Factorized approach to radiative corrections for inelastic lepton-hadron collisions

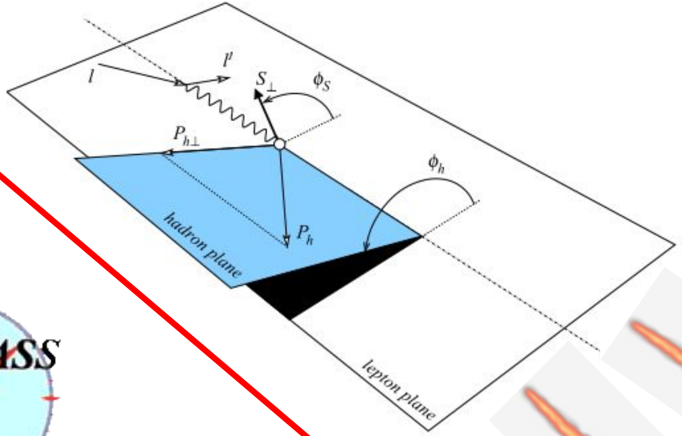
Nobuo Sato

In collaboration with :
Tianbo Liu (Shandong U.), Wally Melnitchouk (JLab), Jianwei Qiu (JLab)

GHP21: TMDs and SIDIS

Jefferson Lab

SIDIS - the next frontier of hadron structure



$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$

$$+\,S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+\,S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

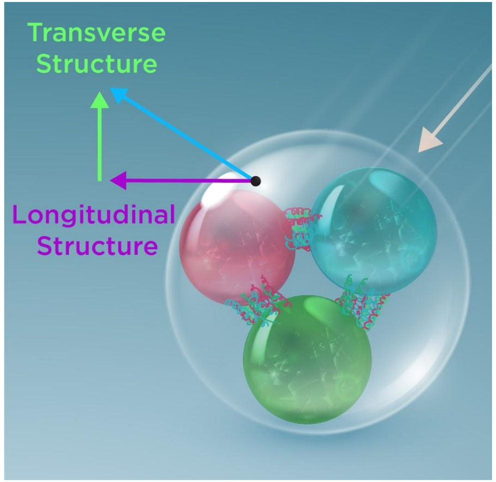
$$+\,|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+\,\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\,\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg]$$

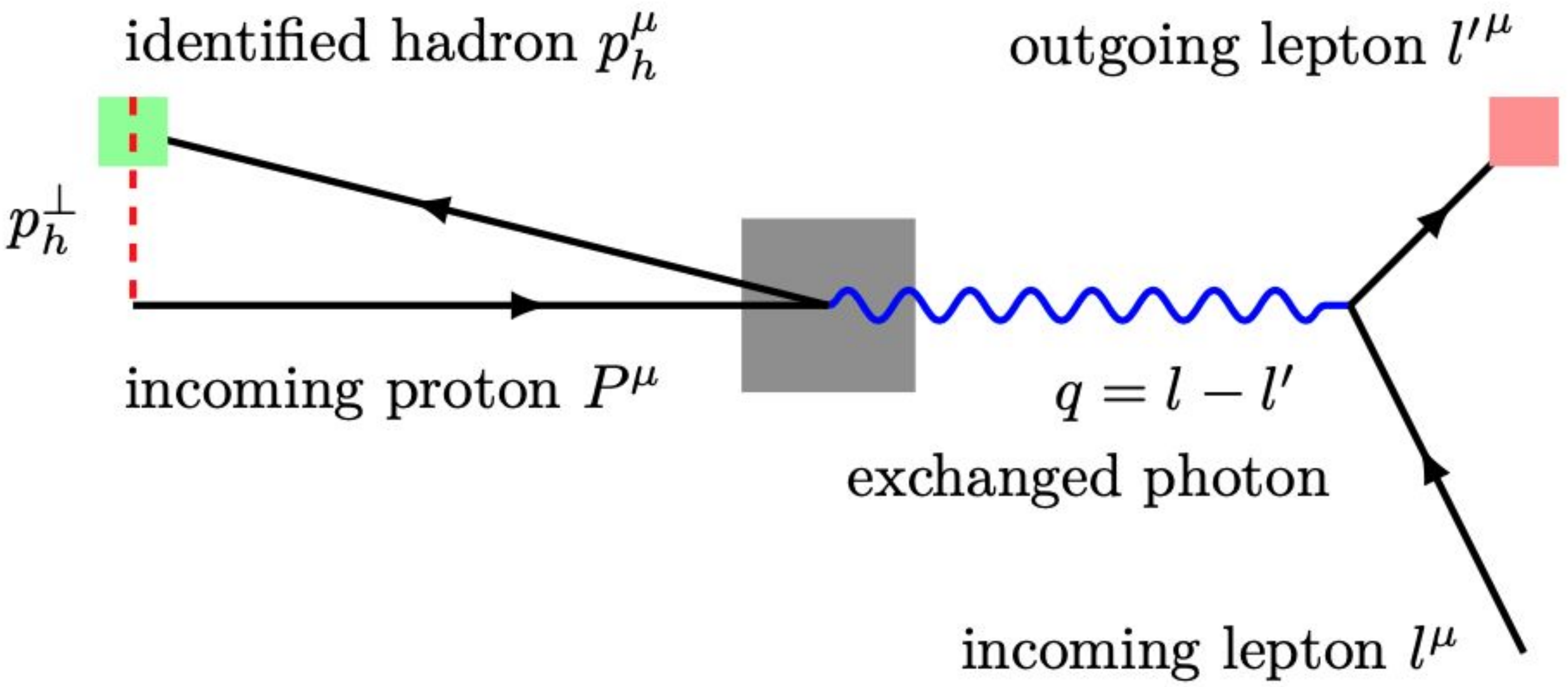
$$+\,|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}$$

