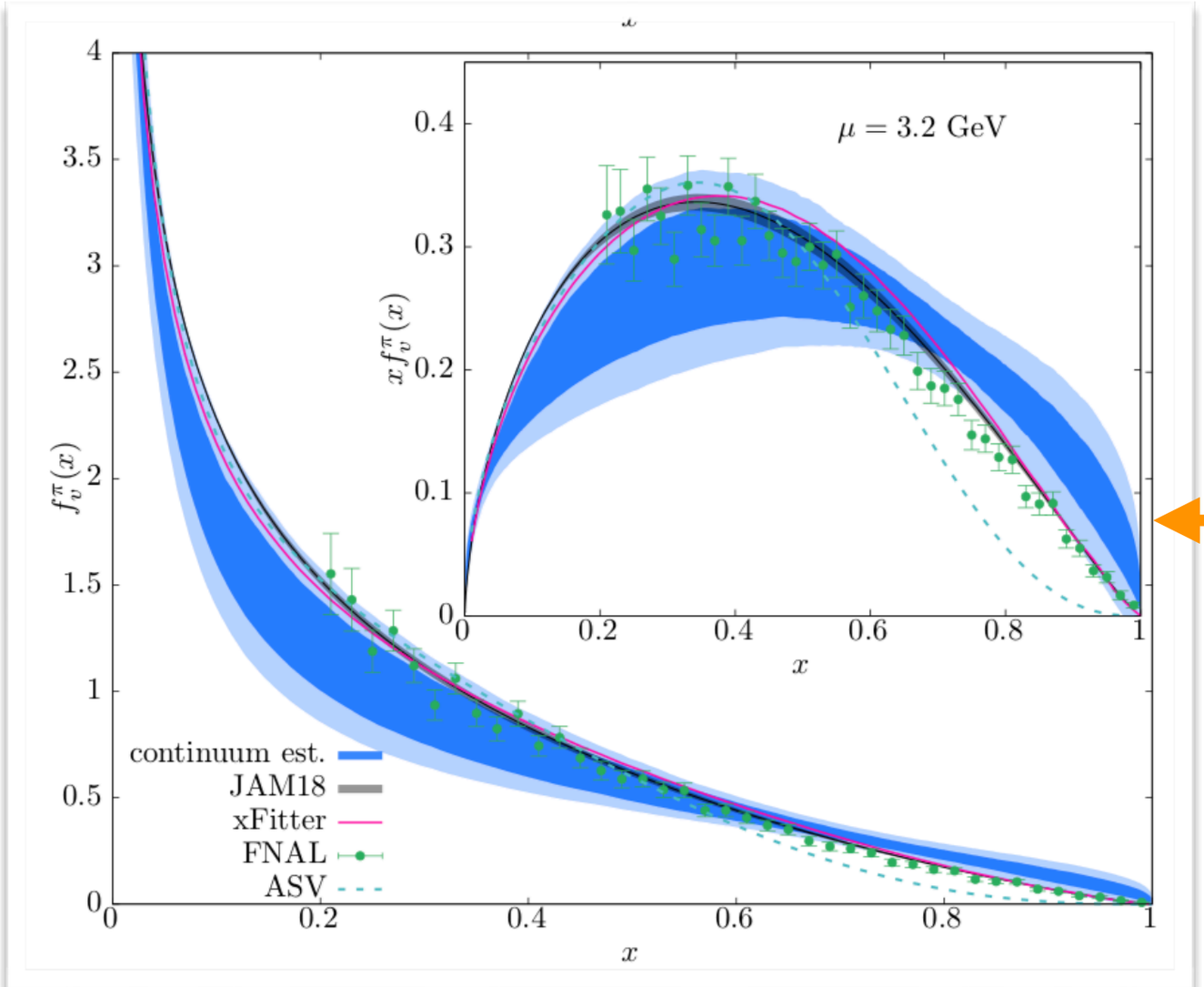

PRD102 (2020), 094513

Methodology leads to estimates of PDFs and Mellin moments that are more-or-less consistent with phenomenological determinations.

Valence pion PDF determination employing LaMET/SDF

LaMET and SDF methodologies

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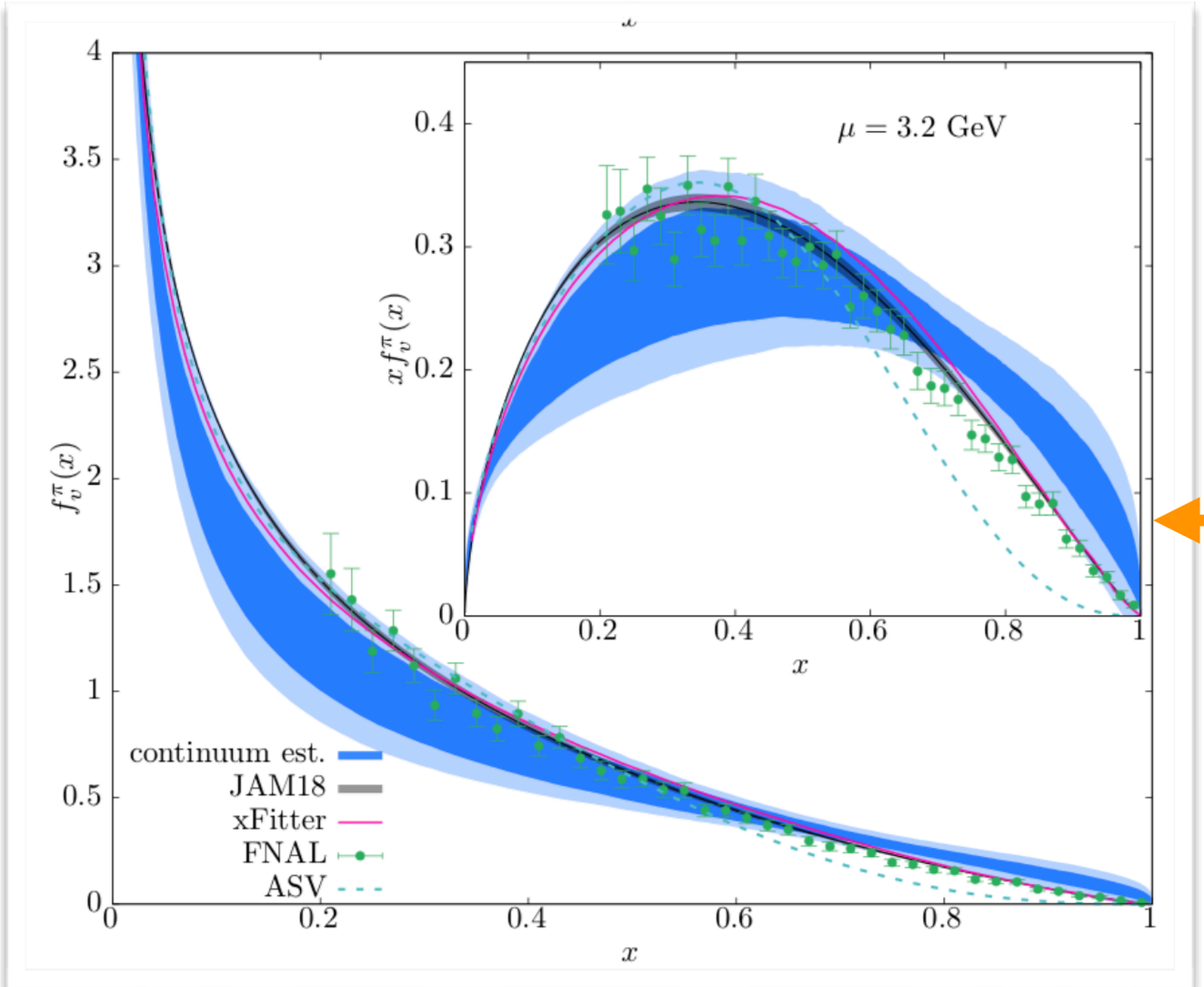

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Valence pion PDF determination employing LaMET/SDF

Progressing towards precision studies

- Control higher-twist
- Lattice-spacing effects at z~O(a)
- Sensitivity to higher Mellin moments
- Inverse problem

....

Motivation for this talk:

Can we use [lattice QCD + LaMET/SDF] methodology as a "theoretical collider" to learn about QCD and the non-perturbative origin of PDFs by studying cases not experimentally accessible ?

pion PDF in QCD and QCD-like theories → chiral symmetry-broken QCD vacuum physics

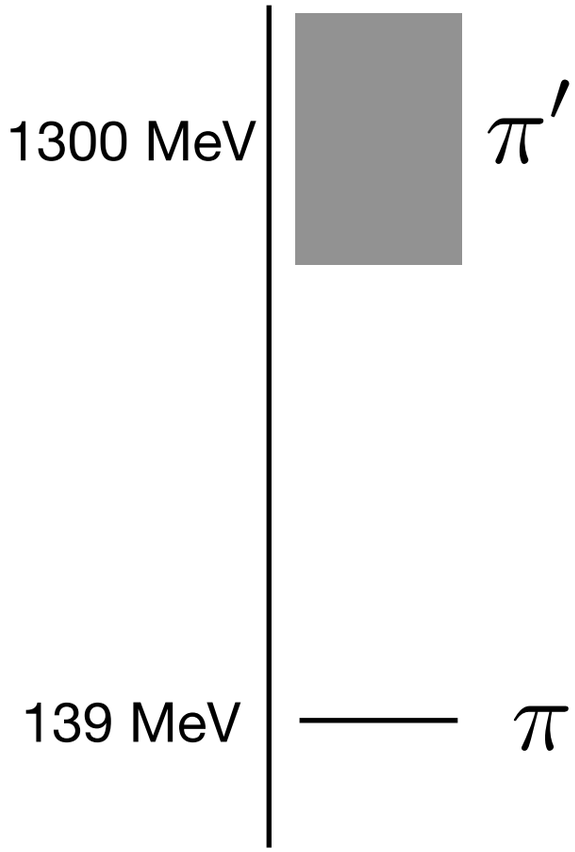
Previous approaches
DSE, BSE, Holography, …

- What is the nature of the large-x $(1-x)^b$ behavior? (X. Gao's talk)

- What happens to the quark-structure when pion is radially excited?

