$\Lambda^\uparrow$ Polarization in $e^+e^-$ collisions

Leonard Gamberg

w/ Zhong-Bo Kang, Ding Yu Shao, John Terry, Fanyi Zhao
arXiv:2102.05553
Outline

• Motivation to study on $\Lambda^{\uparrow}$ physics long standing challenge describe via QCD factz.

• Review “outsized” role of Lambda in studying TSSAs look @ data

• Twist -2 TMD fact. description in terms of PFF. $D_{1T}^{\perp}(z, p_{\perp}, Q^2)$

  ★ Thrust observable $\Lambda$ (Thrust) + $X$ $\quad e^+e^- \rightarrow \Lambda^{\uparrow}(\text{Thrust}) \ X$

  ★ Back to back hadrons $h + \Lambda$ $\quad e^+e^- \rightarrow \Lambda^{\uparrow}h \ X$

• Inclusive process $e^+e^- \rightarrow \Lambda^{\uparrow}X$ possible & interesting to process to study

• Twist -3 fact. description in terms of $D_T(z, Q^2)$

★ Change of ref frame COM of $e^+e^-$ pair $\quad e^+e^- \rightarrow \Lambda^{\uparrow}X$

Test of naive time reversal in QCD
In this note we have pointed out that the asymmetry off a polarized target, and the transverse polarization of a produced quark in $e^+e^-\rightarrow q\bar{q}$, or in $qq\rightarrow qq$ at large $p_T$, or in lepton production, should all be calculable perturbatively in QCD. The result is zero for $m_q=0$ and is numerically small if we calculate $m_q/\sqrt{s}$ corrections for light quarks. We discuss how to test the predictions.

At least for the cases when $P$ is small, tests should be available soon in large-$p_T$ production [where currently $P(\Lambda)=25\%$ for $p_T \approx 2$ GeV/c], and $e^+e^-$ reactions. While fragmentation effects could dilute polarizations, they cannot (by parity considerations) induce polarization. Consequently, observation of significant polarizations in the above reactions would contradict either QCD or its applicability.

TMD factorization says otherwise:
Mulders Tangerman, NPB1996
Boer, Jakob, Mulders NPB1997, 2000
Anselmino Boer, D’Alesio, Murgia. PRD 2001, 2002
Boer, Kang, Vogelsang, Yuan, PRL 2010
QCD is Parity conserving so any final state hadron must be polarised perpendicular to the production plane.

- Proton preferentially emitted along $\Lambda$-spin
- In $\Lambda$ rest frame: pol. decay distribution

\[
\left( \frac{dN}{d\Omega_p} \right)_{\text{pol}} = \left( \frac{dN}{d\Omega_p} \right)_{\text{unpol}} (1 + \alpha P_n^\Lambda \cos(\theta_p))
\]

$P^\Lambda$: Transverse Lambda Polarization

\[
P_{\perp}^\Lambda(z_a, j_\perp) = \frac{d\Delta\sigma}{dz_\Lambda d^2j_\perp} / \frac{d\sigma}{dz_\Lambda d^2j_\perp}.
\]

The Status of Transverse Spin Physics
What does Exp Say …

\[ pA \rightarrow \Lambda^\uparrow X \]

\[ pp \rightarrow \Lambda^\uparrow X \]

\[ \nu N \rightarrow \Lambda^\uparrow X \quad \text{NOMAD} \]

\[ \gamma^* N \rightarrow \Lambda^\uparrow X \quad \text{HERMES} \]

\[ e^+ e^- \rightarrow \Lambda^\uparrow X \]
Transverse $\Lambda$ polarisation: a long history

One of the first transverse spin effects at Fermilab (1976):

$$p + Be \rightarrow \Lambda^\uparrow + X$$

Bunce PRL 76
Heller PRL 78

PRD 89 Lundberg
\[ pA \rightarrow \Lambda^{\uparrow}X \]

Lundberg et al PRD40 (1989) 400 GeV

V. Fanti et al.: NA 48 450 GeV proton energy


**FIG. 4.** The Λ polarization is shown as a function of \( x_F \) for all production angles. Over this range of production angles and within experimental uncertainties, the polarization is angle (or \( p_T \)) independent.
What about LHC? Is it feasible at a high energy collider?