$$+\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},$$

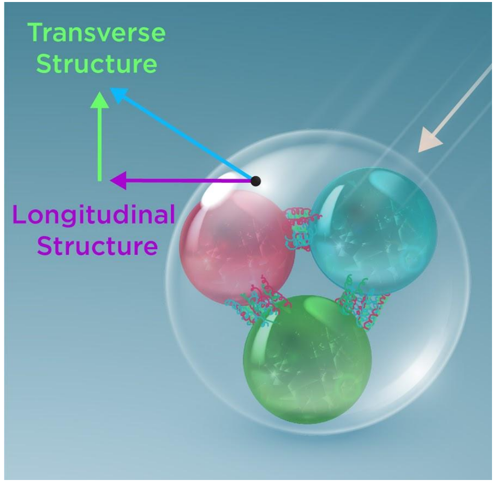


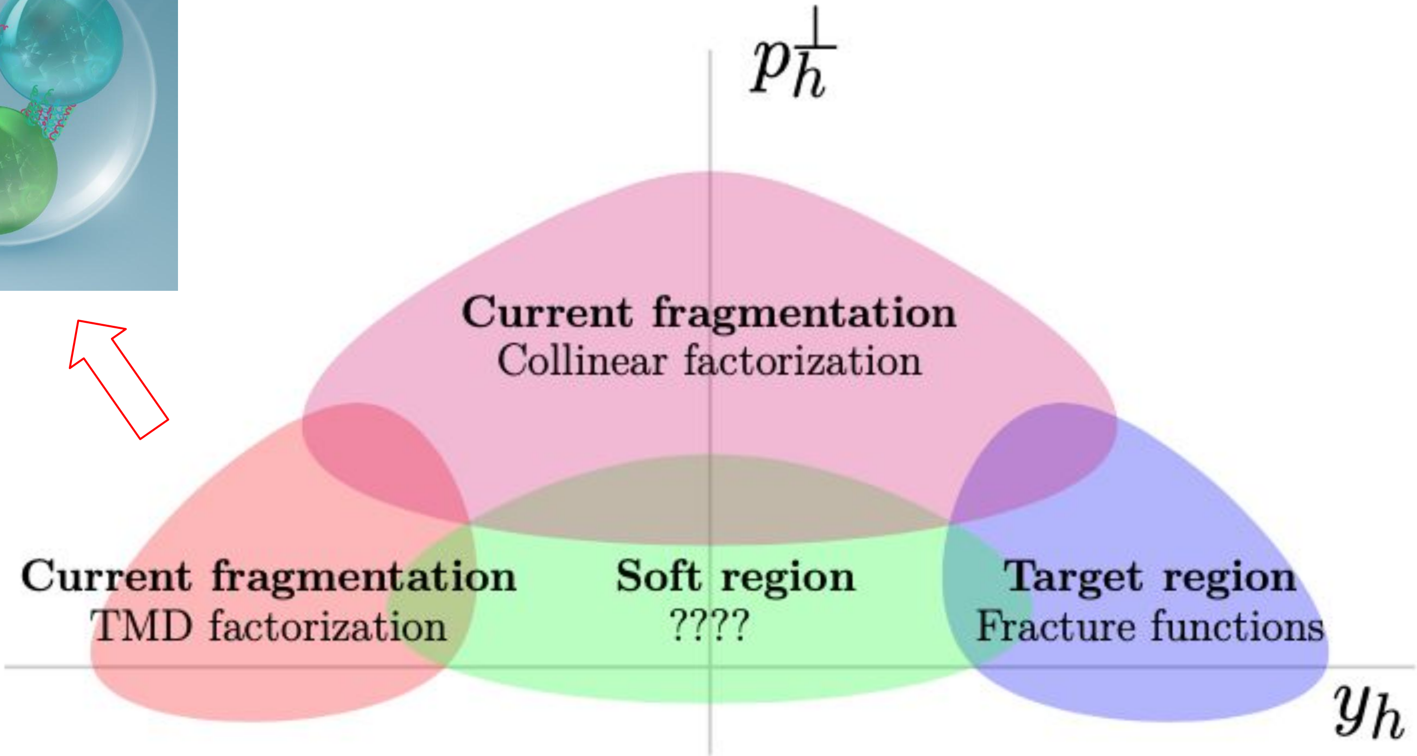
Name	Symbol	meaning
upol. PDF	f_1^q	U. pol. quarks in U. pol. nucleon
pol. PDF	g_1^q	L. pol. quarks in L. pol. nucleon
Transversity	h_1^q	T. pol. quarks in T. pol. nucleon
Sivers	$f_{1T}^{\perp(1)q}$	U. pol. quarks in T. pol. nucleon
Boer-Mulders	$h_1^{\perp(1)q}$	T. pol. quarks in U. pol. nucleon
Boer-Mulders	$h_1^{\perp(1)q}$	T. pol. quarks in U. pol. nucleon
⋮	⋮	⋮
FF	D_1^q	U. pol. quarks to U. pol. hadron
Collins	$H_1^{\perp(1)q}$	T. pol. quarks to U. pol. hadron
⋮	⋮	⋮

