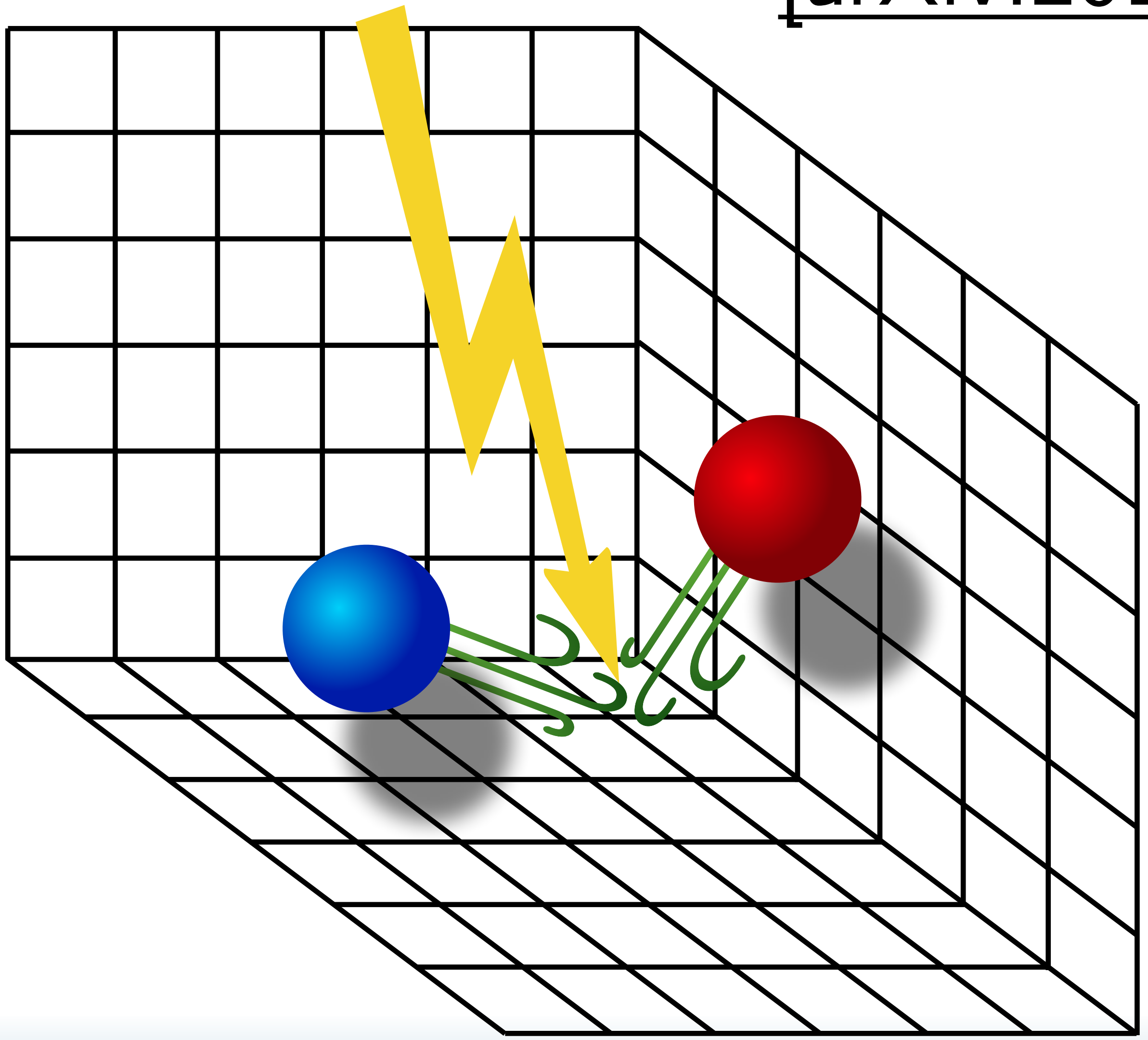
Elastic form-factors of resonances from lattice QCD

[arXiv:2012.13338]



Felipe Ortega-Gama

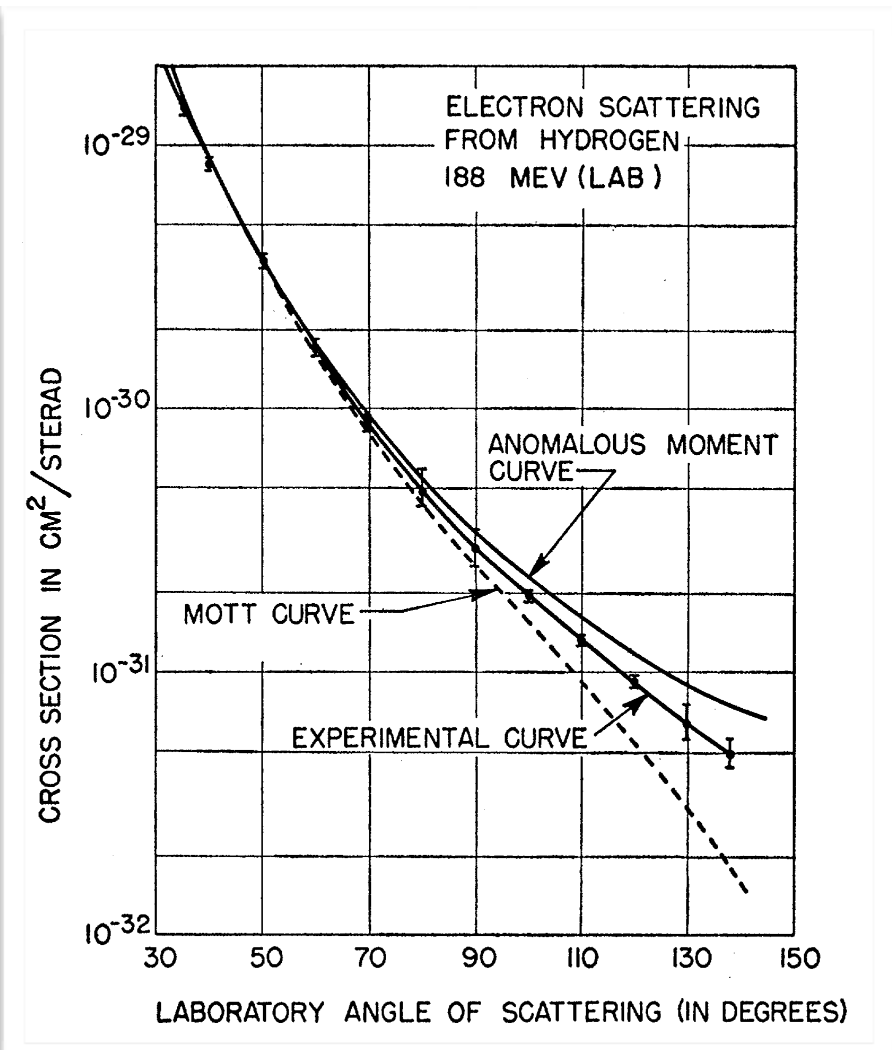
GHP - April 13th 2021

felortga@jlab.org

William & Mary
CHARTERED 1693

Structure of hadrons

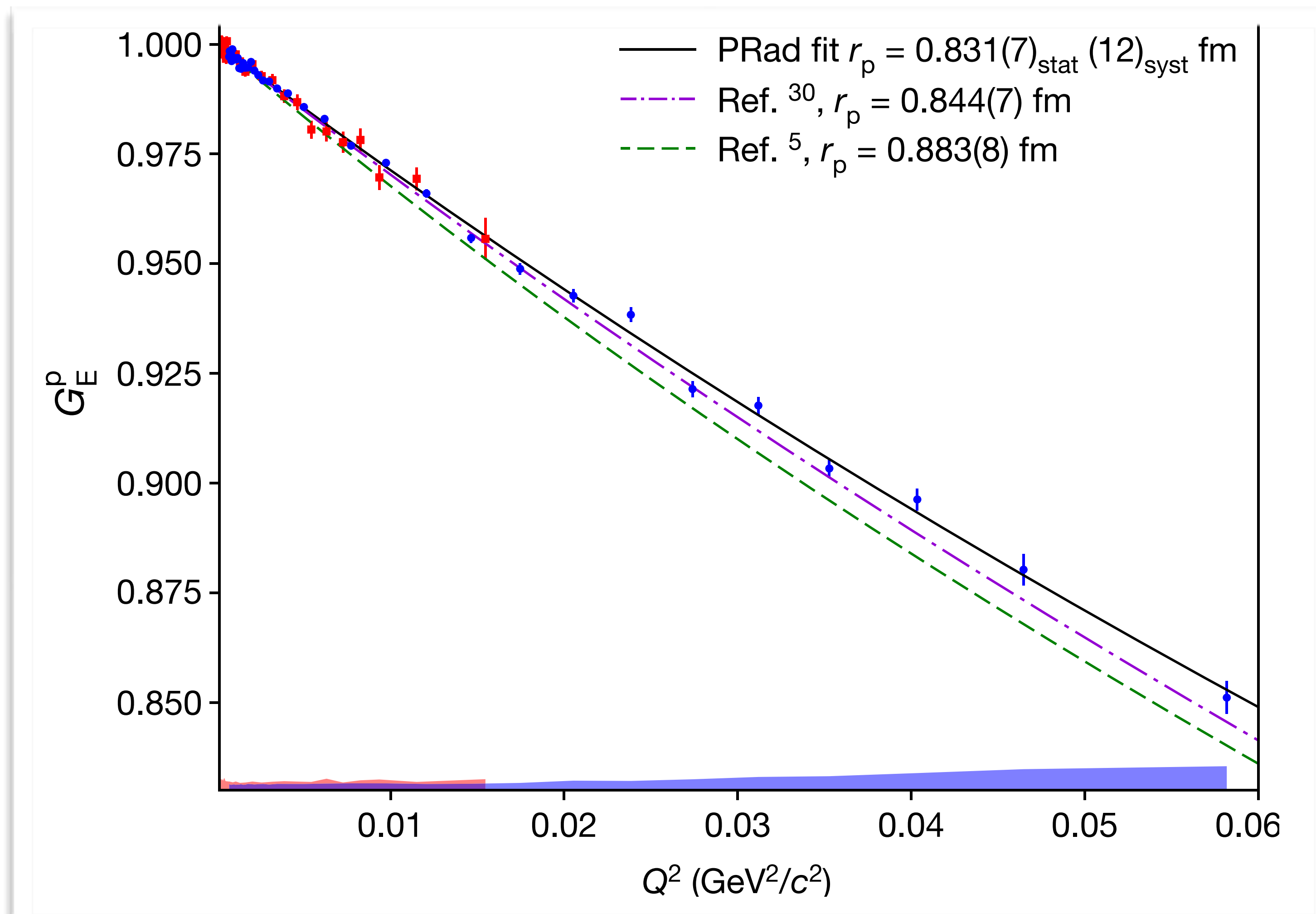
- ✦ 1950's first ep measurements



[Phys. Rev. 98, 217 (1955)]

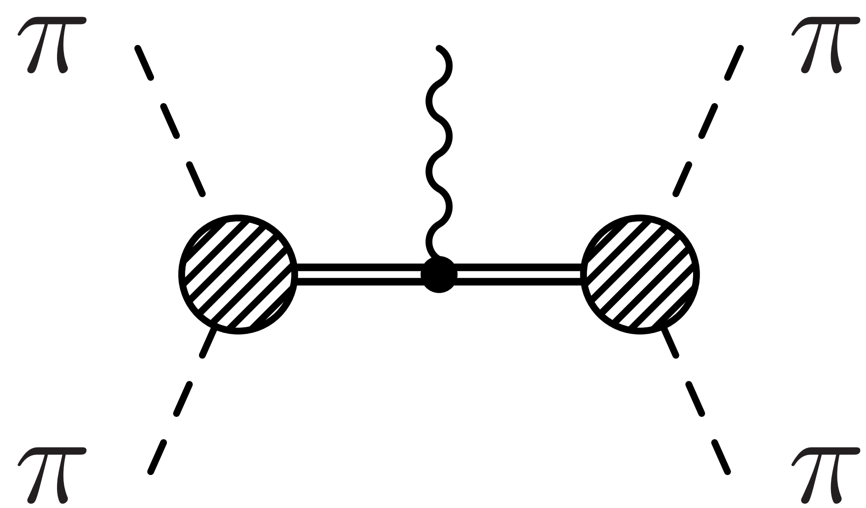
[NY Times, July 12 (2010)]

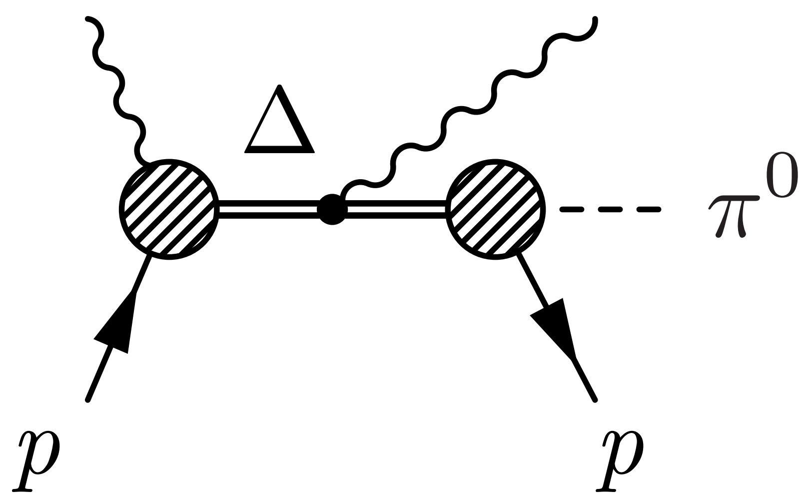
- ✦ Recent resolution of the proton radius puzzle



[Nature 575, 147–150 (2019)]

Structure of resonances



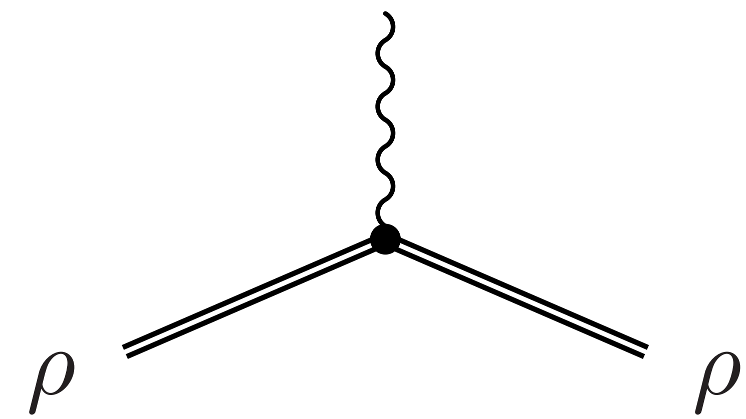


✦ Magnetic Moment of Δ dominated by model unc.

[Phys. Rev. C 64, 065202 (2001),Phys. Rev. Lett. 89, 272001 (2002)]

✦ Beam spin asymmetry ep scattering.

[Phys. Rev. D 96, 113010 (2017)]



✦ Chiral EFT calculation of ρ FFs.