Recent ATLAS measurement at $\sqrt{S} = 7$ TeV

PRD 91, 032004 (2015)

Small Polarisation at mid rapidity but

Such exps. demonstrate feasibility to study $\Lambda^+$ at hi energy
What does Exp Say …

$pA \rightarrow \Lambda^\uparrow X$

$pp \rightarrow \Lambda^\uparrow X$

$\nu N \rightarrow \Lambda^\uparrow X$  NOMAD

$\gamma^* N \rightarrow \Lambda^\uparrow X$  HERMES

$e^+ e^- \rightarrow \Lambda^\uparrow X$

FIG. 1: Schematic diagram of inclusive $\Lambda$ production and decay. The angle $\theta_p$ of the decay proton with respect to the normal $\mathbf{n}$ to the production plane is defined in the $\Lambda$ rest frame.
Simplest and cleanest process: \( e^+ e^- \rightarrow \Lambda^\uparrow \text{(Thrust)} X \)


Longitudinal Polarization, small/zero Transverse Polarization w/ errors

\[ P_T^\Lambda (%) \]

\[ z_\Lambda > 0.15 \]
\[ Q = M_Z \]

Spin orbit

QCD is Parity Conserving \( \text{TSSAs Scattering plane transverse to spin} \)
\( \text{Naively } "T\text{-odd}" \)
\[ \Delta \sigma \sim i S_T \cdot (P \times P_\perp) \otimes ("T - odd" \text{ QCD – phases}) \]
Simplest and cleanest process $\Lambda^\uparrow$ in $e^+e^-$

Belle data: Transverse Polarization

Y. Guan, et al. PRL 122 (2019) → talk by Anselm here @ Jets Workshop

$\Rightarrow$ significant transverse polarization

\[ e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust})\ X \]
\[ e^+e^- \rightarrow \Lambda^\uparrow\ h\ X, \]

Measured w.r.t. thrust axis &
back to back hadrons="bTOb"
The $P_t$ is measured as the transverse momentum of $\Lambda$ relative to the thrust axis.

$$e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust})X$$
Back to back hadrons integrated over $p_\perp$ NOT SMALL
Question for global analysis
& to test Universality Belle BeS BaBar + EIC

\[ e^+ e^- \rightarrow \Lambda^{\uparrow} \text{(Thrust)} \ X \]
\[ e^+ e^- \rightarrow \Lambda^{\uparrow} h \ X, \]

Questions/issues: is “mechanism” the same ??

• TMD factorization “2” two scale fact. Theorems ?
  ✴ TMD factorization formalism ?? for thrust axis measurement
  • ....

\[ \Lambda_{QCD} \lesssim p_\perp \ll Q \]
\[ \Lambda_{QCD} \lesssim j_\perp \ll Q \]
Is it same PFF function in bTOb hadron & hadron + thrust measurements?

The thrust axis defined by vector, \( \hat{n} \) which maximizes the thrust variable \( T \)

\[
T = \frac{\sum_i |\mathbf{p}_i \cdot \hat{n}|}{\sum_i |\mathbf{p}_i|}
\]

• What about “T”-odd universality can we test it with all data?
Explain non trivial $P_{\Lambda^\uparrow}$ via TMD FFs
polarization fragmentation function PFF unsurpressed

TMD framework for bTOb production of
$\Lambda + h$ chiral even, naively T-odd fragmentation function, universal

Boer & Mulders 1997

\[
\hat{D}_{\Lambda/q}(z_{\Lambda}, p_{\perp}, S_{\perp}, Q) = \frac{1}{2} \left[ D_{\Lambda/q}(z_{\Lambda}, p_{\perp}, Q) + \frac{1}{z_{\Lambda} M_{\Lambda}} D_{1T,\Lambda/q}^{\perp}(z_{\Lambda}, p_{\perp}, Q) \epsilon_{\perp \rho \sigma} p_{\perp}^{\rho} S_{\perp}^{\sigma} \right]
\]
Explain via TMD fact.