- How is the quark structure of the Goldstone pion sensitive to the symmetry-broken vacuum structure?

How does quark structure change when pion is radially excited?



PDG '19

$\pi(1300)$ $I^G(J^{PC}) = 1^-(0^{-+})$

Mass $m = 1300 \pm 100$ MeV [l]
Full width $\Gamma = 200$ to 600 MeV

$\pi(1300)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\rho\pi$	seen	404
$\pi(\pi\pi)_{S\text{-wave}}$	seen	–

Pi(1300) : non-Goldstone 0^- meson to compare and contrast with the Goldstone 0^-

Theoretical expectations in the chiral limit : $m_q \to 0$

$$M_\pi \to 0 \qquad M_{\pi\prime} \to \#\Lambda_{\mathrm{QCD}}$$

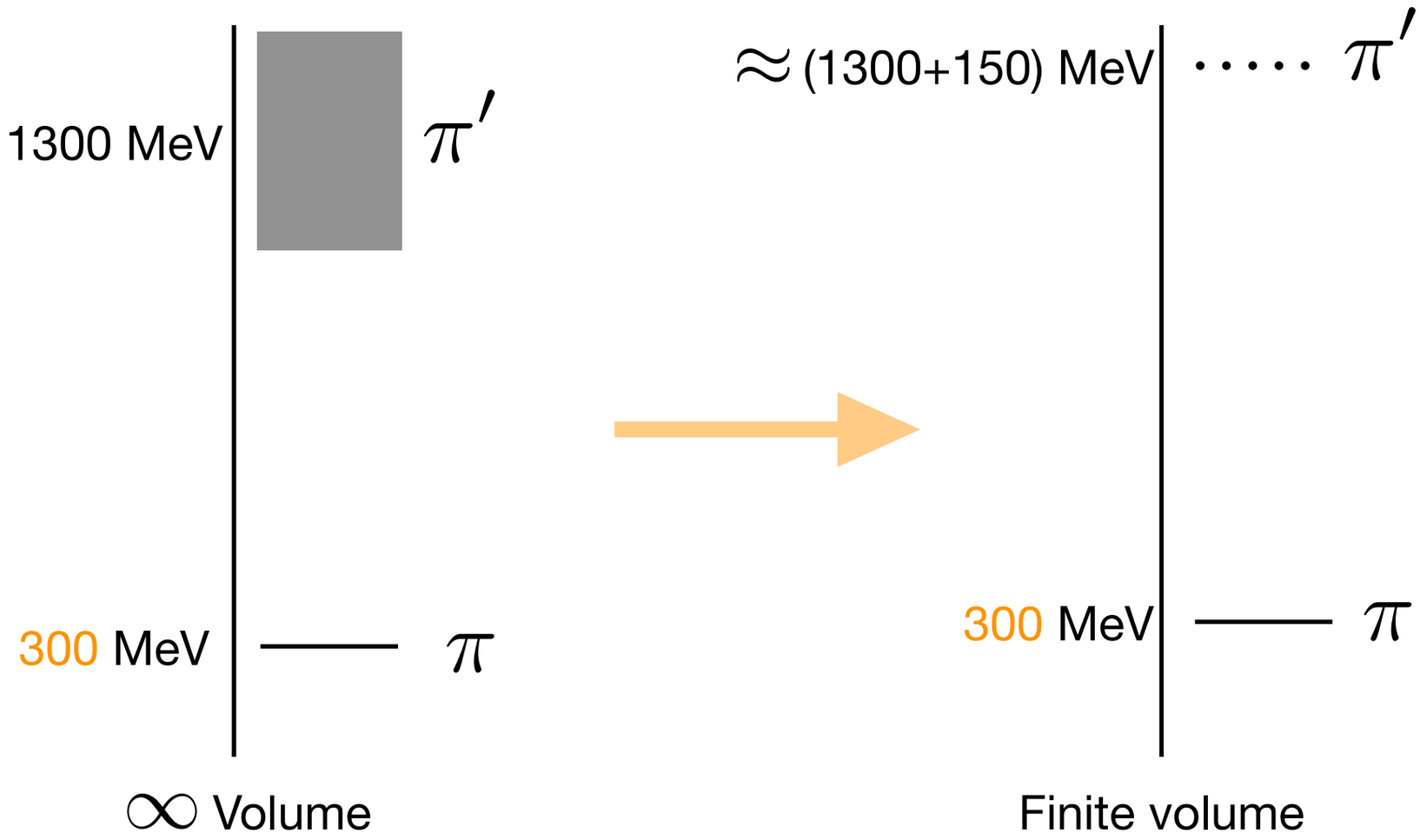
$$F_\pi \to \#\,\Lambda_{\mathrm{QCD}} \qquad F_{\pi\prime} \to 0$$

Previous lattice works:
Mastropas, Richards '14
McNeile, Michael '06

Are there tell-tale differences in their partonic structures?

Method for the first go at this problem:

- Perform the study In fixed finite volume: convert resonance into a finite volume eigenstate



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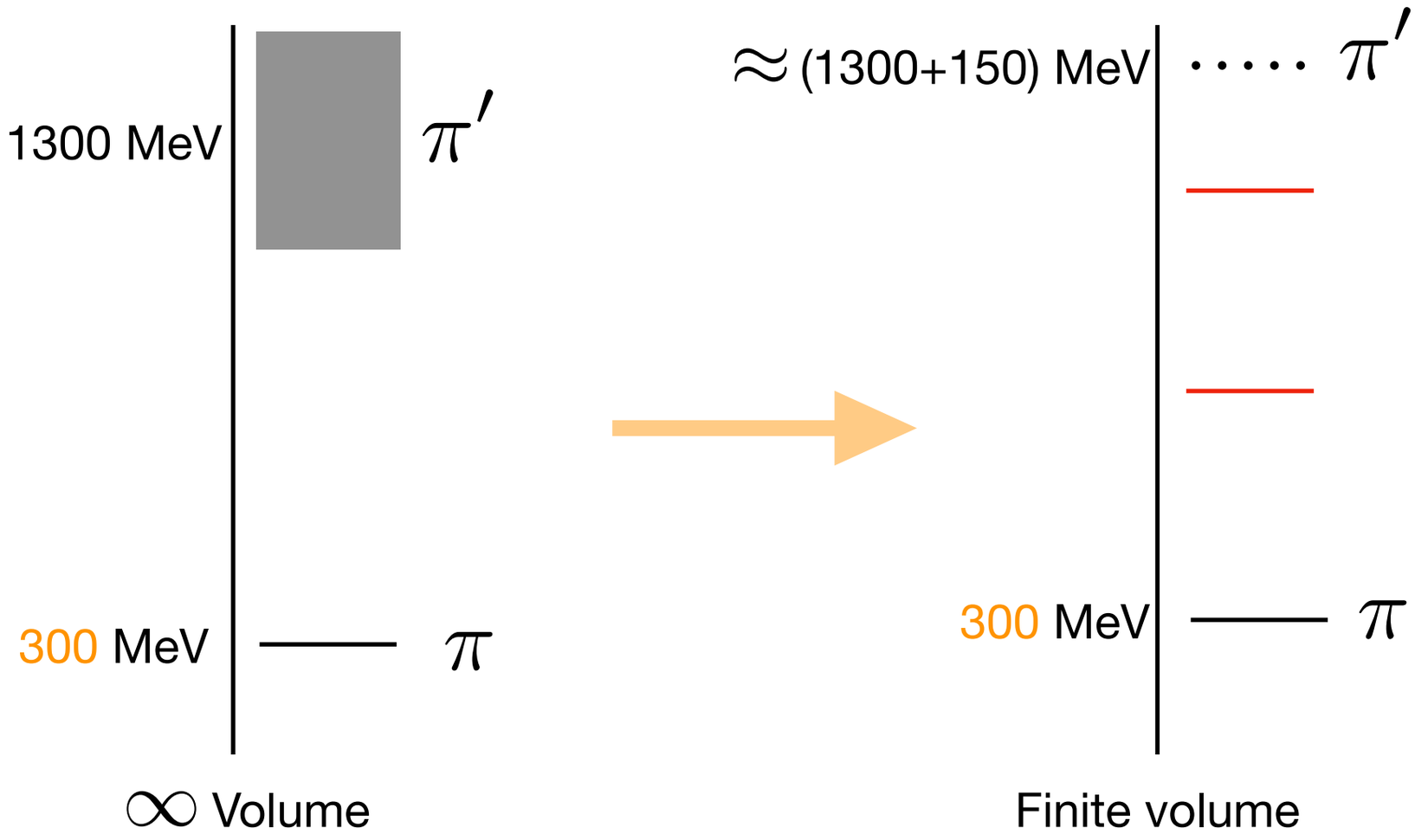

- Use local pion creation operators to suppress contributions from multi-particle states

$$\bar{u}d|0\rangle = c_0|\pi\rangle + c_1|\pi\prime\rangle + c_2|\pi,\rho\rangle + c_3|\pi,\pi,\pi\rangle + \ldots$$