[Phys. Lett. B 730, 115–121 (2014)]

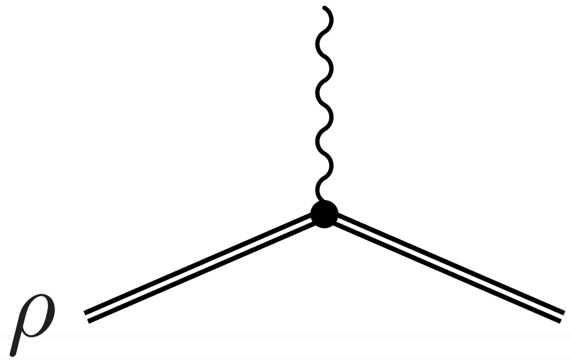
✦ Lattice calculations

[Phys. Rev. D 91, 114501 (2015), Phys. Rev. D 75, 094504 (2007)]

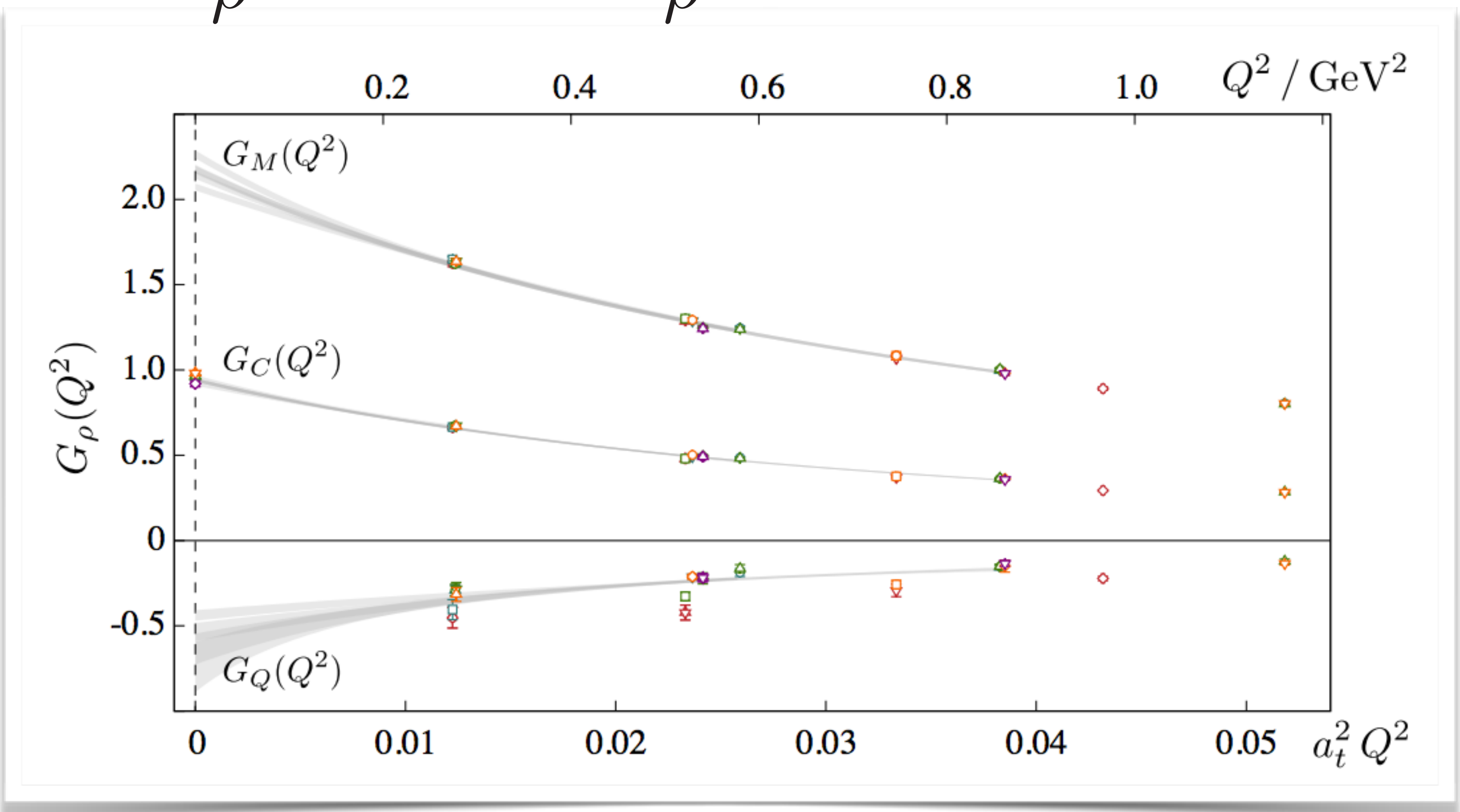
✦ From e^+e^- BaBar data.

[Int. J. Mod. Phys. A 30, 1550114 (2015)]

Vector-meson form-factors in a box



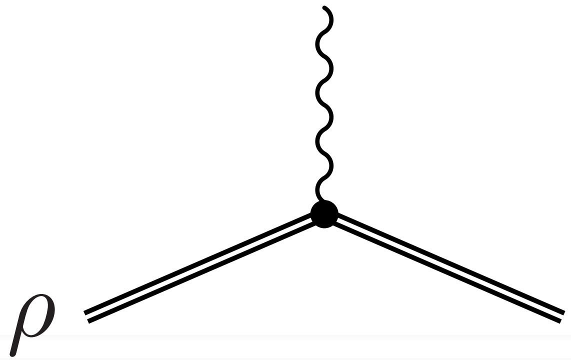
$$= \sum_i C_i^\mu G_i(Q^2)$$



[Phys. Rev. D 91, 114501 (2015)]

$SU(3)$ point, $m_\pi \sim 700$ MeV, $m_\rho \sim 1$ GeV (stable)

Vector-meson form-factors in a box



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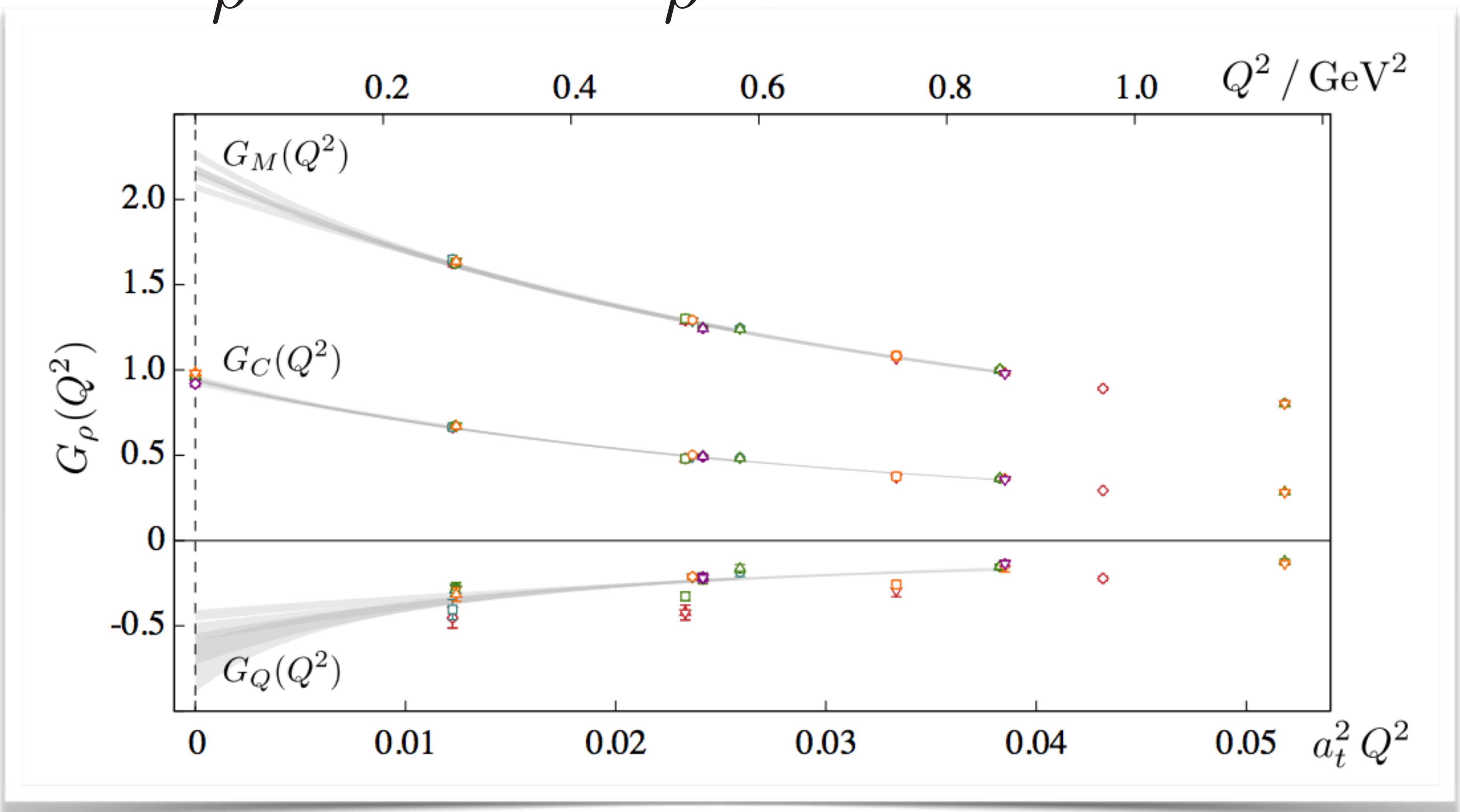
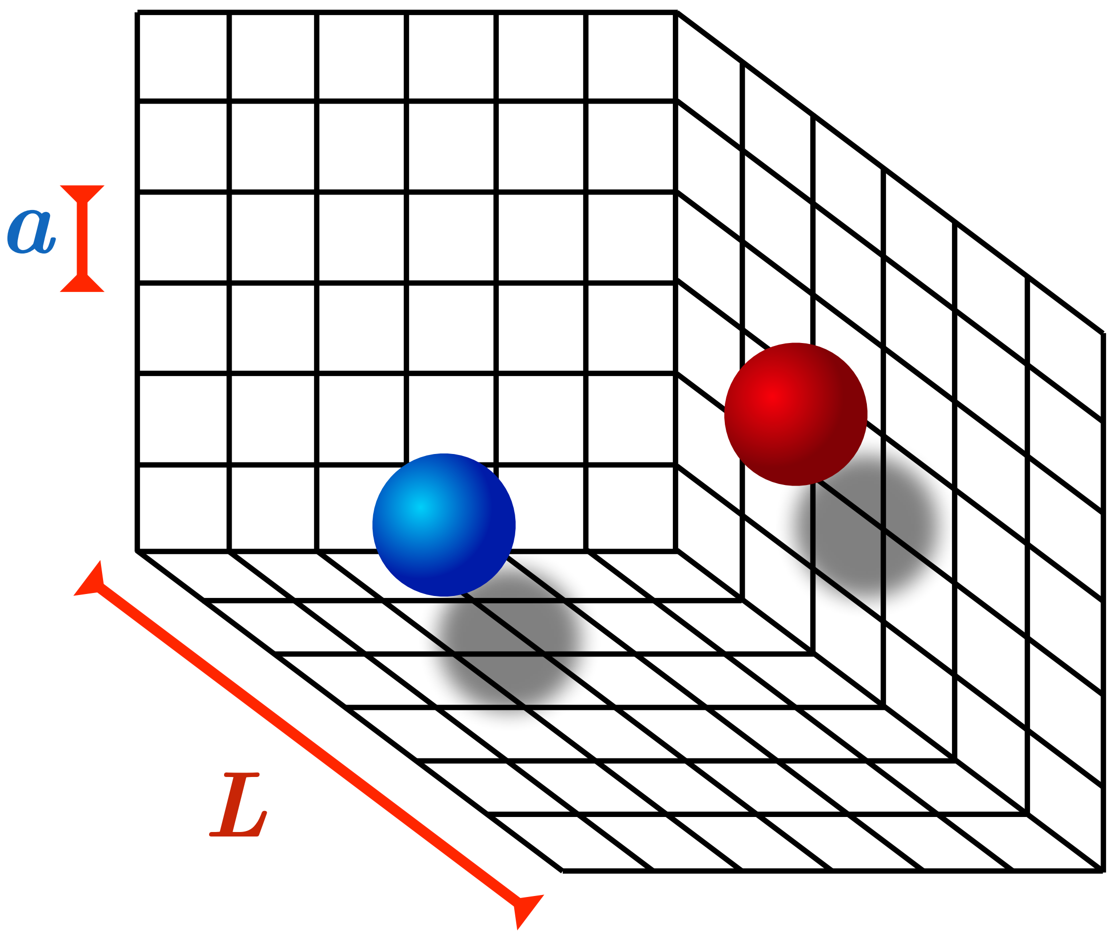


[Phys. Rev. D 91, 114501 (2015)]

$$\frac{\langle r_\rho^2 \rangle^{1/2}}{\langle r_\pi^2 \rangle^{1/2}} = 1.2(2)$$

$SU(3)$ point, $m_\pi \sim 700$ MeV, $m_\rho \sim 1$ GeV (stable)

Physics in a Finite Volume

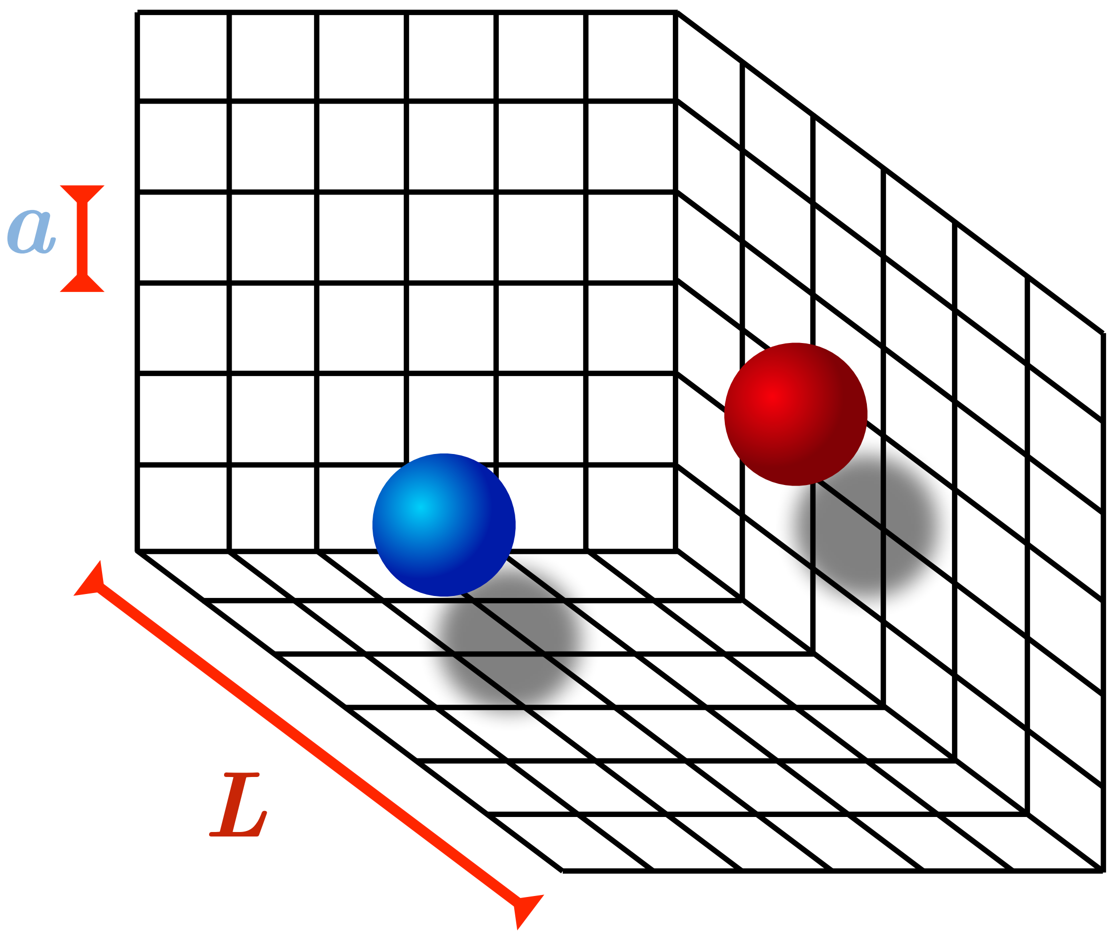


$$e^{-S_{\text{QCD}}} \implies \langle \mathcal{O}_1(\tau_1) \cdots \mathcal{O}_n(\tau_n) \rangle$$

✦LQCD Path Integral:

- ✦ Finite Lattice spacing.
- ✦ Wick rotated: Euclidean spacetime.
- ✦ Finite Volume.