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(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

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(Boer, Jakob, Mulders (1997))
bTOb beyond leading order

TMD Factorization

QCD factorization Collins Soper 1982 NPB,
Collins Foundations of PQCD Cambridge Press 2011

JCC Soft factor further “repartitioned”
This is done to

1) cancel LC divergences in “unsubtracted” TMDs
2) separate “right & left” movers i.e. full factorization
3) remove double counting of momentum regions

\[
\tilde{D}_{H/j}^{\text{sub}}(z_A, b_T; \mu, \zeta) = \lim_{y_A \to +\infty} \frac{\tilde{D}_{H/j}^{\text{unsub}}(z_A, b_T; \mu, y_A - y_B)}{\tilde{D}_{H/j}^{\text{unsub}}(z_A, b_T; \mu, y_A - y_B)} \sqrt{\frac{\tilde{S}(b_T; y_A, y_n)}{\tilde{S}(b_T; y_A, y_B) \tilde{S}(b_T; y_n, y_B)}} \times U_{\text{renorm}}
\]

\[
\tilde{D}_{H/j}^{\text{unsub}}(z_A, b_T; \mu, y_P - y_B) = \frac{1}{z_A} \int \frac{db^+}{2\pi} e^{-ik \cdot b^+} \langle 0|\gamma^- U_{[0,b]} \psi(b) |XPA\rangle \langle PAX|\bar{\psi}(0)|0\rangle \big|_{b^-=0}
\]
Use both data sets to study universality of T-odd fragmentation?

What is prediction of TMD Factorization

Universality of \textbf{T-odd} Collins function: $H^{\perp(1)}_{1,\pi/q}(z, b, Q)$

Metz PLB2002,
Boer Mulders Pijlman NPB2003
Collins Metz PRL 2004,
Gamberg, Mukerjee, Mulders PRD2007,
Meissner Metz PRL 2009,
Gamberg Mukherjee, Mulders PRD 2008

Universality of \textbf{T-odd} PFF prediction from pQCD - $D^{\perp(1)}_{1T,\Lambda/q}(z_{\Lambda}, b, Q)$

phase from FSI but not gluonic/fermionic pole

Boer, Kang, Vogelsang, Yuan PRL 2010
Belle data fall into 2 classes

\[ e^+ e^- \rightarrow \Lambda^+ h X \quad \& \quad \Lambda^+ (\text{Thrust}) X \]

? Is it true that the PFF is the same TMD in both process?

Recent extractions address this
1) D’Alesio & Murgia Zaccheddu PRD 2020 \( bTOb + \text{Thrust} \) assumed same factz here
2) Callos, Kang, Terry PRD 2020 \( bTOb \) only

Other pheno studies
* Anselmino, Kishore, Mukherjee PRD 2019
  single inclusive case and the role of the PFFs twist-2 in place of twist-3 ?
* Earlier Anselmino Boer, D’Alesio, Murgia. PRD 2001, 2002
  TMD factorization applied to inclusive process ?
In TMD factorization framework for production of $\Lambda$ (Thrust) we have non-global observable “right hemisphere” only? chiral even, naively T-odd fragmentation function, universal?

Z.B Kang, D.Y. Shao, F. Zhao 2007.14425

\[
\hat{D}_{\Lambda/q}(z, p, S, Q) = \frac{1}{2} \left[ D_{\Lambda/q}(z, p, Q) + \frac{1}{z M} D_{1T,\Lambda/q}^{\perp}(z, p, Q) e_{\perp \rho \sigma} p_{\perp} S_{\perp}^{\sigma} \right]
\]
Recent work

- M. Boglione & A. Simonelli, 2007.13674
- Z.B Kang, D.Y. Shao, F. Zhao 2007.14425
- M. Boglione & A. Simonelli, 2007.13674

Z.B Kang, D.Y. Shao, F. Zhao 2007.14425—see talk of Dingyu

Derive TMD factorization for unpolarized transverse momentum distribution for the single hadron production with the thrust axis in electron-positron collision
Recent work

