## $\Lambda^{\uparrow}$ Polarization in $e^{+} e^{-}$collisions

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arXiv:2102.05553

## Outline

- Motivation to study on $\Lambda^{\uparrow}$ physics long standing challenge describe via QCD factz.
- Review "outsized" role of Lambda in studying TSSAs look @ data
- Twist -2 TMD fact. description in terms of PFF. $D_{1 T}^{\perp}\left(z, p_{\perp}, Q^{2}\right)$
$\star$ Thrust observable $\Lambda$ (Thrust) $+X \quad e^{+} e^{-} \rightarrow \Lambda^{\uparrow}$ (Thrust) $X$
$\star$ Back to back hadrons $h+\Lambda \quad e^{+} e^{-} \rightarrow \Lambda^{\uparrow} h X$
- Inclusive process $e^{+} e^{-} \rightarrow \Lambda^{\uparrow} X \quad$ possible \& interesting to process to study
- Twist-3 fact. description in terms of $D_{T}\left(z, Q^{2}\right)$
* Change of ref frame COM of $e^{+} e^{-}$pair $\quad e^{+} e^{-} \rightarrow \Lambda^{\uparrow} X$ Test of naive time reversal in QCD


## Dilemma

# Transverse Quark Polarization in Large- $\boldsymbol{p}_{\boldsymbol{T}}$ Reactions, $\boldsymbol{e}^{+} e^{-}$Jets, and Leptoproduction: A Test of Quantum Chromodynamics 

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We point out that the polarization $P$ of a scattered or produced quark is calculable perturbatively in quantum chromodynamics for $e^{+} e^{-} \rightarrow q \bar{q}$, large- $p_{T}$ hadron reactions, and large $-Q^{2}$ leptoproduction, and is infrared finite. The quantum-chromodynamics prediction is that $P=0$ in the scaling limit. Experimental tests are or will soon be possible in $p p \rightarrow \Lambda X$ [where presently $P(\Lambda) \simeq 25 \%$ for $p_{T}>2 \mathrm{GeV} / c$ ] and in $e^{+} e^{-} \rightarrow$ quark jets.

In this note we have pointed out that the asymmetry off a polarized target, and the transverse polarization of a produced quark in $e^{+} e^{-} \rightarrow q \bar{q}$, or in $q q \rightarrow q q$ at large $p_{T}$, or in leptoproduction, should all be calculable perturbatively in QCD. The result is zero for $m_{q}=0$ and is numerically small if we calculate $m_{q} / \sqrt{s}$ corrections for light quarks. We discuss how to test the predictions.

At least for the cases when $P$ is small, tests should be available soon in large $-p_{T}$ production [where currently $P(\Lambda)=25 \%$ for $p_{T} \gtrless 2 \mathrm{GeV} / c$ ], and $e^{+} e^{-}$reactions. While fragmentation effects could dilute polarizations, they cannot (by parity considerations) induce polarization. Consequently, observation of significant polarizations in the above reactions would contradict either QCD or its applicabilitv.

## TMD factorization says otherwise:

Mulders Tangerman, NPB 1996
Boer, Jakob, Mulders NPB1997, 2000
Anselmino Boer, D'Alesio, Murgia. PRD 2001, 2002
Boer, Kang, Vogelsang, Yuan, PRL 2010

Measurement of Lambda-polarization through weak decay $\Lambda^{0} \rightarrow p \pi^{-}$


FIG. 1: Schematic diagram of inclusive $\Lambda$ production and decay. The angle $\theta_{p}$ of the decay proton with respect to the normal $\hat{n}$ to the production plane is defined in the $\Lambda$ rest frame.

- Proton preferentially emitted along $\Lambda$-spin
- In $\Lambda$ rest frame: pol. decay distribution

$$
\left(\frac{d N}{d \Omega_{p}}\right)_{\mathrm{pol}}=\left(\frac{d N}{d \Omega_{p}}\right)_{\text {unpol }}\left(1+\alpha P_{n}^{\Lambda} \cos \left(\theta_{p}\right)\right)
$$

## $P^{\wedge}$ : Transverse Lambda Polarization

$$
P_{\perp}^{\Lambda}\left(z_{a}, j_{\perp}\right)=\frac{d \Delta \sigma}{d z_{\Lambda} d^{2} \boldsymbol{j}_{\perp}} / \frac{d \sigma}{d z_{\Lambda} d^{2} \boldsymbol{j}_{\perp}} .
$$

QCD is Parity conserving so any final state hadron must be polarised perpendicular to the production plane