The Breit Frame



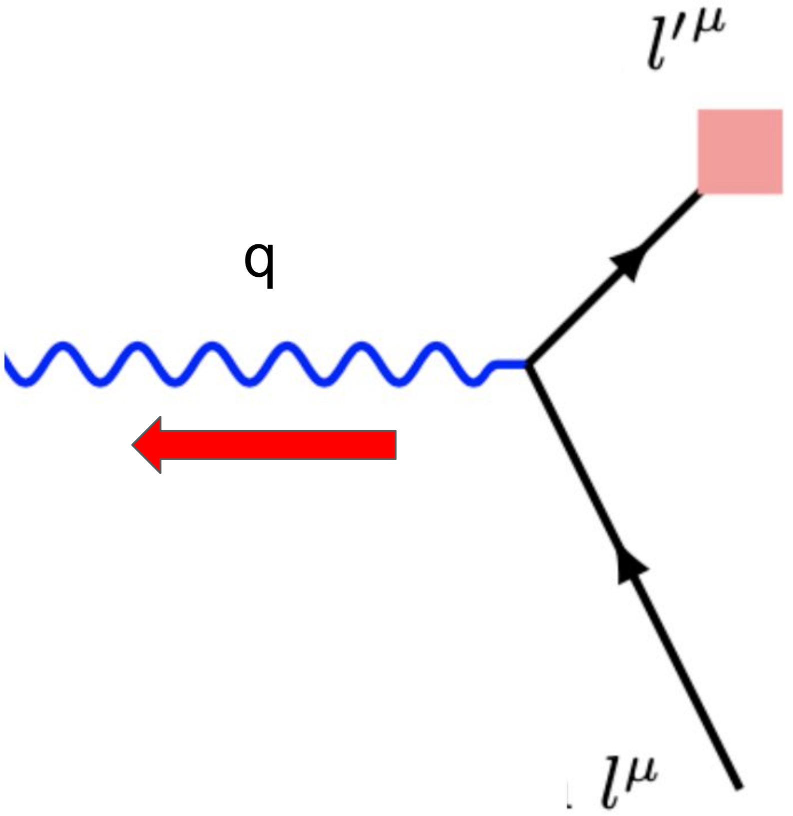
SIDIS regions





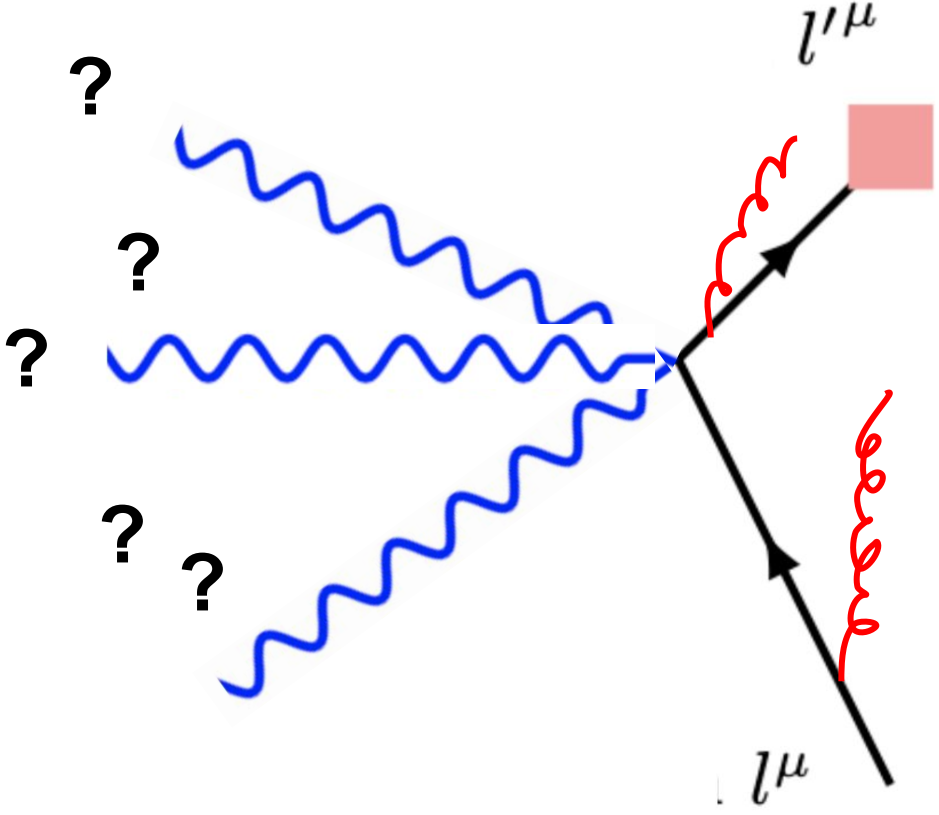
But ....

- Factorization theorem relies on q moving along -z
- How do we know that q is exactly along -z even-by-event?
- Role of QED radiation?



- In the presence of QED radiation, **the q direction is not fixed**

- The experimental Breit Frame **does not need to coincide with the actual Breit-frame** needed in QCD factorization



How to proceed ?


arXiv.org > hep-ph > arXiv:2008.02895

High Energy Physics – Phenomenology

[Submitted on 6 Aug 2020 (v1), last revised 17 Mar 2021 (this version, v3)]

Factorized approach to radiative corrections for inelastic lepton-hadron collisions

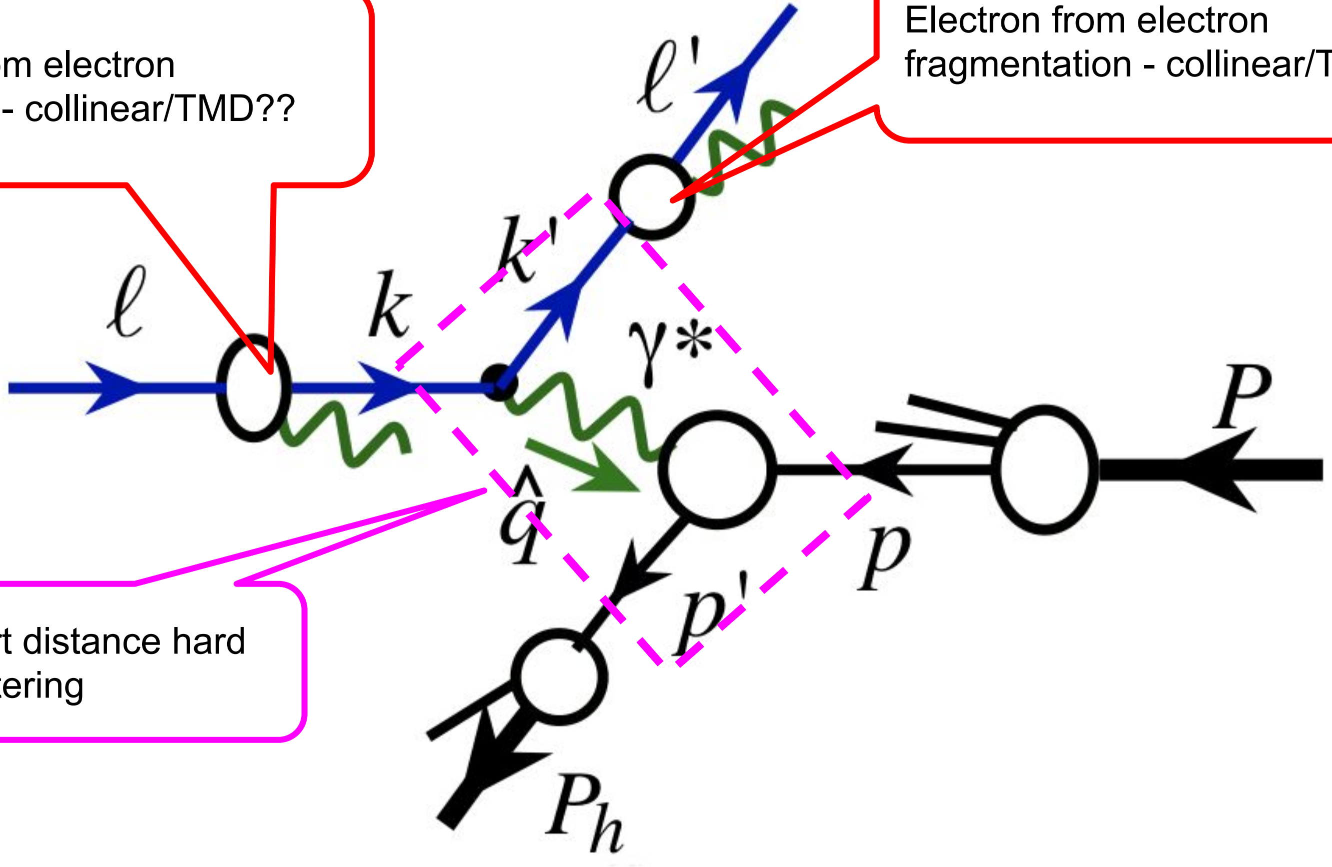
Tianbo Liu, W. Melnitchouk, Jian-Wei Qiu, N. Sato