- Z.-B Kang, D.Y. Shao, F. Zhao 2007.14425

Derive TMD factorization for unpolarized TMD FF for single hadron production with the thrust axis in electron-positron collision $e^+ e^- \rightarrow \Lambda^{\uparrow} \text{(Thrust)}$ non-global observable

$$\frac{d\sigma}{dz\Lambda d^2j_\perp} = \sigma_0 H(Q, \mu) \sum_q e_q^2 \int d^2p_\perp d^2\lambda_\perp \delta^{(2)}(j_\perp - p_\perp - z\Lambda \lambda_\perp) D_{\Lambda/q}(z\Lambda, p_\perp, \mu, \zeta/\nu^2) \sqrt{S_{\text{hemi}}(\lambda_\perp, \mu, \nu)}$$

Calculated to NLO and NLL

$$S_{\text{hemi}}(b, \mu, \nu) = \sqrt{S(b, \mu, \nu)}$$

$$D_{\Lambda/q}^{\text{TMD}}(z\Lambda, b, \mu, \zeta) = D_{\Lambda/q}(z\Lambda, b, \mu, \zeta/\nu^2) \sqrt{S(b, \mu, \nu)}$$

Non-global logs resummed

Factorization theorem $\exists$

Becher Rahn Shao JHEP 2017
M.Dasgupta & G.Salam PLB2001
We extend TMD factorization PFF
\[ e^+e^- \rightarrow \Lambda^\uparrow (\text{Thrust}) \]

Gamberg, Kang, Shao, Terry, Zhao arXiv:2102.05553

\[ \hat{D}_{\Lambda/q}(z_\Lambda, b, S_\perp, Q) = \frac{1}{2} \left[ D_{\Lambda/q}(z_\Lambda, p_{\Lambda\perp}, Q) - i e_{\perp} \rho_{\sigma} b_{\rho} S_{\perp}^{\sigma} M_{\Lambda} D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda, b, Q) \right] \]

Spin dependent
FF Obey CSS equation

\[ \frac{d \Delta \sigma}{dz_\Lambda d^2 j_\perp} = \frac{d \sigma(S_\perp)}{dz_\Lambda d^2 j_\perp} - \frac{d \sigma(-S_\perp)}{dz_\Lambda d^2 j_\perp} \]

\[ = \sigma_0^{\text{TMD}} \sin(\phi_s - \phi_j) \sum_q e_q^2 \int_0^\infty \frac{b^2 \, db}{4 \pi} J_1 \left( \frac{b j_\perp}{z_\Lambda} \right) \]

\[ \times \frac{M_{\Lambda}}{z_{\Lambda}^4} D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda, \mu_{b*}) e^{-S_{\text{NP}}(b, z_\Lambda, Q_0', Q)} - S_{\text{pert}}(\mu_{b*}, Q) U_{\Lambda}^{\text{NG}}(\mu_{b*}, Q) \]

UV & Rapidity subtracted TMD Universal PFF

regarding proper definitions of weighted TMDs and talk in this workshop
Establish factorization for thrust axis factorization
carry out pheno to describe
Belle $P_T$ and OPAL

\[ e^+e^- \rightarrow \Lambda^+(\text{Thrust}) \]
\[
P_{\perp}^{\Lambda}(z_{\Lambda}, j_{\perp}) = \frac{d\Delta \sigma}{dz_{\Lambda} d^{2}j_{\perp}} \bigg/ \frac{d\sigma}{dz_{\Lambda} d^{2}j_{\perp}}.
\]

\[
S_{NP}(b, z_{\Lambda}, Q_{0}, Q) = g_{h} \frac{b^{2}}{z_{\Lambda}^{2}} + \frac{g_{2}}{2} \ln \frac{Q}{Q_{0}} \ln \frac{b}{b_{*}}
\]

\[
g_{h} = 0.042 \text{ GeV}^{2} \quad g_{2} = 0.84 \text{ GeV}^{2}
\]