Physics in a Finite Volume



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- LQCD Path Integral:
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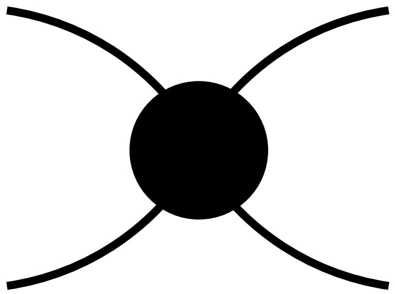


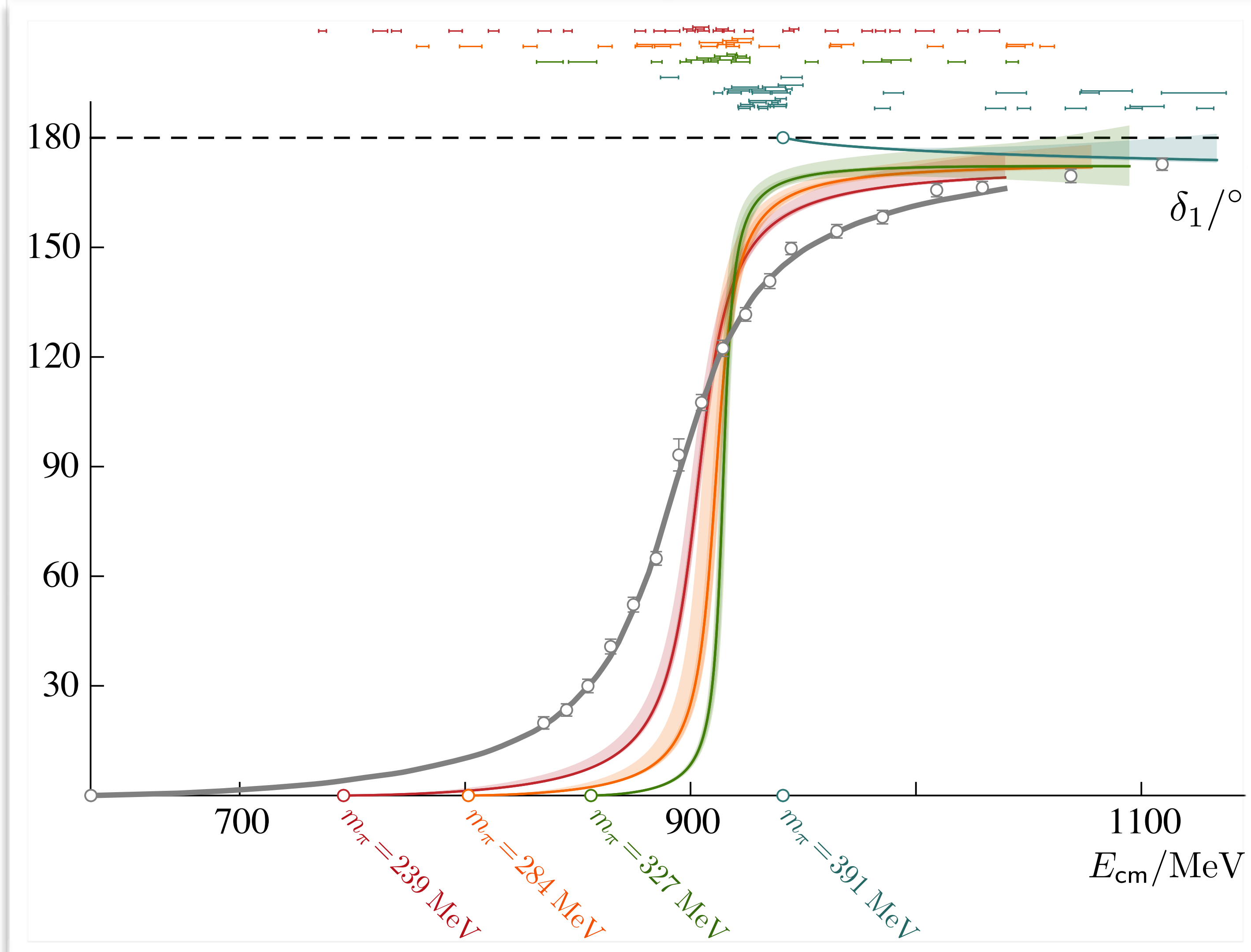
$\mathcal{M} : 2 \to 2$

Lüscher formalism

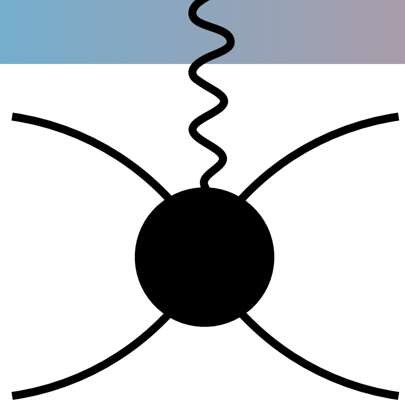
$$\det\left(F^{-1}(E_n, L) + \mathcal{M}(E_n)\right) = 0$$

- D. Wilson plenary [Tue 12pm]:
  - Two-pion systems.
  - Multiple two-particle channels.
  - Multiple partial waves and spin.
    C. Johnson talk [Wed 2:10pm]
  - Channels with exotic resonances, e.g. 1^{-+}.
    J. Dudek talk [Tue 3:30pm]

Determination on the P-wave pion-Kaon scattering.



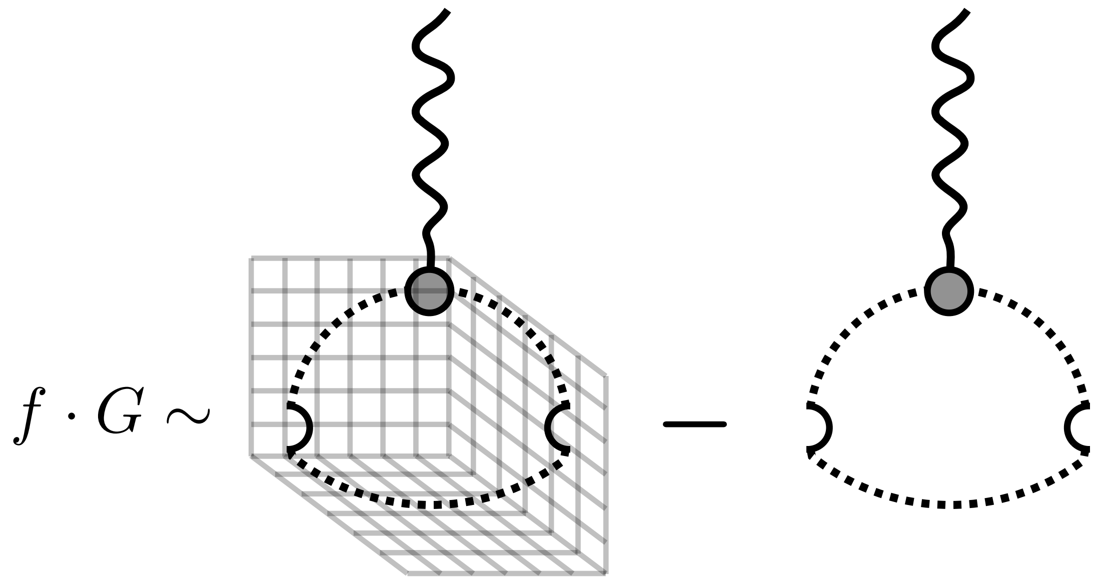
[Phys. Rev. Lett. 123, 042002 (2019)]



$\mathcal{W} : 2 + \mathcal{J} \to 2$

1. Finite $\to$ Infinite Volume

$$\left\langle \mathcal{O}_{\pi\pi}(\tau_2)\mathcal{J}(\tau_1)\mathcal{O}^{\dagger}_{\pi\pi}(0)\right\rangle \sim \sum_{n,m} \langle E_n|\mathcal{J}(Q^2)|E_m\rangle\, e^{-E_n(\tau_2-\tau_1)-E_m\tau_1}$$

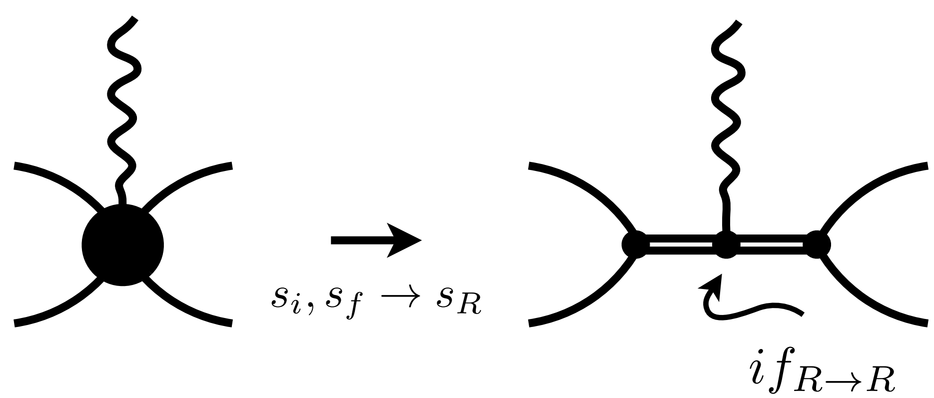


$$\left|\,\langle E_n|\mathcal{J}(Q^2)|E_m\rangle\right|^2_L \to \mathcal{W}_L$$

$$\mathcal{W} = \mathcal{W}_L - \mathcal{M}(E_n)(f\cdot G)\mathcal{M}(E_m)$$

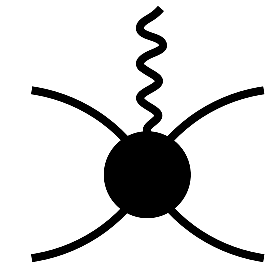
[R. Briceño, M. Hansen (2015)]
[A. Baroni, R. Briceño, M. Hansen, FO (2019)]

2. Amplitude $\to$ Form-Factors

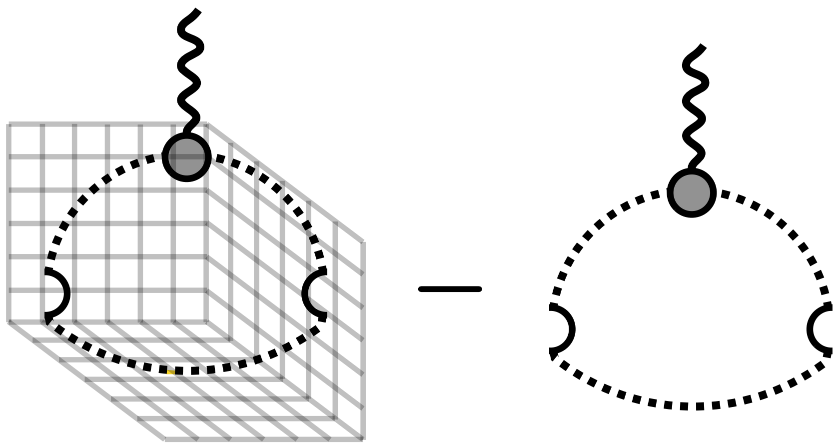


[R. Briceño, A. Jackura, FO, K. Sherman (2020)]

Finite Volume effects: G-function

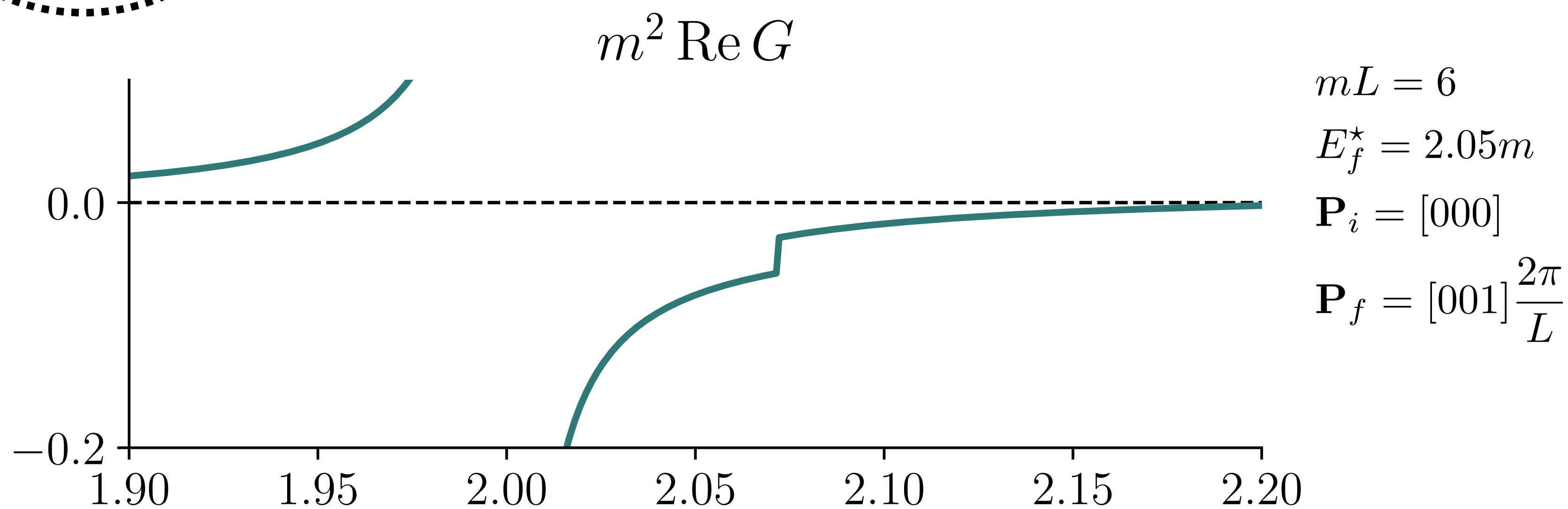


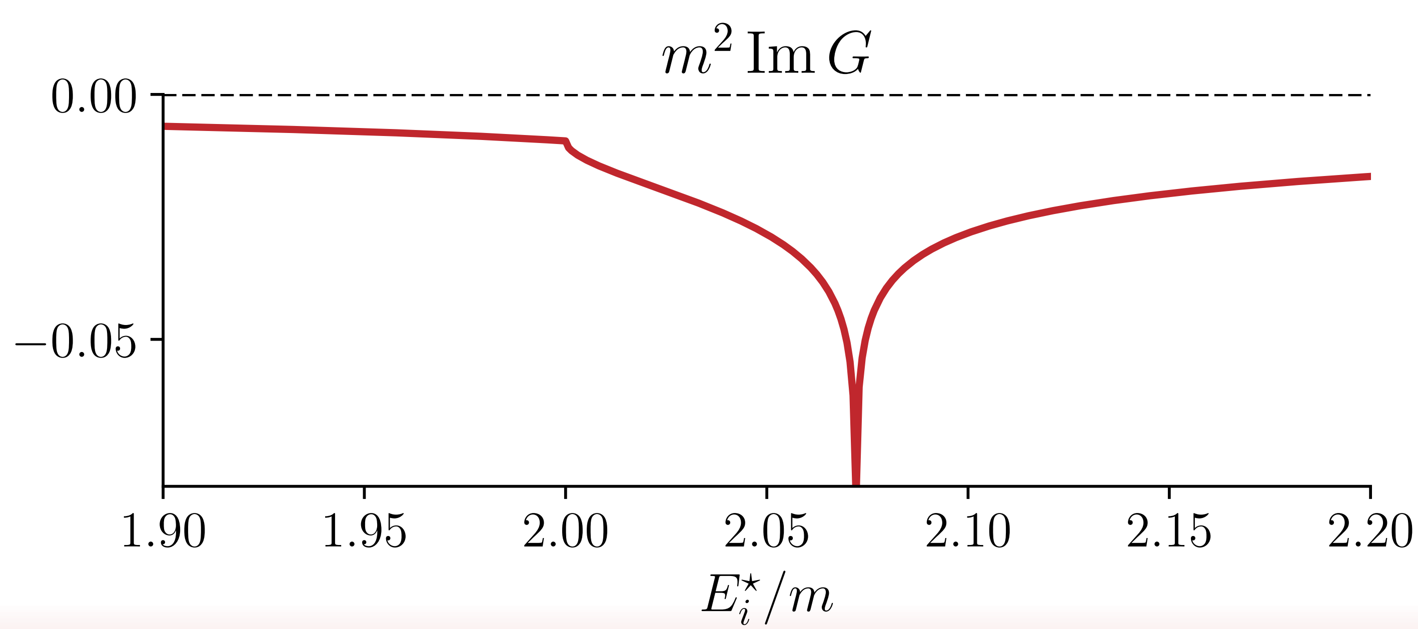
$$\mathcal{W} = \mathcal{W}_L - \mathcal{M}(E_n)(f \cdot G)\mathcal{M}(E_m)$$



$$\sim \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d^3\mathbf{k}}{(2\pi)^3}\right] \frac{i}{(P_f - k)^2 - m^2 + i\epsilon} w(Q^2) \frac{i}{(P_i - k)^2 - m^2 + i\epsilon} = f(Q^2) \cdot G(P_f, P_i)$$

Poles at FV free energies.