## What does Exp Say ...

## $p A \rightarrow \Lambda^{\dagger} X$

$p p \rightarrow \Lambda^{\dagger} X$


FIG. 1: Schematic diagram of inclusive $\Lambda$ production and decay. The angle $\theta_{p}$ of the decay proton with respect to the normal $\hat{n}$ to the production plane is defined in the $\Lambda$ rest frame.

$$
\begin{aligned}
& \nu N \rightarrow \Lambda^{\uparrow} X \text { novad } \\
& \qquad \begin{aligned}
& \gamma^{*} N \rightarrow \Lambda^{\uparrow} X \text { hernes } \\
& e^{+} e^{-} \rightarrow \Lambda^{\uparrow} X
\end{aligned}
\end{aligned}
$$

## Transverse $\Lambda$ polarisation a long history

One of the first transverse spin effects at Fermilab (1976):


Dunce PRL 76
Heller PRL 78


$$
p+B e \rightarrow \Lambda^{\uparrow}+X
$$

ARD 89 Lundberg


## Proton-Nuclei cont ...

## $p A \rightarrow \Lambda^{\dagger} X$

## Lundberg et al PRD40 (1989) 400 GeV

V. Fanti et al.: NA 48450 GeV proton energy

Eur. Phys. J. C 6, 265-269 (1999) CERN SPS


FIG. 4. The $\Lambda$ polarization is shown as a function of $x_{F}$ for all production angles. Over this range of production angles and within experimental uncertainties, the polarization is angle (or $p_{T}$ ) independent.


## What about LHC?

## Is it feasible at a high energy collider?



Recent ATLAS measurement at $\sqrt{ } \mathrm{S}=7 \mathrm{TeV}$

PRD 91, 032004 (2015)
Small Polarisation at mid rapidity but
Such exps. demonstrate feasibility to study $\Lambda^{\uparrow}$ @ hi energy

## What does Exp Say ...

## $p A \rightarrow \Lambda^{\dagger} X$

$p p \rightarrow \Lambda^{\dagger} X$


FIG. 1: Schematic diagram of inclusive $\Lambda$ production and decay. The angle $\theta_{p}$ of the decay proton with respect to the normal $\hat{n}$ to the production plane is defined in the $\Lambda$ rest frame.

$$
\begin{aligned}
\nu N \rightarrow & \Lambda^{\uparrow} X \text { nomad } \\
& \gamma^{*} N \rightarrow \Lambda^{\dagger} X \text { нerues }
\end{aligned}
$$

$$
e^{+} e^{-} \rightarrow \Lambda^{\uparrow} X
$$

## Simplest and cleanest process : $e^{+} e^{-} \rightarrow \Lambda^{\uparrow}($ Thrust $) X$

OPAL at LEP at Z-pole [Eur.Phys.J C2, 49 (1998)]
Longitudinal Polarization, small/zero Transverse Polarization w/ errors


QCD is Parity Conserving TSSAs Scattering plane transverse to spin Naively "T-odd"


Spin orbit

## Simplest and cleanest process $\Lambda^{\uparrow}$ in $e^{+} e^{-}$

Belle data: Transverse Polarization
Y. Guan, et al. PRL 122 (2019) $\rightarrow$ talk by Anselm here @ Jets Workshop
$\Rightarrow$ significant transverse polarization

$$
\begin{aligned}
& e^{+} e^{-} \rightarrow \Lambda^{\uparrow} \text { (Thrust) } X \\
& e^{+} e^{-} \rightarrow \Lambda^{\uparrow} h X,
\end{aligned}
$$

Measured w.r.t. thrust axis \& back to back hadrons="bTOb"



The $P_{t}$ is measured as the transverse momentum of $\Lambda$ relative to the thrust axis

$$
e^{+} e^{-} \rightarrow \Lambda^{\uparrow}(\text { Thrust }) X
$$

## FIRST OBSERVATION BY BELLE bTOb



From Anselm's INT talk

$e^{+} e^{-} \rightarrow \Lambda^{\uparrow} h X$
Back to back hadrons integrated over $p_{\perp}$ NOT SMALL

## Question for gobal analysis \& to test Universality Belle BeS BaBar + EIC

$$
\begin{aligned}
& e^{+} e^{-} \rightarrow \Lambda^{\uparrow} \text { (Thrust) } X \\
& e^{+} e^{-} \rightarrow \Lambda^{\uparrow} h X,
\end{aligned}
$$

Questions/issues: is "mechanism" the same ??
-TMD factorization " 2 " two scale fact. Theorems ?
*TMD factorization formalism ?? for thrust axis measurement

$$
\begin{aligned}
\Lambda_{Q C D} & \lesssim p_{\perp} \ll Q \\
\Lambda_{Q C D} & \lesssim j_{\perp} \ll Q
\end{aligned}
$$

## Global analysis test Universality Belle BeS BaBar + EIC

Is it same PFF function in bTOb hadron \& hadron + thrust measurements?