\[
U_{NG}(\mu_{b_{*}}, Q) = \exp \left[ -C_{A}C_{F} \frac{\pi^{2}}{3} u^{2} \frac{1 + (au)^{2}}{1 + (bu)^{2}} \right]
\]

with \( a = 0.85C_{A}, b = 0.86C_{A}, c = 1.33 \)

\[
D_{1T,h/q}(z, p_{\perp}, Q'_{0}) = \frac{M_{\Lambda}}{\langle M_{D}^{2} \rangle} D_{1T,h/q}(z, Q'_{0}) \frac{e^{-p_{\perp}^{2}/\langle M_{D}^{2} \rangle}}{\pi \langle M_{D}^{2} \rangle}
\]

\[
D_{1T,h/q}(z, Q'_{0}) = N_{q}(z) D_{h/q}(z, Q'_{0}) \quad Q'_{0} = 10.58 \text{ GeV}
\]

\[
S_{NP}'(b, z, Q'_{0}, Q) = \frac{\langle M_{D}^{2} \rangle}{4} \frac{b^{2}}{z^{2}} + \frac{g_{2}}{2} \ln \frac{Q}{Q'_{0}} \ln \frac{b}{b_{*}}
\]

Aidala Field Gamberg Rogers PRD 2014

Implementation Issacson Sun Yuan 2014 MPA

Dasgupta Salam, PLB 2001

\[
u \equiv \int_{\mu_{b_{*}}}^{Q} \frac{d\mu}{\mu} \frac{\alpha_{s}(\mu)}{2\pi} = \frac{1}{\beta_{0}} \ln \left[ \frac{\alpha_{s}(\mu_{b_{*}})}{\alpha_{s}(Q)} \right]
\]

Callos, Kang, Terry PRD2020

Parameters fit from bToB Belle data

Parameters fit from bToB Belle data
Belle data fit $e^+e^- \rightarrow \Lambda^+ h X$

Recent extractions address this
Callos, Kang, Terry PRD2020 bTOb only

FIG. 3. The fit to the experimental data for $\pi$ mesons is shown, with the gray uncertainty band displayed is generated by the replicas at 68% confidence. The left plots are for the production of $\Lambda + \pi^\pm$, while the right plots are for the production of $\bar{\Lambda} + \pi^\pm$.

And for kaons …
Belle data \( e^+e^- \rightarrow \Lambda^\uparrow hX, \)

Recent extractions
Callos, Kang, Terry PRD2020 bTOb only

Exploit Universality to describe \( e^+e^- \rightarrow \Lambda^\uparrow(\text{Thrust}) \)
Compare theory predictions to OPAL & Belle

\[ e^+ e^- \rightarrow \Lambda^\uparrow (\text{Thrust}) \]

Gamberg, Kang, Shao, Terry, Zhao arXiv:2102.05553

- \( P_\perp^\Lambda(z_\Lambda, j_\perp) \) for the Belle data [20]; left to right theory integrated from 0.2 < \( z_\Lambda < 0.3 \), 0.3 < \( z_\Lambda < 0.4 \), 0.4 < \( z_\Lambda < 0.5 \), 0.5 < \( z_\Lambda < 0.6 \)
- The data in red is for \( \Lambda \) production while the data in blue is for \( \bar{\Lambda} \) production
- Data plotted with total exp. uncertainty as vertical error bar & uncertainty on \( j_\perp \) horizontal error bar
- Gray band is the theoretical uncertainty which was generated from the replicas for the TMD PFF, Callos, Kang, Terry PRD2020
Compare theory predictions to OPAL & Belle data

\[ e^+ e^- \rightarrow \Lambda^+(\text{Thrust}) \]

\[ P_\perp^\Lambda(z_{\Lambda}, j_\perp) = \frac{d\Delta\sigma}{dz_{\Lambda}d^2j_\perp} / \frac{d\sigma}{dz_{\Lambda}d^2j_\perp}. \]

\( P_\perp^\Lambda(z_{\Lambda}, j_\perp) \) for OPAL data [19]: Theory curve is integrated over the region 0.2 < \( z_{\Lambda} \) < 0.5. total experimental uncertainty vertical error bar \( j_\perp \) horizontal error bar. Error band, standard deviation of the replicas for TMD PFF in Callos, Kang, Terry PRD2020.

