

9<sup>th</sup> Workshop of the APS Topical Group on Hadronic Physics (GHP)

April 15<sup>th</sup>, 2021

# A spectator-model way to TMD gluon distribution functions

Francesco Giovanni Celiberto

ECT\*/FBK Trento & INFN-TIFPA

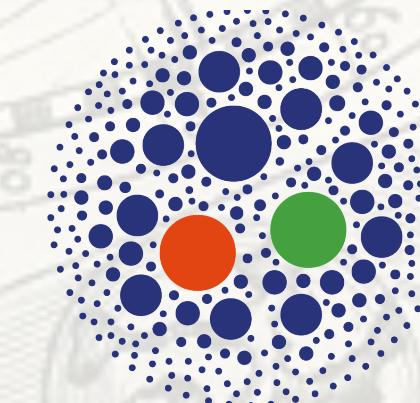
ECT\*

EUROPEAN CENTRE FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS

FONDAZIONE  
BRUNO KESSLER  
FUTURE BUILT  
ON KNOWLEDGE



Trento Institute for  
Fundamental Physics  
and Applications



HAS QCD

HADRONIC STRUCTURE AND  
QUANTUM CHROMODYNAMICS

# Gluon TMDs: a largely unexplored territory



**Theory**: different **gauge-link** structures...

...more diversified kind of **modified universality**!



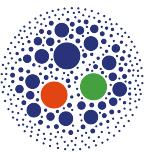
**Pheno**: golden channels for extraction  
of quark TMDs are subleading for gluon TMDs

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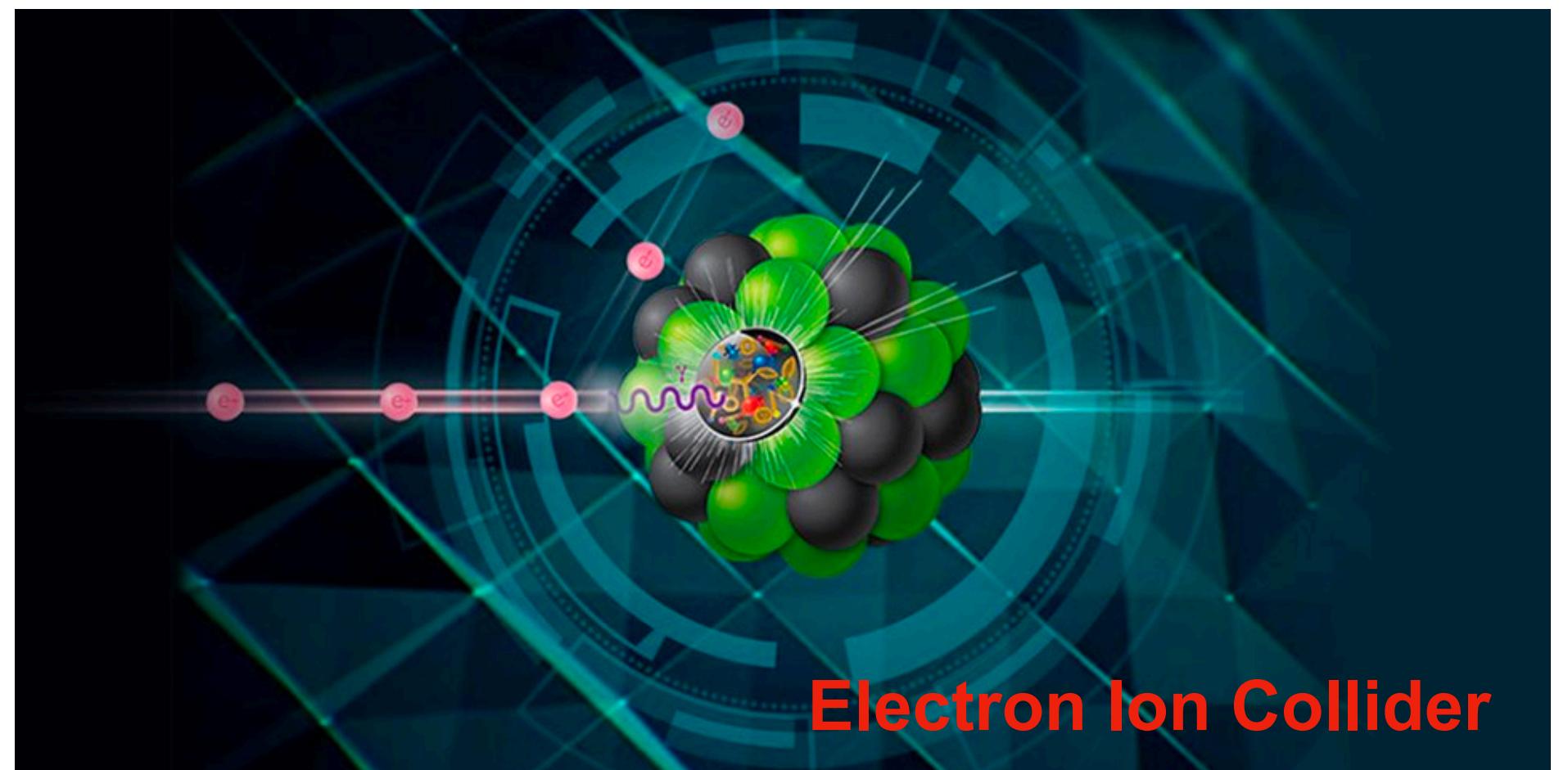


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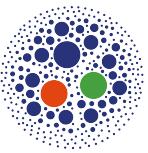
## Motivation



Gluon-TMD PDFs: *core* sector of **EIC** studies



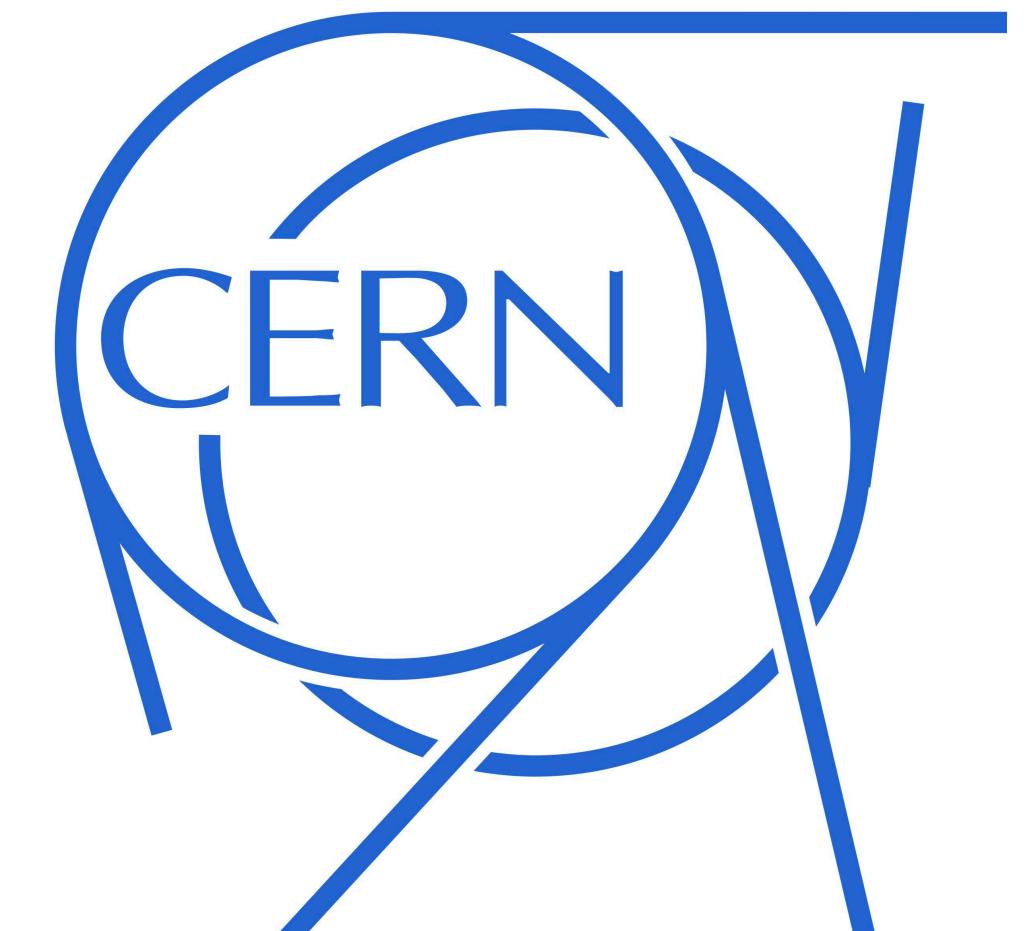
Need for a *flexible* model, suited to *pheno*



**Unpolarized** and **polarized** gluon TMDs



*Consistent* framework for quark TMDs



# **T-even and T-odd gluon TMD PDFs at twist-2**

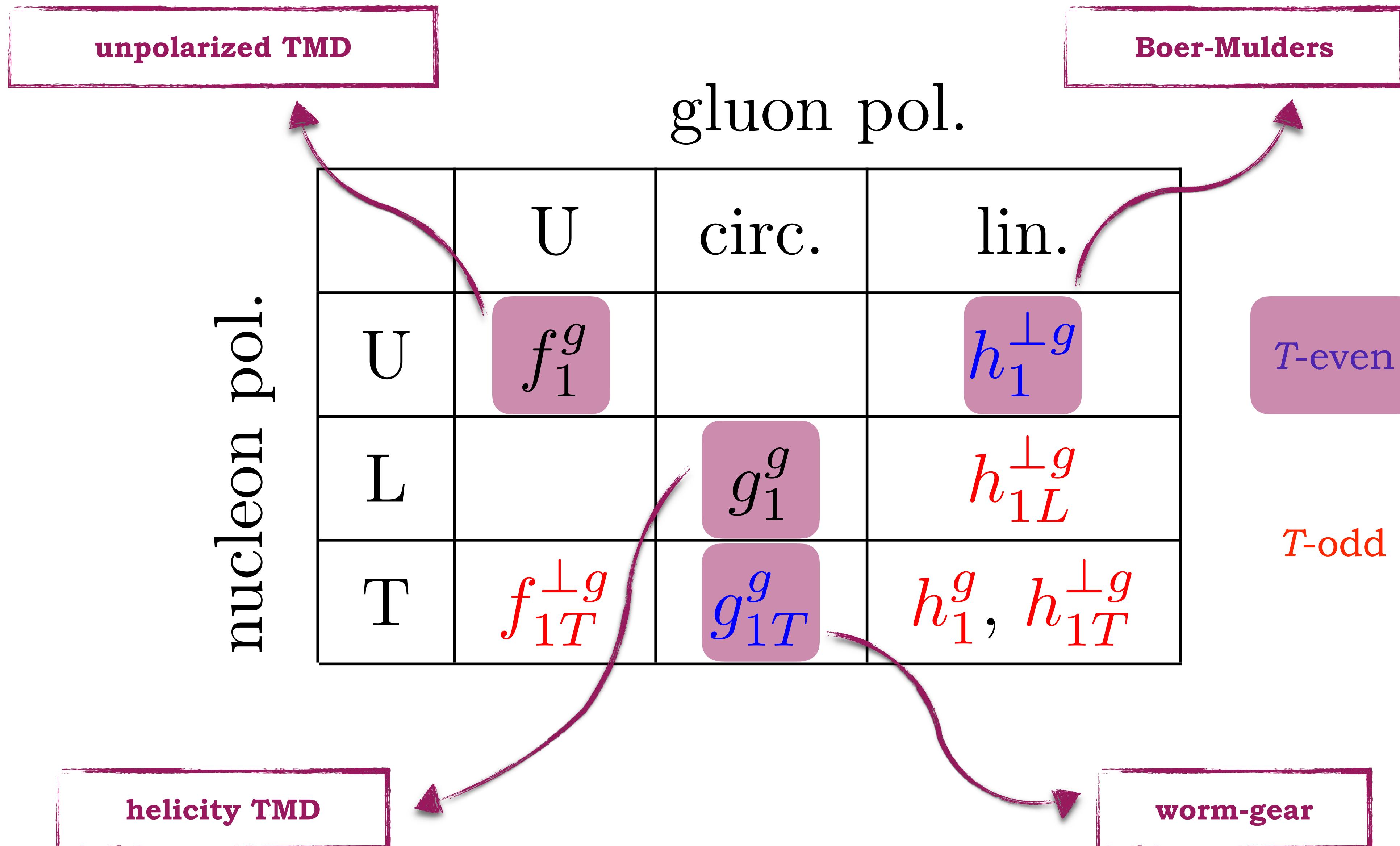
gluon pol.