$T=\frac{\sum_{i}\left|\boldsymbol{p}_{i} \cdot \hat{n}\right|}{\sum_{i}\left|\boldsymbol{p}_{i}\right|} \quad \begin{aligned} & \text { The thrust axis defined by vector, } \hat{n} \\ & \text { which maximizes the thrust variable } T\end{aligned}$

- What about "T"-odd universality can we test it with all data?


## Explain non trivial $P_{\Lambda^{\dagger}}$ via TMD FFs polarization fragmentation function PFF unsurpressed

TMD framework for bTOb production of $\Lambda+h \quad$ chiral even, naively T-odd fragmentation function, universal

Parton Model factorization Mulders \& Tangerman 1996, Boer Jakob Mulders 1996 Boer \& Mulders 1997


$$
\hat{D}_{\Lambda / q}\left(z_{\Lambda}, \mathbf{p}_{\perp}, \mathbf{s}_{\perp}, Q\right)=\frac{1}{2}\left[D_{\Lambda / q}\left(z_{\Lambda}, p_{\Lambda \perp}, Q\right)+\frac{1}{\left.\left.z_{\Lambda} M_{\Lambda} D^{D_{1 T, \Lambda q}}\left(z_{\Lambda}, p_{\perp}, Q\right) \epsilon_{\perp \rho \sigma}\right|_{\perp} ^{p} S_{\perp}^{\sigma}\right]}\right]
$$

## Explain via TMD fact.

| TMD PDFs $\left(x, k_{T}\right)$ |  |  |  |
| :---: | :---: | :---: | :---: |
| H q pol. | $U$ | L | T |
| $\cup$ | $\boldsymbol{f}_{\mathbf{1}}$ |  | $\boldsymbol{h}_{\mathbf{1}}^{\perp}$ |
| L |  | $\boldsymbol{g}_{\mathbf{1 L}}$ | $\boldsymbol{h}_{\mathbf{1 L}}^{\perp}$ |
| T | $\boldsymbol{f}_{\mathbf{1 T}}^{\perp}$ | $\boldsymbol{g}_{\mathbf{1 T}}$ | $\boldsymbol{h}_{\mathbf{1 T}}$ <br> $\boldsymbol{h}_{\mathbf{1 T}}^{\perp}$ |

(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))


(Boer, Jakob, Mulders (1997))


## bTOb beyond leading order TMD Factorization

## QCD factorization Collins Soper 1982 NPB,

Collins Foundations of PQCD Cambridge Press 2011

[^0]
## JCC Soft factor further "repartitioned"

This is done to
I) cancel LC divergences in "unsubtracted" TMDs
2) separate "right \& left" movers i.e. full factorization
3) remove double counting of momentum regions

$$
\begin{gathered}
\tilde{D}_{H / j}^{\text {sub }}\left(z_{A}, b_{T} ; \mu, \zeta\right)=\lim _{\substack{y_{A} \rightarrow+\infty \\
y_{B} \rightarrow-\infty}} \underbrace{\tilde{D}_{H / j}^{\mathrm{unsub}}\left(z_{A}, b_{T} ; \mu, y_{A}-y_{B}\right)}_{\Uparrow} \sqrt{\frac{\tilde{S}\left(b_{T} ; y_{A}, y_{n}\right)}{\tilde{S}\left(b_{T} ; y_{A}, y_{B}\right) \tilde{S}\left(b_{T} ; y_{n}, y_{B}\right)}} \times U V_{\text {renorm }} \\
\tilde{D}_{H / j}^{\mathrm{unsub}}\left(z_{A}, b_{T} ; \mu, y_{P}-y_{B}\right)=\left.\frac{1}{z_{A}} \int \frac{d b^{+}}{2 \pi} e^{-i k_{A}^{-} b^{+}}\langle 0| \gamma^{-} \mathcal{U}_{[0, b]} \psi(b)\left|X P_{A}\right\rangle\left\langle P_{A} X\right| \bar{\psi}(0)|0\rangle\right|_{b^{-}=0}
\end{gathered}
$$



## Use both data sets to study universality of T-odd fragmentation? What is prediction of TMD Factorization

Universality of T-odd Collins function: $H_{1, n / q}^{\perp(1)}(z, b, Q)$
Metz PLB2002,
Boer Mulders Pijlman NPB2003
Collins Metz PRL 2004,
Gamberg, Mukerjee, Mulders PRD2007,
Meissner Metz PRL 2009,
Gamberg Mukherjee, Mulders PRD 2008
Universality of T-odd PFF prediction from pQCD - $D_{1 T, \Lambda / q}^{\perp(1)}\left(z_{\Lambda}, b, Q\right)$ phase from FSI but not gluonic/fermionic pole
Boer, Kang, Vogelsang, Yuan PRL 2010


## $\Lambda$ Belle data fall into 2 classes

$$
e^{+} e^{-} \rightarrow \Lambda^{\uparrow} h X \quad \& \quad \Lambda^{\uparrow}(\text { Thrust }) X
$$

? Is it true that the PFF is the same TMD in both process?