Gamberg, Kang, Shao, Terry, Zhao arXiv:2102.05553

QCD is Parity Conserving TSSAs Scattering plane transverse to spin Naively “T-odd”

\[ \Delta\sigma \sim iS_T \cdot (P \times P_\perp) \otimes ("T - odd" \text{ QCD - phases}) \]

Spin orbit
Fully inclusive process $e^+e^- \rightarrow \Lambda \uparrow X$

$\Rightarrow$ significant transverse polarization ?

$e^+e^- \rightarrow \Lambda \uparrow X$

**Measure** w.r.t. COM in principle can measure at Belle ?

**Questions/issues:**
- QCD prediction of Physics twist-3
  - Twist-3 factorization one hard scale
Simplest and cleanest process \( e^+ e^- \rightarrow \Lambda^\uparrow X \)

\( \Rightarrow \) significant transverse polarization?

\( e^+ e^- \rightarrow \Lambda^\uparrow X \)

\( P_{\Lambda \perp} \sim Q \) twist-3 factorization

And can be measured w.r.t. COM of \( e^+ e^- \) on large scale \( P_T \sim Q \)
Consider Transverse $e^+e^- \rightarrow \Lambda^+X$ polarization

Gamberg, Kang, Pitonyak, Schlegel, Yoshida JHEP 2019, LO & NLO

There are contributions from

‘Intrinsic’ & ‘kinematical’ twist-3 FF

\[
\frac{d\sigma(S_{\Lambda T})}{dz_h \, d\phi} = C \left| S_{\Lambda T} \right| \sin(\phi_S) \sum_q e_q^2 \left[ \frac{D_T^{\Lambda/q}(z_h)}{z_h} - D_{T(1)}^{\Lambda/q}(z_h) + \int_0^1 \frac{d\beta}{1 - \beta} \Re \left[ \hat{D}_{FT} - \hat{G}_{FT} \right]^{\Lambda/q}(z_h, z_h, \beta) \right]
\]

Using the EOMs and LIRs CS can be expressed solely in terms of

\[
\frac{d\sigma(S_{\Lambda T})}{dz_h \, d\phi} = C \left| S_{\Lambda T} \right| \sin(\phi_S) \sum_q e_q^2 \left[ 2 \frac{D_T^{\Lambda/q}(z_h)}{z_h} \right]
\]

‘Dynamical’ twist-3 FF:

Boer, Jakob, Mulders NPB (1997) in TMD framework at twist-3

See talk of F. Aslan on the subtleties of applying LIRs and EOMs
To describe this process, only need a parameterization for $D_{T,Λ/q}(z_Λ, Q)$

Given our lack of knowledge of this fundamental twist-3 T-odd fragmentation function we will employ the approach outlined in Gamberg, Metz, Pitonyak, Prokudin PLB 2017

Re-express the $D_{T,Λ/q}(z_Λ)$ in terms of our knowledge of $D^{⊥(1)}_{1T,Λ/q}(z_Λ)$
\[
\frac{d\Delta \sigma}{dz_{\Lambda} d^2p_{\Lambda \perp}} = -\sin(\phi_s - \phi_{\Lambda})\sigma_0^{\text{Col}} \left(8 \frac{M_{\Lambda}}{Q}\right) \frac{p_{\Lambda \perp}}{Q} \frac{1}{z_{\Lambda}^2} \sum_q e_q^2 D_{T,\Lambda/q}(z_{\Lambda}, Q) / z_{\Lambda}
\]