Lepton Distribution Function (**LDF**)

Electron from electron distribution - collinear/TMD??

Lepton Fragmentation Function (**LFF**)

Electron from electron fragmentation - collinear/TMD??



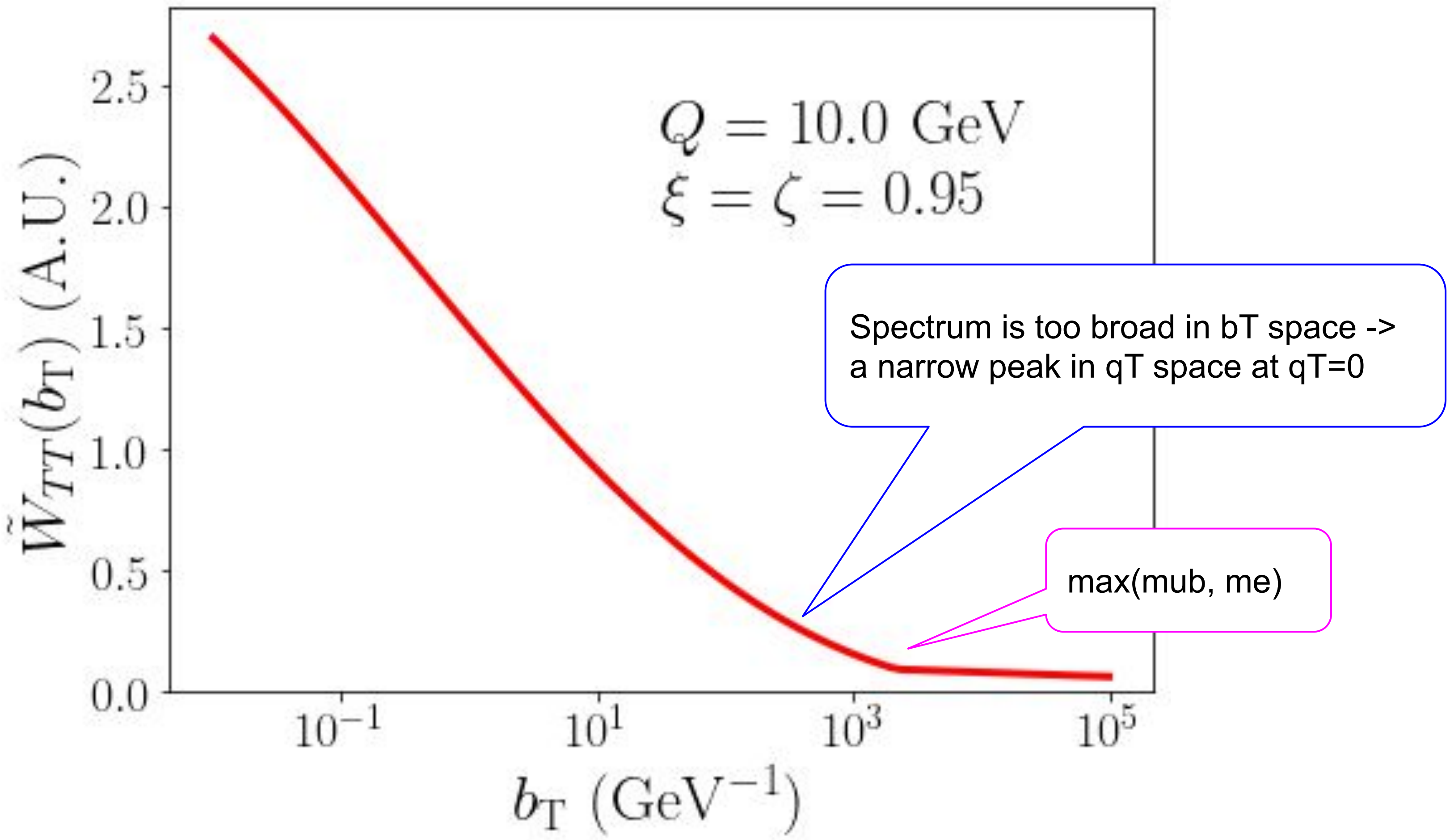
Short distance hard scattering

Key observation

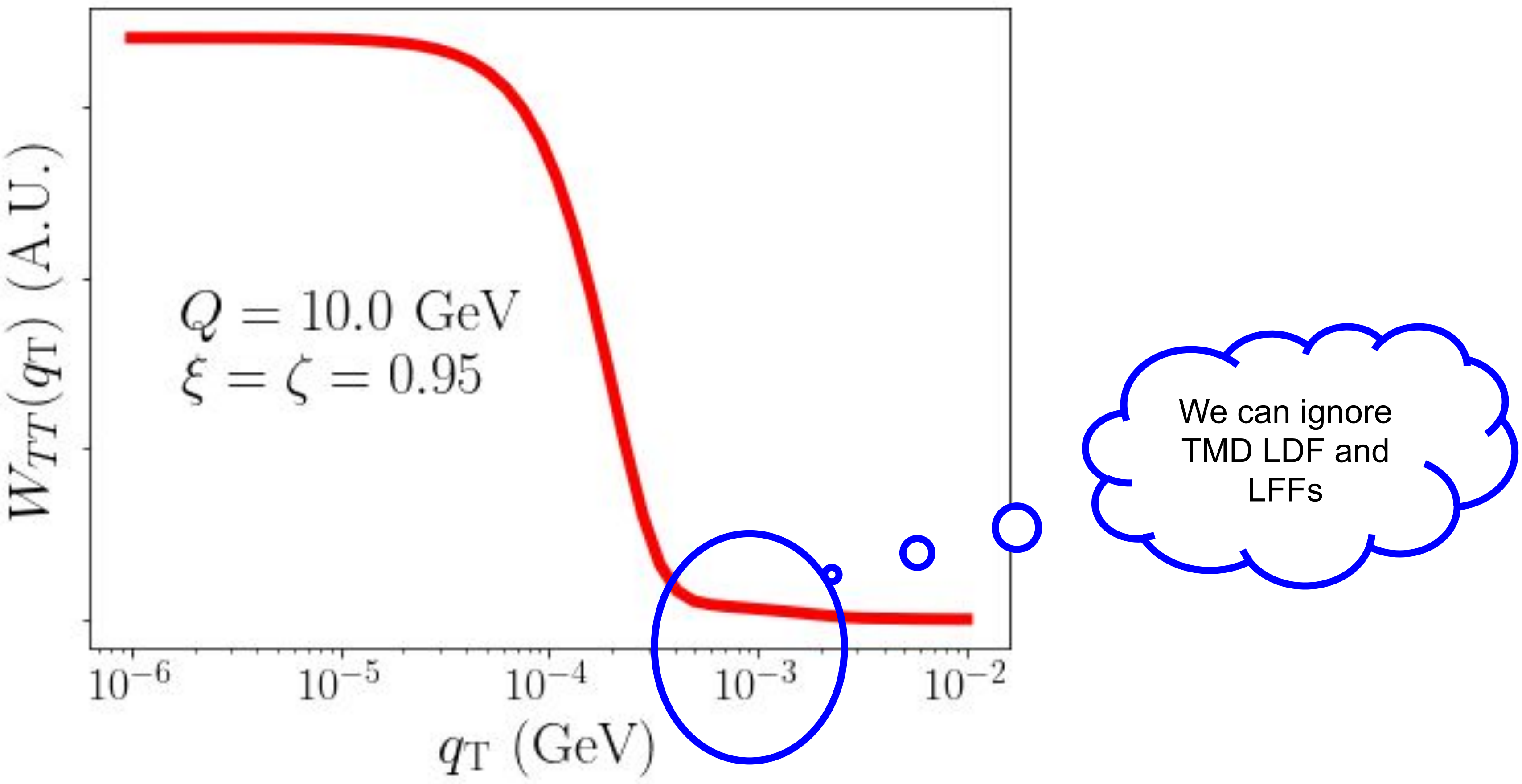
$$L_{\rho\sigma}(\xi_B, \zeta_B, Q^2, \hat{\boldsymbol{q}}_T^2) = \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}} \widetilde{W}_{\rho\sigma}(\xi_B, \zeta_B, Q^2, b) + Y_{\rho\sigma}(\xi_B, \zeta_B, Q^2, \hat{\boldsymbol{q}}_T^2),$$

Collinear LDF and LFFs

$$\widetilde{W}_{TT}(\xi_B, \zeta_B, Q^2, b) = 2\int_{\xi_B}^{1} \frac{d\xi}{\xi} \int_{\zeta_B}^{1} \frac{d\zeta}{\zeta^2} f(\xi) D(\zeta) C_f(\lambda) C_D(\eta) \times \exp\left\{ -\int_{\mu_b^2}^{\mu_Q^2} \frac{d\mu'^2}{\mu'^2} \left[A(\alpha(\mu')) \ln \frac{\mu_Q^2}{\mu'^2} + B(\alpha(\mu')) \right] \right\}$$



$Q = 10.0$ GeV
$\xi = \zeta = 0.95$
$\tilde{W}_{TT}(b_T)$ (A.U.)
b_T (GeV^{-1})
0.0
0.5
1.0
1.5
2.0
2.5
10^{-1}
10^{1}
10^{3}
10^{5}
Spectrum is too broad in bT space -> a narrow peak in qT space at qT=0
max(mub, me)


$W_{TT}(q_T)$ (A.U.)
$Q = 10.0$ GeV
$\xi = \zeta = 0.95$
10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2}
q_T (GeV)
We can ignore TMD LDF and LFFs

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$

$$+\,S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+\,S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

$$+\,|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+\,\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$

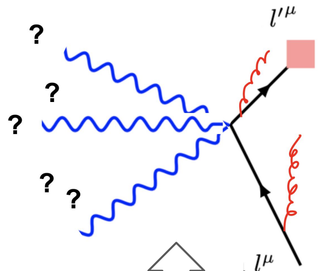
$$+\,\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg]$$

$$+\,|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}$$

$$+\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},$$

+ QED ???