Recent extractions address this

1) D'Alesio \& Murgia ZacchedduPRD2020 bTOb + Thrust assumed same factz.here
2) Callos, Kang, Terry PRD2020 bTOb only

## Other pheno studies

*Anselmino, Kishore, Mukherjee PRD 2019 single inclusive case and the role of the PFFs twist-2 in place of twist-3 ?
*EEarlier Anselmino Boer, D’Alesio, Murgia. PRD 2001, 2002
TMD factorization applied to inclusive process ?

## ? Same PFF ? in $\quad e^{+} e^{-} \rightarrow \Lambda^{\uparrow}$ (Thrust)

In TMD factorization framework for production of
$\Lambda$ (Thrust) we have non-global observable "right hemisphere" only
? chiral even, naively T-odd fragmentation function, universal ?


- Z.B Kang, D.Y. Shao, F. Zhao 2007.14425

$$
\hat{D}_{\Lambda / q}\left(z_{\Lambda}, \mathbf{p}_{\perp}, \mathbf{S}_{\perp}, Q\right)=\frac{1}{2}\left[D_{\Lambda / q}\left(z_{\Lambda}, p_{\Lambda \perp}, Q\right)+\frac{1}{z_{\Lambda} N_{\Lambda_{\perp}}}-D_{1 T, \Lambda / q}^{\perp}\left(z_{\Lambda}, p_{\perp}, Q\right) \epsilon_{\perp \rho \sigma} p_{\perp}^{\rho} S_{\perp}^{\sigma}\right]
$$

## TMD factorization \& Thrust observable

## Recent work

- M. Boglione \& A. Simonelli, 2007.13674
- Z.B Kang, D.Y. Shao, F. Zhao 2007.14425
- M. Boglione \& A. Simonelli, 2007.13674
Z.B Kang, D.Y. Shao, F. Zhao 2007.14425 - seetalk of Dingyu

Derive TMD factorization for unpolarized transverse momentum distribution for the single hadron production with the thrust axis in electron-positron collision

## Lets Drill Down TMD factorization

## Recent work

- Z.-B Kang, D.Y. Shao, F. Zhao 2007.14425

Derive TMD factorization for unpolarized TMD FF for single hadron production with the thrust axis in electron-positron collision $e^{+} e^{-} \rightarrow \Lambda^{\uparrow}$ (Thrust) non-global observable

$$
\begin{aligned}
& \frac{d \sigma}{d z_{\Lambda} d^{2} \boldsymbol{j}_{\perp}}=\sigma_{0} H(Q, \mu) \sum_{q} e_{q}^{2} \int d^{2} p_{\perp} d^{2} \lambda_{\perp} \delta^{(2)}\left(\boldsymbol{j}_{\perp}-\boldsymbol{p}_{\perp}-z_{\Lambda} \boldsymbol{\lambda}_{\perp}\right) D_{\Lambda / q}\left(z_{\Lambda}, p_{\perp}, \mu, \zeta / \nu^{2}\left(S_{\mathrm{hemi}}\left(\lambda_{\perp}, \mu, \nu\right)\right)\right. \\
& \frac{\text { Calculated to NLO and NLL }}{d z_{l a} d^{2} \boldsymbol{j}_{\perp}}=\sigma_{0}^{\mathrm{TMD}} \sum_{q} e_{q}^{2} \int_{0}^{\infty} \frac{b d b}{(2 \pi)} J_{0}\left(\frac{b j_{\perp}}{z_{\Lambda}}\right) \frac{1}{z_{\Lambda}^{2}} D_{\Lambda / q}\left(z_{\Lambda}, \mu_{b_{*}}\right) e^{-S_{\mathrm{NP}}\left(b, z_{\Lambda}, Q_{0}, Q\right)-S_{\text {pert }}\left(\mu_{\left.b_{*}, Q\right)} U_{\mathrm{NG}}\left(\mu_{b_{*}}, Q\right)\right.} \\
& \text { Non-global logs resummed } \\
& \text { Factorization theorem } \exists \\
& \text { Becher Rahn Shao JHEP 2017 }
\end{aligned}
$$