Re-express the \( D_{T,\Lambda/q}(z_{\Lambda}) \) in terms of our knowledge of \( D_{1T,\Lambda/q}^{(1)}(z_{\Lambda}) \)

\[
\frac{1}{z_{\Lambda}} D_{T,\Lambda/q}(z_{\Lambda}) = -\left(1 - z_{\Lambda} \frac{d}{dz_{\Lambda}}\right) D_{1T,\Lambda/q}^{(1)}(z_{\Lambda}) - 2 \int_0^1 d\beta \Im \left[ \hat{D}_{FT}^{gg}(z_{\Lambda}, \beta) \right] \frac{\hat{D}_{FT}^{qg}(z_{\Lambda}, \beta)}{(1 - \beta)^2}
\]

\[
\frac{1}{z_{\Lambda}} D_{T,\Lambda/q}(z_{\Lambda}) \approx -\left(1 - z_{\Lambda} \frac{d}{dz_{\Lambda}}\right) D_{1T,\Lambda/q}^{(1)}(z_{\Lambda})
\]
Prediction for Belle

Gamberg, Kang, Shao, Terry, Zhao arXiv:2102.05553

\[ P^\Lambda_{CM}(z_\Lambda, p_{\Lambda\perp}) = \frac{d\Delta\sigma}{dz_\Lambda \; d^2p_{\Lambda\perp}} / \frac{d\sigma}{dz_\Lambda \; d^2p_{\Lambda\perp}}. \]

\[ P^\Lambda_{CM} \] — 3-D plot of the polarization in \( z_\Lambda \) and \( p_{\Lambda\perp} \).

Center: Plot of the polarization as a function of only \( z_\Lambda \),
Right: Plot of the polarization as a function of \( p_{\Lambda\perp} \): polarization in our scheme is \( \sim 1-2\% \)

Plots are generated only using the central fit.
The red and blue curves are generated using the central fit, gray band is the theoretical uncertainty.
• Interesting that while these two measurements probe different distribution functions, they differ only by the definition of the measurement axis.

• That is, a measurement the polarization as a function of $j_\perp$ is a useful process for probing the properties of the PFF $D_{1T}^\perp$ with respect to the thrust axis.

• While a measurement if polarization as a function of $p_{\Lambda\perp}$, the transverse momentum of the $\Lambda$ in the lepton center-of-mass (COM) frame, is a useful process for probing the $D_T$ function.

• Therefore the polarization in the COM frame can in principle be studied from the existing Belle data by reanalyzing the data for the inclusive $e^+e^- \rightarrow \Lambda\text{(Thrust)} X$ measurement in COM $e^+e^- \rightarrow \Lambda X$.

Comments …$e^+e^- \rightarrow \Lambda\text{(Thrust)} X$ and $e^+e^- \rightarrow \Lambda X$
Unique effect driven by a single fragmentation function absent in DIS ($1 \gamma$)

Consider crossing this process to inclusive DIS for transverse polarised target

Would have the function $f_T^{q/\Lambda}(x)$, $d\sigma(S_{\Lambda T})/dxd\phi \sim \sin(\phi_S) \sum q e_q^2 f_T^{q/\Lambda}(x) = 0$  

Constraints from time reversal on quark correlation function
A unique test of time reversal in QCD: Non-zero intrinsic

Unique effect driven by a single fragmentation function absent in DIS (1 γ)

Single-Transverse $\Lambda^\uparrow$ spin asymmetry

\[
\frac{d\sigma(S_{\Lambda T})}{dz_n \, d\phi} = C \, |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[ 2 \frac{D_T^{\Lambda/q}(z_n)}{z_n} \right]
\]

\[
\frac{d\sigma(S_{\Lambda T})}{dx \, d\phi} \sim \sin(\phi_S) \sum_q e_q^2 f_T^{q/\Lambda}(x) = 0 \quad !!! \quad f_T^{q/\Lambda}(x)
\]

Constraints from time reversal on quark correlation function
Take aways II

• Non-zero $e^+e^- \rightarrow \Lambda^\uparrow X$ inclusive result is an indication that there are no gluonic poles in ffs, ie time reversal is not a constraint on FFs: the simplest process is an interesting a test of time reversal in QCD, $D_T^{\Lambda/q} \neq 0$

• We are performing a test of twist-3 factorisation at NLO in $e^+e^- \rightarrow \Lambda^\uparrow X$

• Would be great if Belle carried out a fully inclusive measurement to directly test $D_T^{\Lambda/q} \neq 0$