	U	circ.	lin.
U	$f_1^g$		$h_1^{\perp g}$
L		$g_1^g$	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_1^g, h_{1T}^{\perp g}$

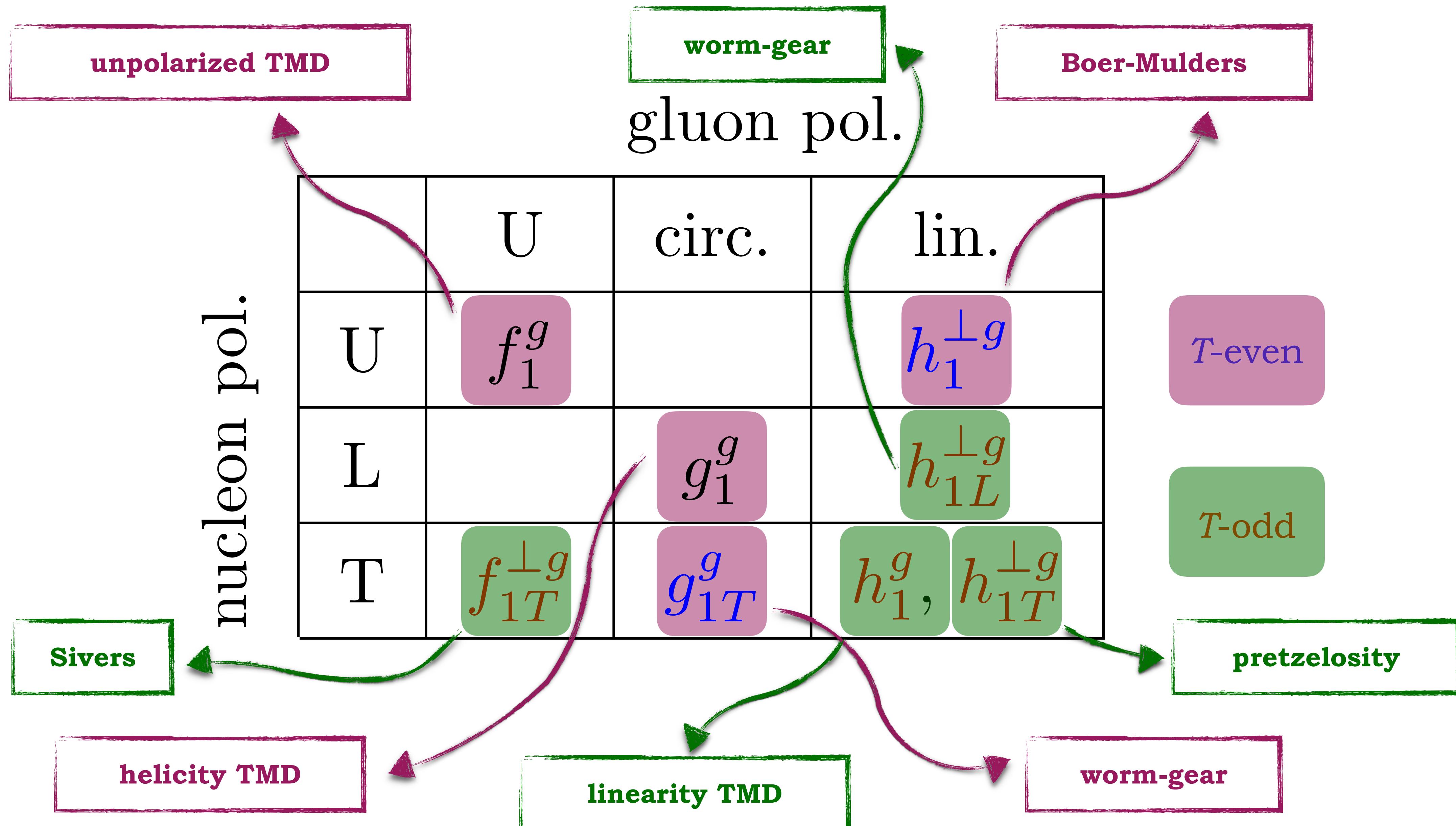
*T-even*

*T-odd*

# T-even and T-odd gluon TMD PDFs at twist-2



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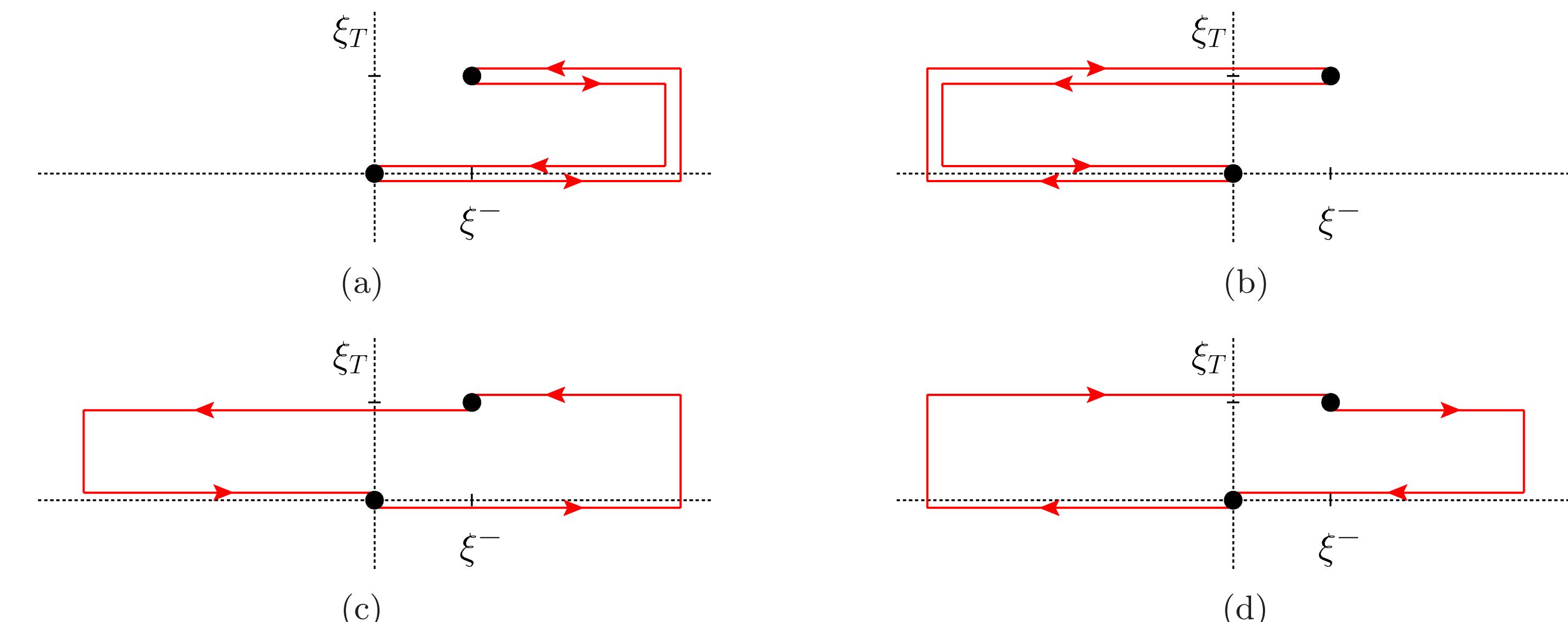


# Gluon TMDs: gauge links and modified universality

- \* **Single-spin asymmetries** → process dependence of TMDs via **gauge links**
- \* **Color flow** → integration paths of gauge links calculable
- \* Gluon TMDs → more complicated structure with respect to quark **staple links**
- \* **Factorization-preserving** processes → two main kinds of **modified universality**
- \* Different classes of processes → distinct gluon TMDs, **not related** to each other

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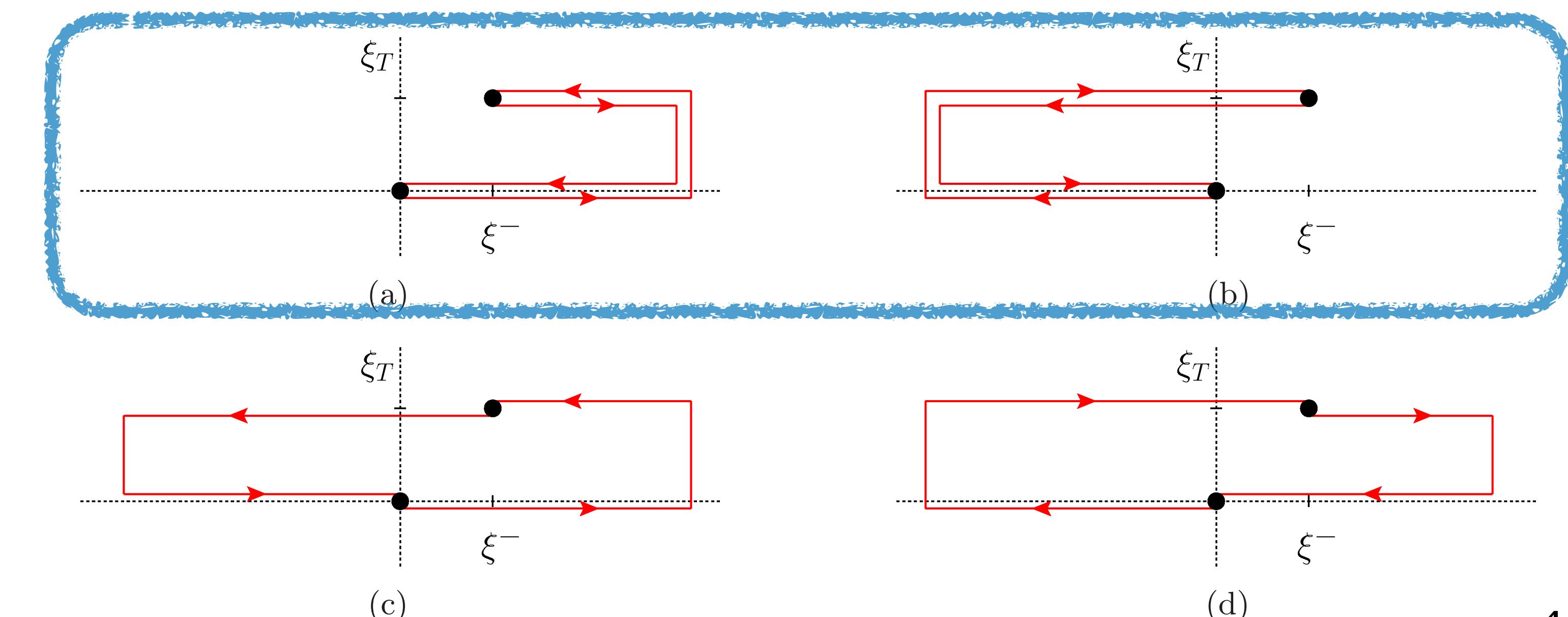


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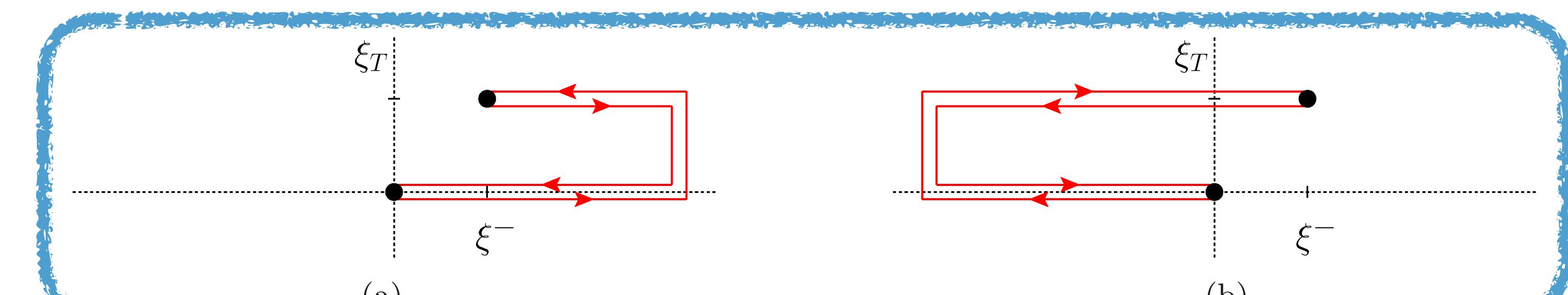


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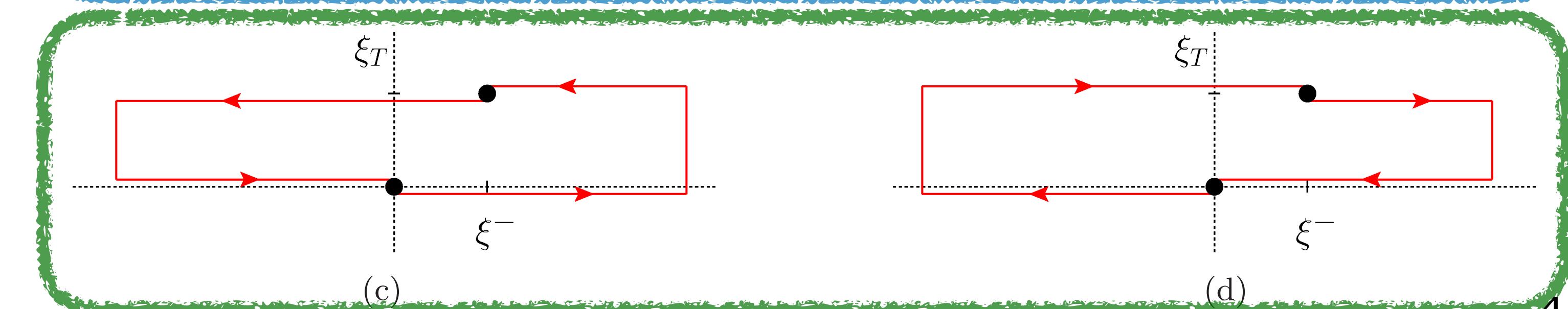
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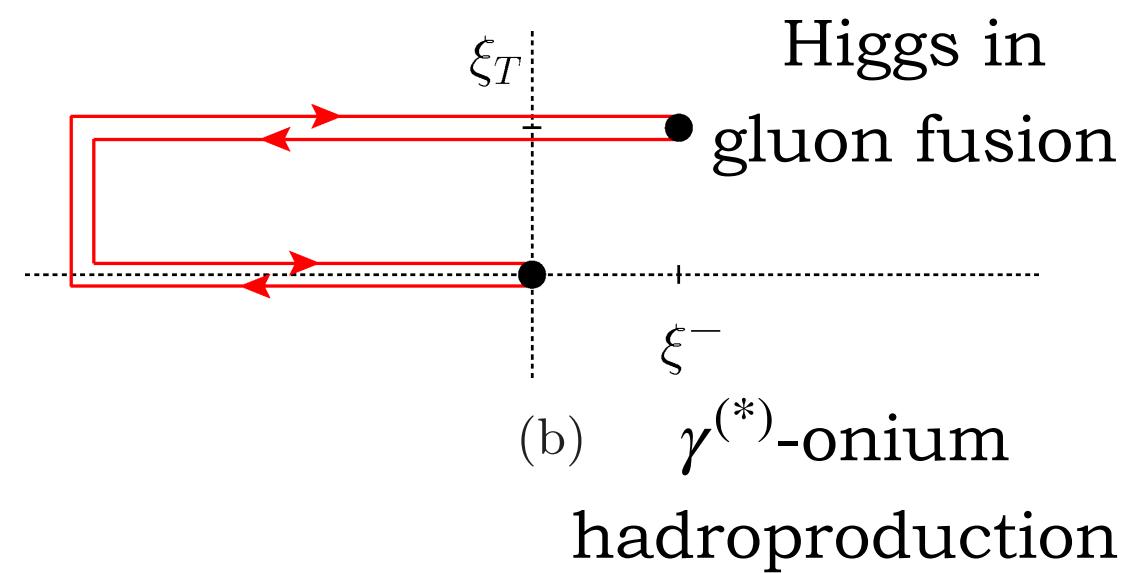
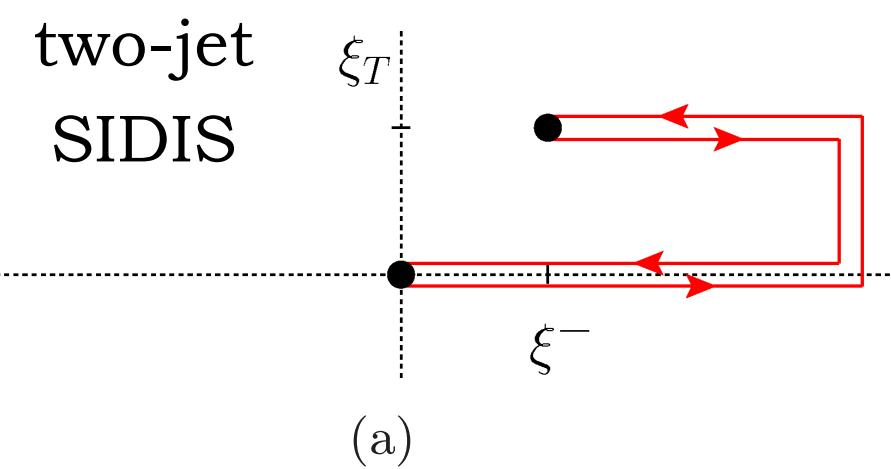
(c) [ + , - ] or (d) [ - , + ]



# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

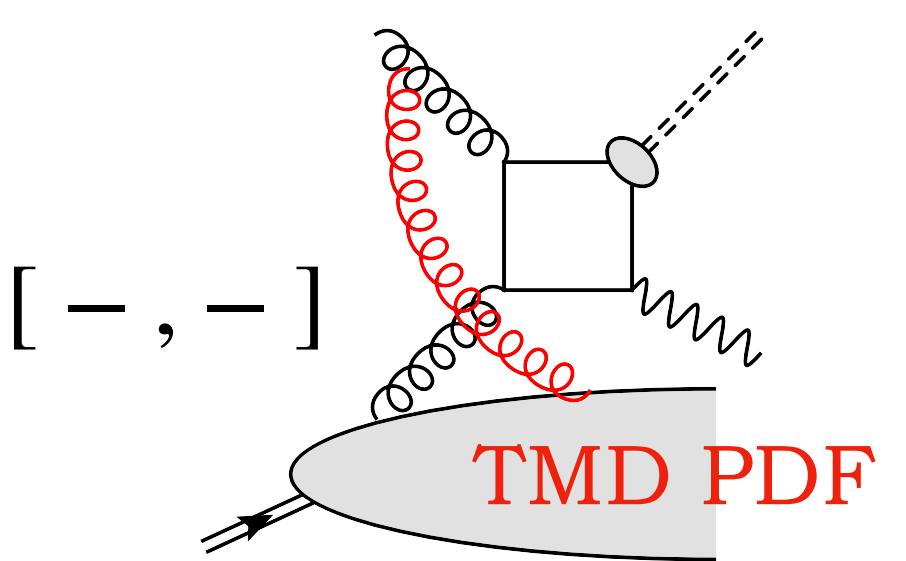
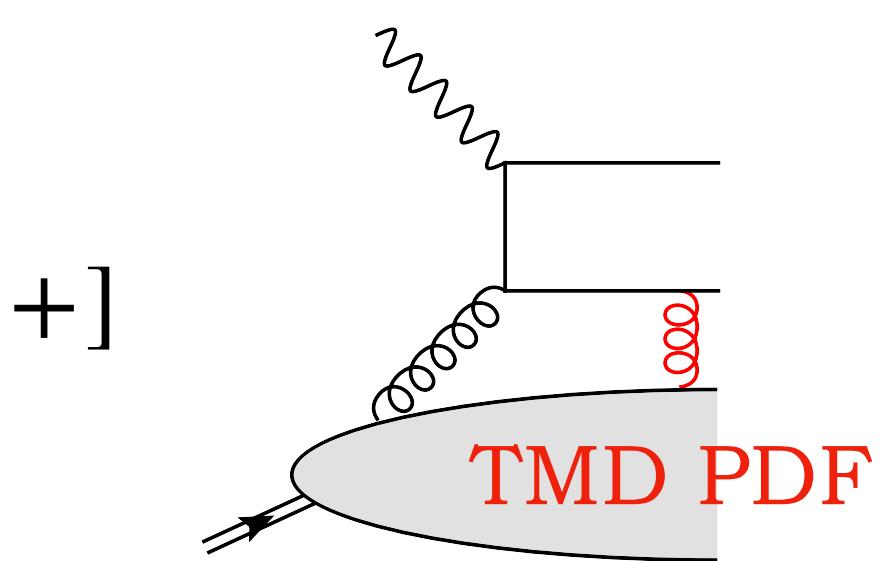
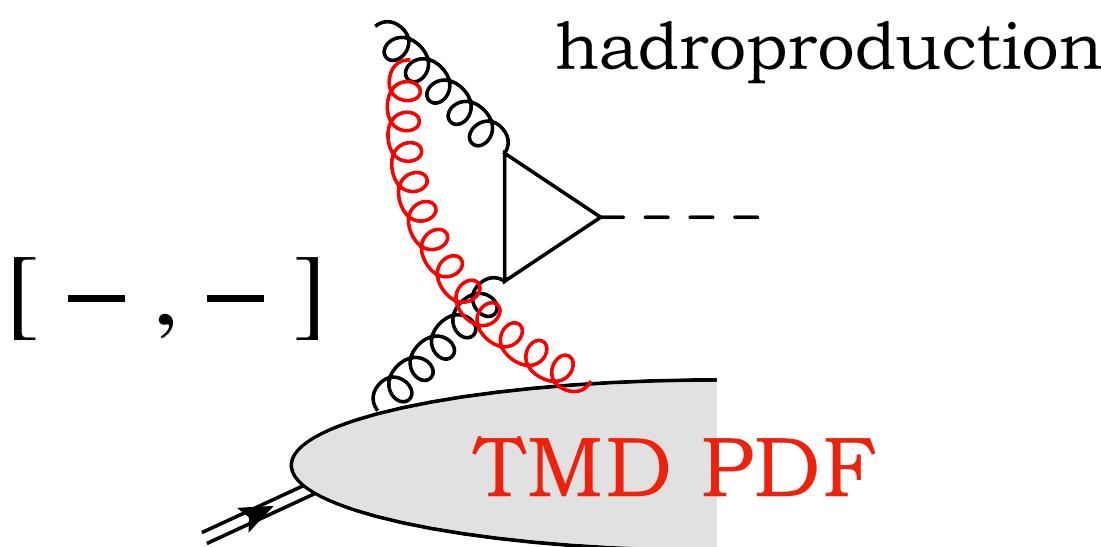
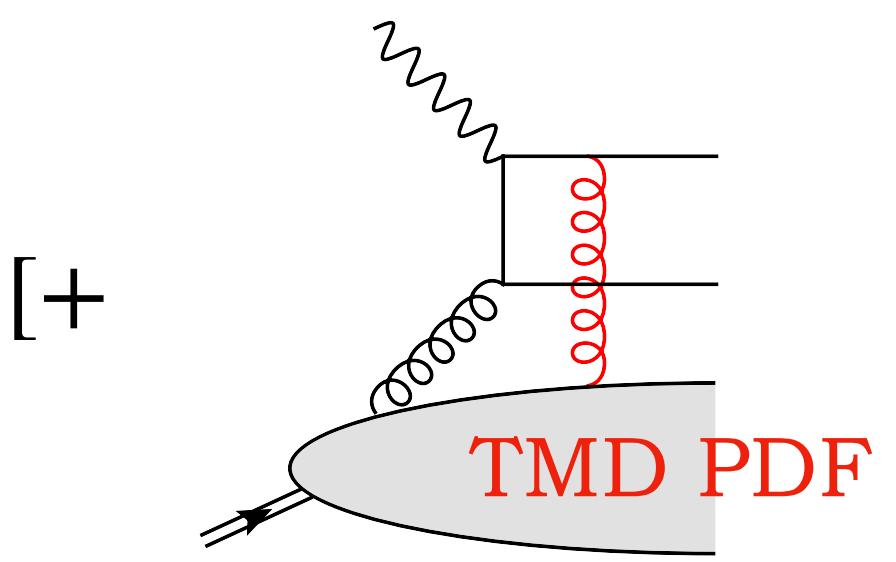
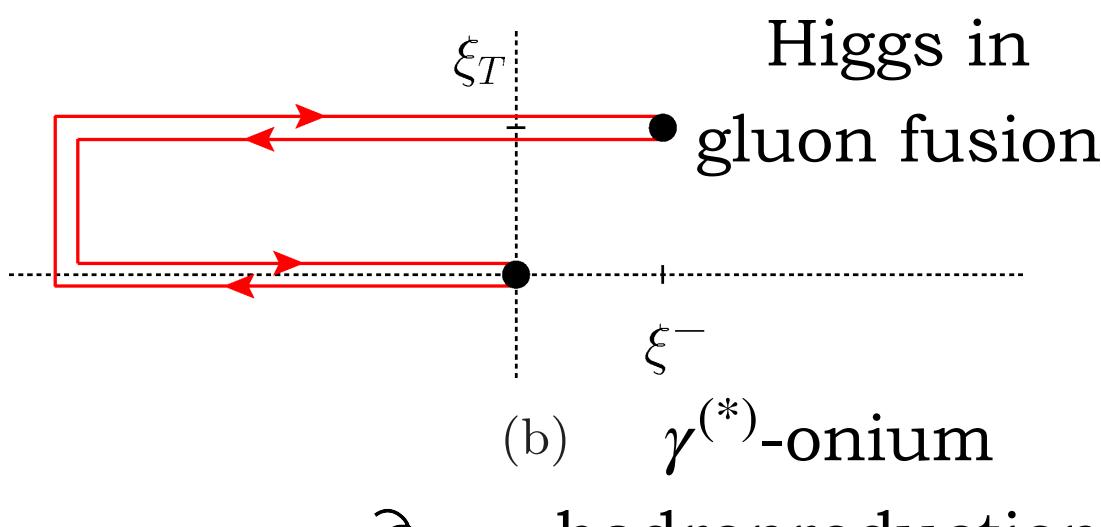
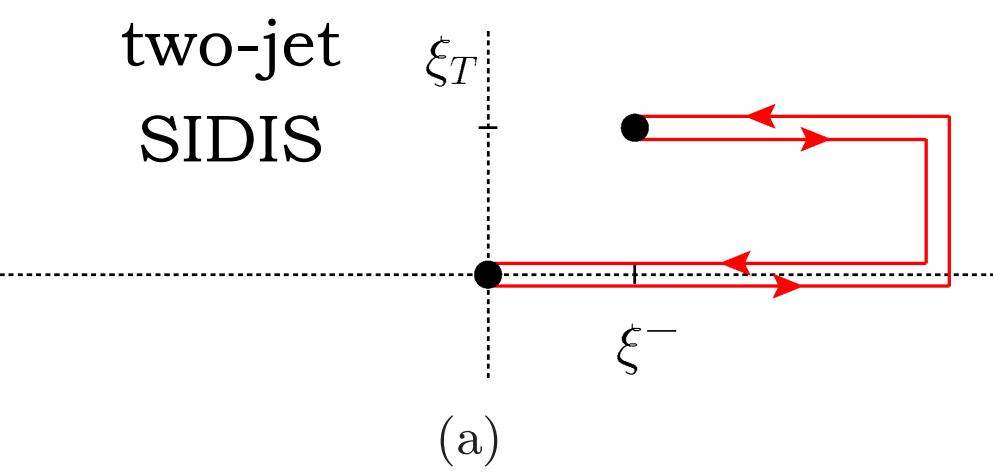
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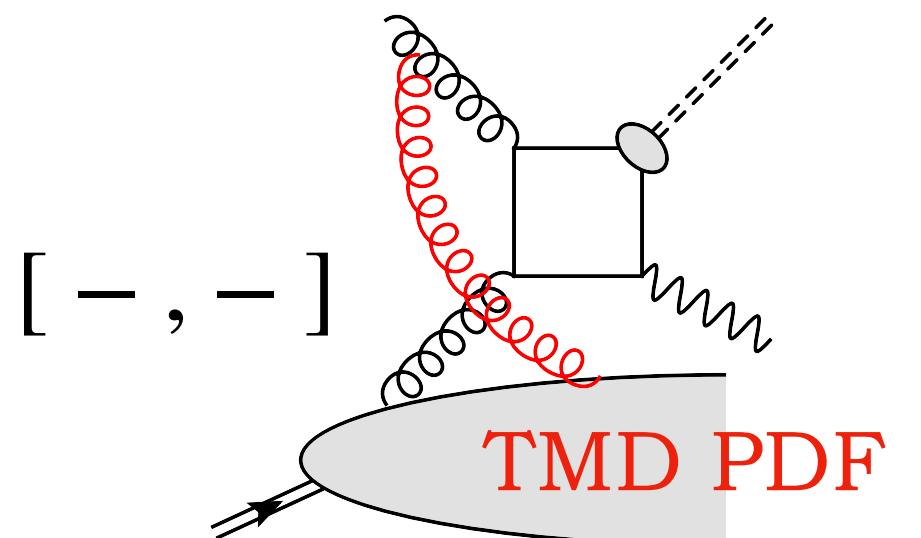
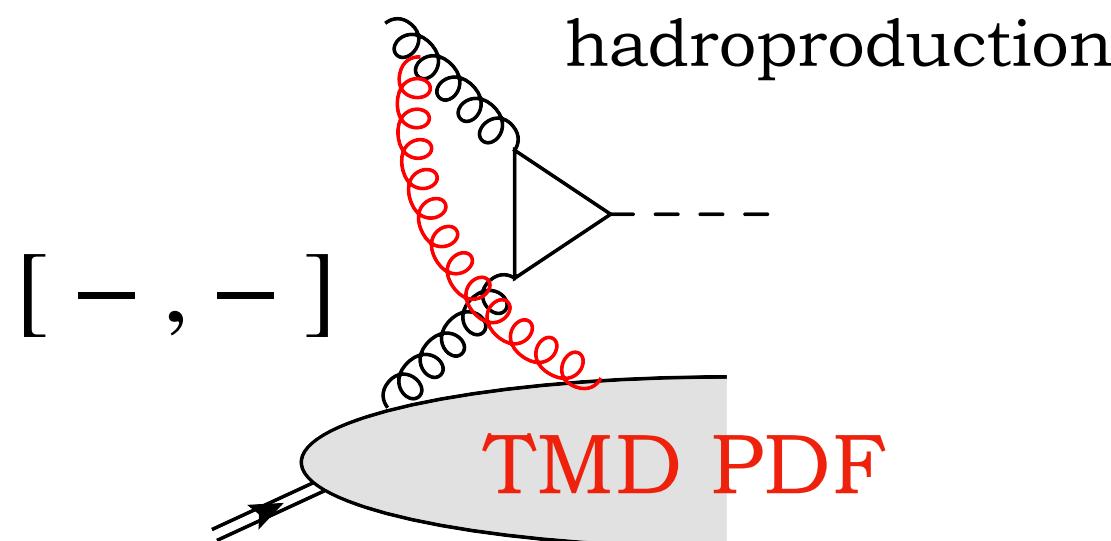
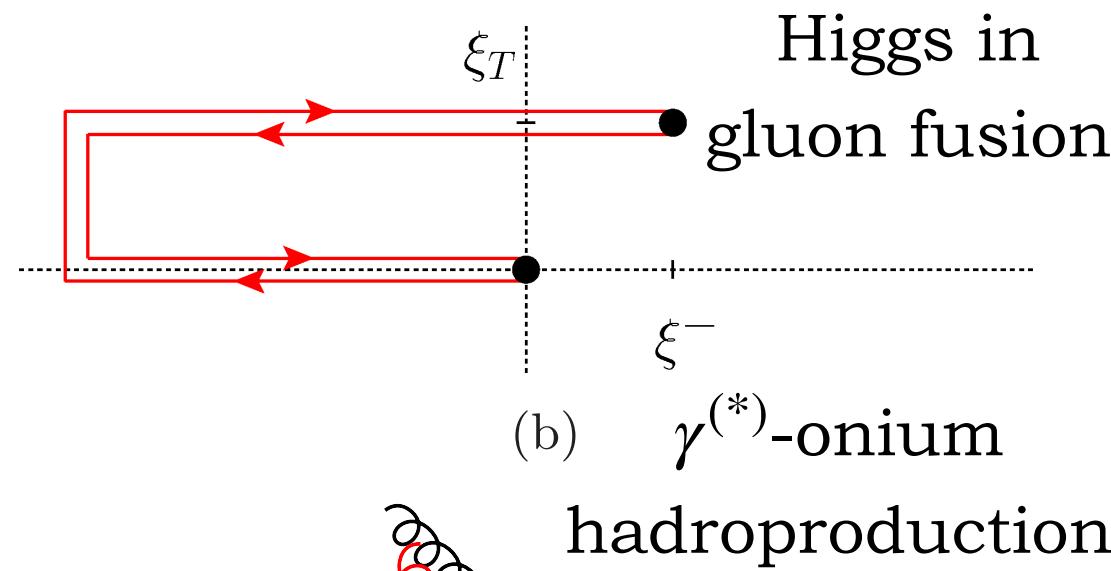
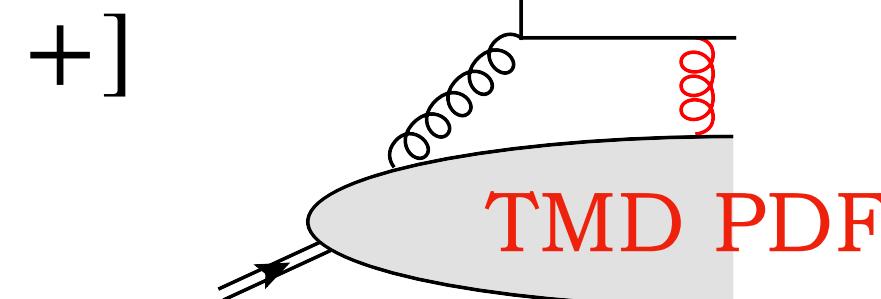