## We extend TMD factorization PFF $e^{+} e^{-} \rightarrow \Lambda^{\uparrow}$ (Thrust)

Gamberg, Kang, Shao,Terry, Zhao arXiv:2102.05553


$$
\begin{aligned}
& \frac{d \Delta \sigma}{d z_{\Lambda} d^{2} \boldsymbol{j}_{\perp}}=\frac{d \sigma\left(\boldsymbol{S}_{\perp}\right)}{d z_{\Lambda} d^{2} \boldsymbol{j}_{\perp}}-\frac{d \sigma\left(-\boldsymbol{S}_{\perp}\right)}{d z_{\Lambda} d^{2} \boldsymbol{j}_{\perp}} \\
& \quad=\sigma_{0}^{\mathrm{TMD}} \sin \left(\phi_{s}-\phi_{j}\right) \sum_{q} e_{q}^{2} \int_{0}^{\infty} \frac{b^{2} d b}{4 \pi} J_{1}\left(\frac{b j_{\perp}}{z_{\Lambda}}\right) \\
& \quad \times \frac{M_{\Lambda}}{z_{\Lambda}^{4}} D_{1 T, \Lambda / q}^{\perp(1)}\left(z_{\Lambda}, \mu_{b_{*}}\right) e^{-S_{\mathrm{NP}}^{\perp}\left(b, z_{\Lambda}, Q_{0}^{\prime}, Q\right)-S_{\mathrm{pert}}\left(\mu_{b_{*}}, Q\right)} U_{\mathrm{NG}}\left(\mu_{b_{*}}, Q\right)
\end{aligned}
$$

UV \& Rapidity subtracted TMD Universal PFF

## Establish factorization for thrust axis factorization carry out pheno to describe Belle $P_{T}$ and OPAL

$$
e^{+} e^{-} \rightarrow \Lambda^{\uparrow}(\text { Thrust })
$$

## Postage stamp of input for Pheno

$$
P_{\perp}^{\Lambda}\left(z_{\Lambda}, j_{\perp}\right)=\frac{d \Delta \sigma}{d z_{\Lambda} d^{2} j_{\perp}} / \frac{d \sigma}{d z_{\Lambda} d^{2} j_{\perp}} .
$$

$S_{\mathrm{NP}}\left(b, z_{\Lambda}, Q_{0}, Q\right)=g_{h} \frac{b^{2}}{z_{\Lambda}^{2}}+\frac{g_{2}}{2} \ln \frac{Q}{Q_{0}} \ln \frac{b}{b_{*}} \quad$ Aidala Field Gamberg Rogers PRD 2014

$$
g_{h}=0.042 \mathrm{GeV}^{2} \quad g_{2}=0.84 \mathrm{GeV}^{2} \quad \text { Implementation Issacson Sun Yuan } 2014 \text { MPA }
$$

$U_{\mathrm{NG}}\left(\mu_{b_{*}}, Q\right)=\exp \left[-C_{A} C_{F} \frac{\pi^{2}}{3} u^{2} \frac{1+(a u)^{2}}{1+(b u)^{c}}\right] \quad$ Dasgupta Salam, PLB 2001
with $a=0.85 C_{A}, b=0.86 C_{A}, c=1.33$

$$
D_{1 T, h / q}^{\perp}\left(z, p_{\perp}, Q_{0}^{\prime}\right)=\frac{M_{\Lambda}}{\left\langle M_{D}^{2}\right\rangle} D_{1 T, h / q}^{\perp}\left(z, Q_{0}^{\prime}\right) \frac{e^{-p_{\perp}^{2} /\left\langle M_{D}^{2}\right\rangle}}{\pi\left\langle M_{D}^{2}\right\rangle}
$$

$$
u \equiv \int_{\mu_{b_{*}}}^{Q} \frac{d \mu}{\mu} \frac{\alpha_{s}(\mu)}{2 \pi}=\frac{1}{\beta_{0}} \ln \left[\frac{\alpha_{s}\left(\mu_{b_{*}}\right)}{\alpha_{s}(Q)}\right]
$$

$$
\begin{gathered}
D_{1 T, h / q}^{\perp}\left(z, Q_{0}^{\prime}\right)=\mathcal{N}_{q}(z) D_{h / q}\left(z, Q_{0}^{\prime}\right) \quad Q_{0}^{\prime}=10.58 \mathrm{GeV} \\
\mathcal{N}_{q}(z)=N_{q} z^{\alpha_{q}}(1-z)^{\beta_{q} \alpha_{q}} \frac{\left(\alpha_{q}+\beta_{q}-1\right)^{\alpha_{q}+\beta_{q}-1}}{\left(\alpha_{q}-1\right)^{\alpha_{q}-1} \beta_{q}^{\beta_{q}}}
\end{gathered}
$$