SIDIS with QED



Collinear LDF and LFFs

QED rotational effects

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h,\mathrm{T}}^2} = \int_{\zeta_{\min}}^{1} d\zeta \int_{\xi_{\min(\zeta)}}^{1} d\xi\, f(\xi)\, d(\zeta)\, \frac{\hat{x}}{x\xi\zeta}$$

$$\times \left[\frac{\alpha_{\mathrm{EM}}^2}{\hat{x}\hat{y}\hat{Q}^2}\frac{\hat{y}^2}{2(1-\hat{\epsilon})}\left(1+\frac{\hat{\gamma}}{2\hat{x}}\right) F_{UU}(\hat{x},\hat{Q}^2,\hat{z},\hat{P}_{h,\mathrm{T}})\right]$$

Hatted variables all depend on xi and zeta

Hadron pT in the true Breit frame

Collinear LDFs and LFFs

$$f_{i/e}(\xi) = \int \frac{dz^-}{4\pi}\, e^{i\xi\ell^+ z^-} \langle e|\, \overline{\psi}_i(0)\gamma^+ \Phi_{[0,z^-]}\, \psi_i(z^-)|e\rangle$$

$$D_{e/j}(\zeta) = \frac{\zeta}{2}\sum_X \int \frac{dz^-}{4\pi} e^{i\ell'^+ z^-/\zeta}\, \mathrm{Tr}\big[\gamma^+ \langle 0|\, \overline{\psi}_j(0)\Phi_{[0,\infty]}\, |e, X\rangle\langle e, X|\psi_j(z^-)\Phi_{[z^-,\infty]}|0\rangle\big].$$

Attention: These objects are perturbatively calculable

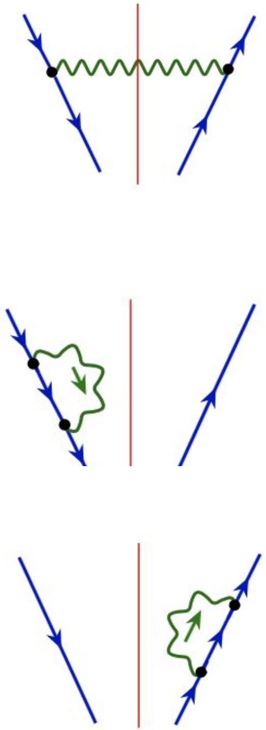
Input scale LDFs/LFFs

$$f_{e/e}^{(0)}(\xi) = \delta(\xi - 1)$$

$$f_{e/e}^{(1)}(\xi, \mu^2) = \frac{\alpha}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2\, m_e^2}\right]_+$$

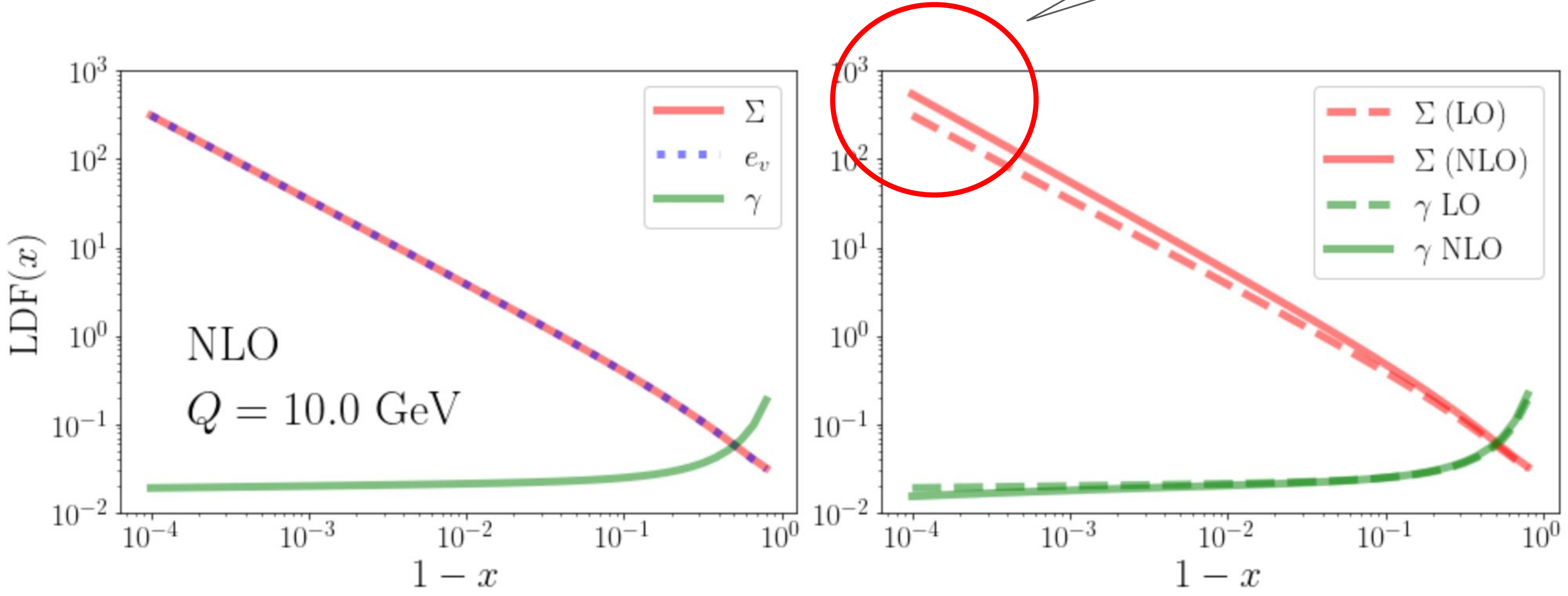
$$D_{e/e}^{(0)}(\zeta) = \delta(\zeta - 1)$$

$$D_{e/e}^{(1)}(\zeta, \mu) = \frac{\alpha}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2\, m_e^2}\right]_+$$



Evolved LDFs

LDFs peaks at the endpoint



Subtraction trick

$$\sigma = \int_{\zeta_{\min}}^{1} d\zeta \int_{\xi_{\min(\zeta)}}^{1} d\xi f(\xi) d(\zeta) H(\xi, \zeta)$$