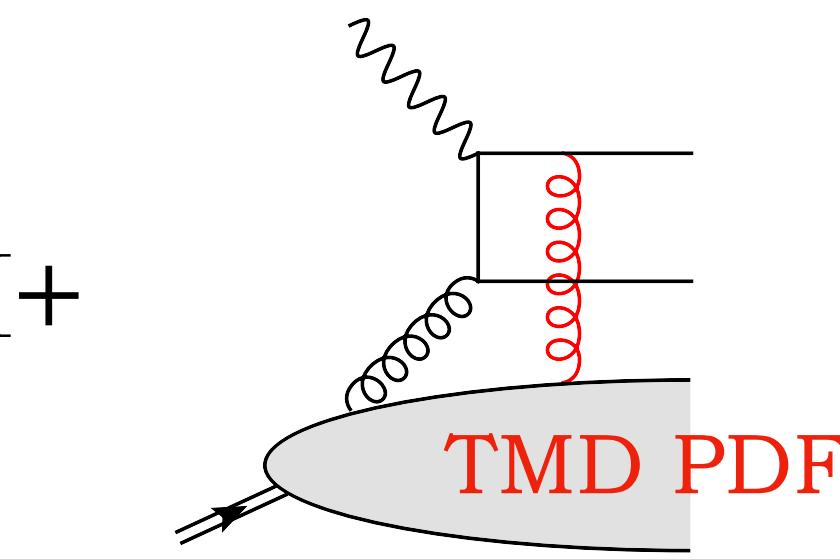
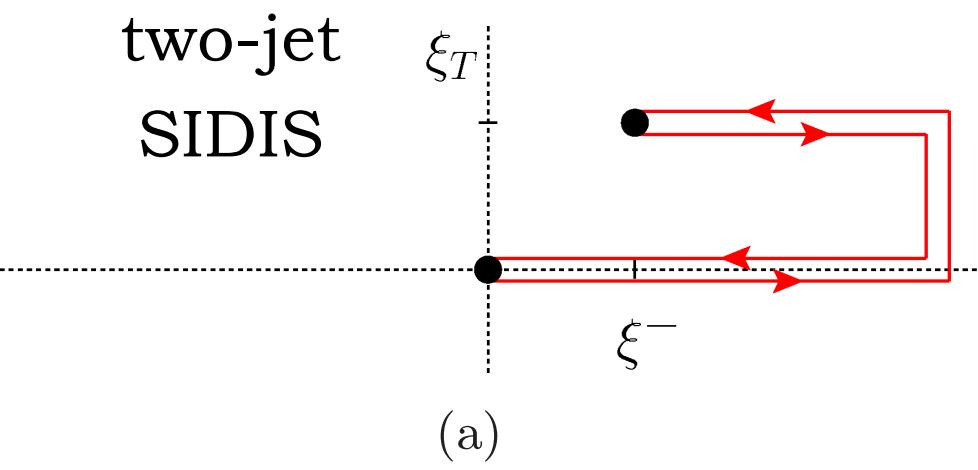
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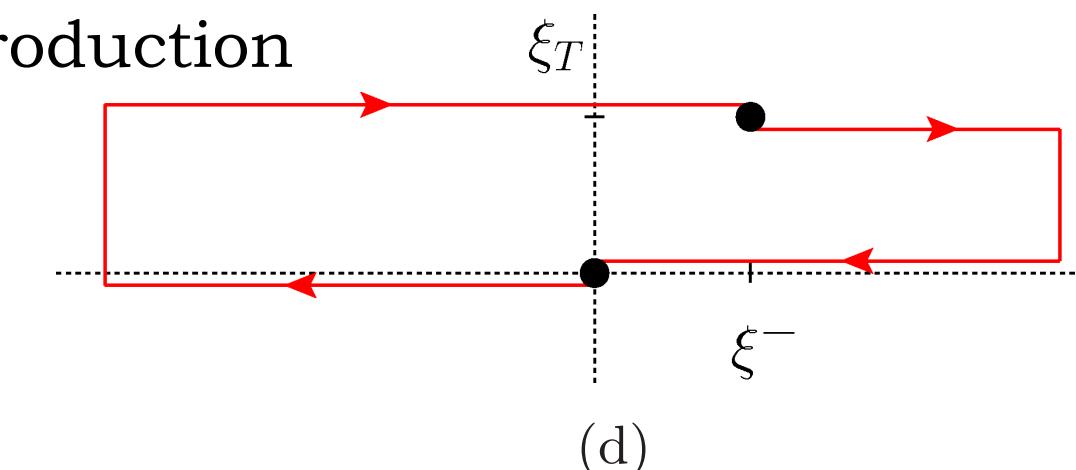
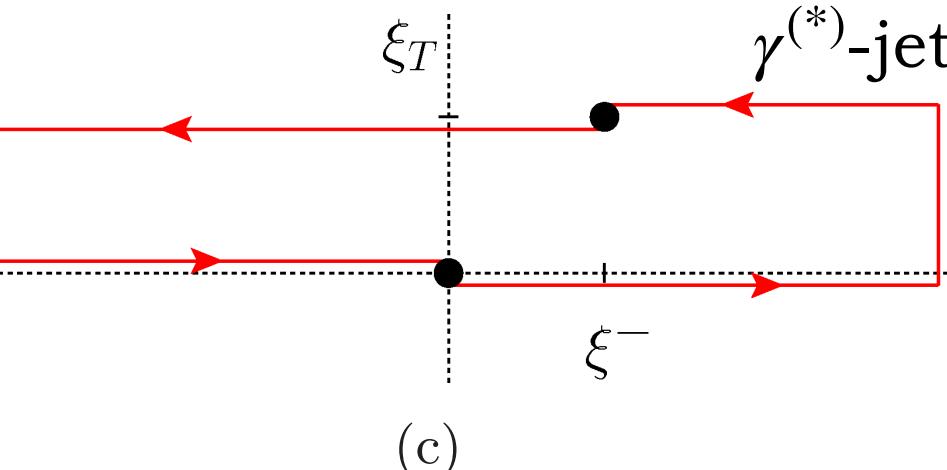
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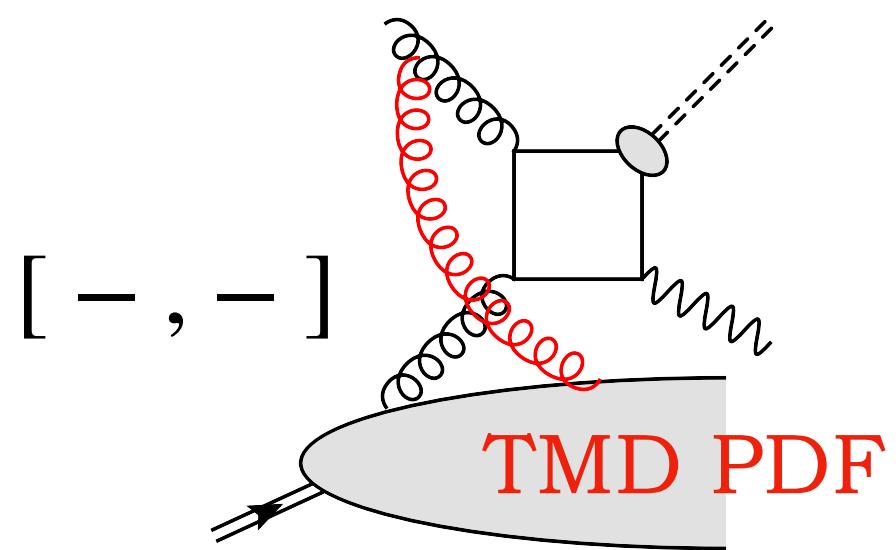
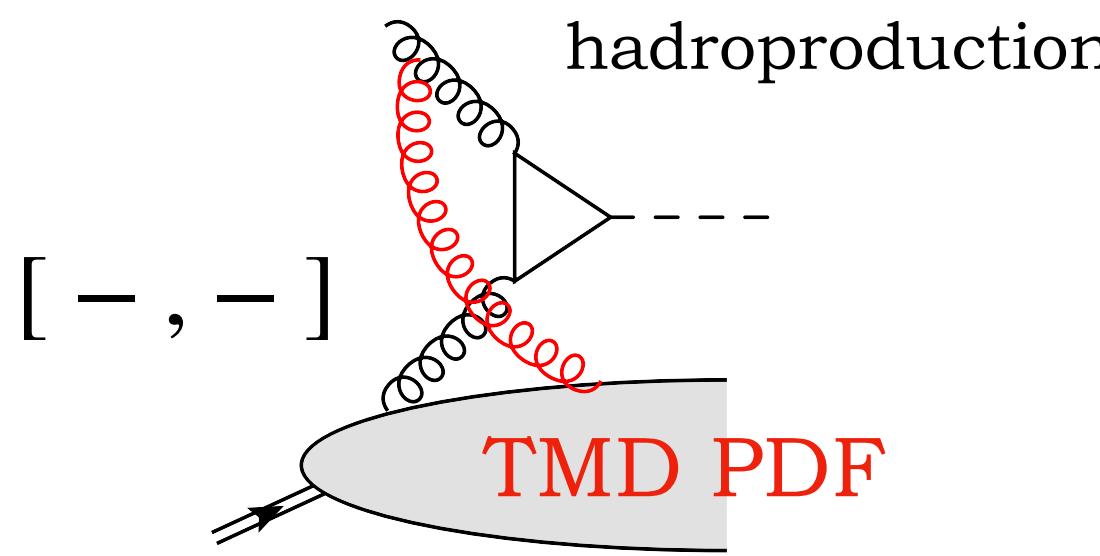
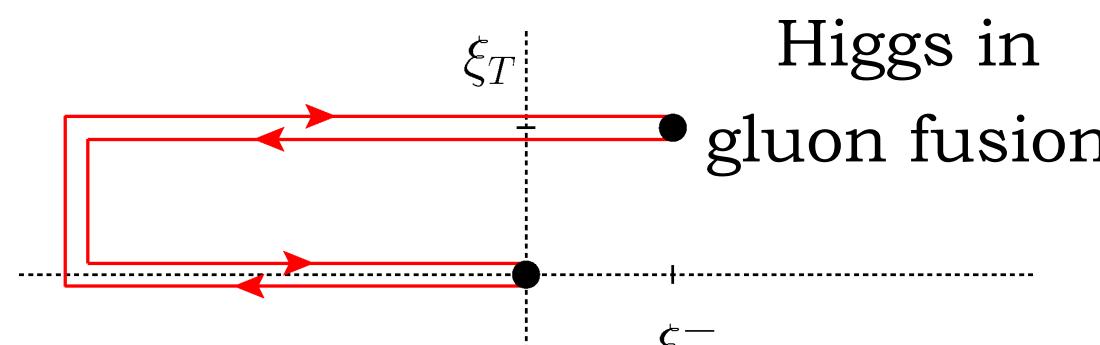
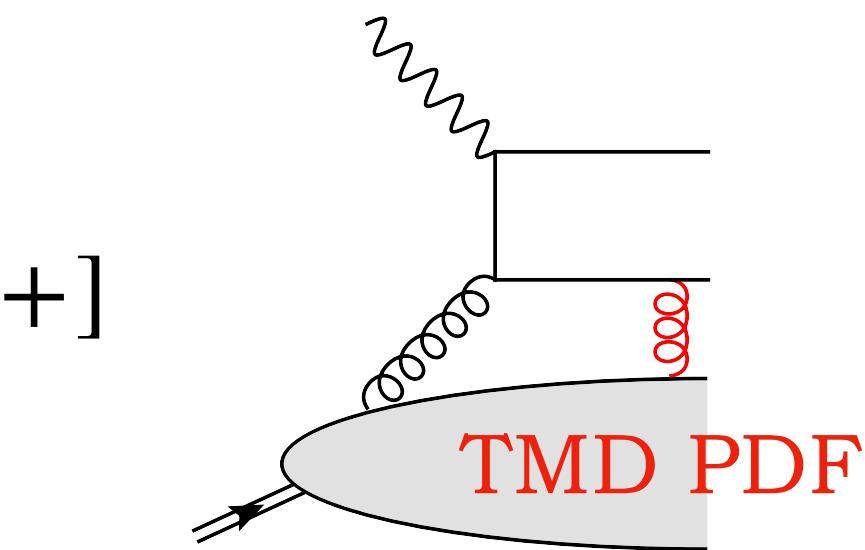
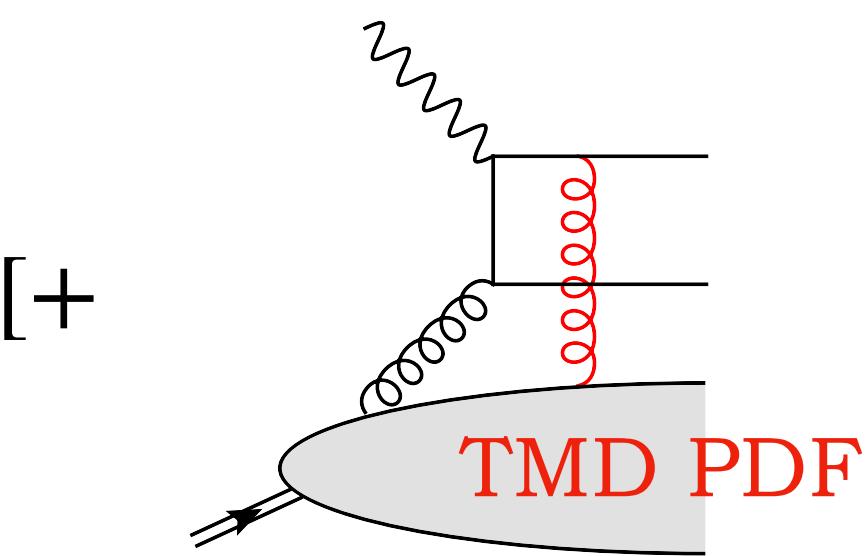
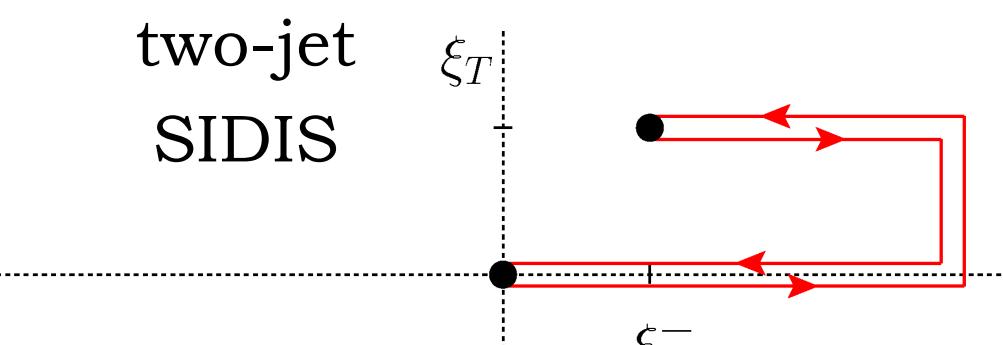