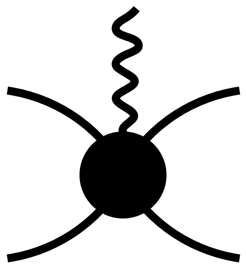




Threshold singularity.

Triangle singularity.

Finite Volume effects: G-function



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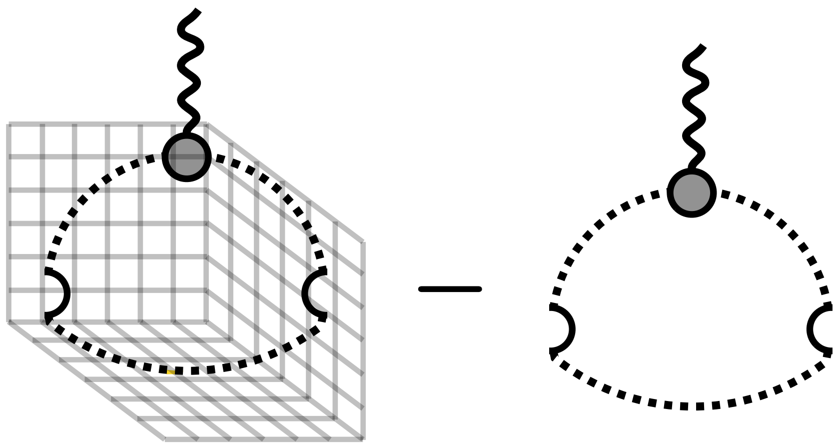

$$\sim \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d^3\mathbf{k}}{(2\pi)^3}\right] \frac{i}{(P_f-k)^2 - m^2 + i\epsilon} w(Q^2) \frac{i}{(P_i-k)^2-m^2+i\epsilon} = f(Q^2)\cdot G(P_f, P_i)$$

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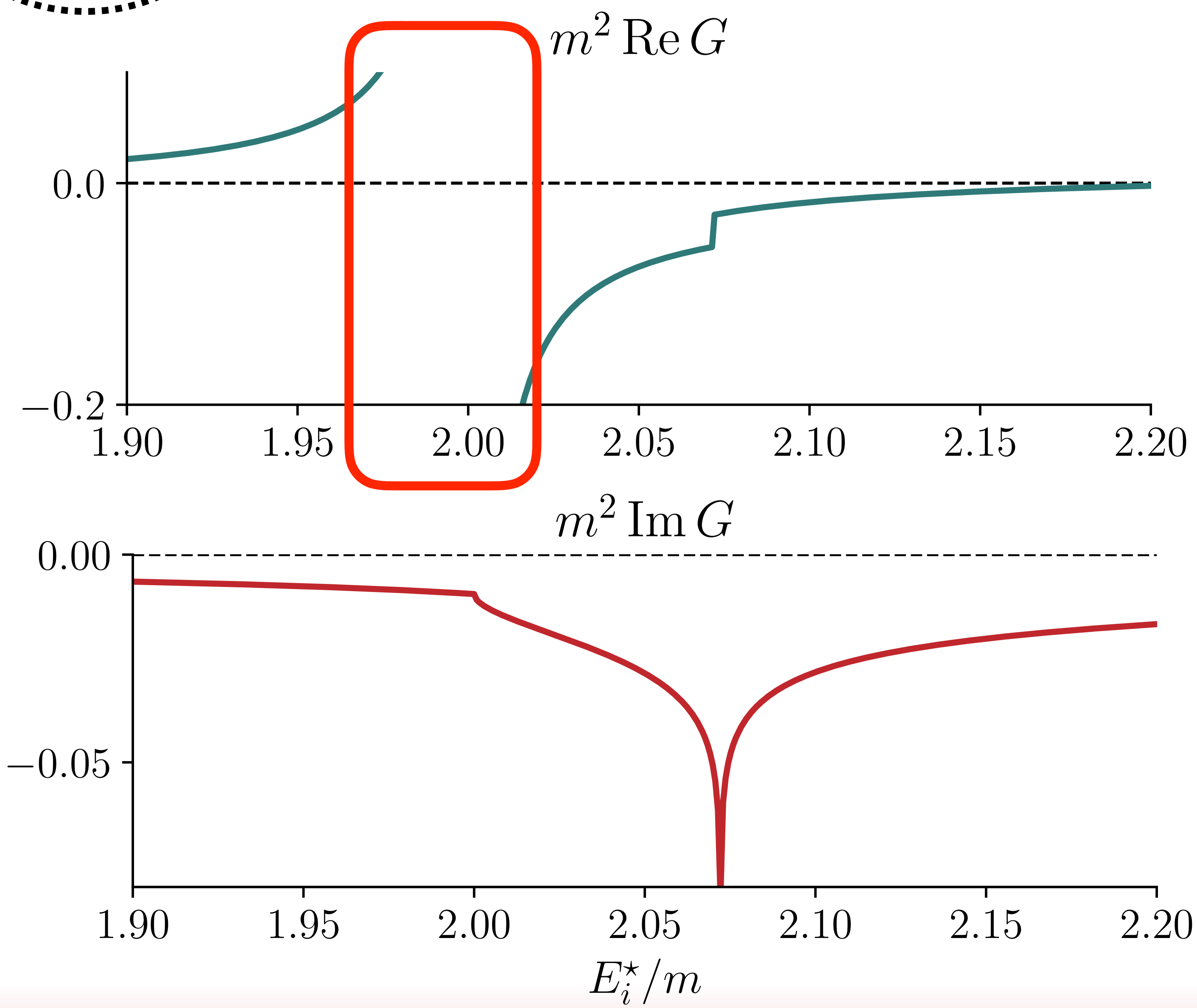


$mL = 6$

$E_f^\star = 2.05m$

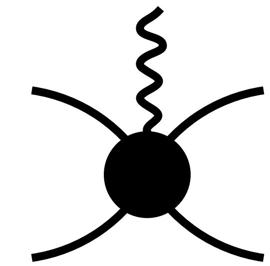
$\mathbf{P}_i = [000]$

$\mathbf{P}_f = [001]\frac{2\pi}{L}$

Threshold singularity.

Triangle singularity.

Finite Volume effects: G-function



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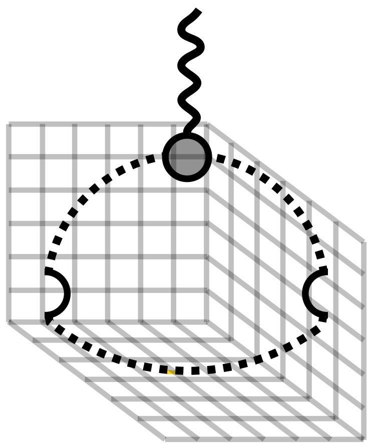
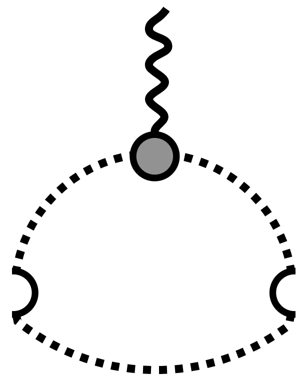
 $-$ 

$$\sim \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d^3\mathbf{k}}{(2\pi)^3}\right] \frac{i}{(P_f-k)^2 - m^2 + i\epsilon} w(Q^2) \frac{i}{(P_i-k)^2 - m^2 + i\epsilon} = f(Q^2)\cdot G(P_f, P_i)$$

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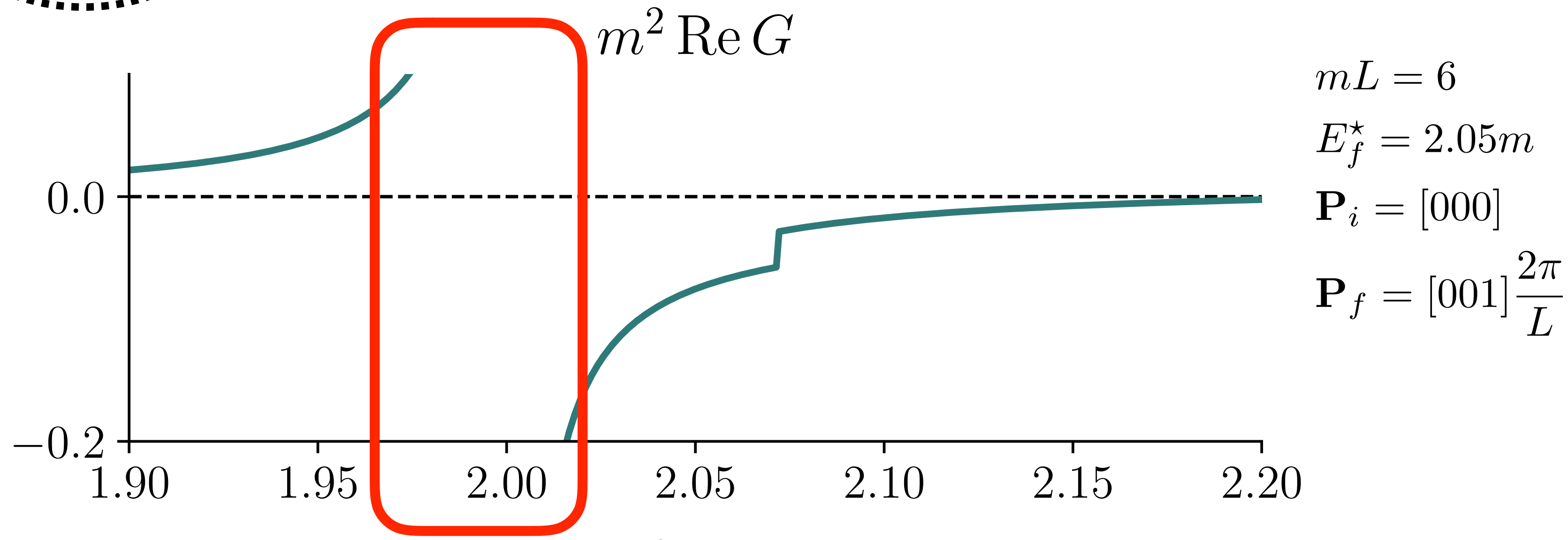

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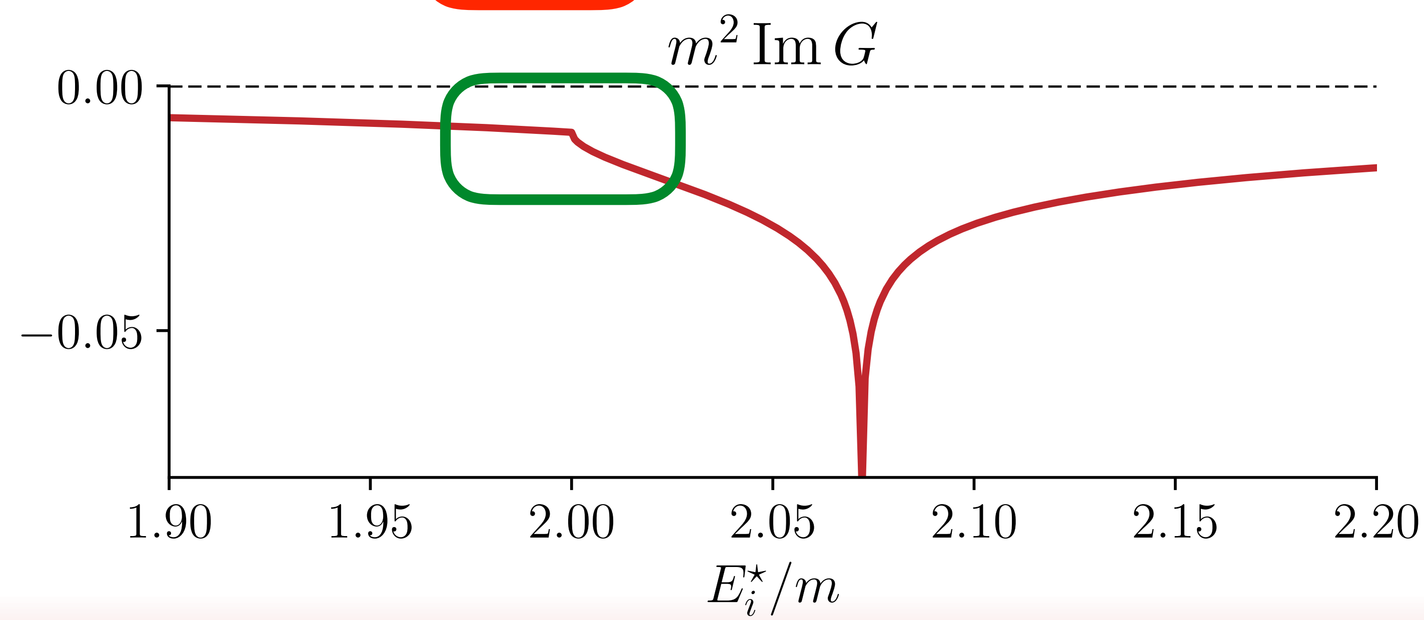

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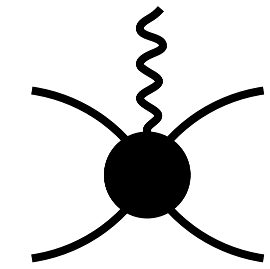

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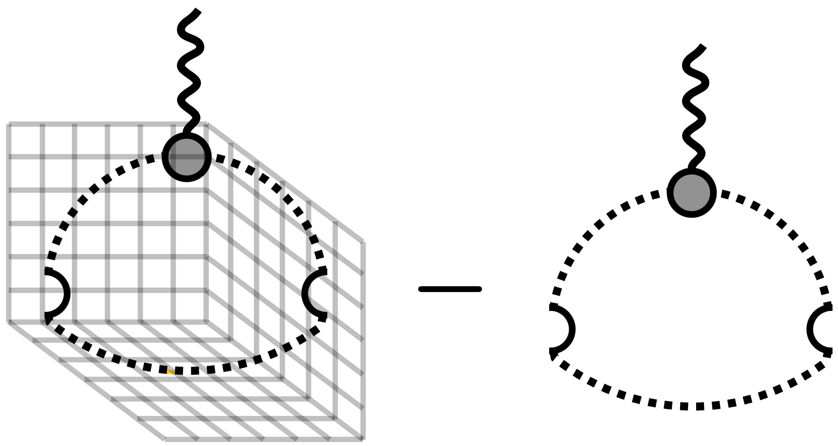