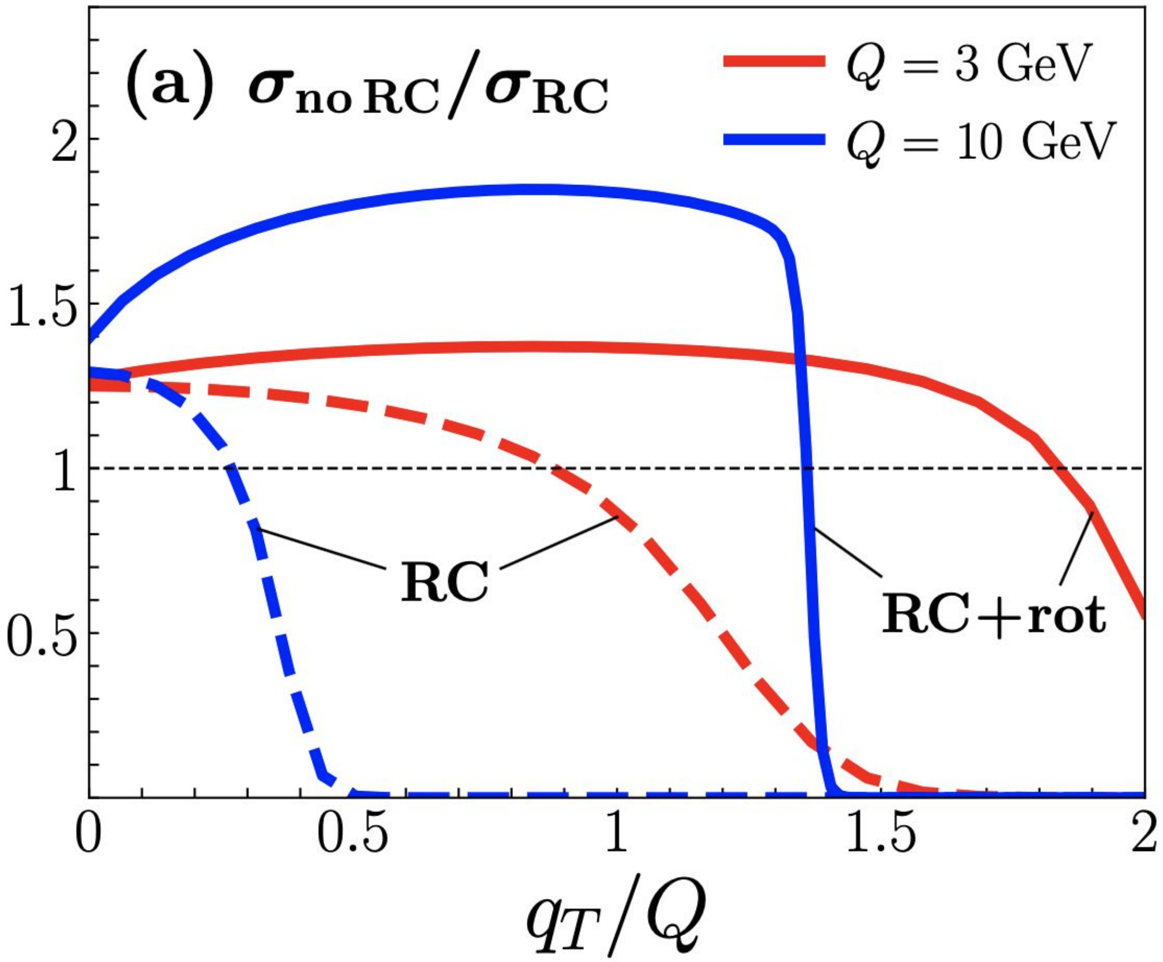
$$D_N = \int_0^1 d\zeta \zeta^{N-1} d(\zeta)$$

$$\sigma = \int_{\zeta_{\min}}^{1} d\zeta\, d(\zeta)[g(\zeta) - g(1)] + g(1) \frac{\zeta_{\min}}{2\pi i} \int dN \zeta_{\min}^{-N} \frac{D_N}{N-1}$$

$$F_N = \int_0^1 d\xi \xi^{N-1} f(\xi)$$

$$g(\zeta) = \int_{\xi_{\min}(\zeta)}^{1} d\xi\, f(\xi)[H(\xi, \zeta) - H(1, \zeta)] + H(1, \zeta) \frac{\xi_{\min}(\zeta)}{2\pi i} \int dN \xi_{\min}(\zeta)^{-N} \frac{F_N}{N-1}$$

We remove the numerically problematic region and compute the difference accurately in Mellin space



$\sqrt{s} = 140$ GeV

$y = 0.4$

$z = 0.5$

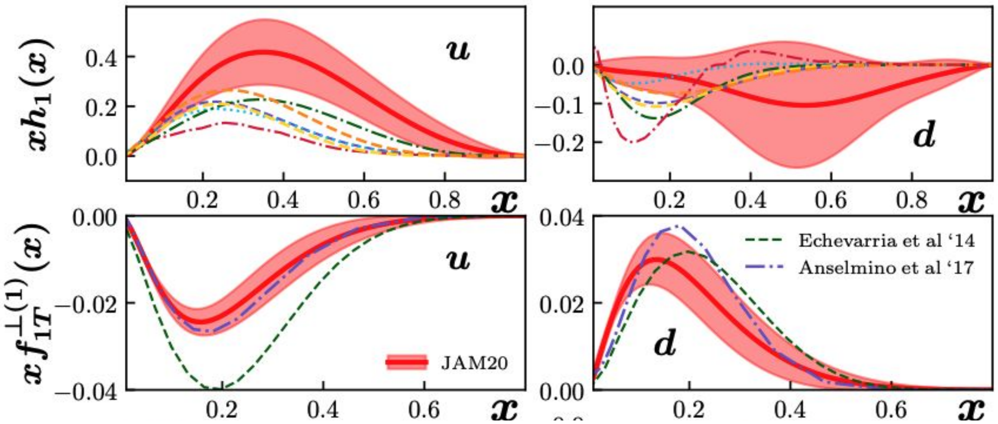
What about spin structures?

arXiv:2002.08384 (hep-ph)

