The x-dependence of twist-3 PDFs from lattice QCD

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TEMPLE UNIVERSITY

based on: arXiv: 2004.04130 (PRD) & article in preparation

In collaboration with

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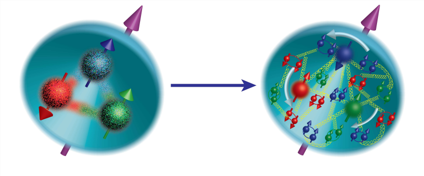
Outline

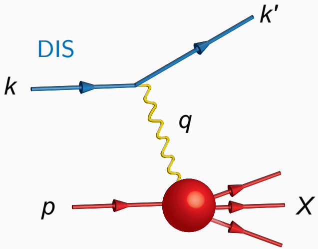
1. Why twist-3 PDFs?
2. Lattice procedure: Quasi-PDF approach
3. Results: $g_T(x)$ and $h_L(x)$
4. Summary & Outlook

Why twist-3 PDFs?

Studying hadron structure

- Hadron structure typically studied through scattering processes





[EIC, Eur.Phys.J.A 52 (2016) 9, 268]

- Factorization theorems apply (due to asymptotic freedom)

$$\sigma_{DIS}(x,Q^2) = \sum_i \left[H^i_{DIS} \otimes f_i \right](x,Q^2)$$

  - H^i_{DIS}: perturbative part (short distance interaction)
  - $f_i(x,Q^2)$: non-perturbative part (PDF of a given parton i, for forward kinematic)

Twist classification of PDFs

- Infinite number of PDFs inside a hadron

$$f^0 + \frac{1}{Q} f^1 + \frac{1}{Q^2} f^2 + \ldots$$

twist-2 twist-3

- PDFs classified according to their twist - order in $1/Q$ at which PDFs appear in factorization theorem of structure functions

Leading twist-2 PDFs:

⁎ probability density of finding a parton with momentum fraction x of the hadron's momentum

⁎ some of them very well known

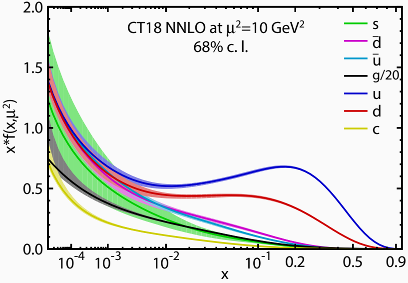
Twist-3 PDFs:

⁎ information about *qgq* correlations

⁎ kinematically suppressed

⁎ challenging to probe experimentally (JLAB 12 GeV upgrade, EIC)

Unpolarized twist-2 PDFs



[Kovarik et al., 2019, Rev.Mod.Phys. 92 (2020) 4, 045003]

Our lattice work on twist-3 PDFs

We focus on:

1. Isovector $g_T^{u-d}(x)$
2. Isovector $h_L^{u-d}(x)$

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$$\Phi^\Gamma(x,S) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle P,S|\bar\psi\left(-\frac{z}{2}\right)\Gamma W\left(-\frac{z^-}{2},\frac{z^-}{2}\right)\psi\left(\frac{z}{2}\right)|P,S\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

- $\Gamma=\gamma_\perp^i\gamma_5$ (for g_T) and $\Gamma=\sigma^{-+}\gamma_5$ (for h_L) - all indices perpendicular to the proton boost
- Light-cone distances not accessible on Euclidean lattice $\Rightarrow$ *Quasi-PDF approach*

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* g_T and h_L contaminated by leading twist-contributions, e.g. $g_T = g_T^{tw2} + g_T^{tw3}$

- We don't attempt to disentangle $tw2$ from $tw3$ (*qgq* matrix elements would be needed)
- Test of the *Wandzura-Wilczek approximation* **[S. Wandzura and F. Wilczek, Phys. Lett.72B, 195(1977)]** **[R.L. Jaffe and Xiang-Dong Ji, Phys.Rev.Lett. 67 (1991) 552-555]**