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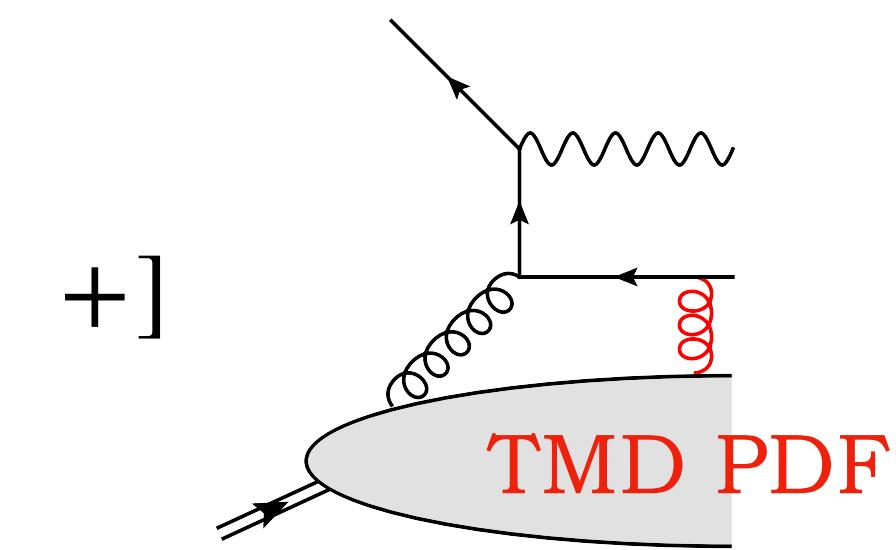
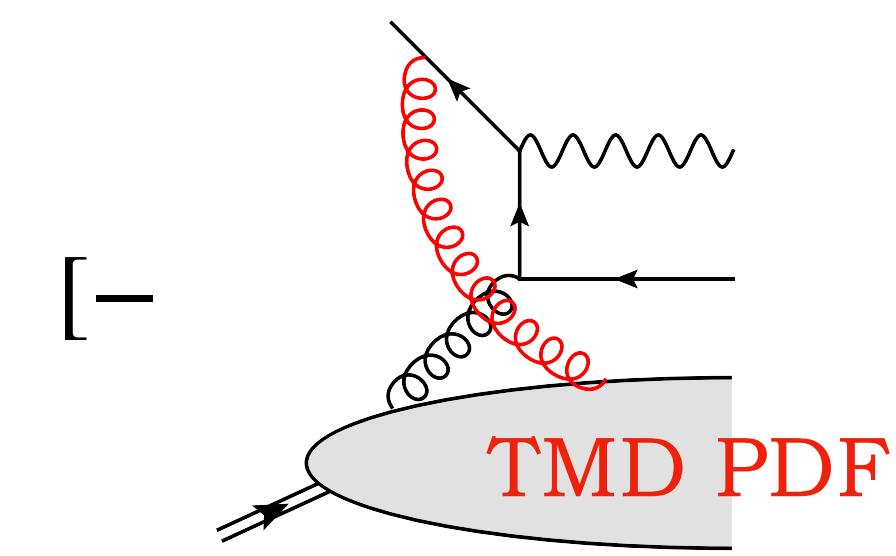
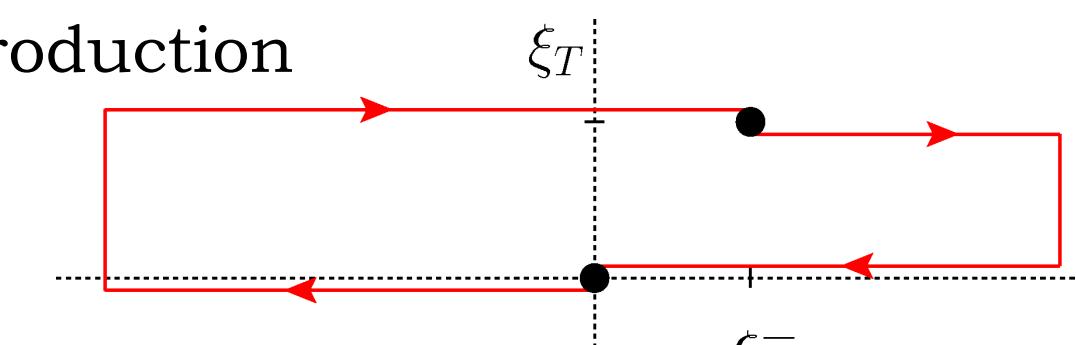
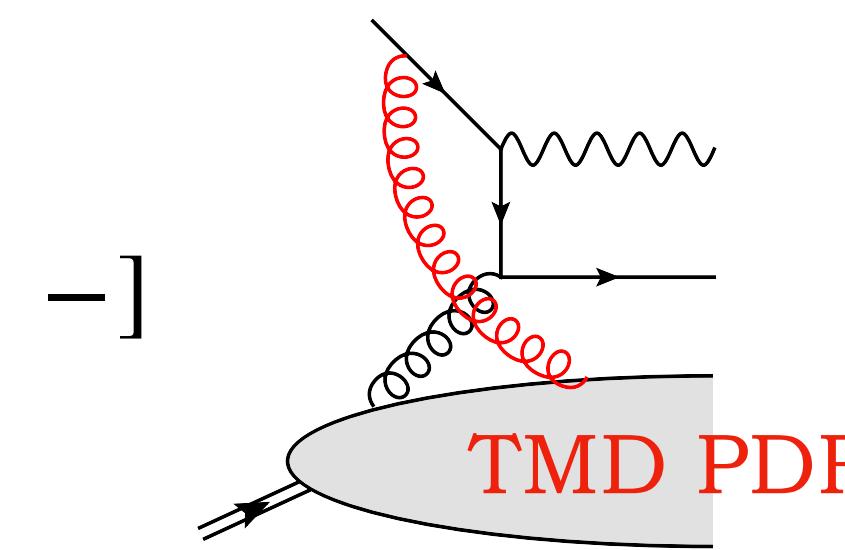
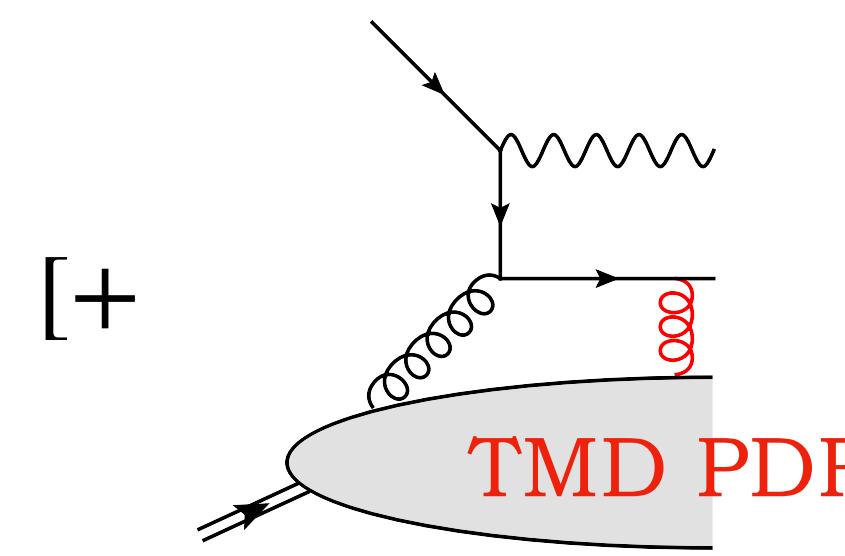
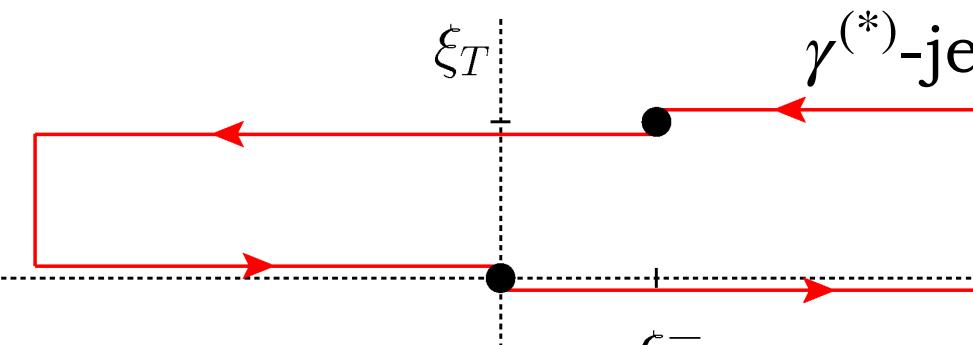
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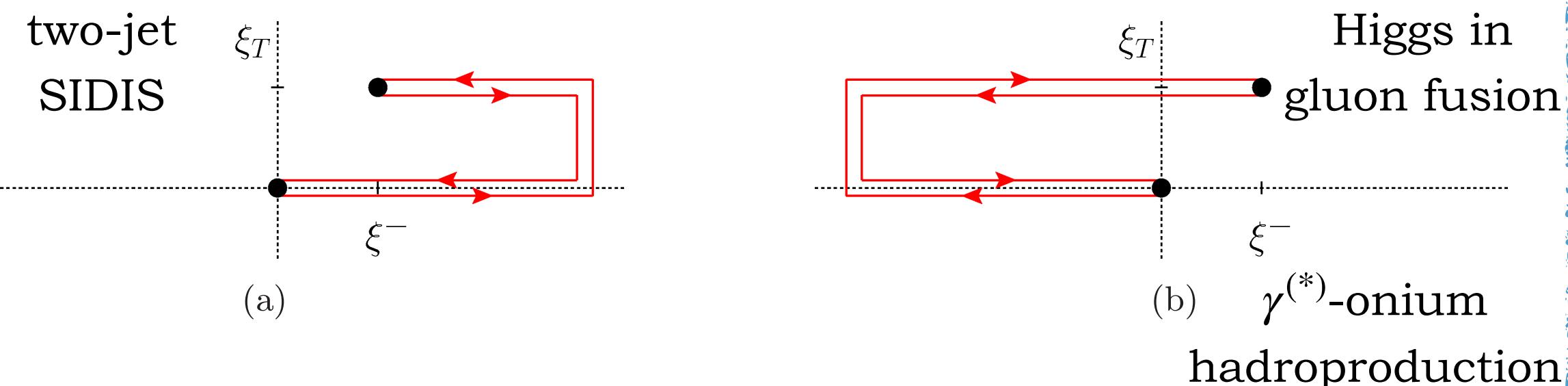
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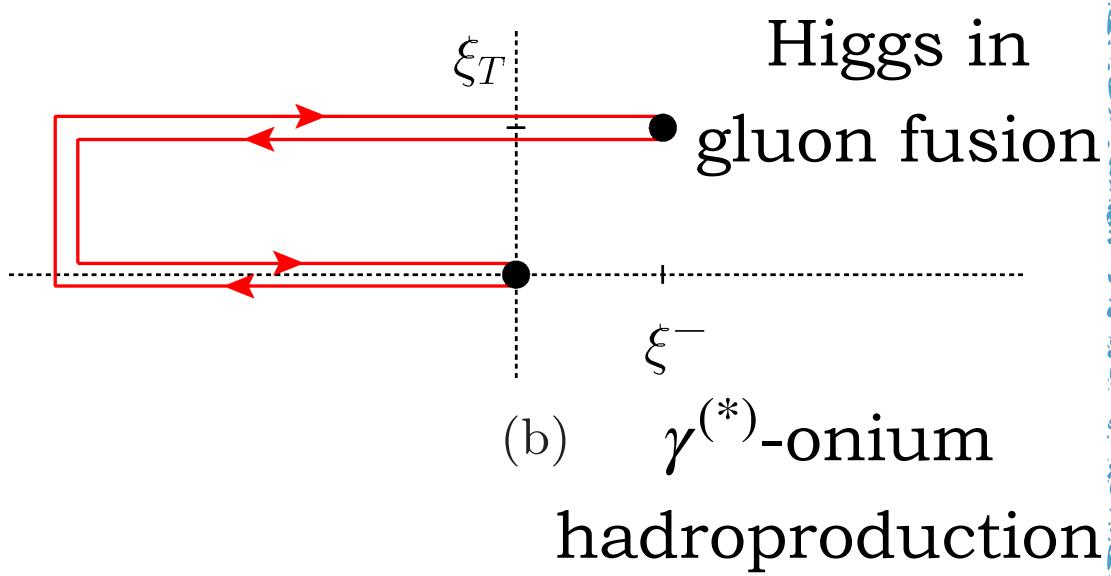
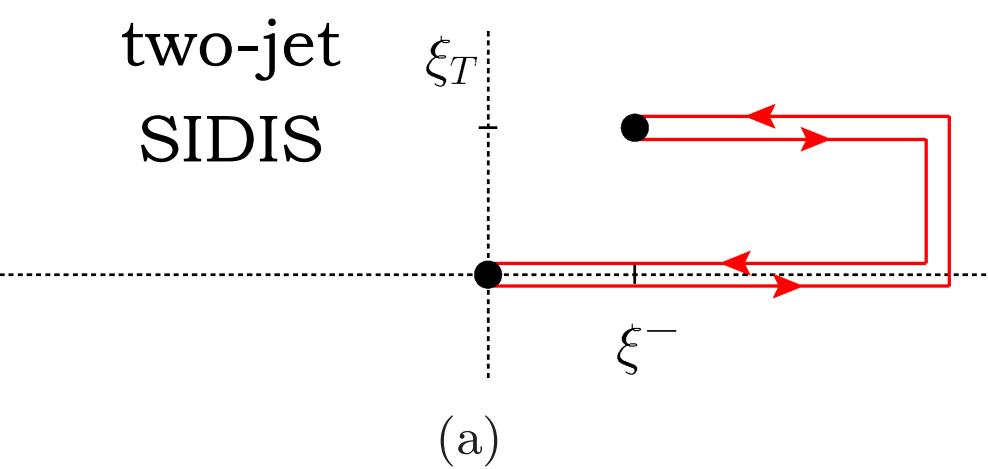