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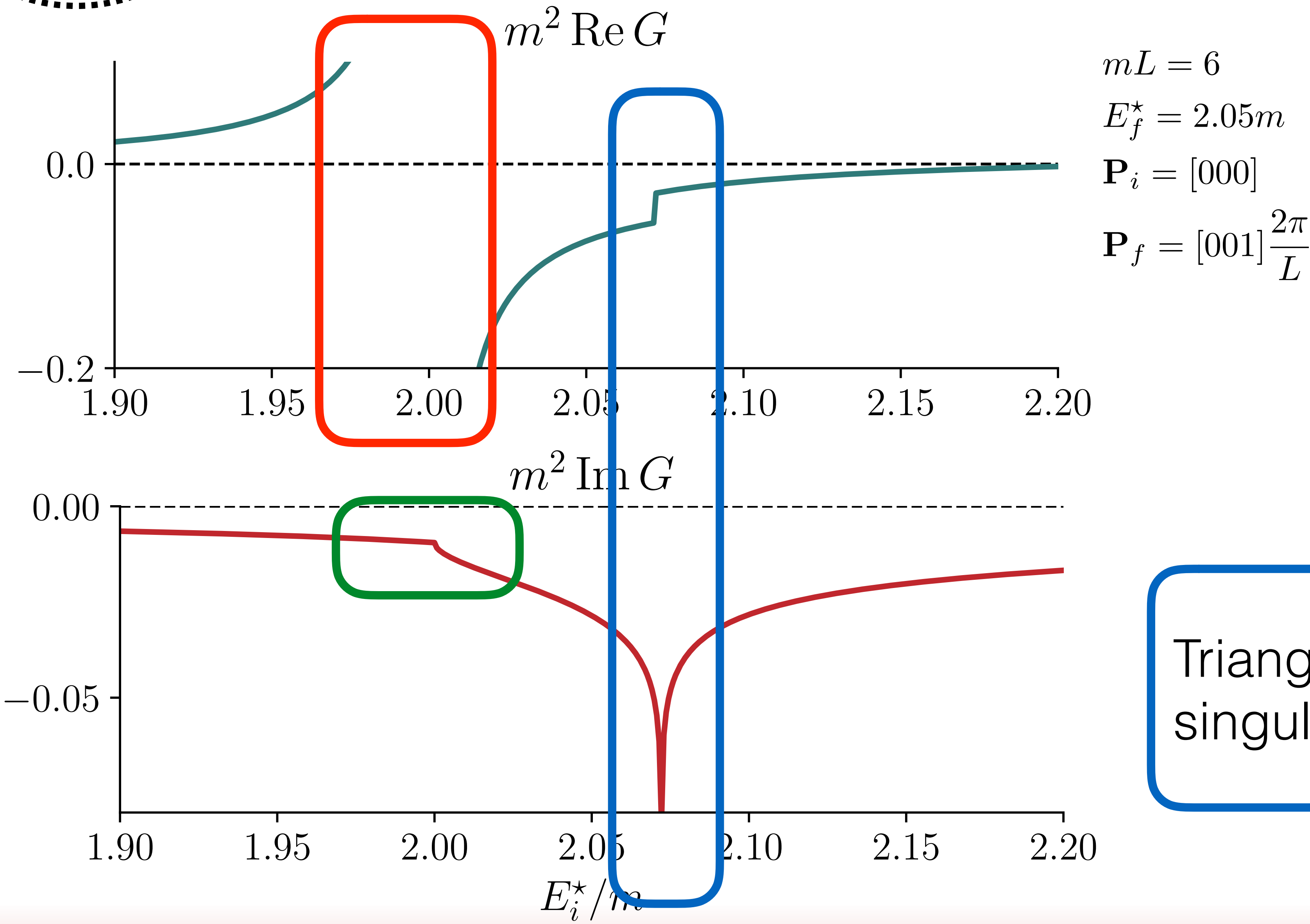
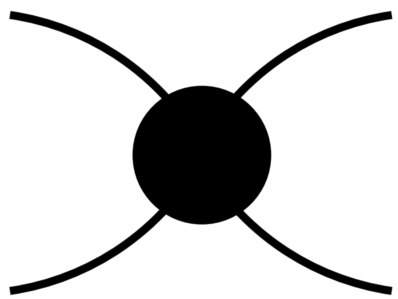
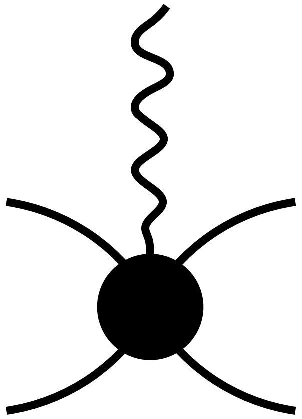
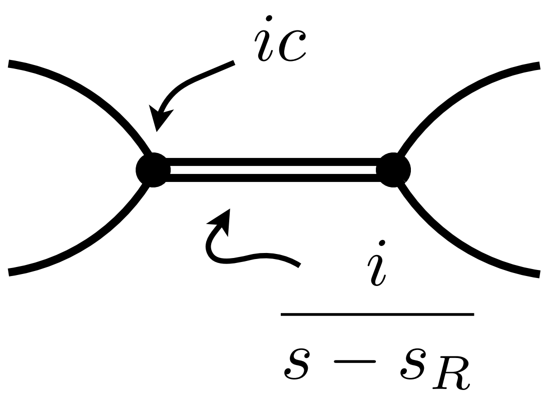


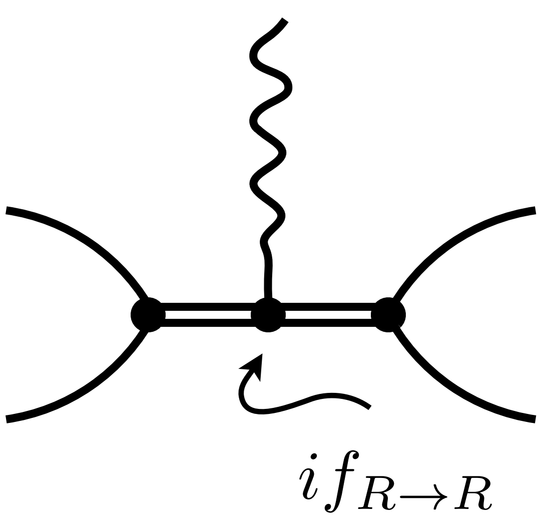
Triangle singularity.

Infinite Volume Amplitude properties

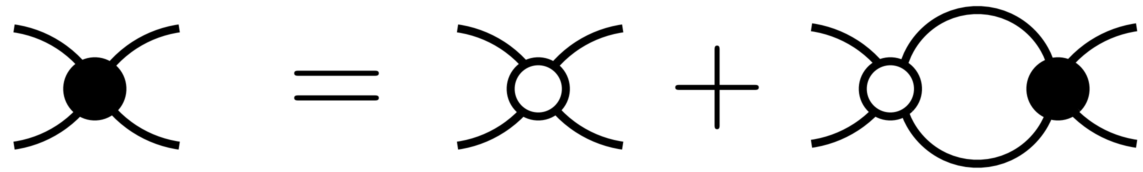




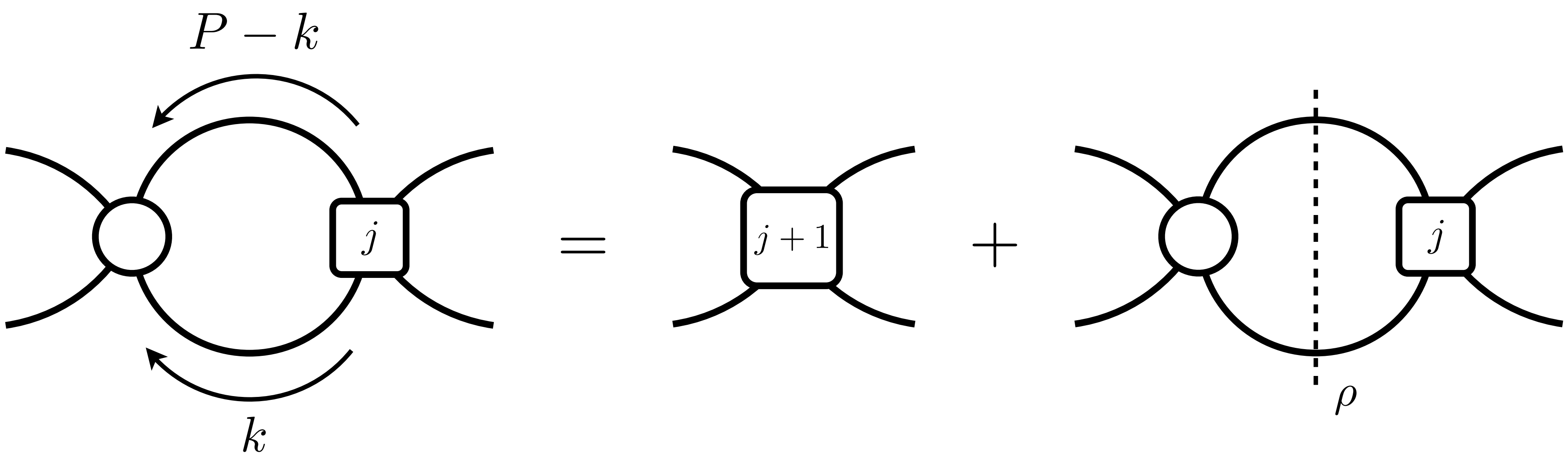




Infinite volume analytic structure

$i\mathcal{M} =$ 

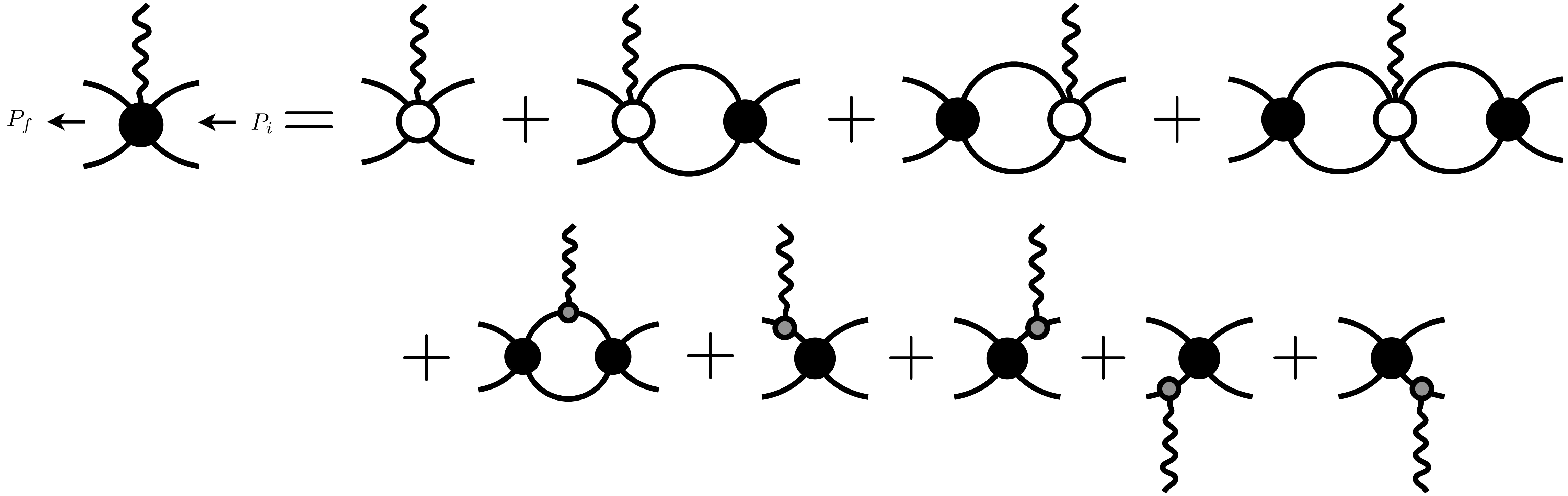
Split s channel loops depending on analytic properties



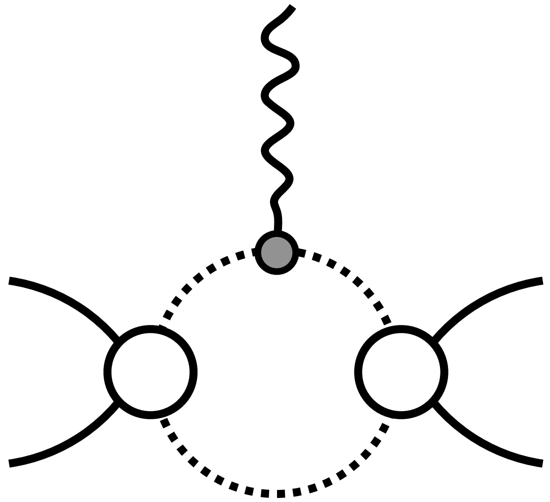
$$i\mathcal{M} = i\mathcal{K}\frac{1}{1 - i\rho\mathcal{K}}$$

TBD dynamic (analytic) function

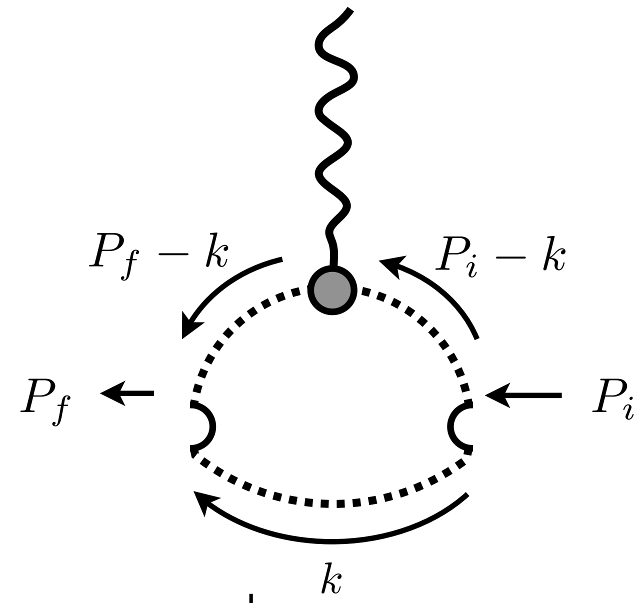
Infinite volume analytic structure: $\mathcal{W}$

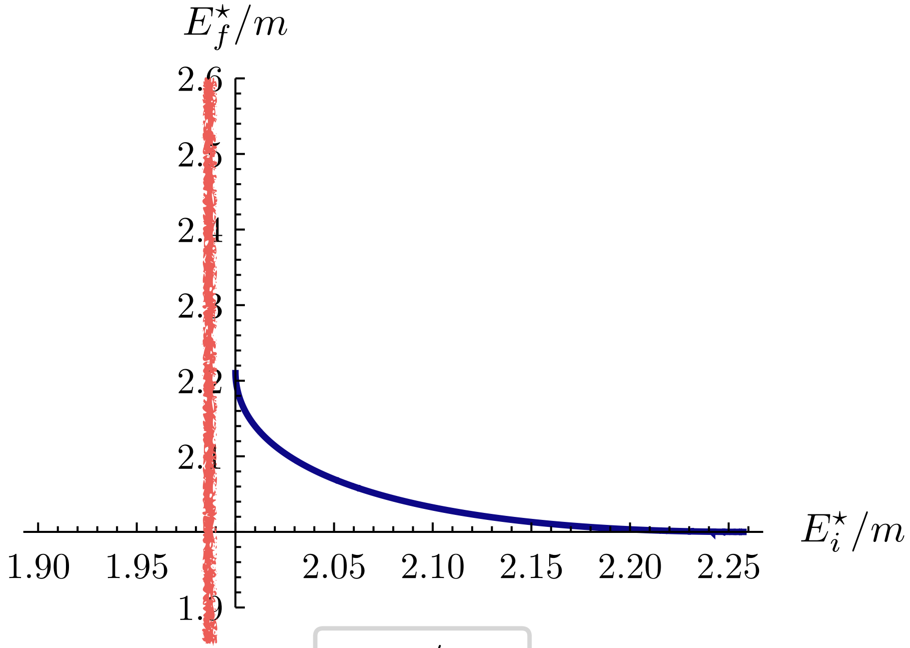


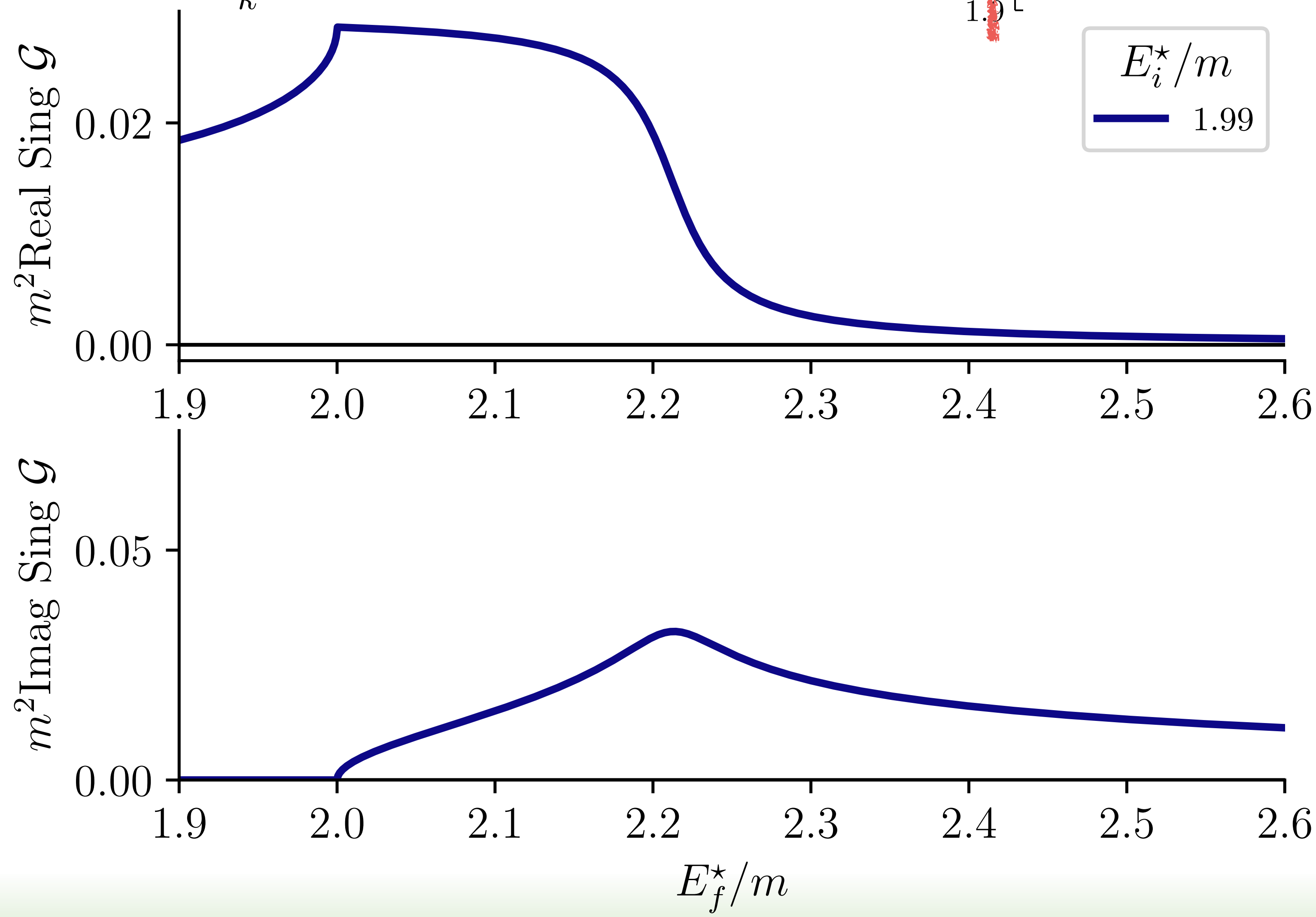
New type of singularity structure in s channel loops