E.g.: $g_T(x) \stackrel{?}{=} \int_x^1 dy\, \frac{g_1(y)}{y}$ $\qquad g_1(y)$: helicity twist-2 PDF

Lattice procedure: Quasi-PDF approach

Access twist-3 PDFs through the quasi-distribution approach [X.Ji, Phys. Rev. Lett.110(2013) 262002]

- PDFs computed through purely space-like correlation functions

$$\tilde{\Phi}^{\Gamma}(x,\mu,P_3) = \int \frac{dz}{4\pi} e^{-ixP_3 z} \langle \underbrace{N(P_3)|\bar{\psi}(0)\Gamma W(0,z)\psi(z)|N(P_3)}_{\text{matrix elements of fast moving nucleons}} \rangle$$

$\ast$ μ: renormalization scale $\quad$ $\ast$ $z=(0,0,0,z)$ $\quad$ $\ast$ $\vec{P}=(0,0,P_3)$: proton boost

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- For sufficiently large P_3 ($P_3 >> M$), quasi-PDFs matched onto light-cone PDFs
  - ⁎ twist-2 case: factorization to all orders
    many works between 2013-2020
  - ⁎ twist-3 case: proved factorization to 1-loop order
    [S.Bhattacharya, K.Cichy, M.Constantinou, A.Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 3, 034005, Phys.Rev.D 102 (2020) 114025]
    [V.M. Braun et al., 2021, arXiv: 2103.12105]

$$\boxed{\tilde{\Phi}^{\Gamma}(x,\mu,P_3) = \int_{-1}^{+1} \frac{dy}{|y|} C\left(\frac{x}{y},\mu\right) \Phi(y,\mu) + \mathcal{O}\left(\frac{1}{P_3^2}\right)}$$

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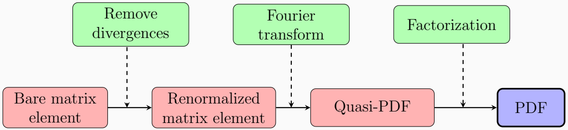
- Road map



Lattice setup

- Configurations of $N_f = 2+1+1$ flavors & clover term [ETM collaboration]

N_f	$L^3 \times T$	lattice spacing a	m_π	$m_\pi L$
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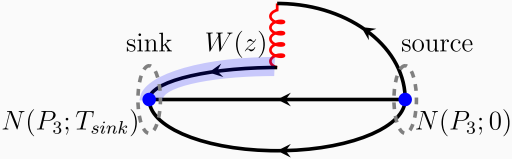
- Matrix elements extracted from:

$$\frac{C^{3pt}(T_{sink};\tau;\Gamma;P_3)}{C^{2pt}(T_{sink};P_3)} \stackrel{T_{sink} \ll \tau \ll 0}{=\!=} \mathcal{M}(P_3,\Gamma)$$



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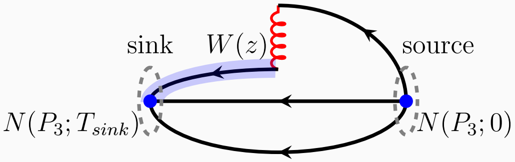

Lattice techniques:

$\ast$ sequential inversions for all-to-all propagator (here $T_{sink} = 12a \simeq 1.12$ fm)
$\ast$ Momentum smearing to improve overlap with proton boosted state [G. Bali et al., Phys.Rev.D 93 (2016) 9, 094515]

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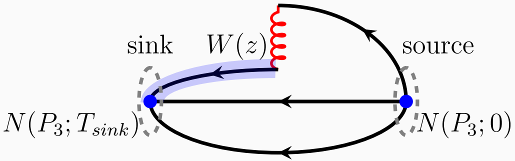