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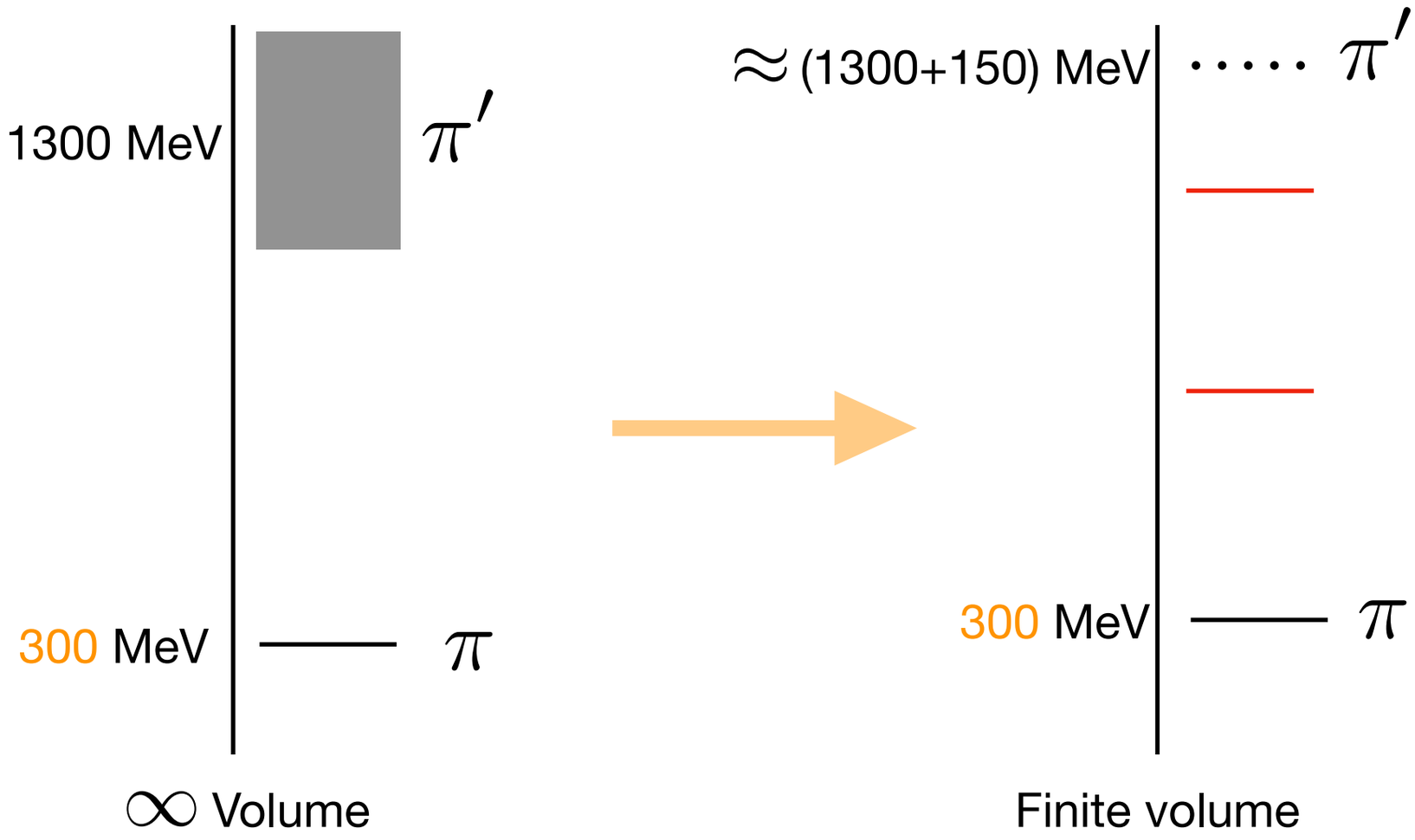

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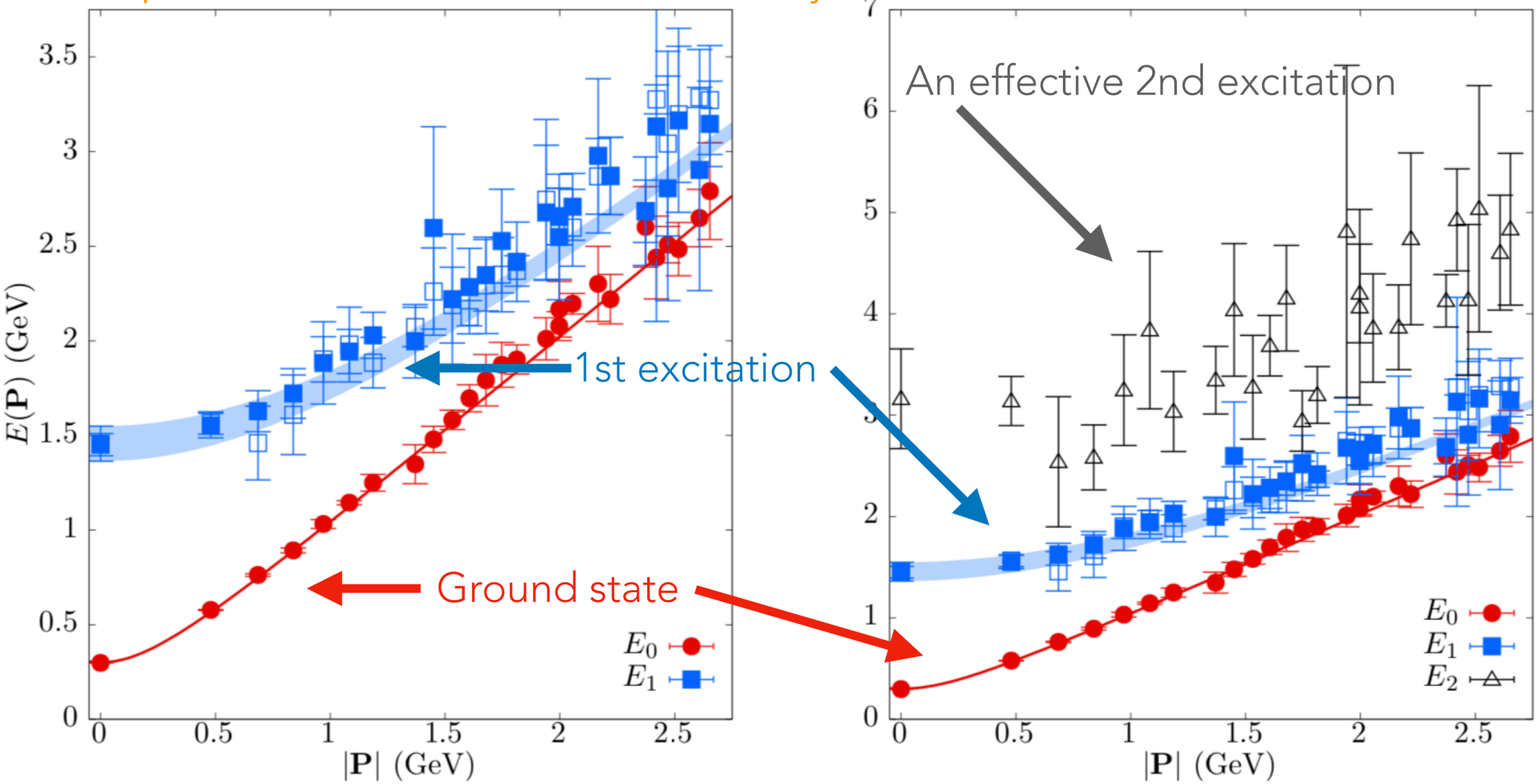
- Room for lot of improvements: Luscher method, GEVP with extended operator basis,…

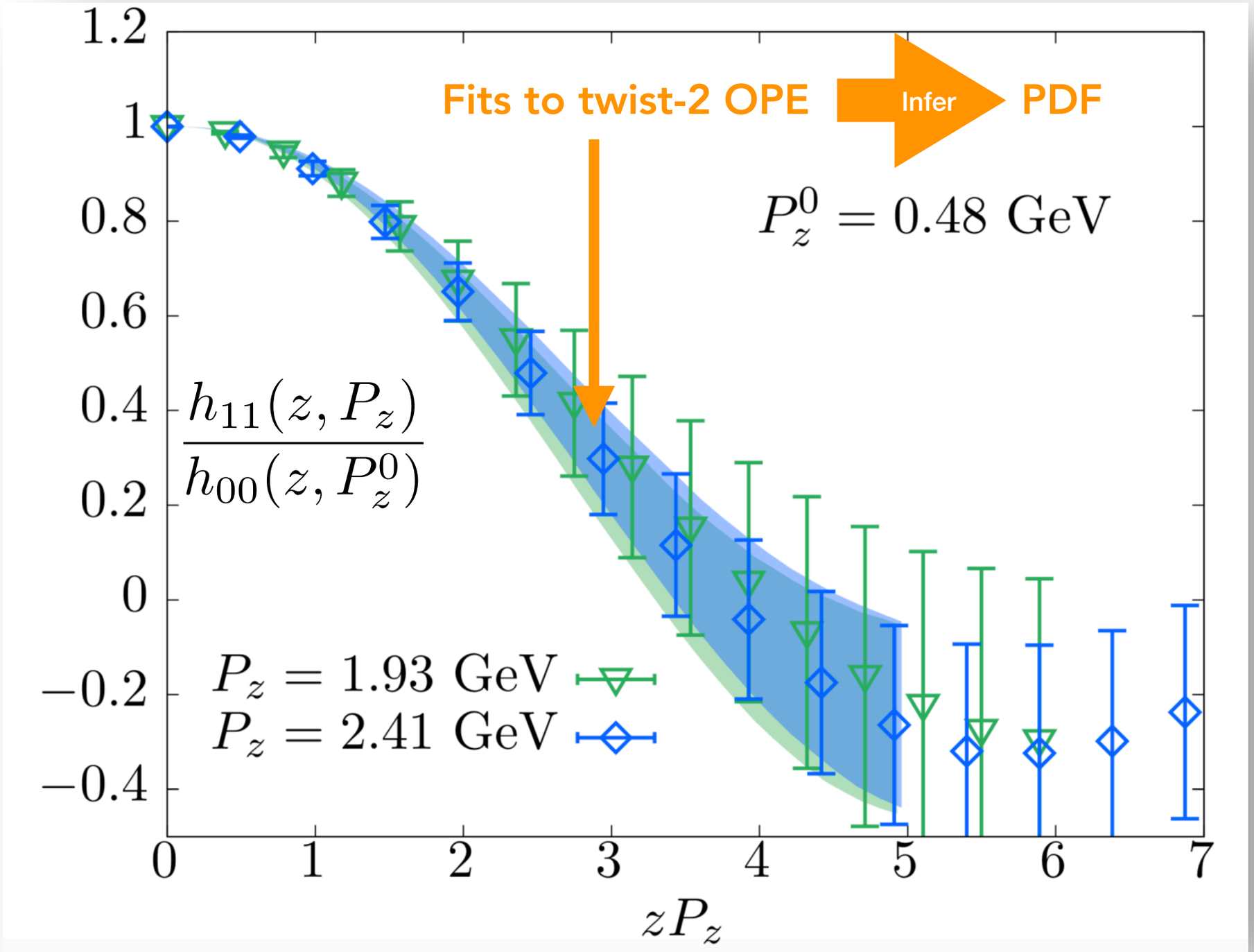
The 1st excited state in pion 2-pt function follows single-particle dispersion with mass close to that of pi(1300)