$S_{N P}^{\perp}\left(b, z, Q_{0}^{\prime}, Q\right)=\frac{\left\langle M_{D}^{2}\right\rangle}{4} \frac{b^{2}}{z^{2}}+\frac{g_{2}}{2} \ln \frac{Q}{Q_{0}^{\prime}} \ln \frac{b}{b_{*}}$

Parameters fit from bTOb Belle data Callos, Kang, Terry PRD2020

## Belle data fit $e^{+} e^{-} \rightarrow \Lambda^{\uparrow} h X$



## Recent extractions address this Callos, Kang, Terry PRD2020 bTOb only



FIG. 3. The fit to the experimental data for $\pi$ mesons is shown, with the gray uncertainty band displayed is generated by the replicas at $68 \%$ confidence. The left plots are for the production of $\Lambda+\pi^{ \pm}$, while the right plots are for the production of $\bar{\Lambda}+\pi^{ \pm}$.

## And for kaons

## Belle data $e^{+} e^{-} \rightarrow \Lambda^{\uparrow} h X$,

$D_{1 T, \Lambda / q}^{\perp(1)}\left(z_{\Lambda}, Q\right)$

Recent extractions
Callos, Kang, Terry PRD2020 bTOb only


Exploit Universality to describe $\quad e^{+} e^{-} \rightarrow \Lambda^{\uparrow}$ (Thrust)

## Compare theory predictions to OPAL \& Belle

$$
e^{+} e^{-} \rightarrow \Lambda^{\uparrow} \text { (Thrust) }
$$

Gamberg, Kang, Shao,Terry, Zhao arXiv:2102.05553


- $P_{\perp}^{\Lambda}\left(z_{\Lambda}, j_{\perp}\right)$ for the Belle data [20]; left to right theory integrated from
$0.2<z_{\Lambda}<0.3,0.3<z_{\Lambda}<0.4,0.4<z_{\Lambda}<0.5,0.5<z_{\Lambda}<0.6$
- The data in red is for $\Lambda$ production while the data in blue is for $\bar{\Lambda}$ production
- Data plotted with total exp. uncertainty as vertical error bar \& uncertainty on $j_{\perp}$ horizontal error bar
- Gray band is the theoretical uncertainty which was generated from the replicas for the TMD PFF, Callos, Kang, Terry PRD2020


## Compare theory predictions to OPAL \& Belle data

$$
e^{+} e^{-} \rightarrow \Lambda^{\uparrow} \text { (Thrust) }
$$



Gamberg, Kang, Shao,Terry, Zhao arXiv:2102.05553

$$
P_{\perp}^{\Lambda}\left(z_{a}, j_{\perp}\right)=\frac{d \Delta \sigma}{d z_{\Lambda} d^{2} \boldsymbol{j}_{\perp}} / \frac{d \sigma}{d z_{\Lambda} d^{2} \boldsymbol{j}_{\perp}} .
$$


$P_{\perp}^{\Lambda}\left(z_{\Lambda}, j_{\perp}\right)$ for OPAL data [19]: Theory curve is integrated over the region $0.2<z_{\Lambda}<0.5$. total experimental uncertainty vertical error bar $j_{\perp}$ horizontal error bar. Error band, standard deviation of the replicas for TMD PFF in Callos, Kang, Terry PRD2020.

## Fully inclusive process $e^{+} e^{-} \rightarrow \Lambda^{\uparrow} X$

$\Rightarrow$ significant transverse polarization ?

$$
e^{+} e^{-} \rightarrow \Lambda^{\uparrow} X
$$

Measure w.r.t. COM in principle can measure at Belle ?

Questions/issues:
QCD prediction of Physics twist-3

- Twist-3 factorization one hard scale


## Simplest and cleanest process $\quad e^{+} e^{-} \rightarrow \Lambda^{\uparrow} X$

$\Rightarrow$ significant transverse polarization ?

$$
e^{+} e^{-} \rightarrow \Lambda^{\uparrow} X
$$

$P_{\Lambda \perp} \sim Q$ twist-3 factorization
And can be measured w.r.t. com of $e^{+} e^{-}$on large scale $P_{T} \sim Q$


## Consider Transverse $e^{+} e^{-} \rightarrow \Lambda^{\uparrow} X$ polarization

Gamberg, Kang, Pitonyak, Schlegel, Yoshida JHEP 2019, LO \& NLO
There are contributions from
'Intrinsic' \& 'kinematical' twist-3 FF