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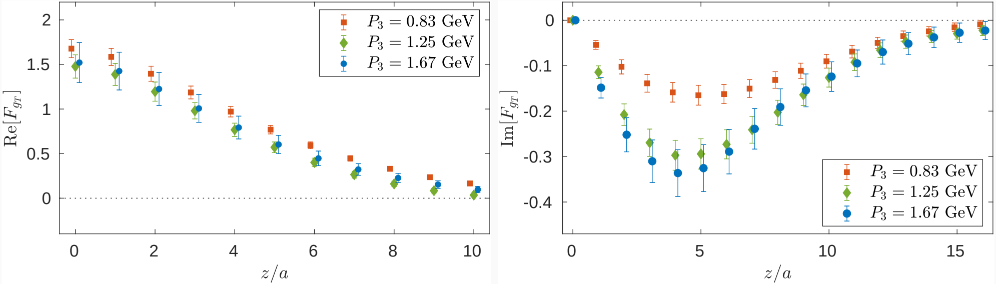
- Momentum dependence of the lattice PDFs studied at three proton boosts

P_3[GeV]	N_{conf}	N_{meas}
0.83	194	1552
1.25	731	11696
1.67	1644	105216

Results: $g_T(x)$ and $h_L(x)$

Bare matrix elements of g_T^{u-d}

$$F_{g_T}(P_3,z) = -i\frac{E}{m}\langle N(P_3)|\bar{\psi}(0)\tau_3\Gamma W(0,z)\psi(z)|N(P_3)\rangle \qquad (\Gamma = \gamma_x\gamma_5, \gamma_y\gamma_5)$$



- Statistical uncertainties under control (statistics compared to 0.83 GeV: X 7 1.25 GeV, X 67 1.67 GeV)
- Convergence between the two largest momenta (not guaranteed after matching)

Towards the light-cone $g_T^{u-d}(x)$

- Non-perturbative renormalization of the matrix elements at each z/a-value
  - ⁎ removal of power-like divergences
  - ⁎ results presented at 2 GeV in the modified $\overline{\text{MS}}$ scheme (same used for matching)

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- Fourier transform to x-space

$$\tilde{g}_T(x,P_3,\mu) = \sum_{z=-\infty}^{+\infty} dz\, e^{-iP_3 z}\, \mathcal{R}[F_{g_T}(z,P_3,\mu)] \xrightarrow{\text{on the lattice}} \sum_{z=-z_{max/a}}^{+z_{max/a}} [\ldots\ldots]$$

Challenge: reconstruction of continuous distributions from finite data sets

- ⁎ Backus-Gilbert method (in our work) [G. Backus and F.Gilbert, Geophys. J. R. astr. Soc. (1968) 16, 169–205]
- ⁎ Neural network, fitting and Bayesian reconstructions [J. Karpie, K. Orginos, A. Rothkopf, S. Zafeiropolous (2019) 1901.05408]

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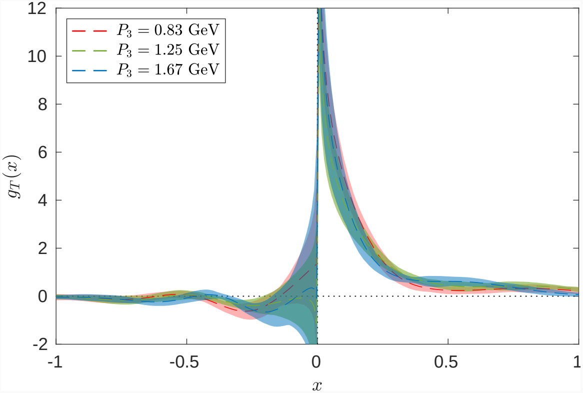
- Perturbative matching on quasi-PDFs

$$g_T(x, P_3) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{g}_T\left(\frac{x}{\xi}, \mu, P_3\right)$$

[S.Bhattacharya, K.Cichy, M.Constantinou, A.Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 3, 034005]