Infinite volume analytic structure: $\mathcal{W}$

$f \cdot \mathcal{G} =$ 





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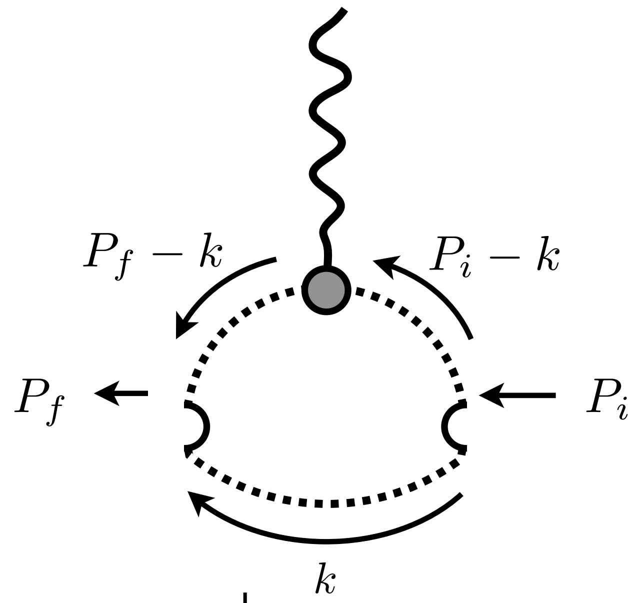

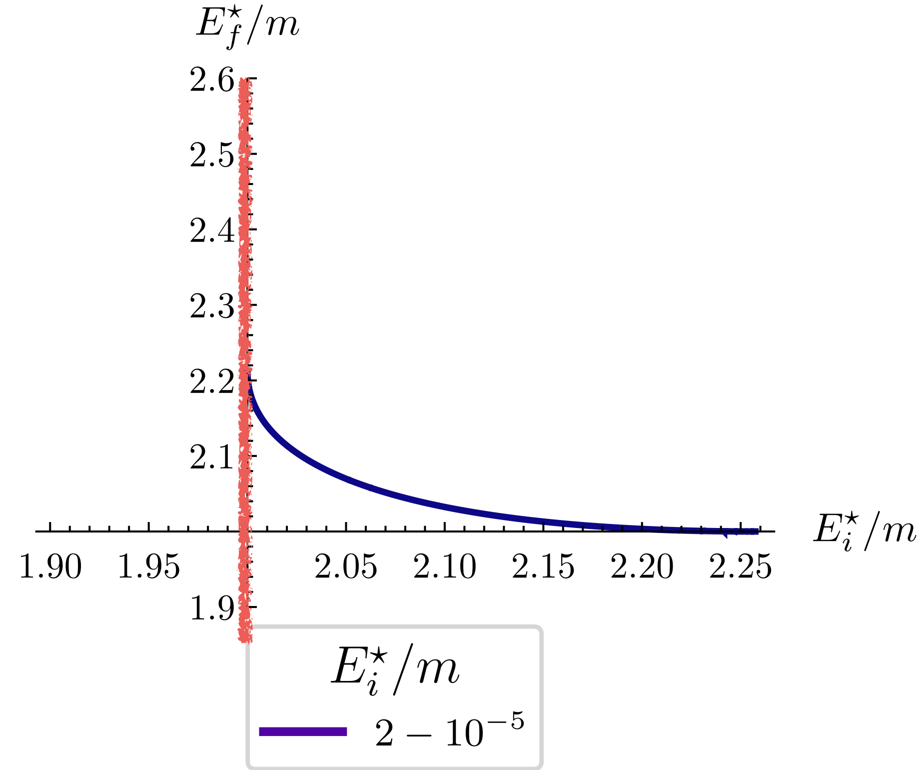

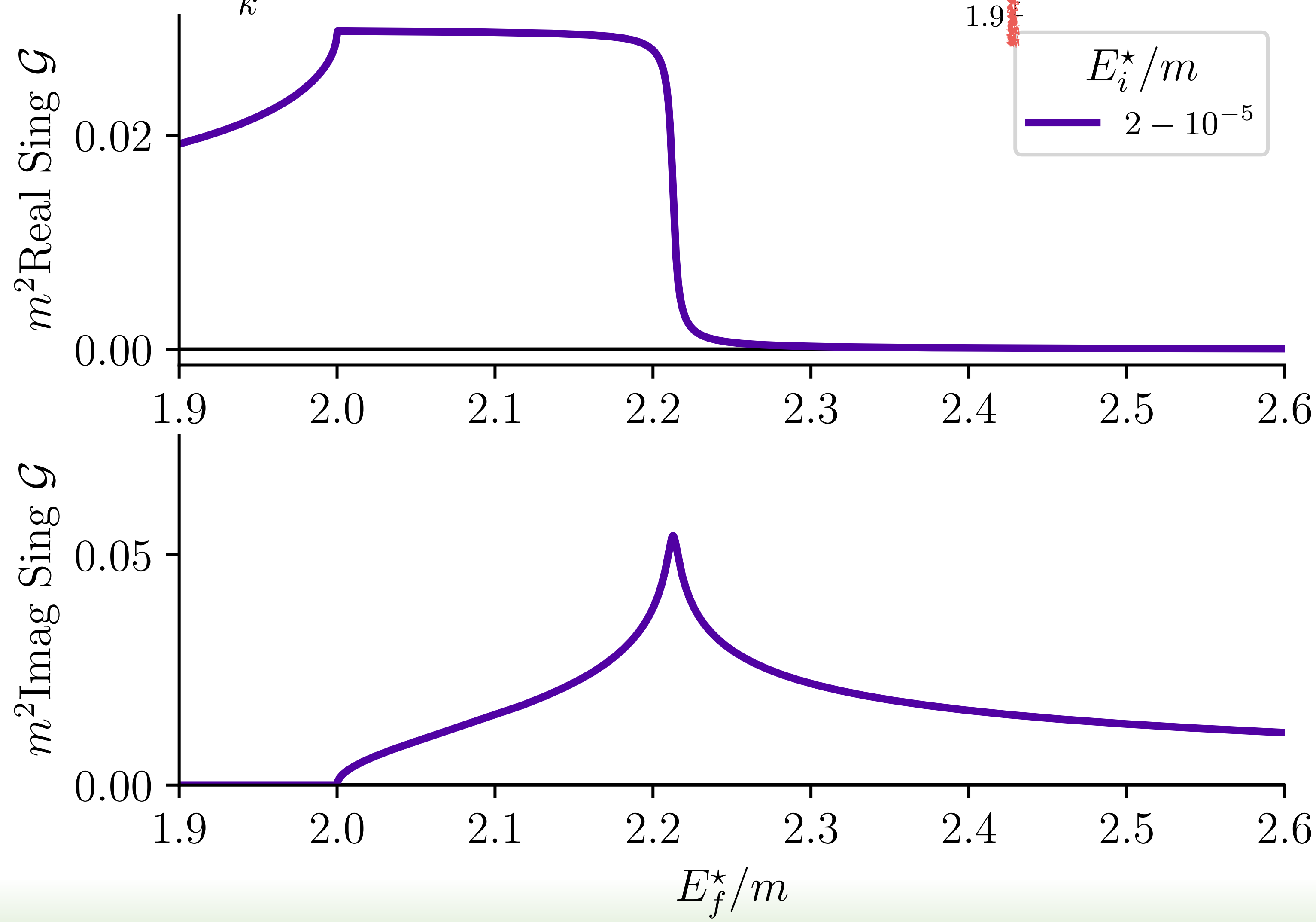

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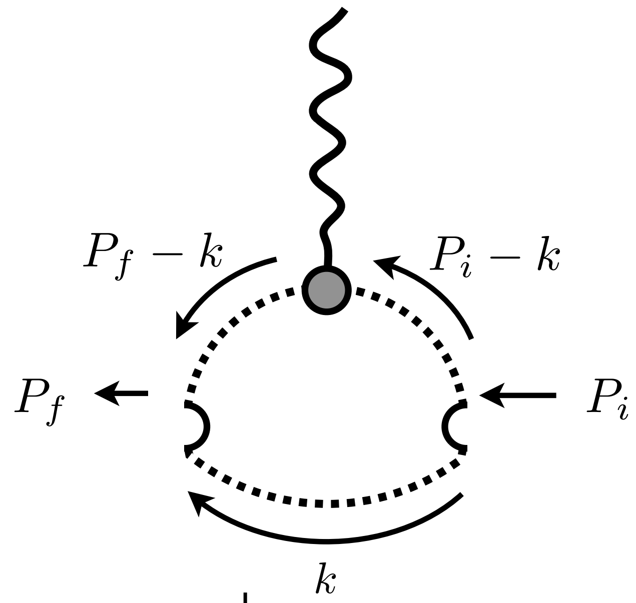

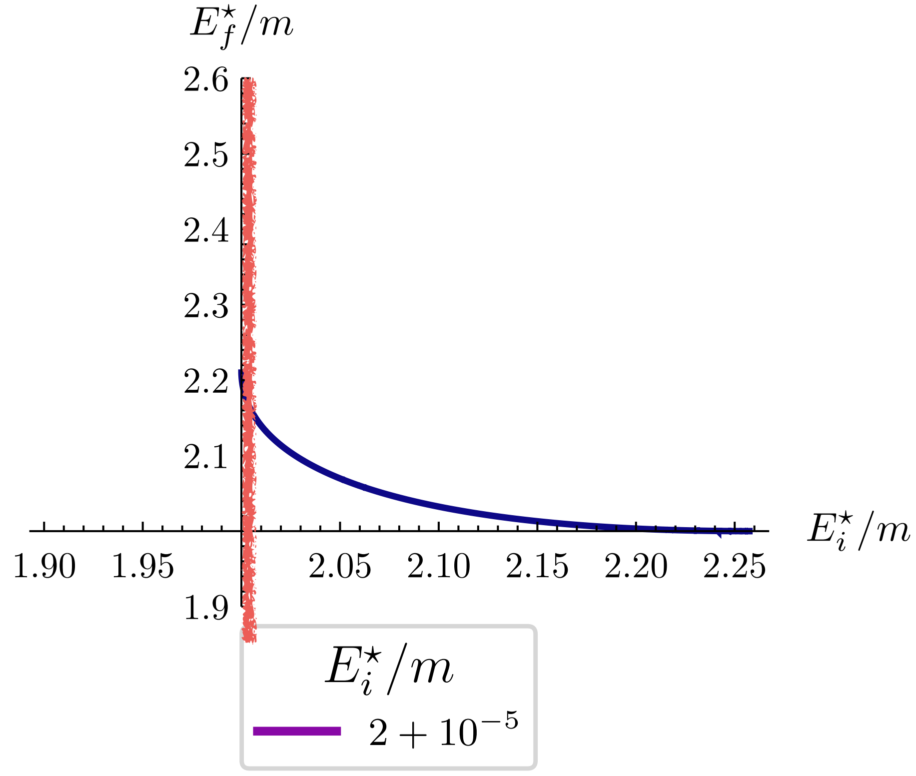

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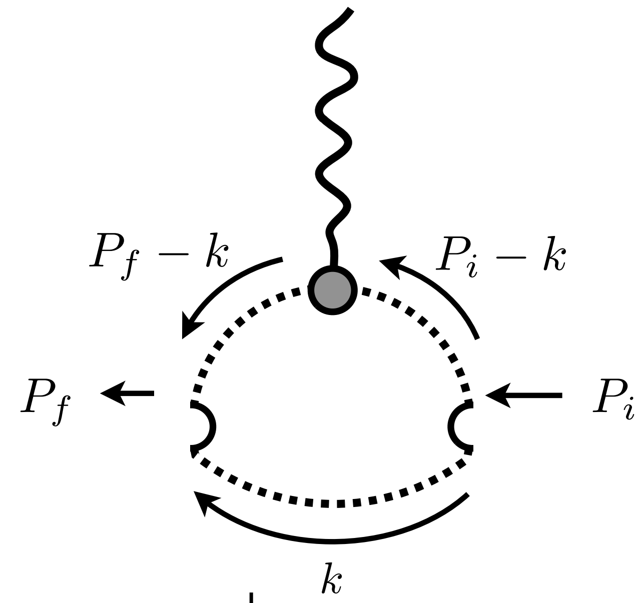