Intrinsic Kinematical
$\frac{d \sigma\left(S_{\Lambda T}\right)}{d z_{h} d \phi}=C\left|S_{\Lambda T}\right| \sin \left(\phi_{S}\right) \sum_{q} e_{q}^{2}\left[\frac{D_{T}^{\Lambda / q}\left(z_{h}\right)}{z_{h}}-D_{1 T}^{\perp(1) \Lambda / q}\left(z_{h}\right)+\int_{0}^{1} d \beta \frac{\Im\left[\hat{D}_{F T}-\hat{G}_{F T}\right]^{\Lambda / q}\left(z_{h}, z_{h} / \beta\right)}{1-\beta}\right]$
Using the EOMs and LIRs CS can be expressed soley in terms of $\| D_{T}^{\Lambda / q}(z)$

$$
\frac{d \sigma\left(S_{\Lambda T}\right)}{d z_{h} d \phi}=C\left|S_{\Lambda T}\right| \sin \left(\phi_{S}\right) \sum_{q} e_{q}^{2}\left[2 \frac{D_{T}^{\Lambda / q}\left(z_{h}\right)}{z_{h}}\right]
$$

Boer, Jakob, Mulders NPB (1997)
in TMD framework at twist-3

## Twist - 3 Pheno

$$
\frac{d \Delta \sigma}{d z_{\Lambda} d^{2} p_{\Lambda \perp}}=-\sin \left(\phi_{s}-\phi_{\Lambda}\right) \sigma_{0}^{\mathrm{Col}}\left(\frac{8 M_{\Lambda}}{Q}\right) \frac{p_{\Lambda \perp}}{Q} \frac{1}{z_{\Lambda}^{3}} \sum_{q} e_{q}^{2} \frac{D_{T, \Lambda / q}\left(z_{\Lambda}, Q\right)}{z_{\Lambda}}
$$

To describe this process, only need a parameterization for $D_{T, \Lambda / q}\left(z_{\Lambda}, Q\right)$
Given our lack of knowledge of this fundamental twist-3 T-odd fragmentation function we will employ the approach outlined in Gamberg, Metz, Pitonyak, Prokudin PLB 2017

Re-express the $D_{T, \Lambda / q}\left(z_{\Lambda}\right)$ in terms of our knowledge of $D_{1 T, \Lambda / q}^{\perp(1)}\left(z_{\Lambda}\right)$

## Twist - 3 Pheno

$$
\frac{d \Delta \sigma}{d z_{\Lambda} d^{2} p_{\Lambda \perp}}=-\sin \left(\phi_{s}-\phi_{\Lambda}\right) \sigma_{0}^{\mathrm{Col}}\left(\frac{8 M_{\Lambda}}{Q}\right) \frac{p_{\Lambda \perp}}{Q} \frac{1}{z_{\Lambda}^{3}} \sum_{q} e_{q}^{2} \frac{D_{T, \Lambda / q}\left(z_{\Lambda}, Q\right)}{z_{\Lambda}}
$$

Re-express the $D_{T, \Lambda / q}\left(z_{\Lambda}\right)$ in terms of our knowledge of $D_{1 T, \Lambda / q}^{\perp(1)}\left(z_{\Lambda}\right)$

$$
\begin{aligned}
& \frac{1}{z_{\Lambda}} D_{T, \Lambda / q}\left(z_{\Lambda}\right)=-\left(1-z_{\Lambda} \frac{d}{d z_{\Lambda}}\right) D_{1 T, \Lambda / q}^{\perp(1)}\left(z_{\Lambda}\right)-2 \int_{0}^{1} d \beta \frac{\Im\left[\hat{D}_{F T}^{q g}\left(z_{\Lambda}, \beta\right)\right]}{(1-\beta)^{2}} \\
& \frac{1}{z_{\Lambda}} D_{T, \Lambda / q}\left(z_{\Lambda}\right) \approx-\left(1-z_{\Lambda} \frac{d}{d z_{\Lambda}}\right) D_{1 T, \Lambda / q}^{\perp(1)}\left(z_{\Lambda}\right) \\
& \\
&
\end{aligned}
$$

## Prediction for Belle

Gamberg, Kang, Shao,Terry, Zhao arXiv:2102.05553

$$
P_{\mathbf{C M}}^{\Lambda}\left(z_{\Lambda}, p_{\Lambda \perp}\right)=\frac{d \Delta \sigma}{d z_{\Lambda} d^{2} p_{\Lambda \perp}} / \frac{d \sigma}{d z_{\Lambda} d^{2} p_{\Lambda \perp}}
$$


$P_{\mathrm{CM}}^{\Lambda}-3$-D plot of the polarization in $z_{\Lambda}$ and $p_{\Lambda \perp}$
Center: Plot of the polarization as a function of only $z_{\Lambda}$,
Right: Plot of the polarization as a function of $p_{\Lambda \perp}$ : polarization in our scheme is $\sim 1-2 \%$
Plots are generated only using the central fit
The red and blue curves are generated using the central fit, gray band is the theoretical uncertainty