Momentum dependence of lattice $g_T^{u-d}(x)$



Antiquarks **Quarks**

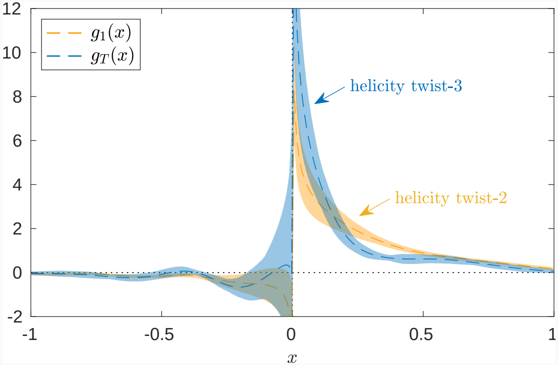
- Bands include statistical errors + systematic uncertainity due to the choice of the cutoff z_{max}/a
- Convergence between 1.25 GeV and 1.67 GeV

[S.Bhattacharya, K.Cichy, M. Constantinou,A. Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 11]

Comparison between $g_1(x)$ and $g_T(x)$

[S.Bhattacharya, K.Cichy, M. Constantinou,A. Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 11]

$P_3 = 1.67$ GeV



Antiquarks

Quarks

- $g_1(x)$ extracted on the same gauge ensemble
- g_1 and g_T compatible for antiquarks, $\bar{u} - \bar{d}$
- g_T dominant at small positive x ($x \lesssim 0.2$)
  $\rightarrow$ twist-3 contributions may be sizable

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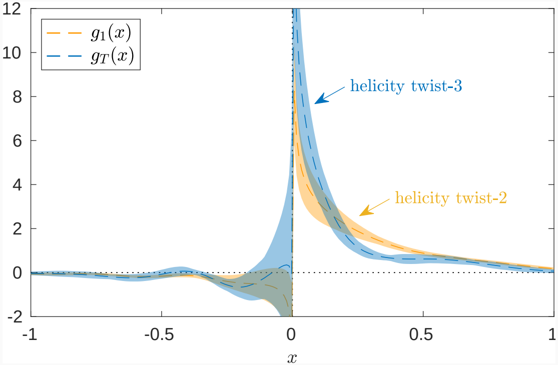

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Burkhardt-Cottingham sum rule:

[H. Burkhardt and W. N. Cottingham, Annals Phys.56,453 (1970)]

$$\int_{-1}^{1} dx\, g_1(x) = \int_{-1}^{1} dx\, g_T(x)$$

(lattice) $$\int_{-1}^{1} dx\, g_1(x) - \int_{-1}^{1} dx\, g_T(x) = 0.01(20)$$

Wandzura-Wilczek approximation for g_T^{u-d}

- Wandzura-Wilczek relation

On the light cone: $$g_T(x) = \int_x^1 dy\, \frac{g_1(y)}{y} + g_T^{twist-3}(x)$$

∗ Mellin moments of $g_T^{twist-3}$ seem to be very small (models calculations and experiments)

- Approximation: $\boxed{\mathrm{g}_T(x) \stackrel{?}{=} \int_x^1 dy\, \frac{g_1(y)}{y} = g_T^{WW}(x)}$

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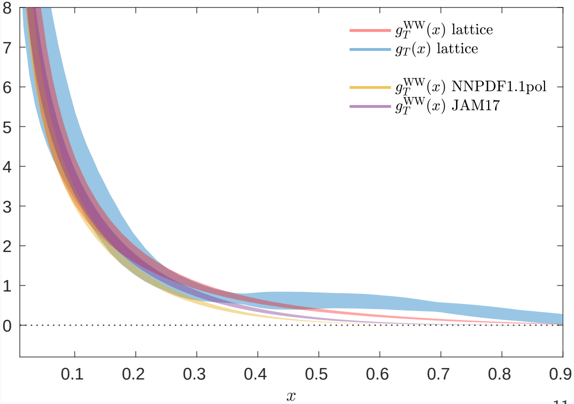
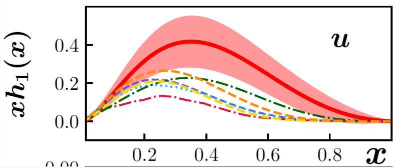