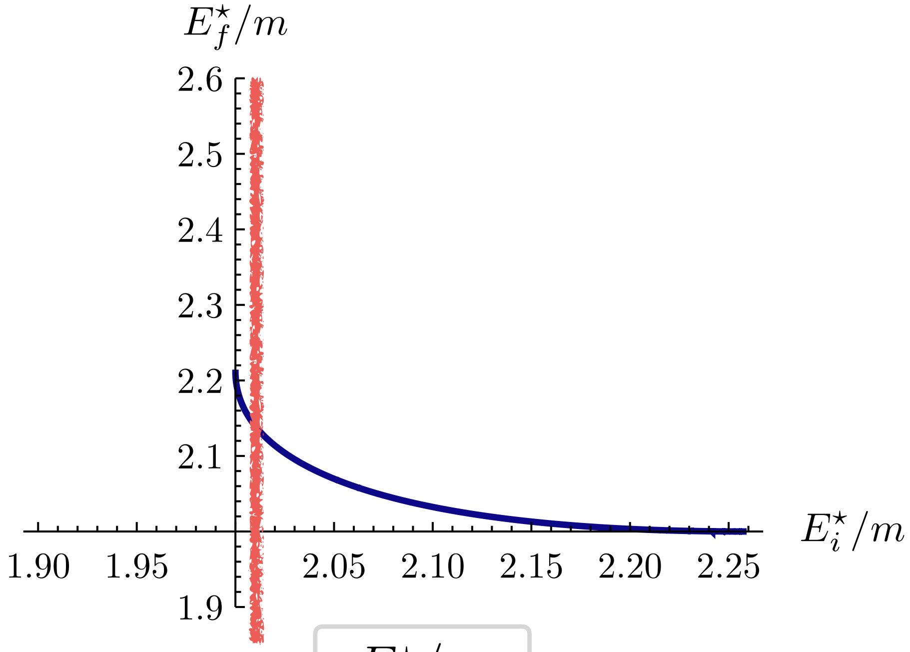

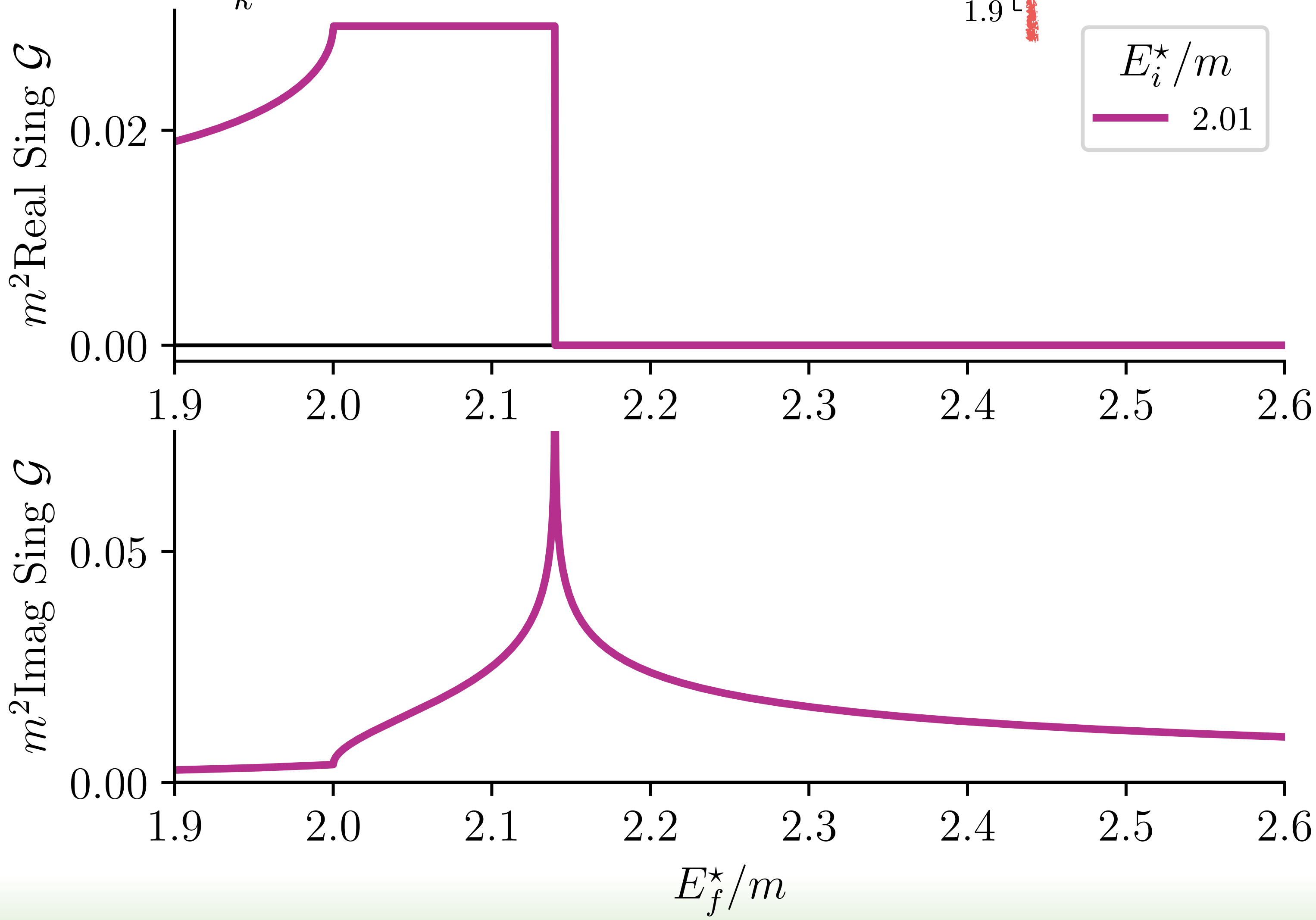

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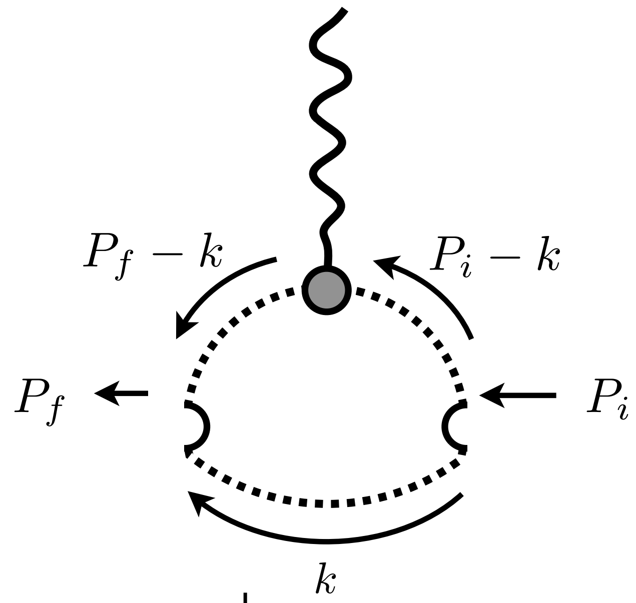

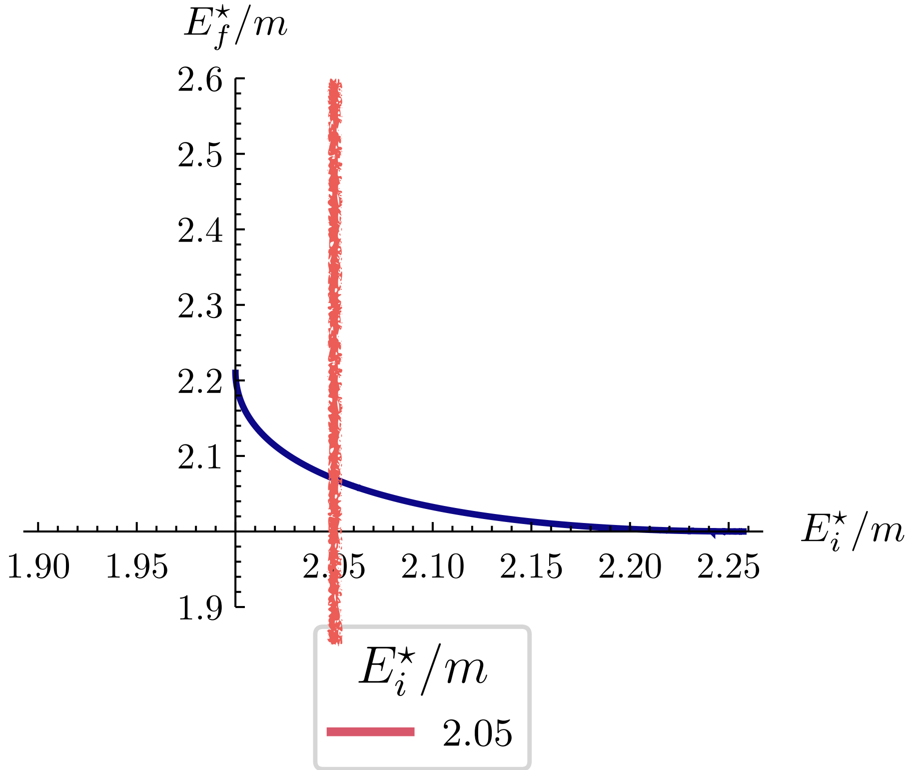

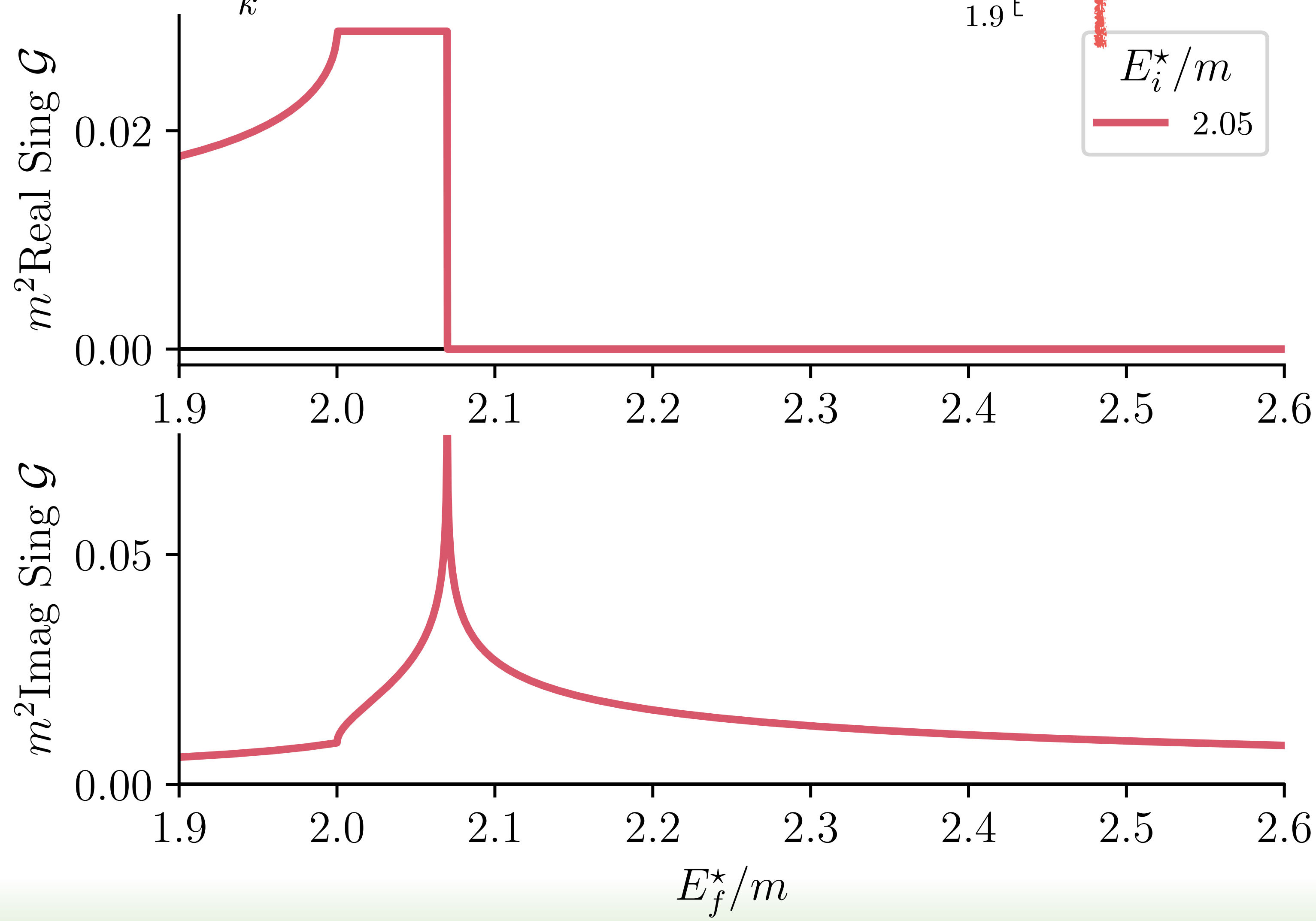

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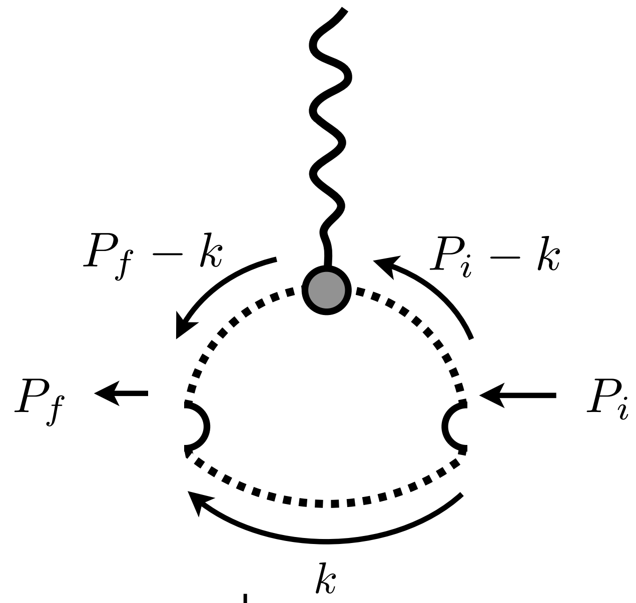
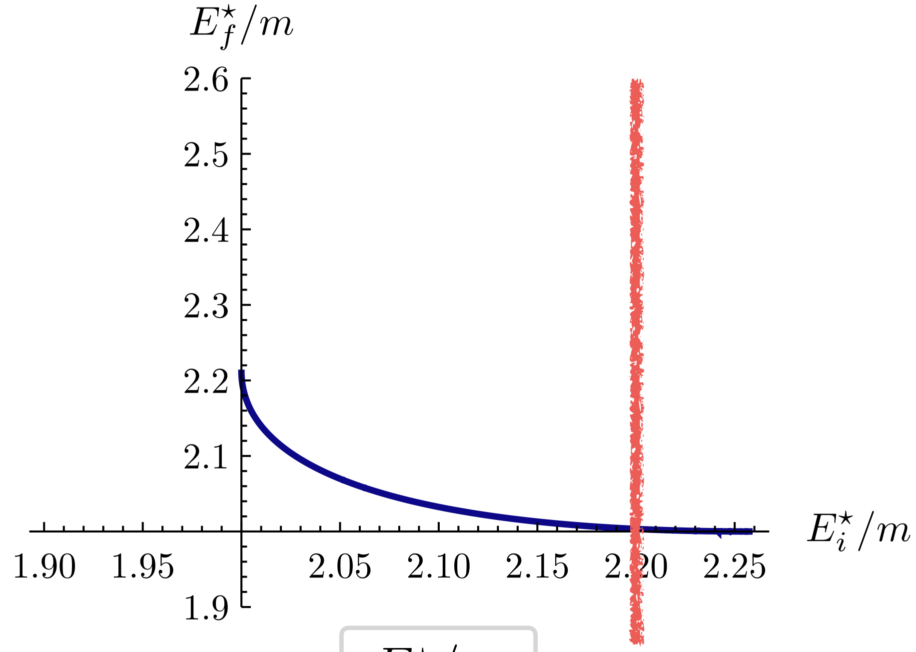
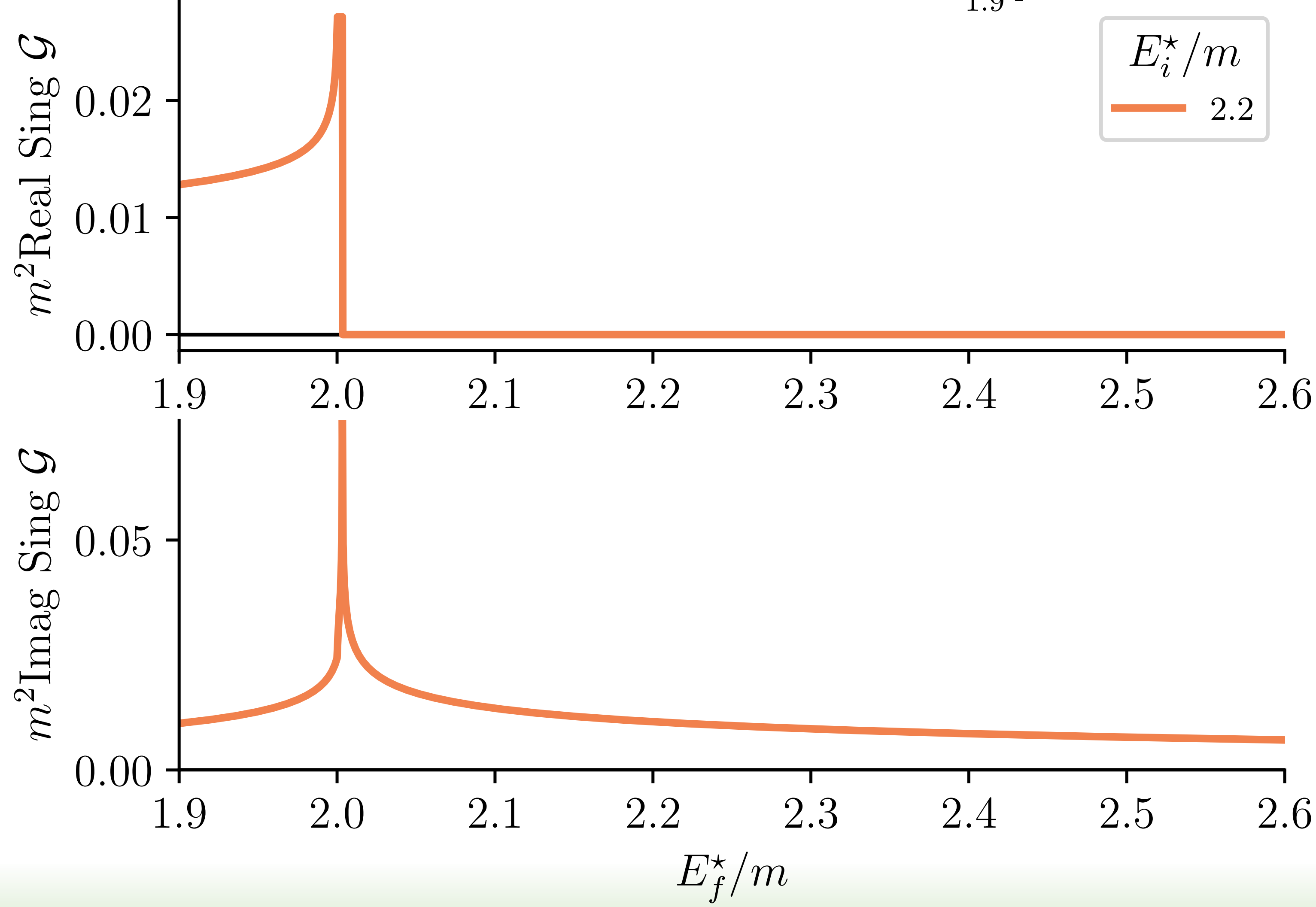
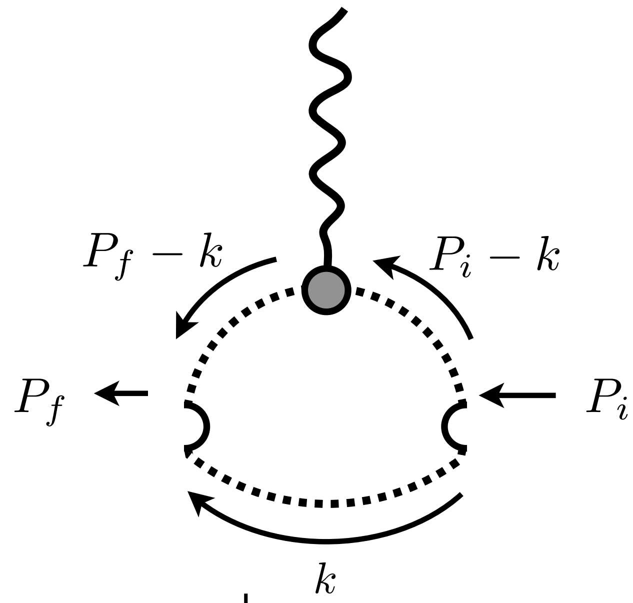
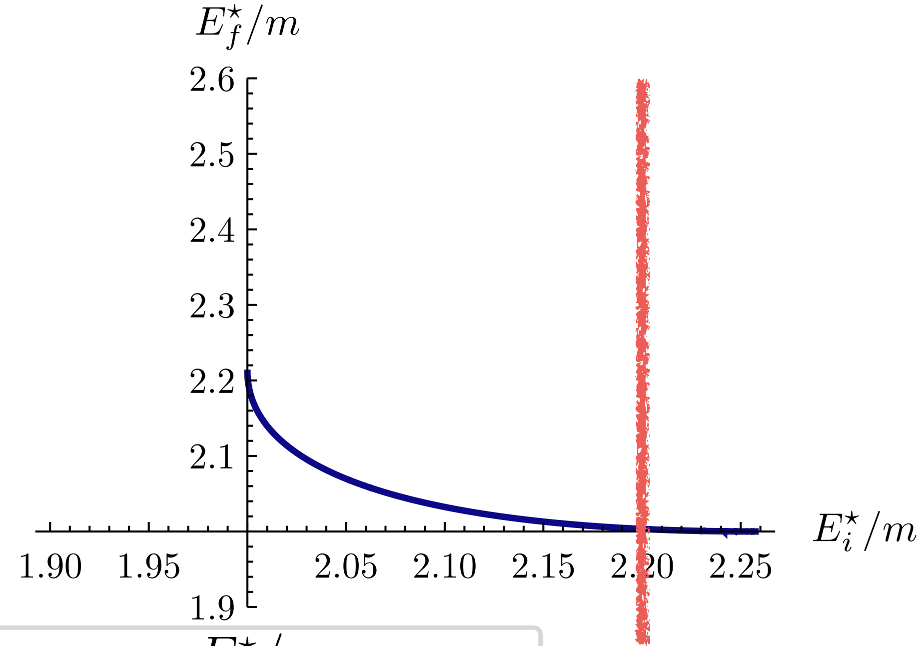
$f \cdot \mathcal{G} =$ 