- \* Color flow annihilated within final/initial state
- \*  $f$ -type gluon TMDs  $\rightarrow f^{abc}$  color structure
- \* Modified universality:
$$f_1^{[+,+]} = f_1^{[-,-]},$$
$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$
- \* Phenomenology: Higgs, quarkonia or  $\gamma\gamma$  in  $pp$ , two-jet SIDIS, heavy-quark pair SIDIS

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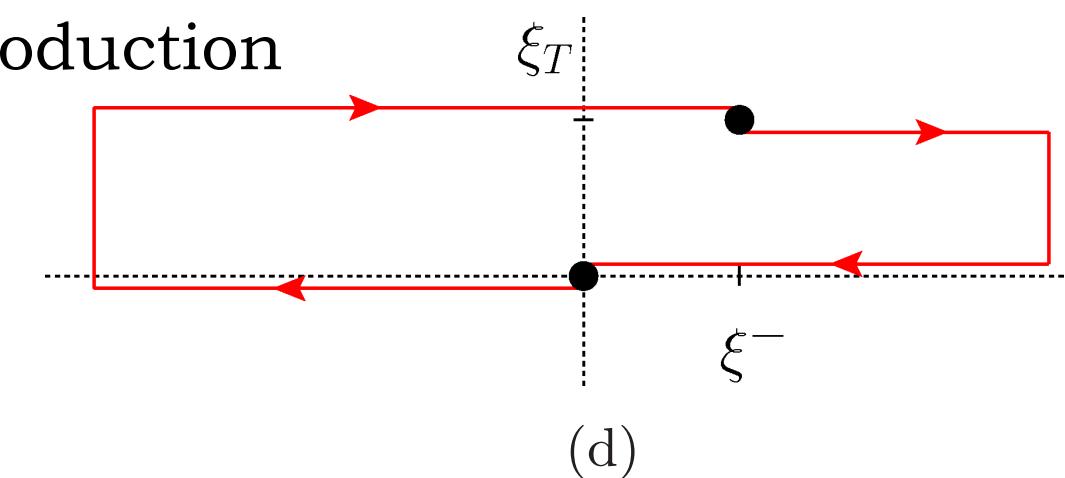
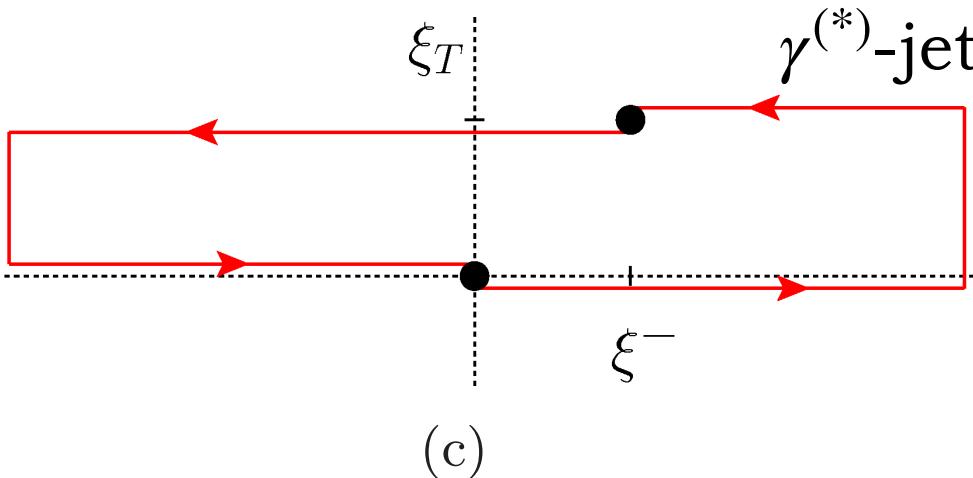
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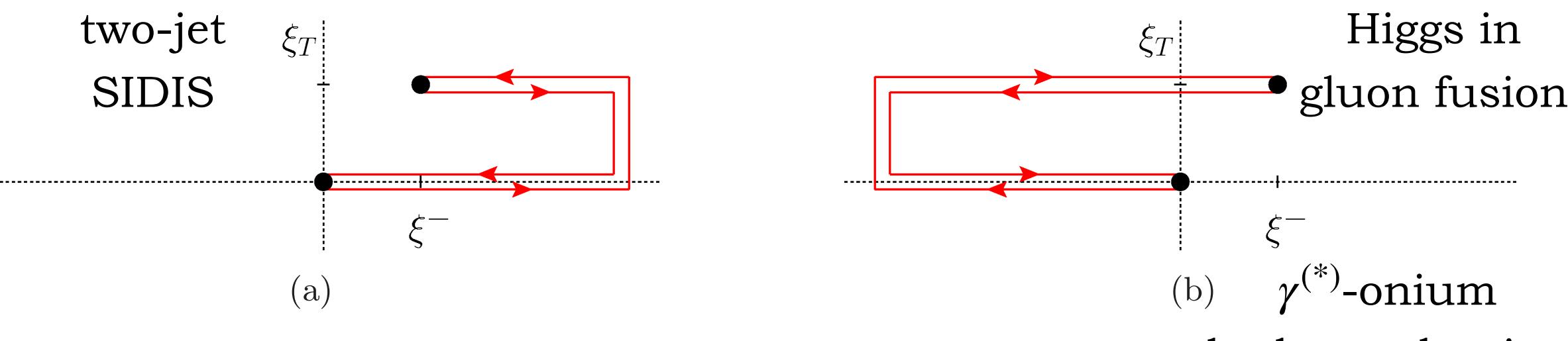
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- \* Phenomenology: single hadron or  $\gamma^{(*)}$ -jet hadroproduction, SIDIS or Drell-Yan (subleading)

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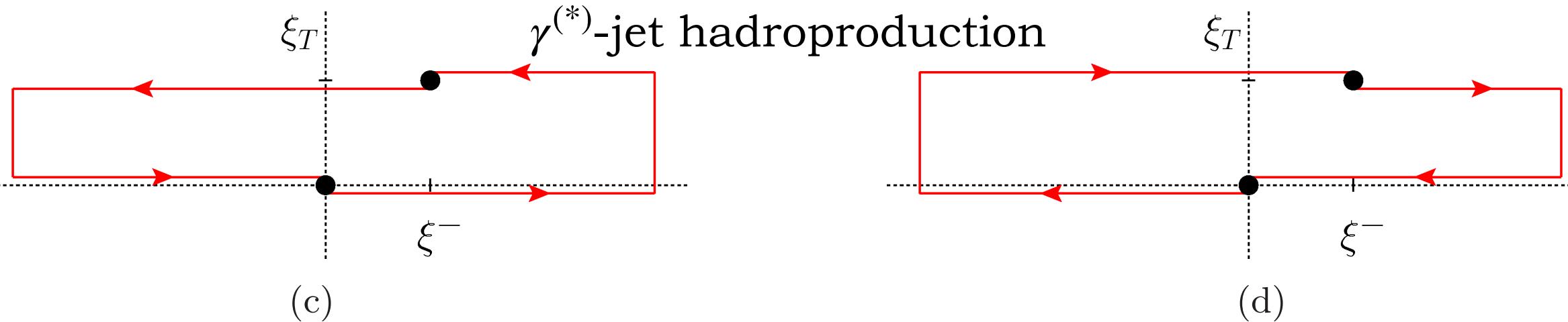


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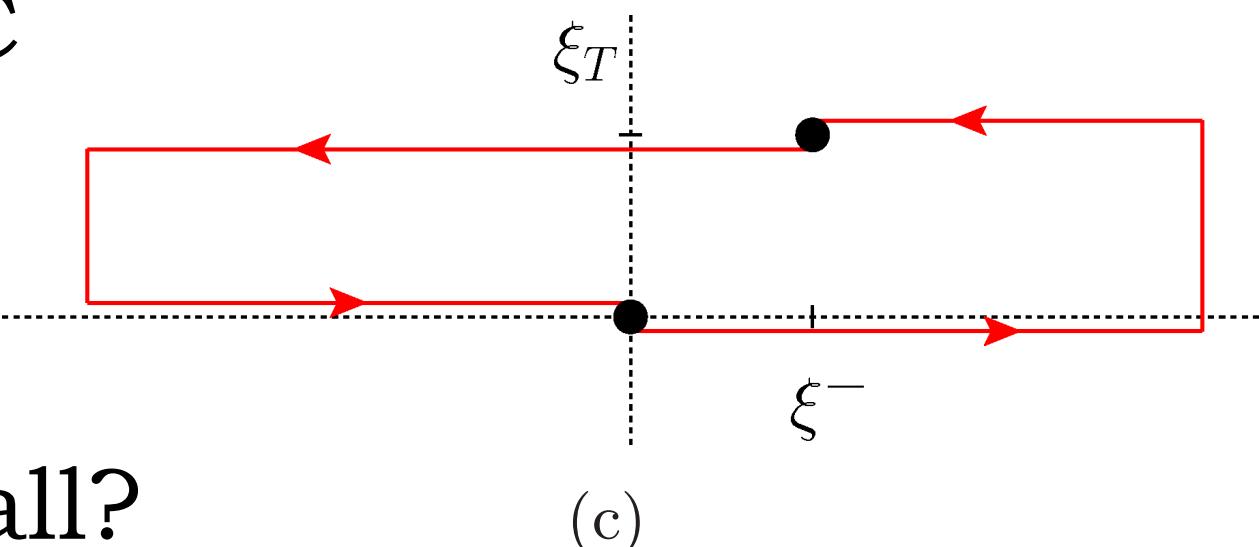
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Gauge link  $\rightarrow$  two main independent sets of TMDs, **not related** to each other