## Take aways I

## Comments $\ldots e^{+} e^{-} \rightarrow \Lambda$ (Thrust) $X$ and $e^{+} e^{-} \rightarrow \Lambda X$

- Interesting that while these two measurements probe different distribution functions, they differ only by the definition of the measurement axis.
- That is, a measurement the polarization as a function of $j_{\perp}$ is a useful process for probing the properties of the PFF $D_{1 T}^{\perp}$ with respect to the thrust axis
- While a measurement if polarization as a function of $p_{\Lambda \perp}$, the transverse momentum of the $\Lambda$ in the lepton center-of-mass (COM) frame, is a useful process for probing the $D_{T}$ function.
- Therefore the polarization in the COM frame can in principle be studied from the existing Belle data by reanalyzing the data for the inclusive $e^{+} e^{-} \rightarrow \Lambda$ (Thrust) $X$ measurement in $\mathrm{COM} e^{+} e^{-} \rightarrow \Lambda X$


## Single-Transverse $\Lambda^{\uparrow}$ spin asymmetry

## Unique effect driven by a single fragmentation function $\left|D_{r}^{N_{r}^{\prime} q(z)}\right| \rightarrow$

 absent in DIS (1 $\gamma$ )

Intrinsic

$$
\frac{d \sigma\left(S_{\Lambda T}\right)}{d z_{h} d \phi}=C\left|S_{\Lambda T}\right| \sin \left(\phi_{S}\right) \sum_{q} e_{q}^{2}\left[2 \frac{D_{T}^{\Lambda / q}\left(z_{h}\right)}{z_{h}}\right]
$$

See also Boer, Jakob, Mulders NPB (1997)
n.b. some intuition ...

Consider crossing this process to inclusive DIS for transverse polarised target
Would have the function $f_{T}^{q / \Lambda}(x), \frac{d \sigma\left(S_{\Lambda T}\right)}{d x d \phi} \sim \sin \left(\phi_{S}\right) \sum_{q} e_{q}^{2} f_{T}^{q / \Lambda}(x)=0 \quad$ !!!
Constraints from time reversal on quark correlation function Goeke, Metz, Schlegel PLB 2006, Bacchetta et al JHEP 2007, Christ \& Lee 1960

## A unique test of time reversal in QCD: Non-zero intrinsic

## Unique effect driven by a single fragmentation function ${D_{r}^{1 / q}(z)} \rightarrow$

 absent in DIS (1 $\gamma$ )
## Single-Transverse $\Lambda^{\uparrow}$ spin asymmetry

$$
\frac{d \sigma\left(S_{\Lambda T}\right)}{d z_{h} d \phi}=C\left|S_{\Lambda T}\right| \sin \left(\phi_{S}\right) \sum_{q} e_{q}^{2}\left[2 \frac{D_{T}^{\Lambda / q}\left(z_{h}\right)}{z_{h}}\right]
$$

$$
\frac{d \sigma\left(S_{\Lambda T}\right)}{d x d \phi} \sim \sin \left(\phi_{S}\right) \sum_{q} e_{q}^{2} f_{T}^{q / \Lambda}(x)=0 \quad!!!\quad f_{T}^{q / \Lambda}(x)
$$

## Take aways II

- Non-zero $e^{+} e^{-} \rightarrow \Lambda^{\uparrow} X$ inclusive result is an indication that there are no gluonic poles in ffs, ie time reversal is not a constraint on FFs: the simplest process is an interesting a test of time reversal in QCD, $D_{T}^{\Lambda / q} \neq 0$
- We are performing a test of twist-3 factorisation at NLO in $e^{+} e^{-} \rightarrow \Lambda^{\uparrow} X$
- Would be great if Belle carried out a fully inclusive measurement to directly test $D_{T}^{\Lambda / q} \neq 0$


[^0]:    Collins Soper (81,82), Collins, Soper, Sterman (85), Boer (0I) (09) (I3), Ji,Ma, Yuan (04,05,06),
    Collins-Cambridge University Press (II),Aybat Rogers PRD (II), Abyat, Collins, Qiu, Rogers (I I), Aybat, Prokudin, Rogers (I I), Bacchetta, Prokudin (I3), Sun, Yuan (I3),Echevarria, Idilbi, Scimemi JHEP 20I2, Collins Rogers 2015 ...
    SCET: Bauer, Flemming Pirjol Rothstein, Stewart PRD 2002, Chiu, Jain, Neill, Rothstein JHEP 2013, Rothstein \& Stewart JHEP 2016,