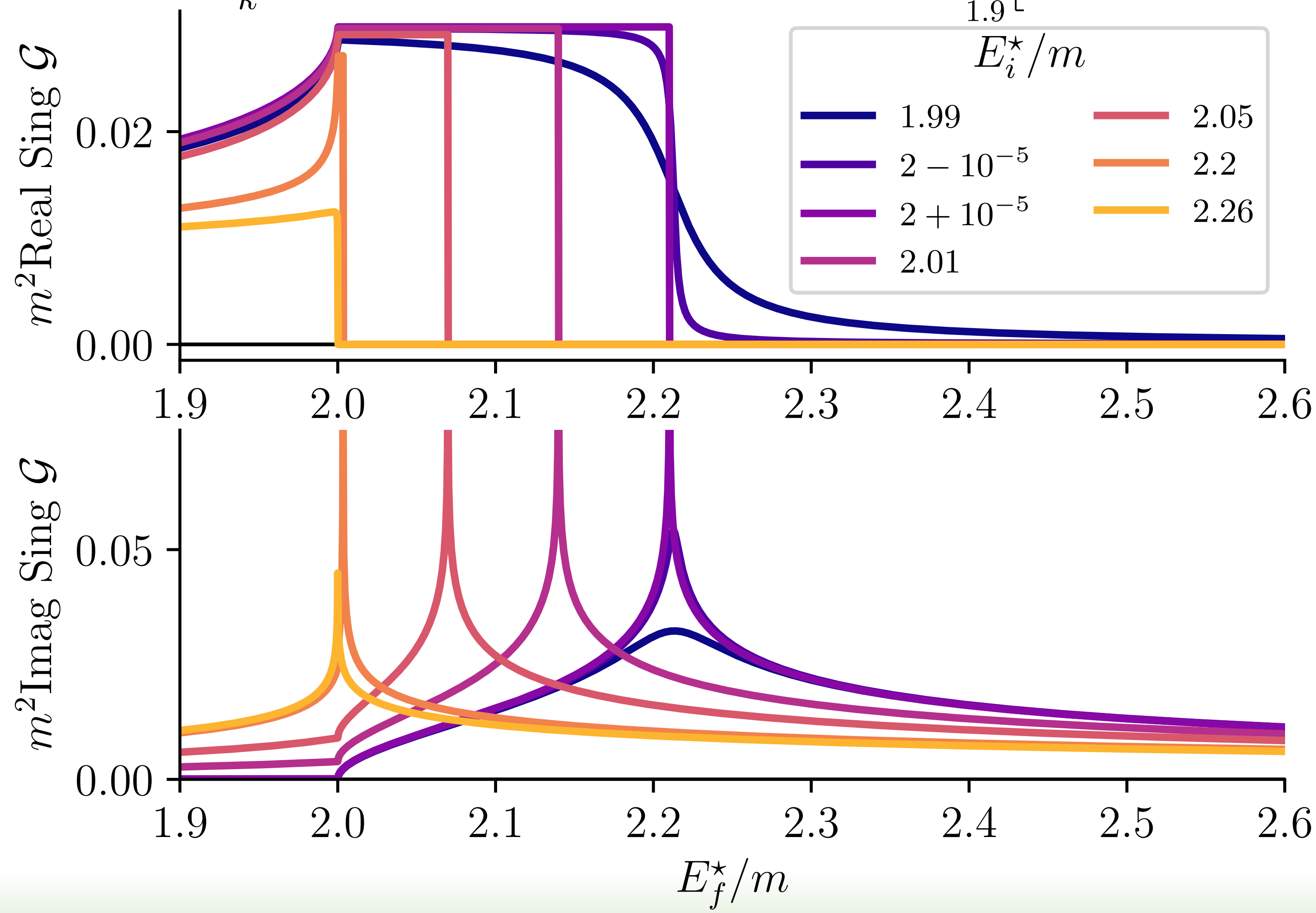




Infinite volume analytic structure: $\mathcal{W}$

$f \cdot \mathcal{G} =$ 





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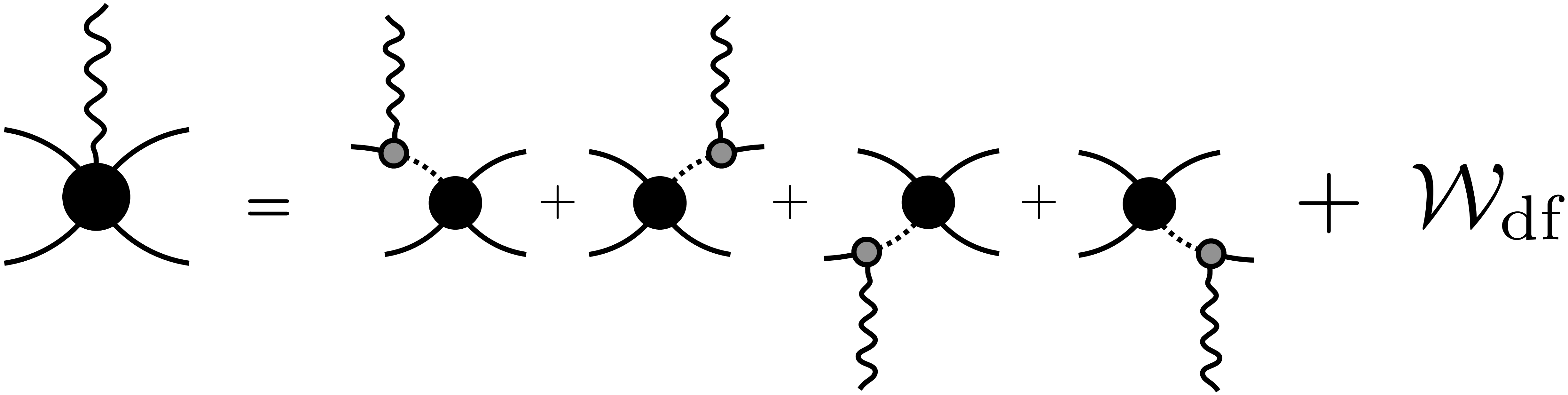
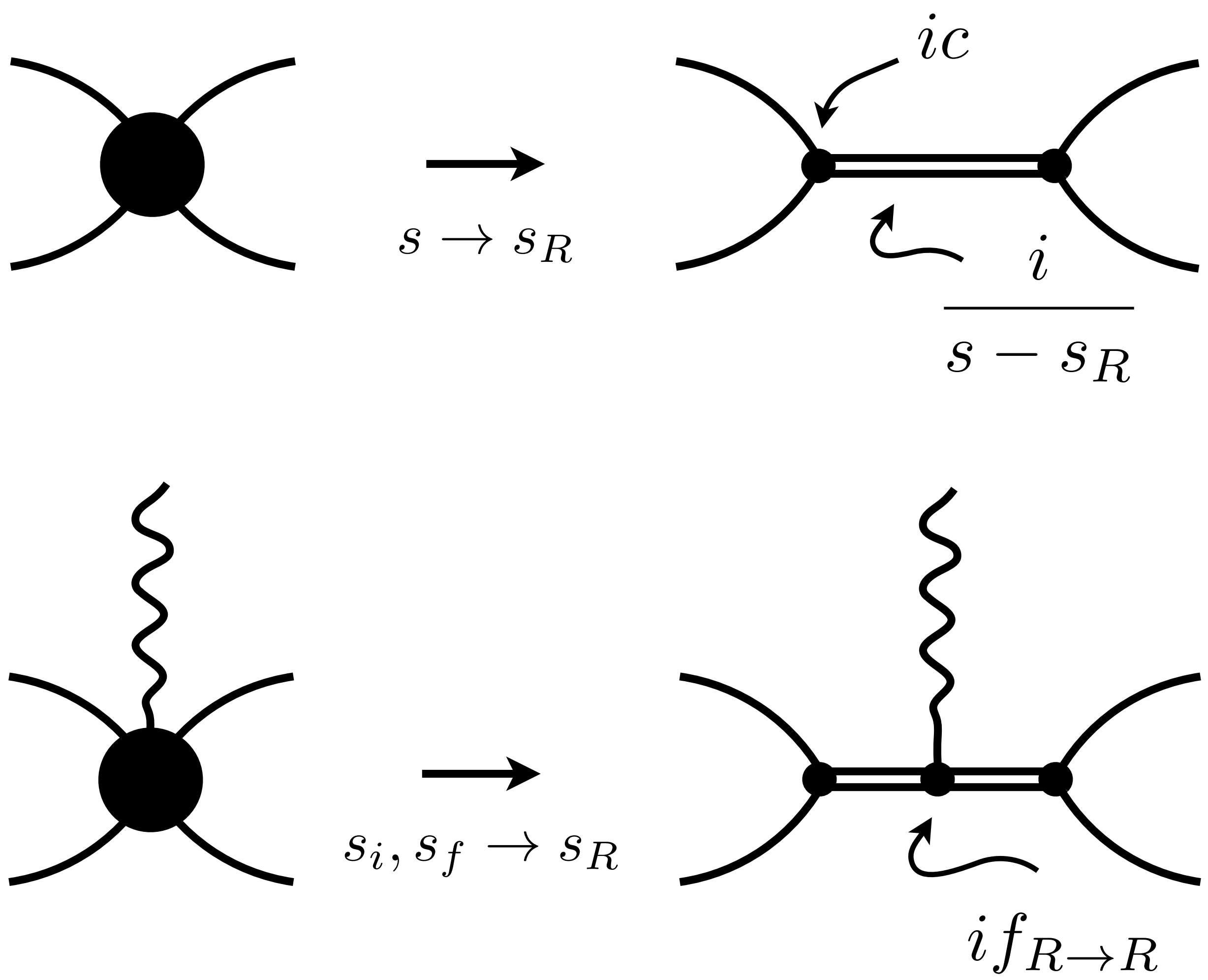


$$\mathcal{W}_{\text{df}} = \mathcal{M}(\mathcal{A}_{22} + f \cdot \mathcal{G})\mathcal{M}$$

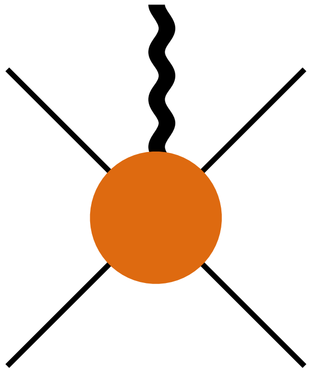
TBD dynamic function

Form factors of **resonances**



Knowing the analytic structure of the amplitudes allows a rigorous extraction of the resonance properties

Summary and overview



- LQCD formalism to calculate two-body transitions.
  - Consistency checks: NW-limit, bound states, and WT identities.
    [Phys. Rev. D 100, 114505 (2019), [Phys. Rev. D 101, 094508 (2020)]
- The isovector channel is the most amenable for a proof of principle calculation.
- Characterize scalar resonances by their response to external currents.
- How do we generalize for external spinors and vectors?

$\mathcal{G}$ analytic structure

$$\mathcal{G}_{00;00}(P_f, P_i) = \frac{i}{32\pi\sqrt{(P_f \cdot P_i)^2 - P_i^2 P_f^2}} \left[\log\left(\frac{1 + z_f^\star + i\epsilon}{1 - (z_f^\star + i\epsilon)} \right) + \log\left(\frac{1 + z_i^\star + i\epsilon}{1 - (z_i^\star + i\epsilon)} \right) \right] + \ldots ,$$