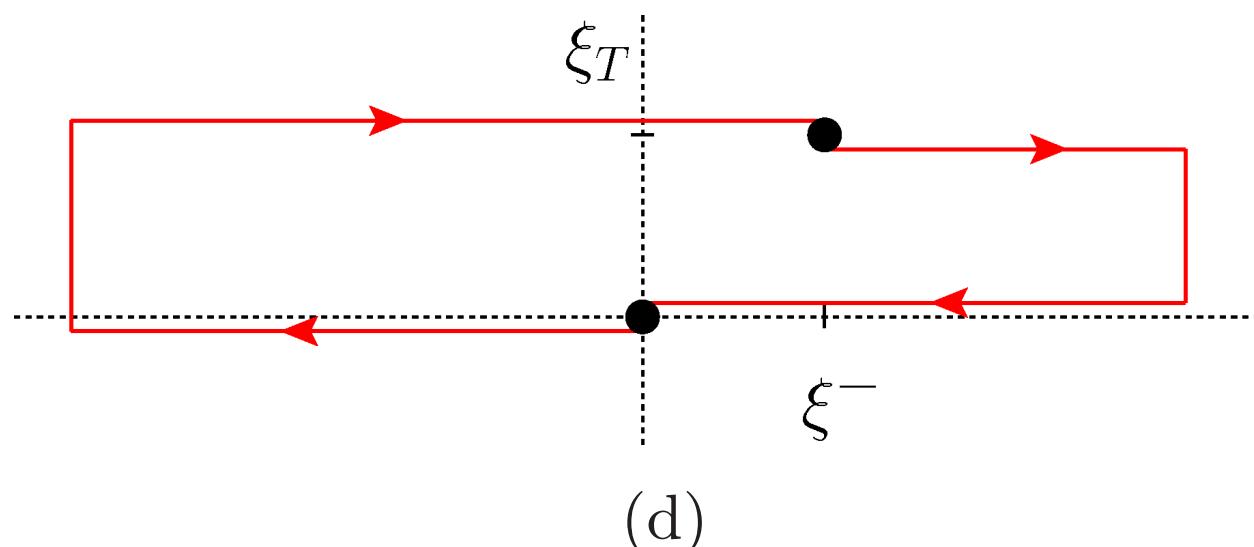
# Dihadron hadroproduction and factorization breaking

- \* Proof of factorization violation  [T. J. Rogers, P. J. Mulders (2010)]

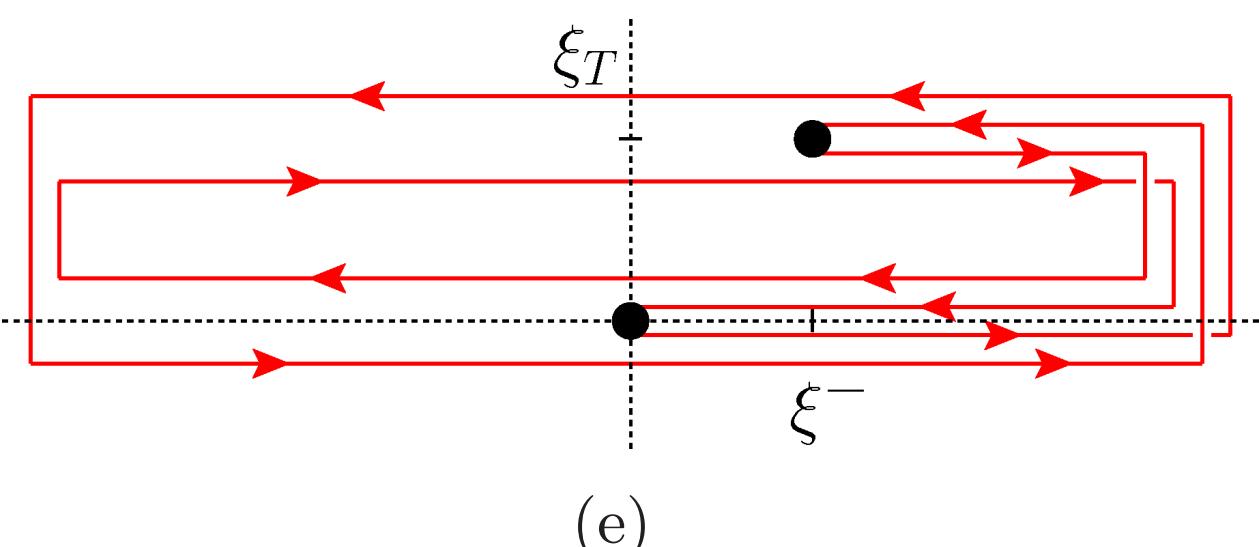
- \* Assumed factorization in SCET and CGC



- \* Significance of low- $x$  studies

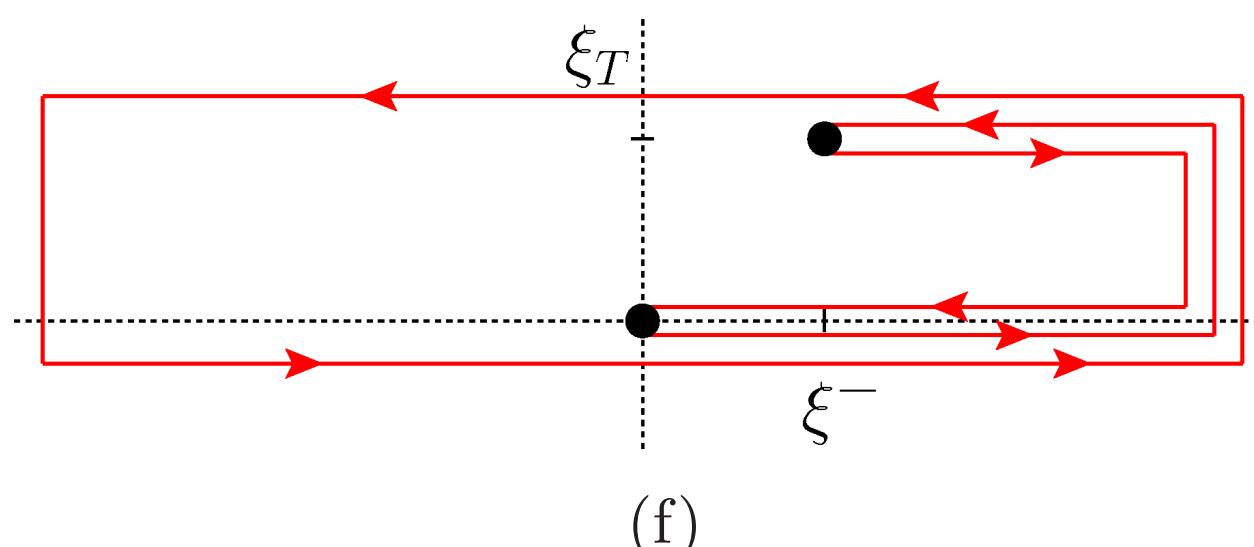


- \* Size of factorization-breaking effects small?



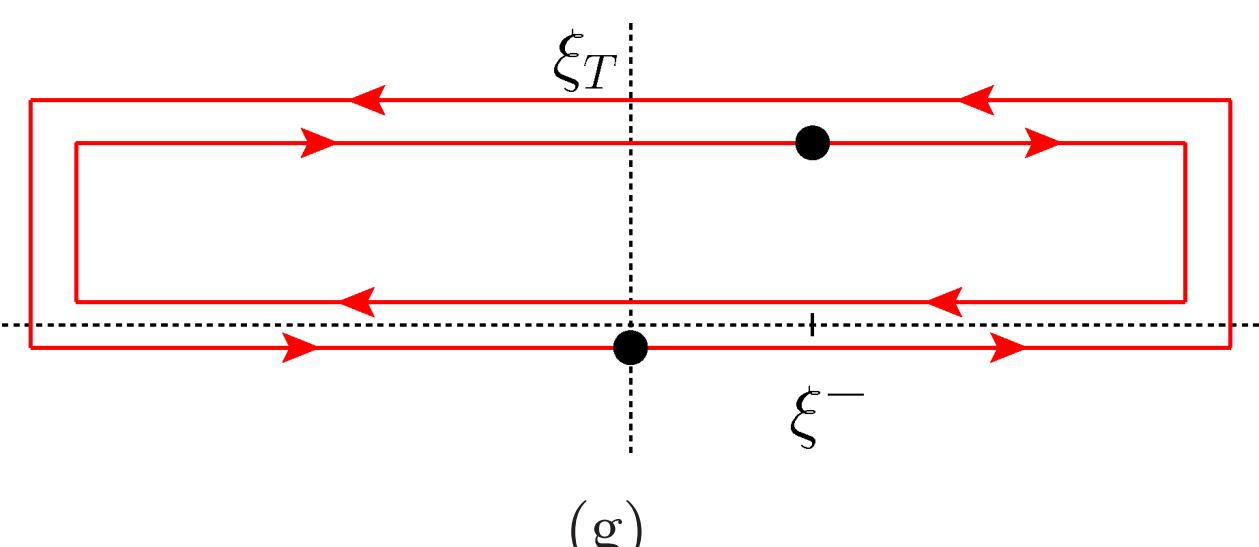
- \* DP TMDs:

(c)  $[+, -]$  and (d)  $[-, +]$

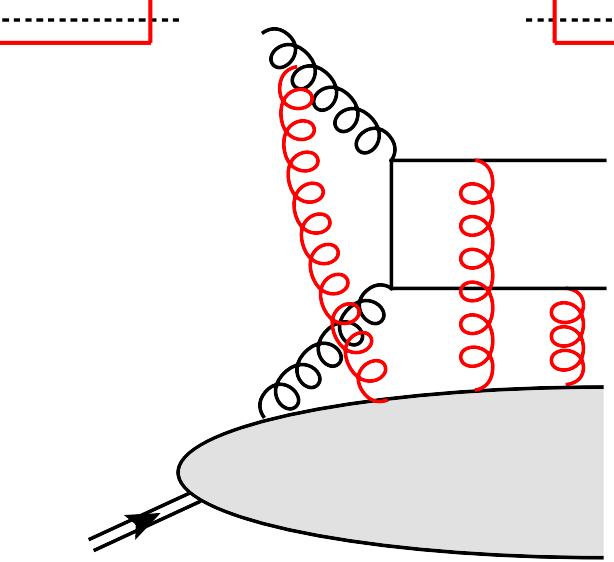


- \* Appearance of new gauge **loop links**:

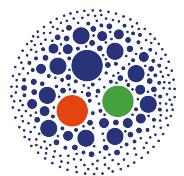
(e)  $[+ \square, + \square]$ , (f)  $[+, + \square]$ ,



(g)  $[\square, \square]$ , and (h)  $[\square, \square]$

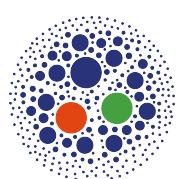
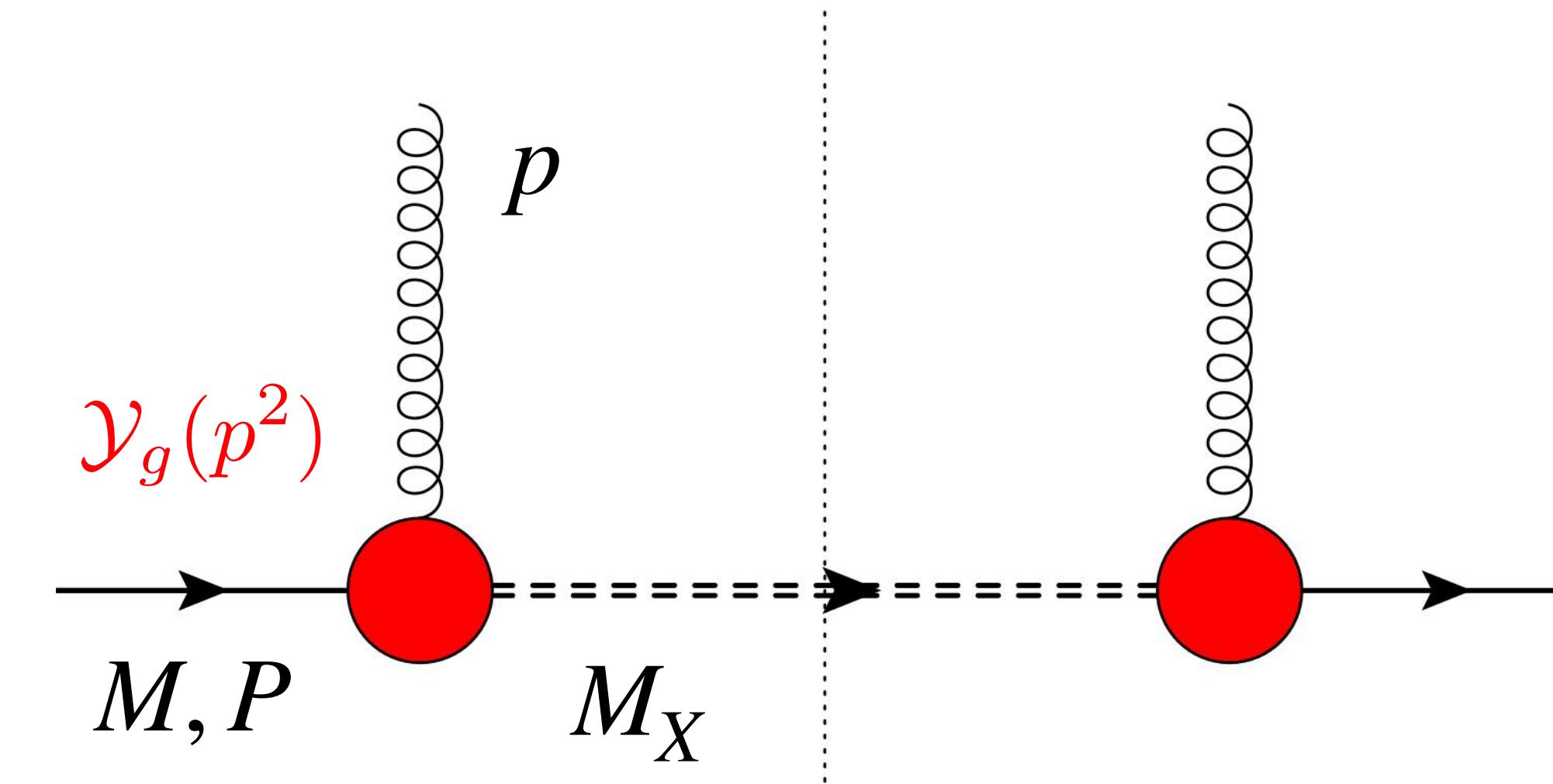