- here focus on the quark region $(x > 0)$
- agreement between g_T and g_T^{WW} for $x \lesssim 0.5$ within uncertainties
- still violations up to $30 - 40\%$
  similar violations observed in experimental analysis at JLAB[A. Accardi et al., JHEP11 (2009) 093]

- $h_L(x)$ contains the leading twist-2 h_1 transversity PDF
- No experimental data on $h_L(x)$

Twist-2 h_1



[JAM, Phys.Rev.D 102 (2020) 5, 054002]

Beyond $g_T(x)$... The $h_L(x)$ case

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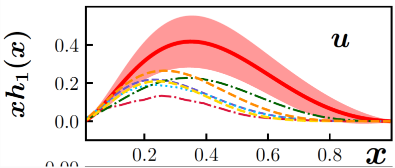
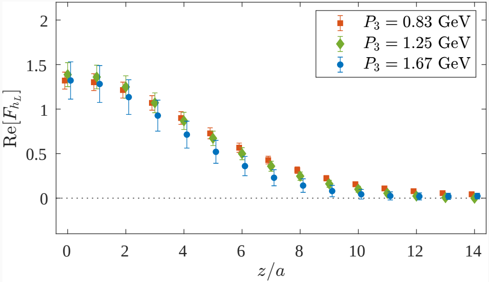
Twist-2 h_1

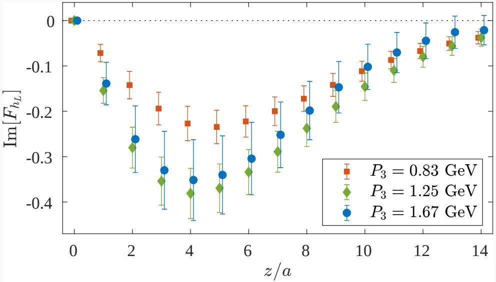


[JAM, Phys.Rev.D 102 (2020) 5, 054002]

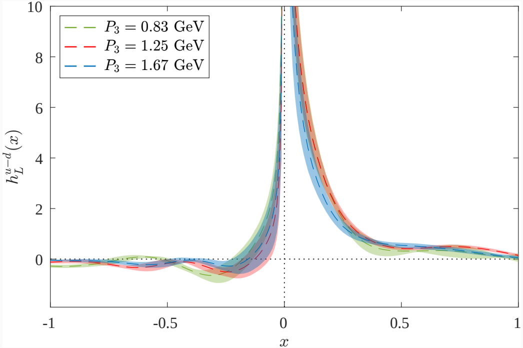
h_L from quasi-PDFs $\longrightarrow$

$$F_{h_L}(P_3, z) = \langle N(P_3)|\bar{\psi}(0)\sigma^{12}\tau_3 W(0,z)\psi(z)|N(P_3)\rangle$$





Proton boost dependence



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- perturbative matching in M$\overline{\text{MS}}$ scheme

[S.Bhattacharya, K.Cichy, M. Constantinou,A. Metz, AS, F. Steffens, Phys.Rev.D 102 (2020) 114025] (see Shohini's talk!)

Results for h_L and h_1

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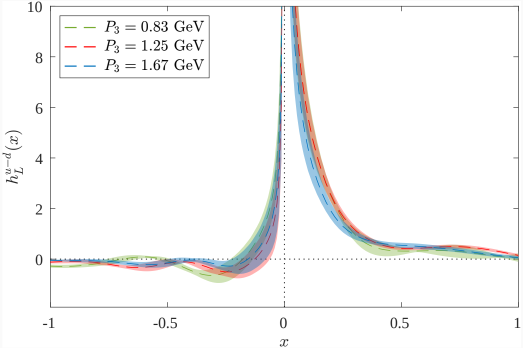
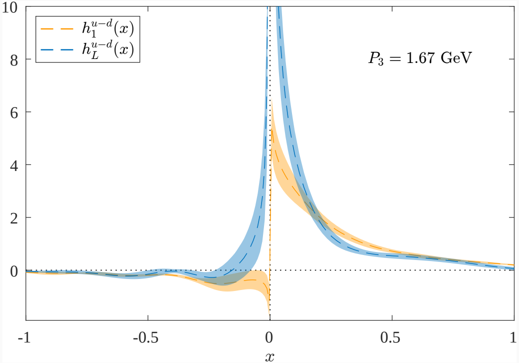