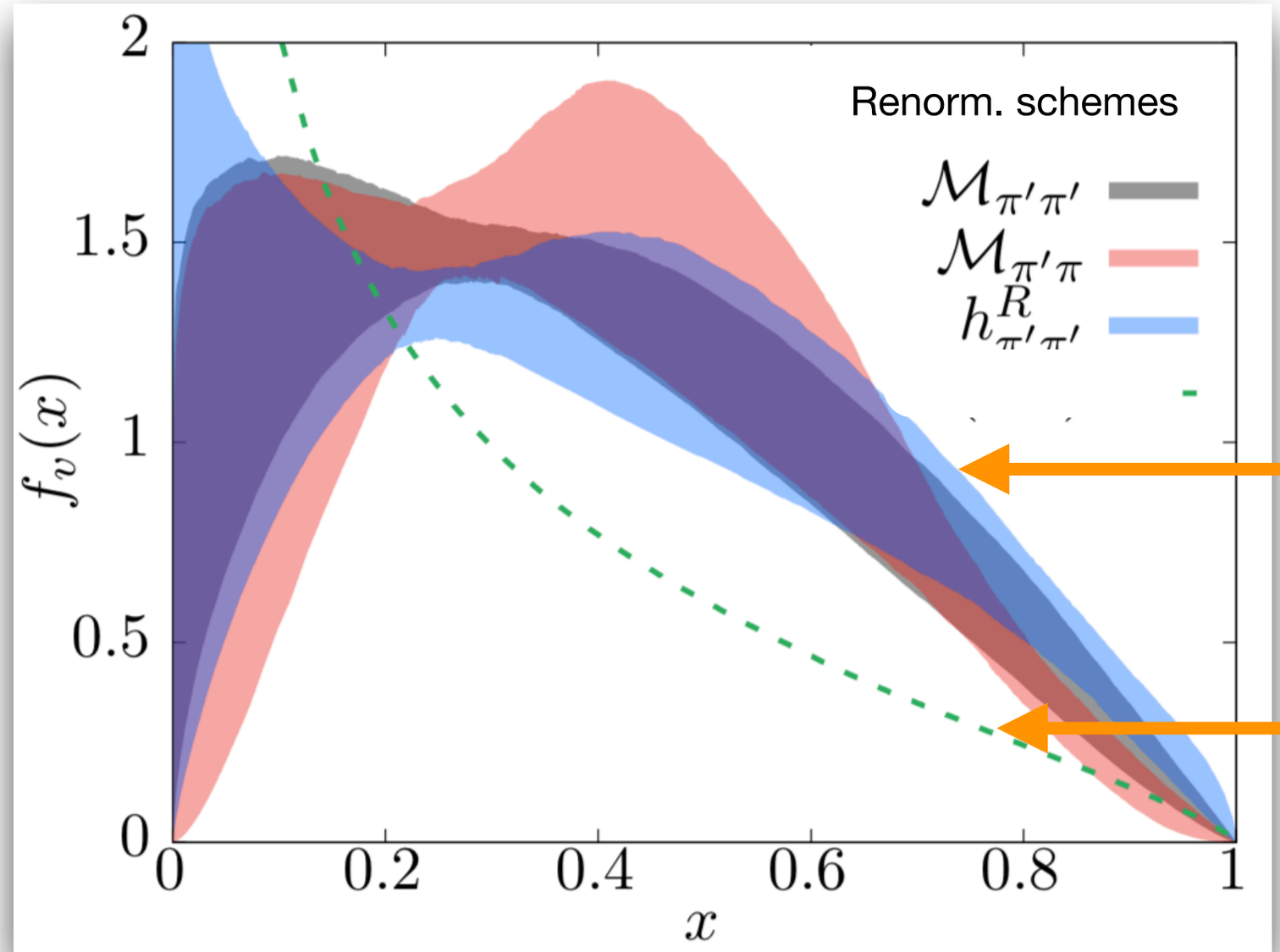
Dispersion relation obtained numerically:



$$\left\langle \hat{\pi}(t_s) \; \overset{\overline{\psi}(0)}{\bullet}\!\!\!-\!\!\!-\!\!\!\underset{W(0,z)}{-}\!\!\!-\!\!\!\bullet\overset{\psi(z)}{} \; \hat{\pi}^\dagger(0) \right\rangle = \sum_{i,j} A_i^* A_j h_{ij}(z, P_z) e^{-E_i(t_s-\tau)-E_j\tau}$$

Fit ground state h_{00} and excited state h_{11}





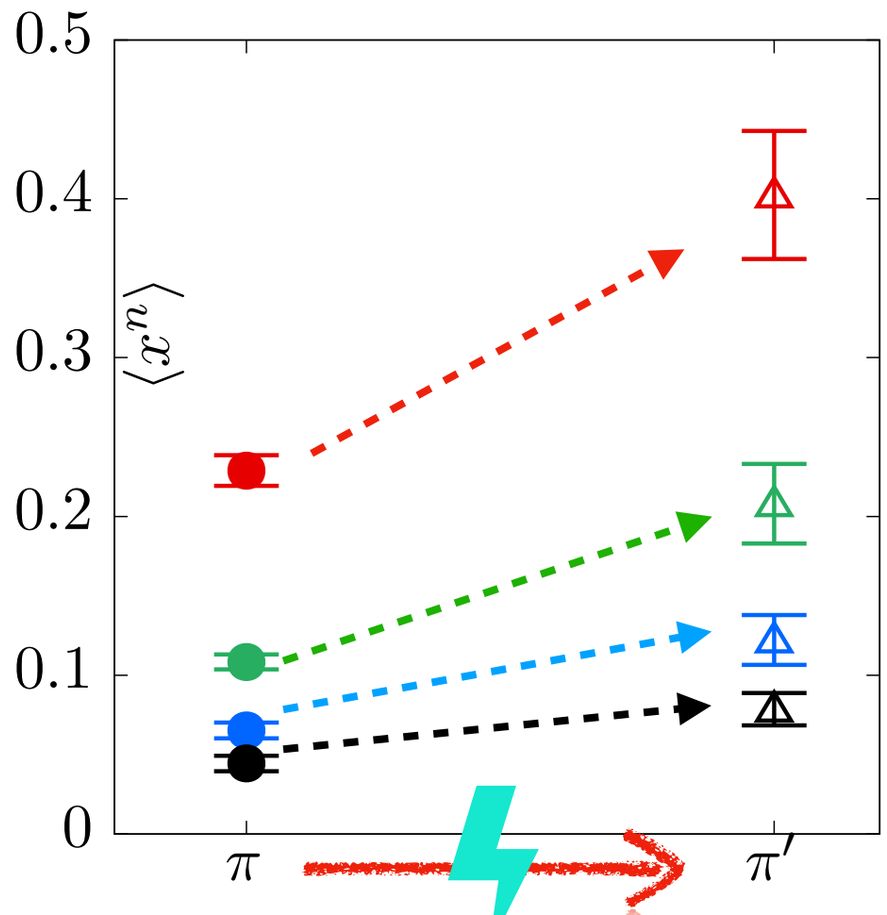
PDF reconstructed based on $x^a(1-x)^b$ Ansatz

Pi(1300) valence PDF

pion valence PDF in the same ensemble

A ratio of momentum differentials:

$$\zeta = \frac{2M_{\pi'}\langle x\rangle_{\pi'} - 2M_{\pi}\langle x\rangle_{\pi}}{M_{\pi'} - M_{\pi}}$$
$$\approx 80 - 99\%$$



How is the internal structure of Nambu-Goldstone mode affected by the long-distance vacuum structure?

What aspects of pion ITD and PDF are determined by (or sensitive to) the symmetry-breaking physics?

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Slowly change strength of symmetry-breaking and record how PDFs change?

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Embed in class of theories: QCD(N) → Confinement physics on sliding scale

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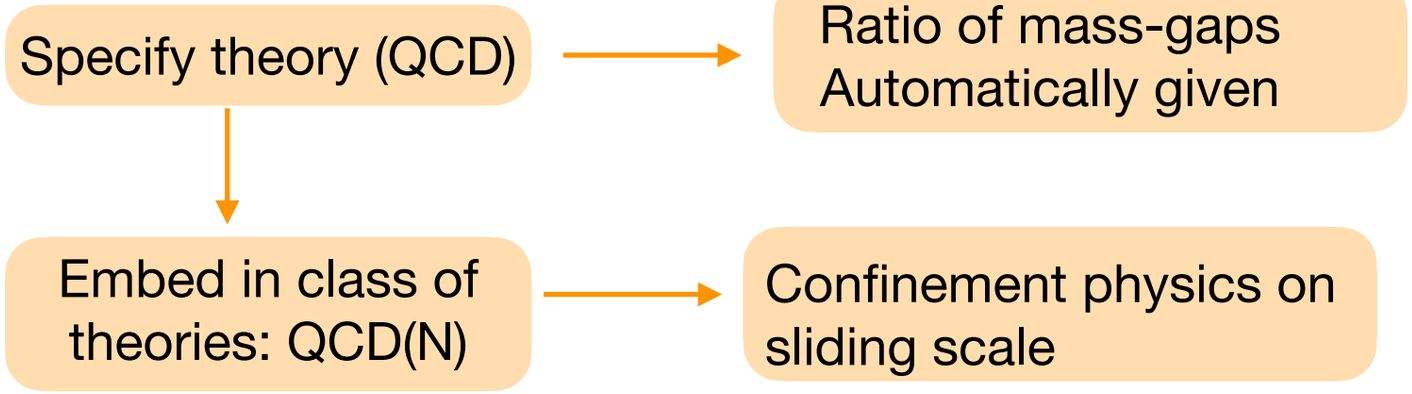