[Submitted on 19 Feb 2020 (v1), last revised 2 Sep 2020 (this version, v2)]

Origin of single transverse-spin asymmetries in high-energy collisions

Justin Cammarota, Leonard Gamberg, Zhong-Bo Kang, Joshua A. Miller, Daniel Pitonyak, Alexei Prokudin, Ted C. Rogers, Nobuo Sato



Transversity

Sivers

$$
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =
$$

$$
\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}
$$

$$
+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}
$$

$$
+\,S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]
$$

$$
+\,S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]
$$

$$
+\,|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)
$$

$$
+\,\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}
$$

$$
+\,\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg]
$$

$$
+\,|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}
$$

$$
+\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},
$$

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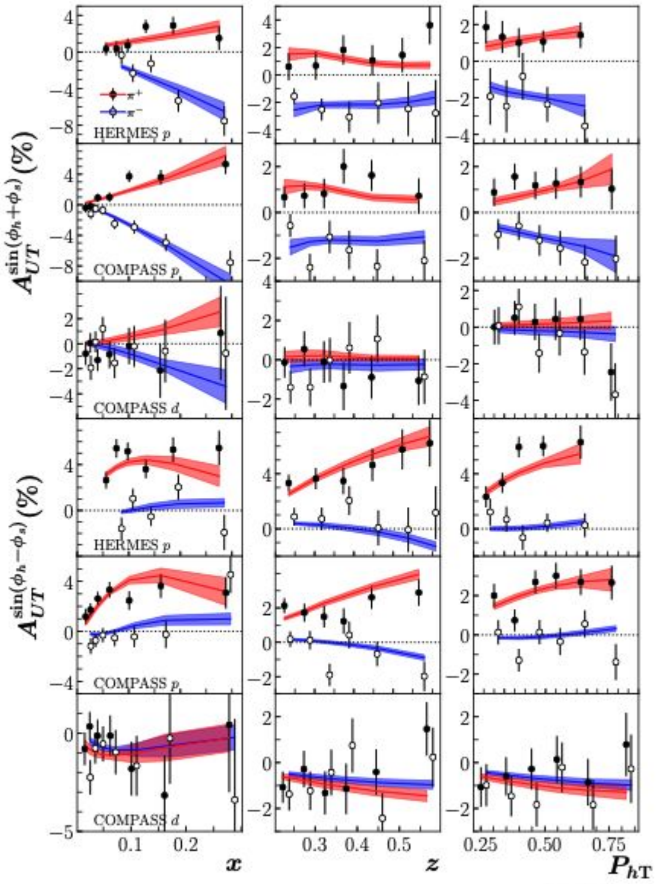
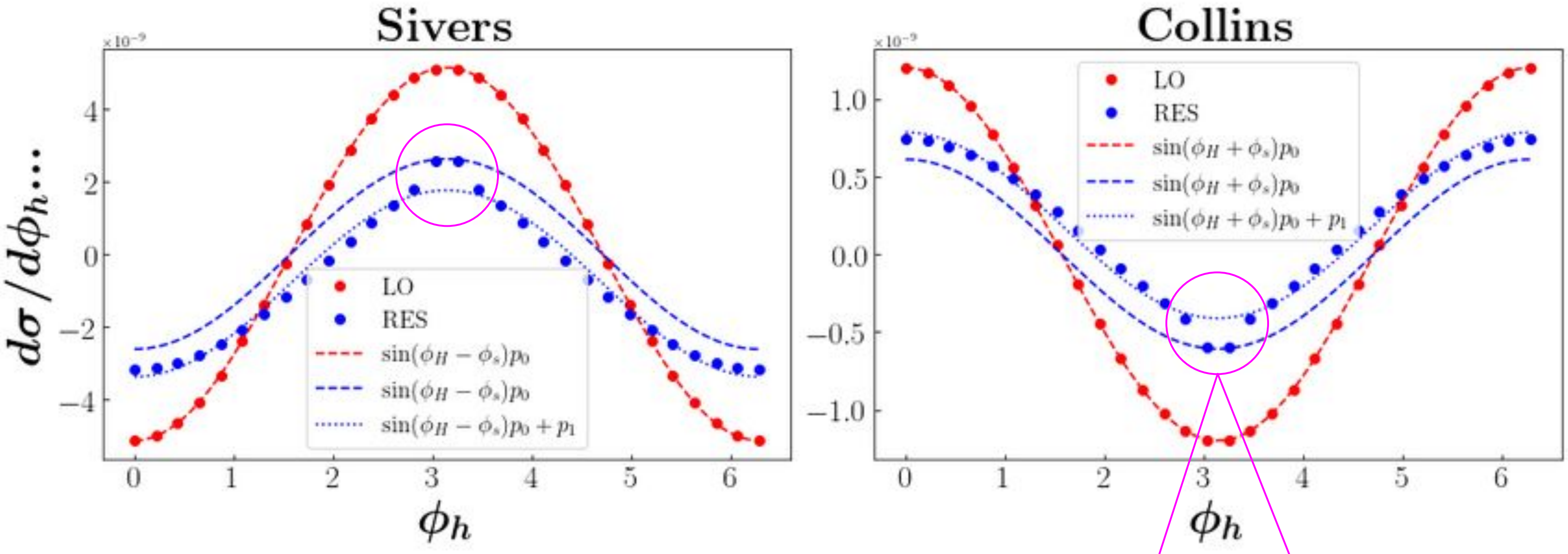
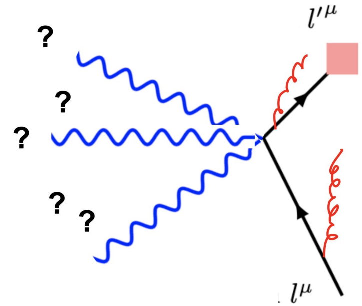




QED corrections generates non trivial signal around phi_h=pi

- Curves are a fits
- Dots are full theory calculations

Summary/Outlook



- In the presence of QED radiation, **the q direction is not fixed**

- The experimental Breit Frame **does not need to coincide with the actual Breit-frame** needed in QCD factorization

- QED effects **needs** to take into account for the next frontier