Simultaneous Monte Carlo analysis of parton densities and fragmentation functions

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Introduction

- Significant tension between large transverse momentum data and Fixed Order (FO) predictions using existing collinear Parton Distribution Functions (PDFs) and Fragmentation Functions (FFs).
- Resolving this tension is crucial for the study of Transverse Momentum Dependent (TMD) PDFs and FFs and Generalized PDFs (GPDs).
- To facilitate exploring the reasons for this tension in SIDIS, performed a new fit using Jefferson Lab Angular Momentum Collaboration (JAM) methodology:
  - Multi-Step Monte Carlo fit utilizing Bayesian Inference
  - Simultaneously fit PDFs and charged pion, kaon, and unidentified hadron FFs.
    - First such fit involving charged hadrons
Theory

\[ \frac{d\sigma_{\text{DIS}}}{dQ^2dx_Bj} = \sum_{i \in \text{flavors}} \mathcal{H}_i^{\text{DIS}} \otimes f_i, \]

inclusive DIS

\[ \frac{d\sigma_{\text{SIDIS}}}{dQ^2dx_Bjdz_h} = \sum_{ij \in \text{flavors}} \mathcal{H}_{ij}^{\text{SIDIS}} \otimes f_i \otimes D_j^h, \]

semi-inclusive DIS

\[ \frac{d\sigma_{\text{DY}}}{dQ^2dx_F} = \sum_{ij \in \text{flavors}} \mathcal{H}_{ij}^{\text{DY}} \otimes f_i \otimes f_j, \]

Drell-Yan lepton-pair production

\[ \frac{d\sigma_{\text{SIA}}}{dQ^2dz_h} = \sum_{j \in \text{flavors}} \mathcal{H}_j^{\text{SIA}} \otimes D_j^h, \]

semi-inclusive annihilation

- Using NLO perturbative calculations
JAM fit

- Simultaneously fit PDFs and charged pion, kaon, and unidentified hadron FFs
  - Functional form:
    \[
    T(x; \mathbf{a}) = \mathcal{M} \frac{x^\alpha (1 - x)^\beta (1 + \gamma \sqrt{x} + \delta x)}{\int_0^1 dx \; x^{\alpha+1} (1 - x)^\beta (1 + \gamma \sqrt{x} + \delta x)}
    \]
  - Unidentified charged hadron fragmentation function:
    \[
    D_i^{h+} = D_i^{\pi+} + D_i^{K+} + D_i^{res+}
    \]
  - Total: Fitting 24 functions, 129 parameters
JAM fit

- Multi-Step Monte Carlo fit utilizing Bayesian Inference:

\[ \mathcal{P}(\mathbf{a}|\text{data}) \sim \mathcal{L}(\mathbf{a}, \text{data}) \pi(\mathbf{a}) \quad \mathcal{L}(\mathbf{a}, \text{data}) = \exp \left( -\frac{1}{2} \chi^2(\mathbf{a}, \text{data}) \right) \]

\[ \chi^2(\mathbf{a}) = \sum_{i,e} \left( \frac{d_{i,e} - \sum_k r^k e \beta^k_{i,e} - T_{i,e}(\mathbf{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k (r^k_e)^2 + \left( \frac{1 - N_e}{\delta N_e} \right)^2 \]

\[ \mathbb{E}[\mathcal{O}] = \int d^d \mathbf{a} \mathcal{P}(\mathbf{a}|\text{data}) \mathcal{O}(\mathbf{a}) , \quad \mathbb{E}[\mathcal{O}] = \frac{1}{n} \sum_{k=1}^n \mathcal{O}(\mathbf{a}_k) , \]

\[ \mathbb{V}[\mathcal{O}] = \int d^d \mathbf{a} \mathcal{P}(\mathbf{a}|\text{data}) (\mathcal{O}(\mathbf{a}) - \mathbb{E}[\mathcal{O}])^2 \quad \mathbb{V}[\mathcal{O}] = \frac{1}{n} \sum_{k=1}^n (\mathcal{O}(\mathbf{a}_k) - \mathbb{E}[\mathcal{O}])^2 \]

- Use least squares to obtain maximum likelihood (minimum chi squared) for each replica.
Multi-step process
Data sets

- Data Sets:
  - Inclusive Deep Inelastic Scattering (DIS)
    - BCDMS, NMC, SLAC, HERA
  - Semi-Inclusive DIS (SIDIS)
    - COMPASS
  - Single-Inclusive $e^+/e^-$ Annihilation (SIA)
    - TASSO, TPC, TOPAZ, BELLE, BABAR, ARGUS, DELPHI, ALEPH, OPAL, SLD
  - Drell-Yan Scattering (DY)
    - E866
\[ \chi_{\text{red}}^2 = \frac{1}{N} \sum_{i,e} \frac{1}{\alpha_{i,e}^2} \left( d_{i,e} - E \left[ \sum_k r_{i,e}^k \beta_{i,e}^k + T_{i,e}/N_e \right] \right)^2 \]

residual \((e, i) = \frac{1}{\alpha_{i,e}} \left( d_{i,e} - E \left[ \sum_k r_{i,e}^k \beta_{i,e}^k + T_{i,e}/N_e \right] \right) \]
SIDIS Data and theory comparison: pions
SIDIS Data and theory comparison: kaons
SIDIS Data and theory comparison: hadrons
SIA Data over theory comparison: pions
SIA Data over theory comparison: kaons
SIA Data over theory comparison: hadrons
PDFs

$x u_v$

$x d_v$

$x (\bar{d} - \bar{u})$

$d/u$

$x (\bar{d} + \bar{u})$

$R_s$

$x g$

$\mu^2 = 10 \text{ GeV}^2$
FFs

\[ \mu^2 = 100 \text{ GeV}^2 \]
Strange PDF suppression

- Best fits to kaon SIA data favor smaller strange PDFs
- Consistent with JAM19 findings
Transverse momentum dependent SIDIS predictions

- FO predictions for large transverse momentum SIDIS
- Discrepancy between FO predictions and data can be reduced significantly by including the transverse momentum dependent data in the fit.
Conclusions

- Successfully performed simultaneous global extraction of PDFs and pion, kaon, and hadron FFs.
- Observed similar strange PDF suppression as was observed in JAM19.
- Comparison of FO predictions using these results to COMPASS transverse momentum dependent data shows there is still a large discrepancy
  - This discrepancy is reduced when transverse momentum dependent data is included in the fit.