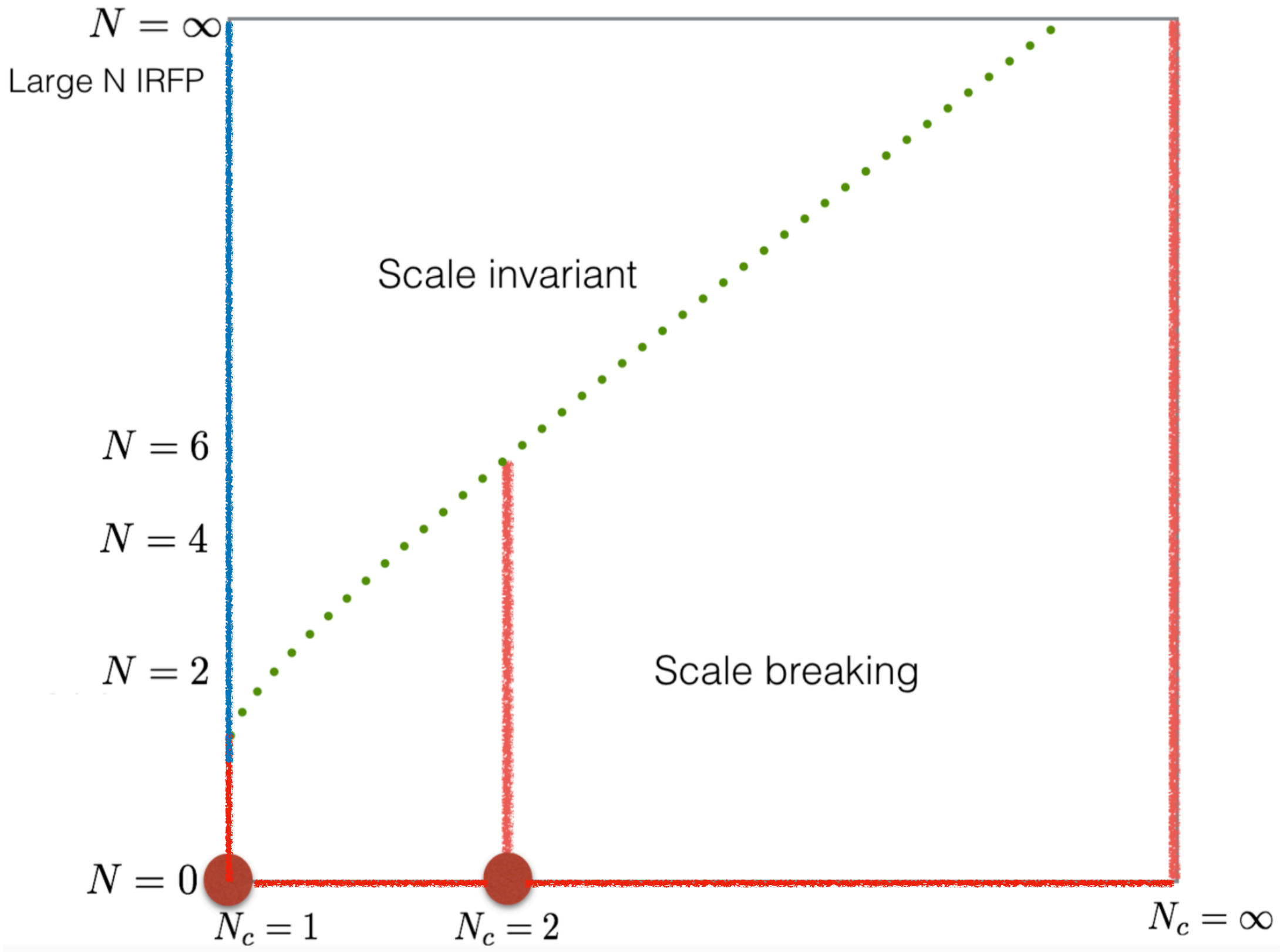
R. Pisarski '84
T. Appelquist et al, '86
Lot of literature till now

A model QFT: 2+1d SU(2) gauge theory coupled to massless N fermion flavors

- Computationally cheaper to do.

- Most important physics reason: parallel theoretical developments in 2+1d in understanding confinement and mass-gap via classification of RG flows; e.g., conformal bootstrap.

Mapping the IR phase diagram of 2+1d massless QCD



NK, R.Narayanan ’15, ’16, ’18

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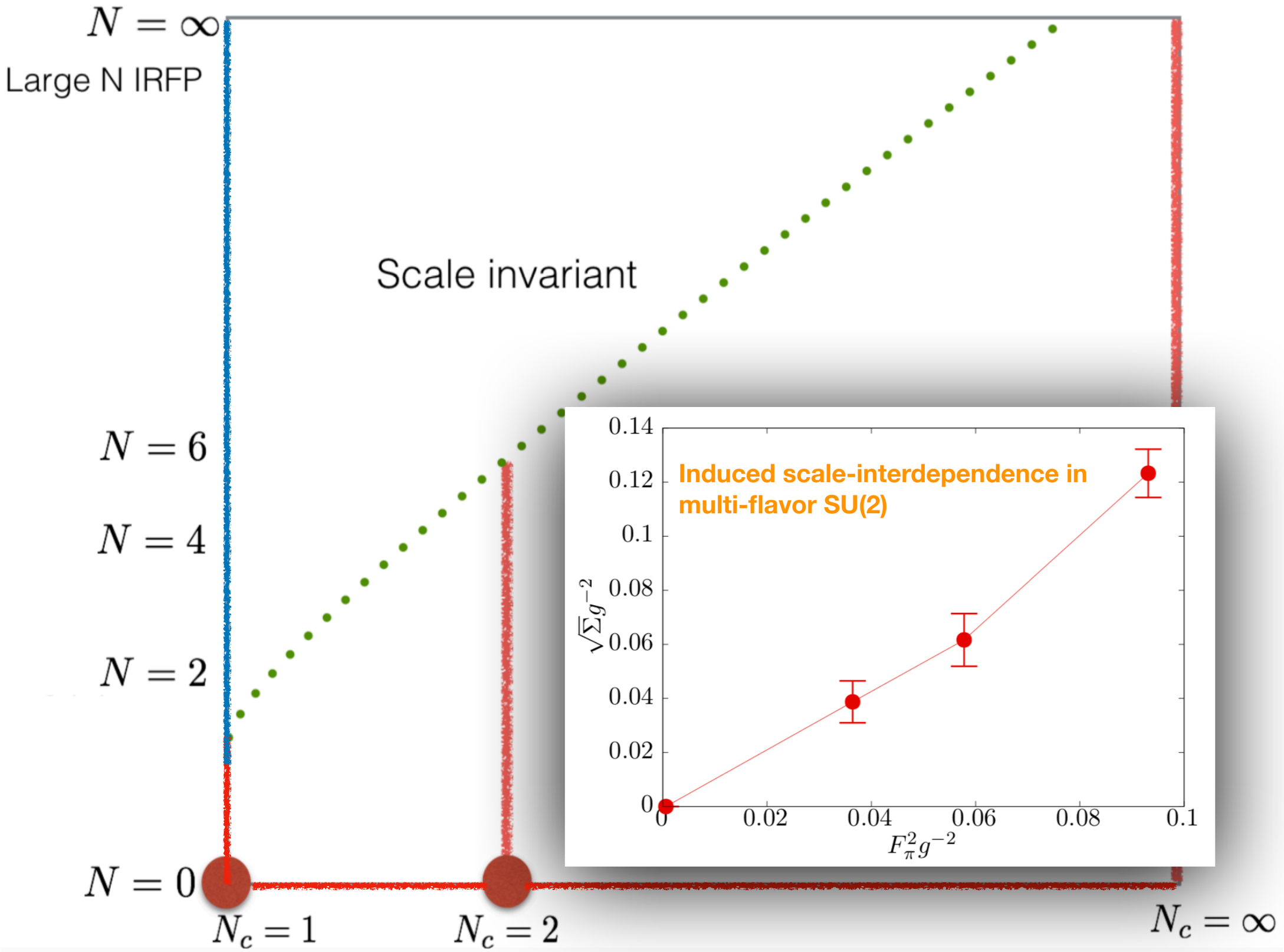