Twist-3 h_L vs. twist-2 h_1



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- $h_L(x)$ suppressed only in a limited interval
- similar contributions at large $|x|$

Wandzura-Wilczek approximation for h_L

- In analogy to g_T

$$h_L(x) = 2x \int_x^1 \frac{h_1(y)}{y^2} dy + h_L^{twist-3}(x)$$

[R.L. Jaffe and Xiangdong Ji, Nucl. Phys. B 375 (1992) 527—560]

$\ast$ $h_L^{twist-3}(x)$ might be numerically very small (instanton model)

Test: $\boxed{\mathsf{h}_L(x) \stackrel{?}{=} 2x \int_x^1 dy\, \frac{h_1(y)}{y^2} = h_L^{WW}(x)}$

Wandzura-Wilczek approximation for h_L

- In analogy to g_T

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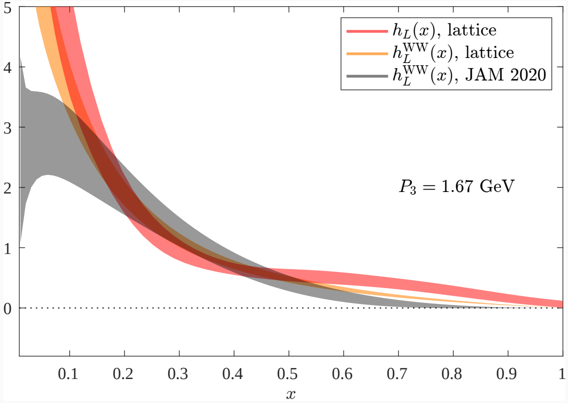
largest boost



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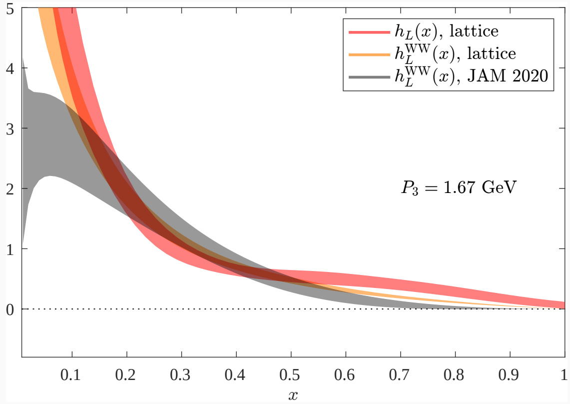

Summary & Outlook

Study of proton twist-3 PDFs

- Lattice investigations can be pursued using quasi-distribution formalism (and also other approaches)
- For the first time, a qualitative comparison between twist-2 and twist-3 PDFs can be made

  $\rightarrow$ test of the Wandzura-Wilczek approximation for g_T and h_L for $x > 0$

- Several challenges in front of us:
  - ⁎ can we disentangle the twist-2 and twist-3 parts of g_T and h_L?

    $\rightarrow$ calculation of quark-gluon-quark matrix elements

    $\rightarrow$ to what extent numerical results are affected?

    [V.M. Braun, Y. Ji , A. Vladimirov, arXiv: 2103.12105, 2021]
  - ⁎ Systematics on the lattice: finite lattice spacing and volume effects, pion mass dependence, ...
- Lots of work, but of great impact in phenomenology in the next few years

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Thank you!