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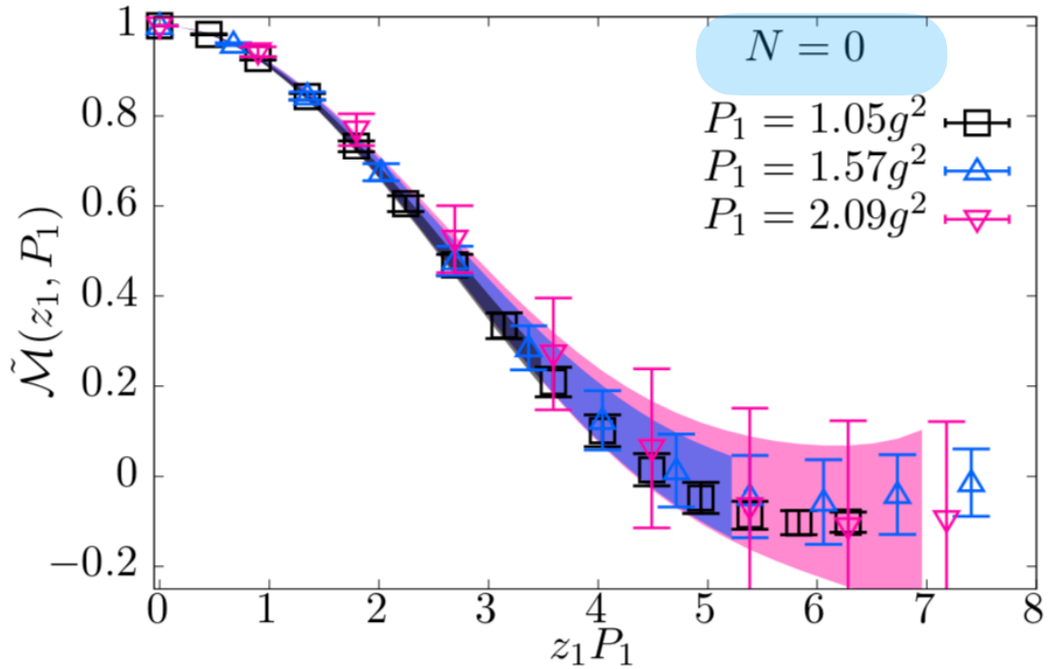
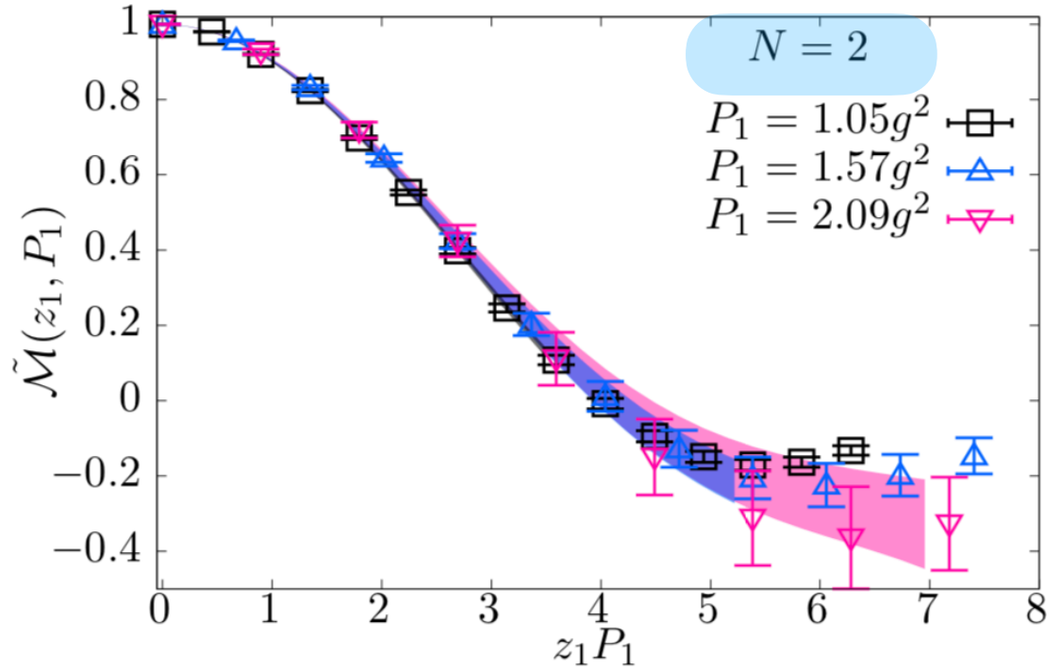
Analysis

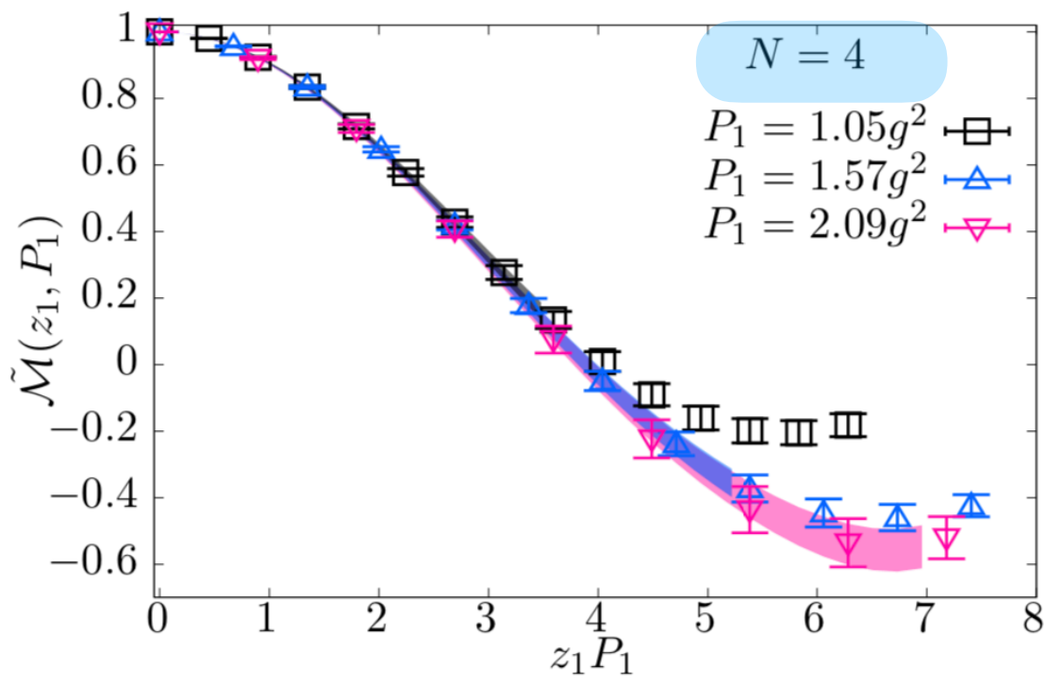
$$\tilde{\mathcal{M}}(z_1, P_1) = 1 + \left[\sum_{k=1}^{N_{\max}} (-1)^k \frac{(z_1 P_1)^{2k}}{(2k)!} \langle x^{2k} \rangle_v \right] + \text{H.T.}$$

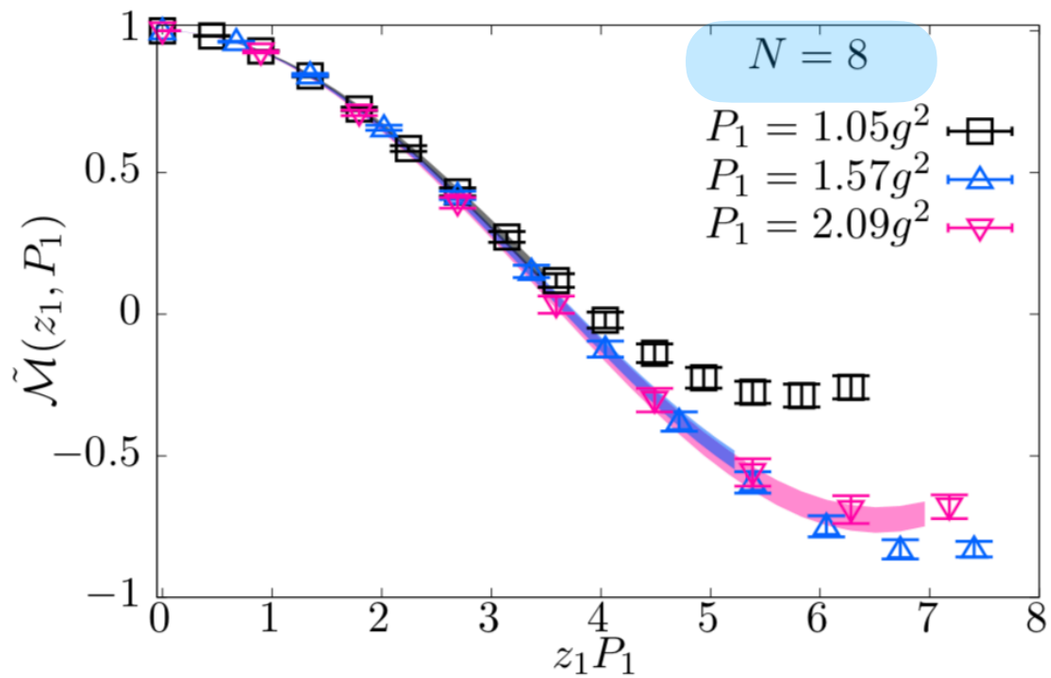
- Fit moments as fit parameters
- PDF Ansatz fit assuming $f(x) \sim x^{\alpha}(1-x)^{\beta}$

reduced ITD

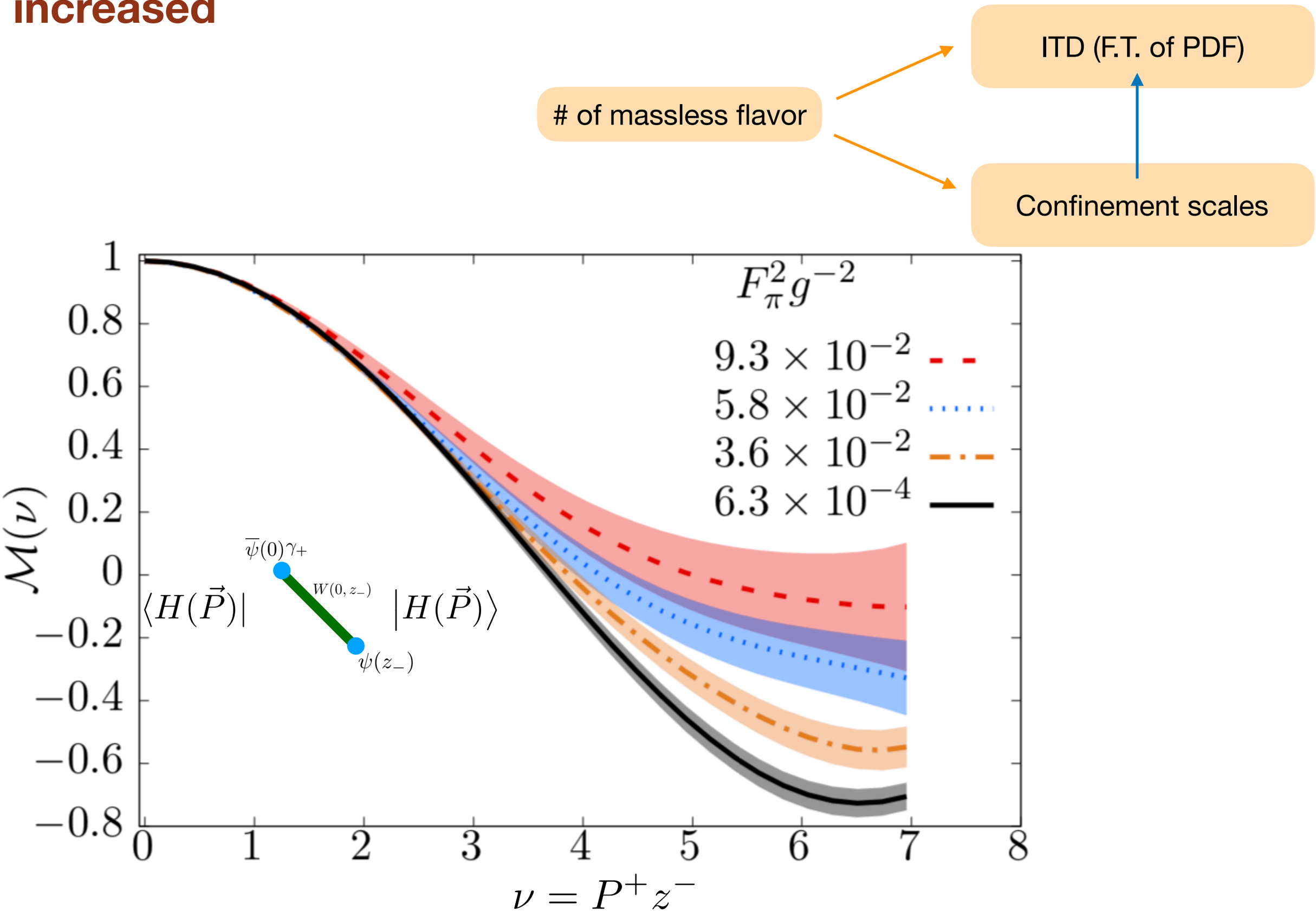




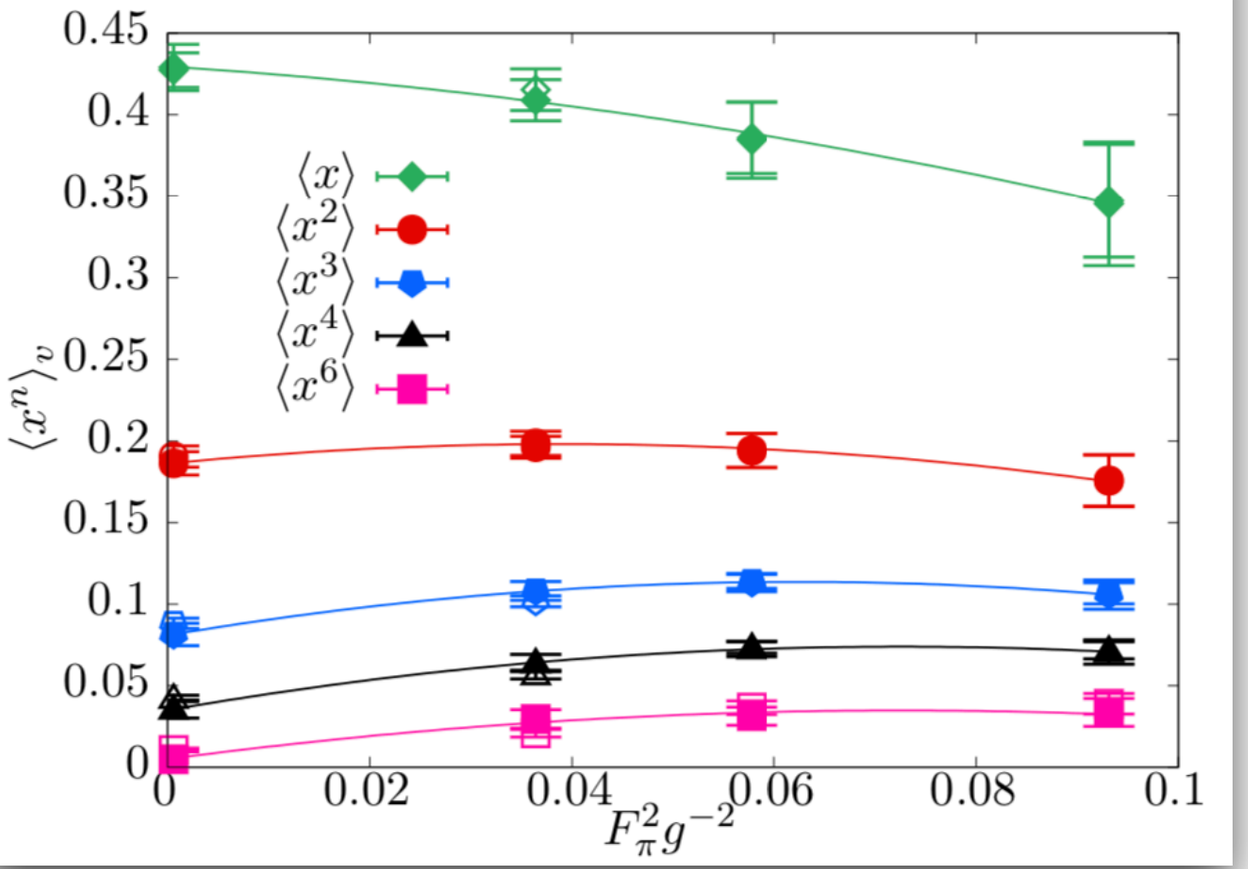




How ITD/LF-correlation changes when decay-constant is increased



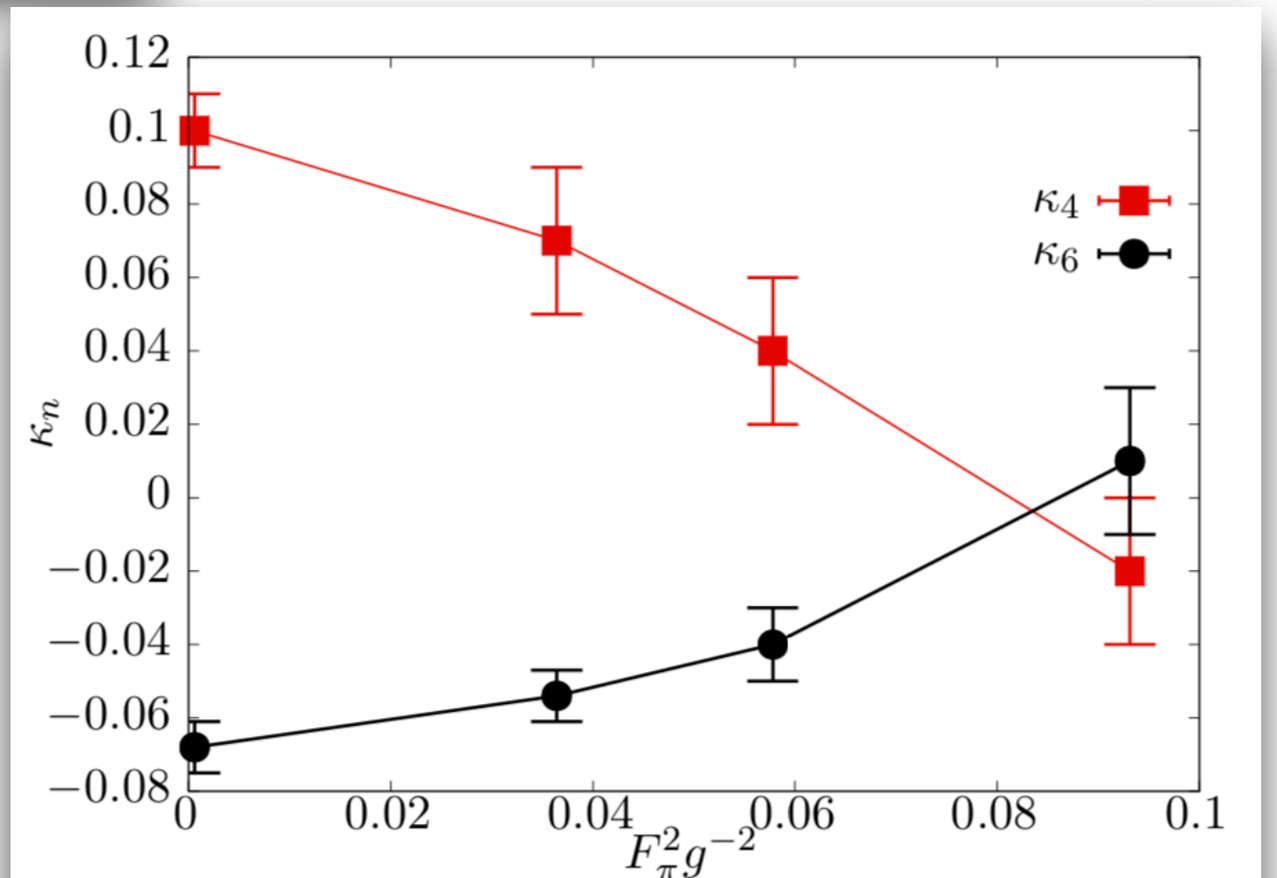
Effect on moments



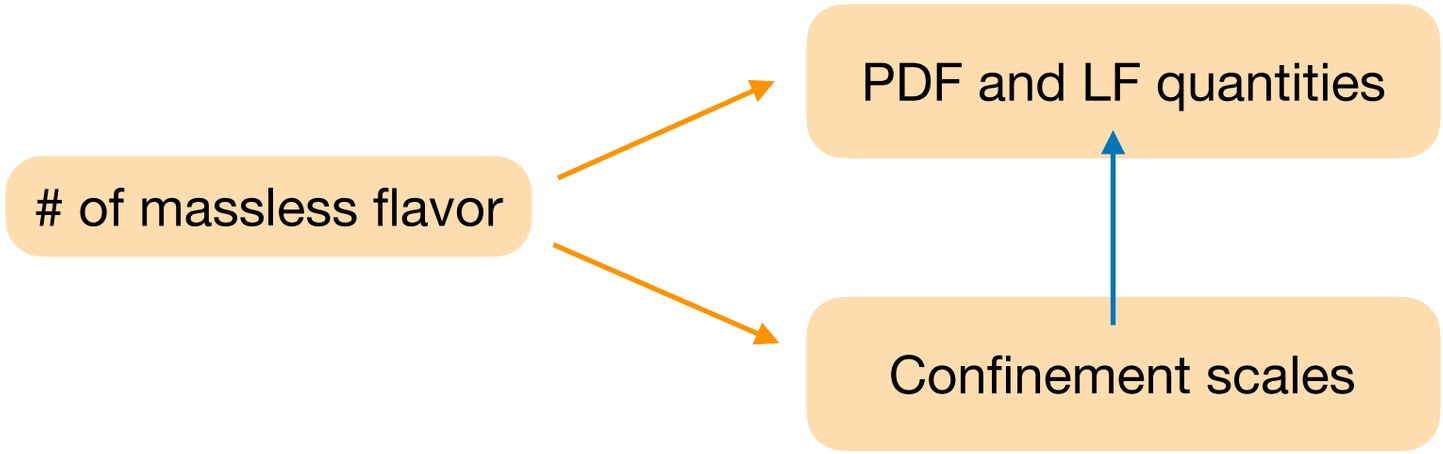
Increased IR sensitivity of PDF shape observables

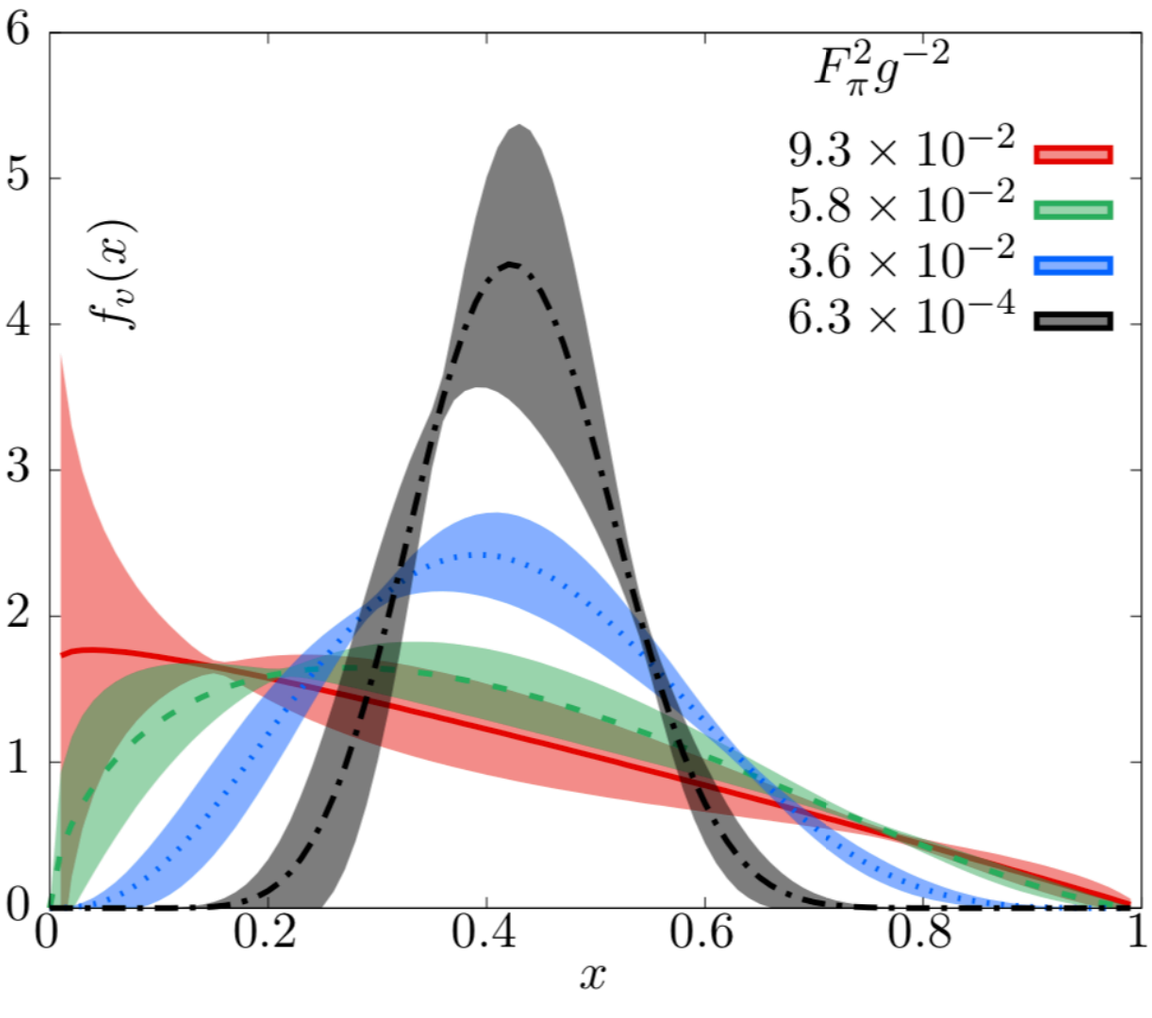
Cumulants of PDF more sensitivity to IR

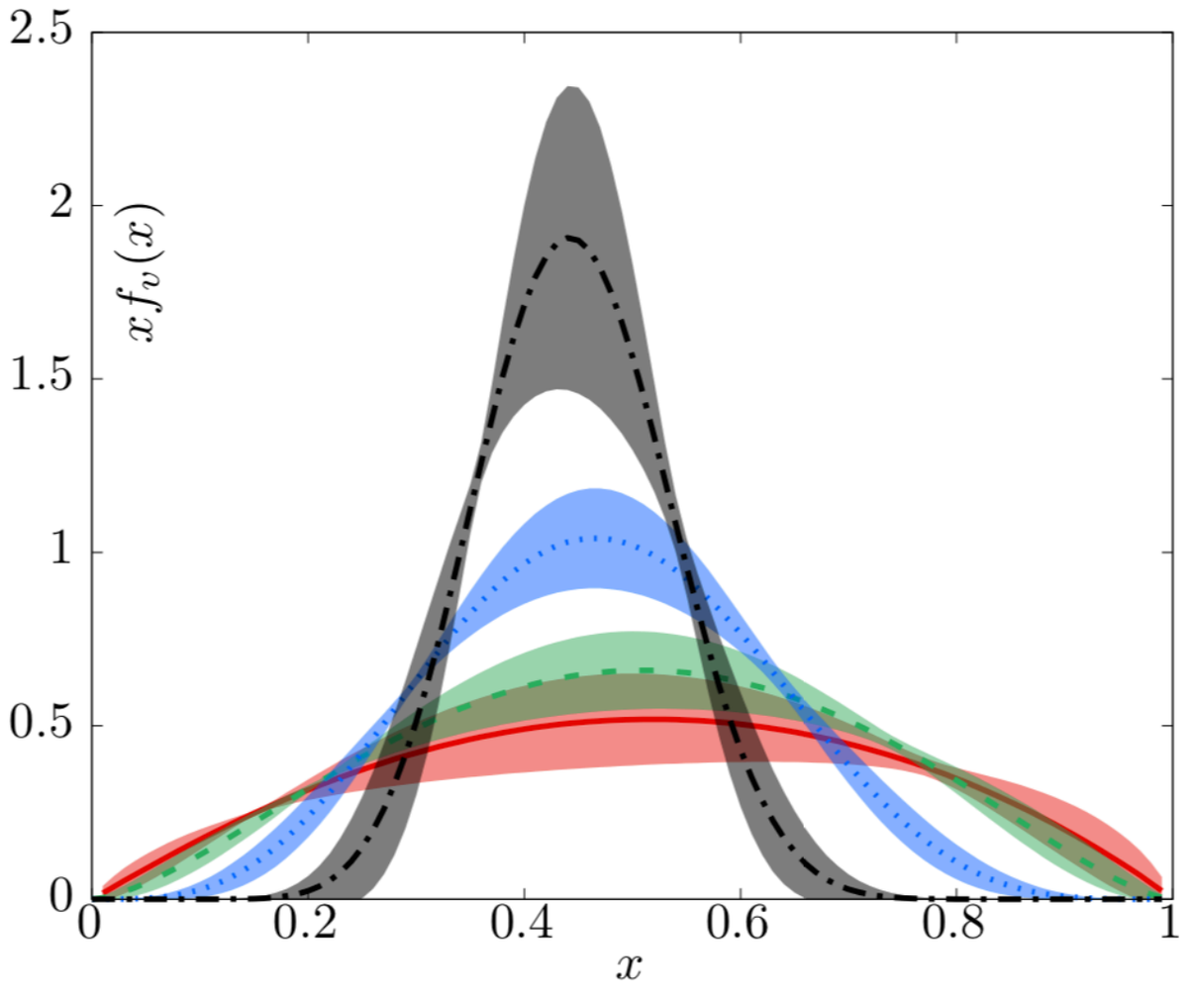
$$\kappa_n \equiv \frac{\partial^n}{\partial s^n} \log \left(\int_{-1}^{1} f_{u-d}(x) e^{sx} dx \right) \Big|_{s=0}$$
$$\kappa_4 = \langle x^4 \rangle_v - 3\langle x^2 \rangle_v^2,$$
$$\kappa_6 = \langle x^6 \rangle_v - 15\langle x^2 \rangle_v \langle x^4 \rangle_v + 30\langle x^2 \rangle_v^3.$$



The main observation: broadening of pion PDF when the strength of symmetry-breaking is *dialed-up*







Summary and outlook

- Ab intio lattice determination of PDF using LaMET/SDF successful

Precision PDF determination
as pheno input and global analysis

A new theoretical tool to understand QCD

Structure of all low-lying hadrons and resonances

example-1:
Low moments of pi(1300) ~2 x pion

Extension to QCD-like theories

Structure of Large N_c mesons

example-2:
Establish correlation of NG boson structure with IR by controlling SSB:
Couple N flavors of massless quarks