# Assumptions of the model



## Effective vertex

Lowest Fock state:  
**tri-quark spectator**  
on-shell and  
with mass  $M_X$



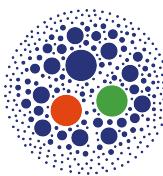
## Spin-1/2 spectator (gluon)

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} Tr \left[ (\not{P} + M) \frac{1 + \gamma^5 \$}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \gamma_g^{\rho*} \gamma_{g\sigma}(\not{P} - \not{p} + M) \right]$$

$$\gamma_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M} \sigma^{\mu\nu} p_\nu$$

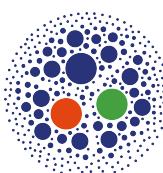
mimics proton form factors  
(conserved EM current  
of a free nucleon)

# Assumptions of the model



## Link with collinear factorization

$p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution

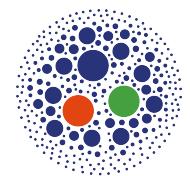


## Dipolar form factor(s)

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

1. Cancels singularity of gluon propagator
2. Suppresses effects of high  $p_T$
3. Compensates log divergences arising from  $p_T$ -integration
4. Adds three more parameters:  $\kappa_{1,2}$  and  $\Lambda_X$

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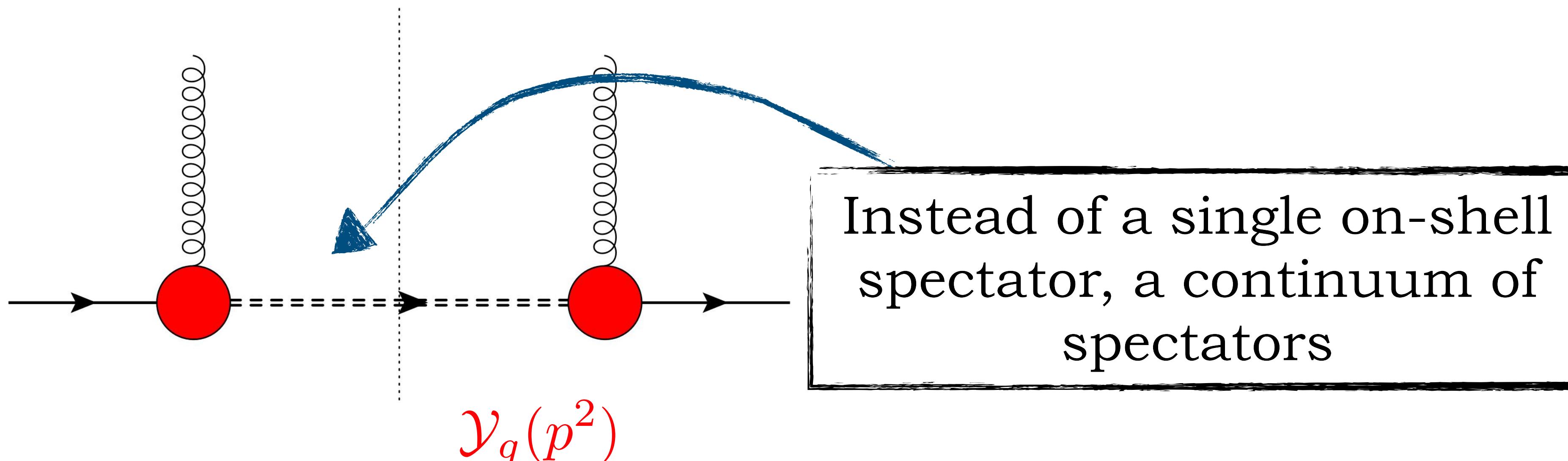
## Spectator-system spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectral-mass function

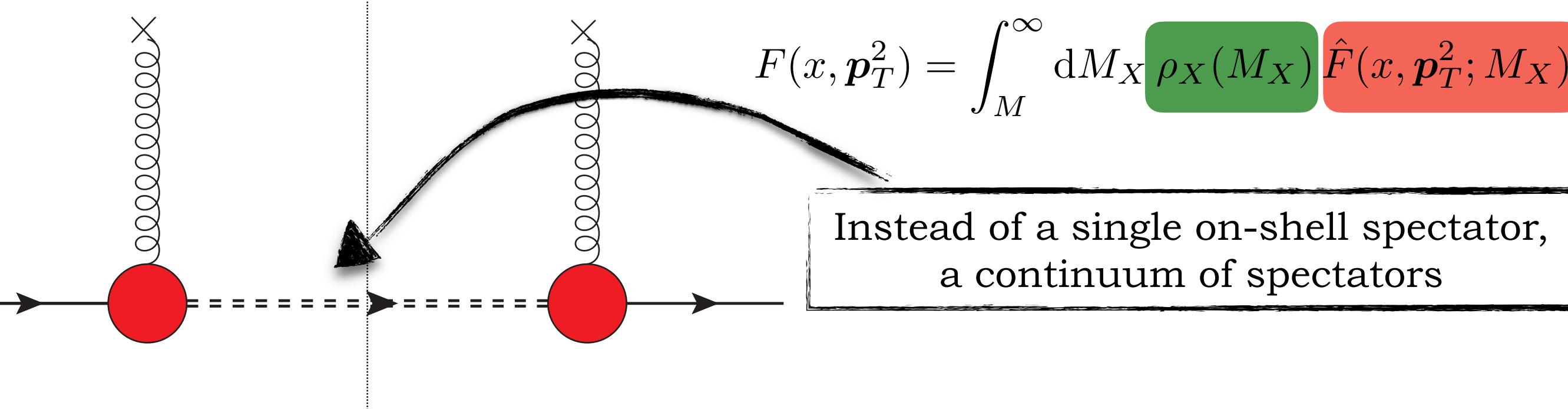
spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



# Our model at a glance

## Spectator-system spectral-mass function



Spectral function **learns** small- and moderate- $x$  info  
encoded in **NNPDF** collinear parametrizations

(NNPDF3.1sx + NNPDFpol1.1)

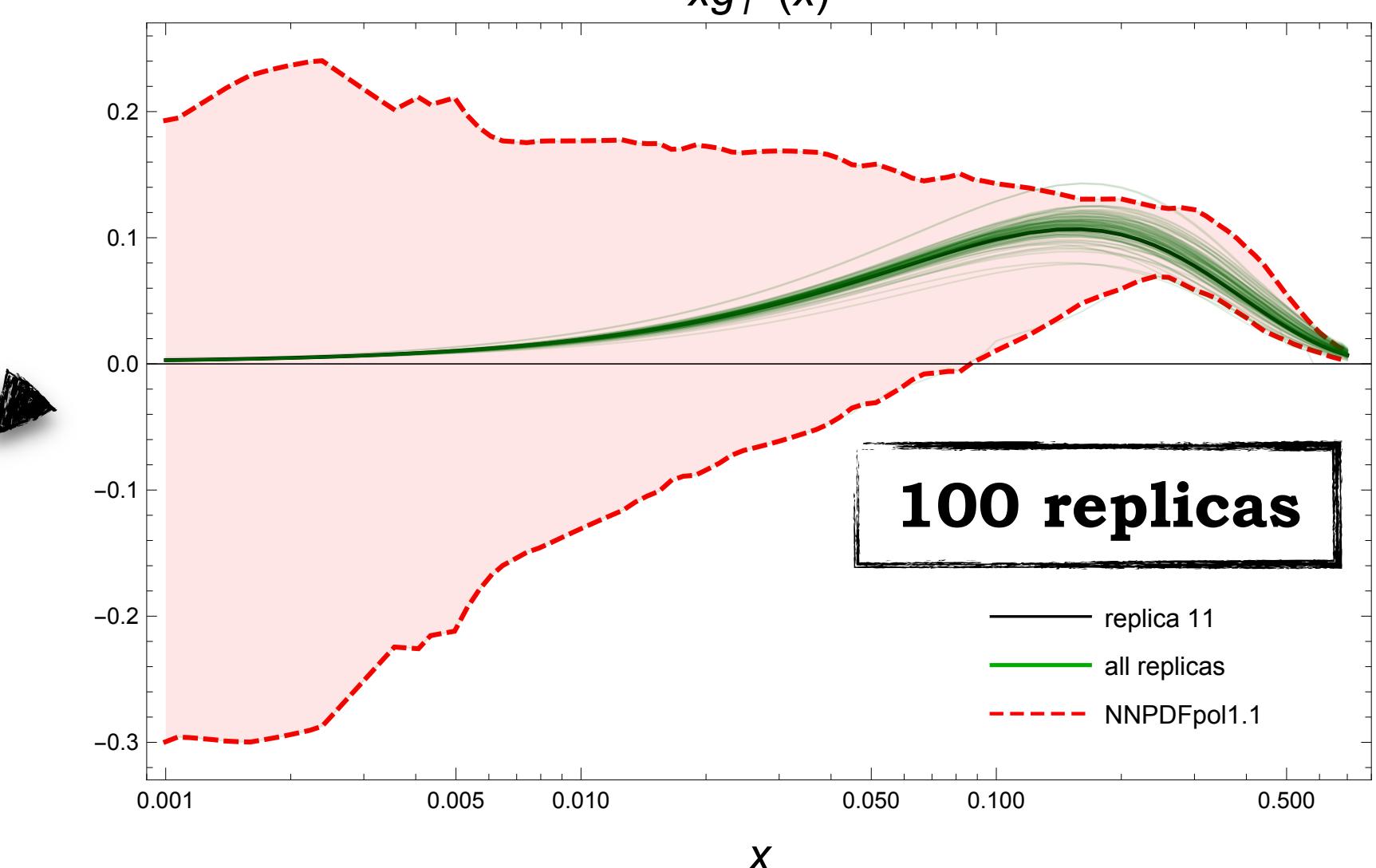
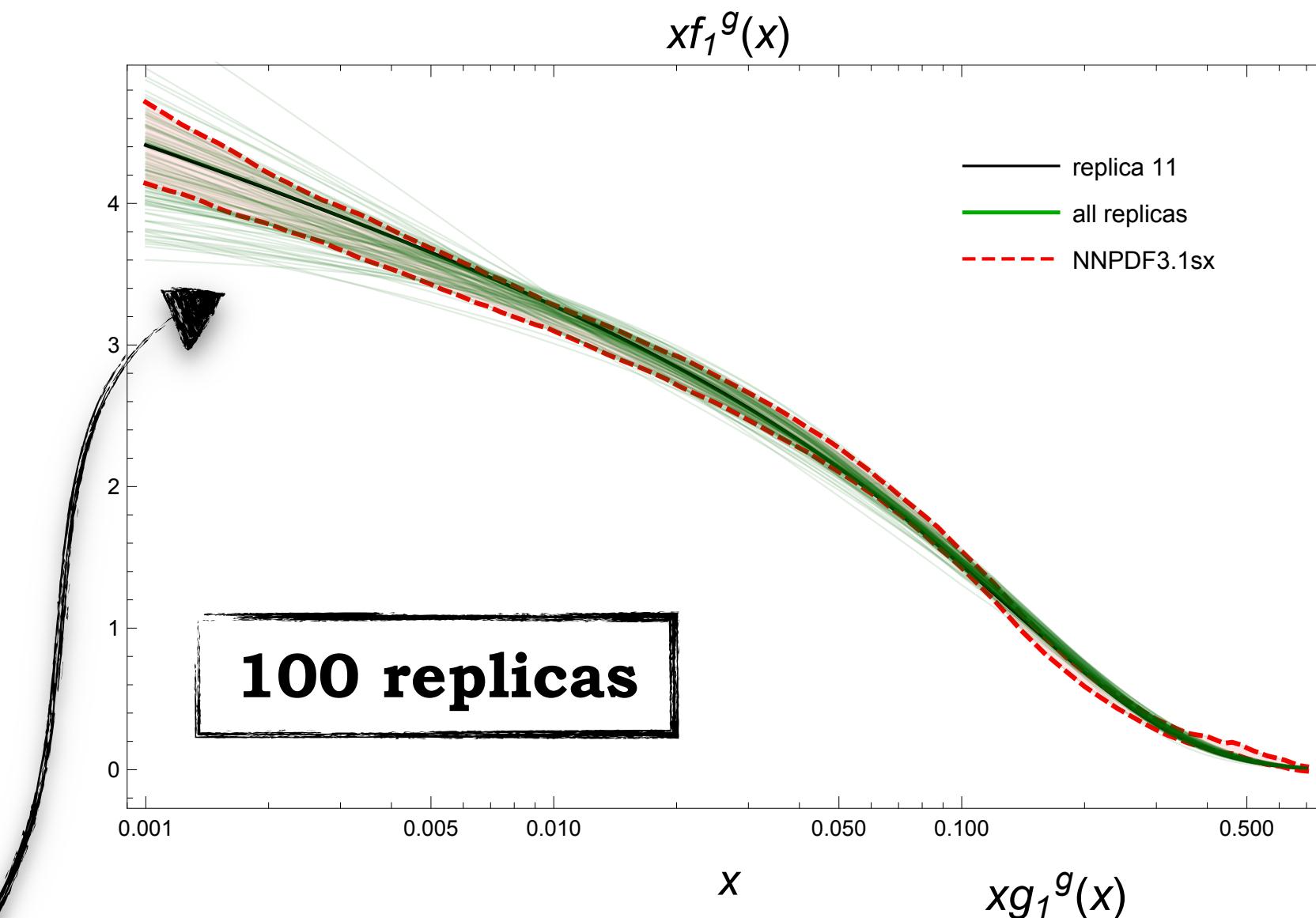
**Simultaneous fit** of  $f_1$  and  $g_1$  PDFs

Inclusion of small- $x$  resummation effects (**BFKL**)

Calculation of all twist-2  $T$ -even gluon TMDs

## Link with collinear factorization

$p_T$ -integrated TMDs **have to** reproduce PDFs  
at the lowest scale ( $Q_0$ ) *before* evolution



# Fit specifics

$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for  $g_1$

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$$S_g = \frac{1}{2} \langle 1 \rangle_{\Delta g} = \int_0^1 dx g_1^g(x, Q_0)$$

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Our model @  $Q_0 = 1.64$  GeV

$$\langle x \rangle_g = 0.424(9)$$

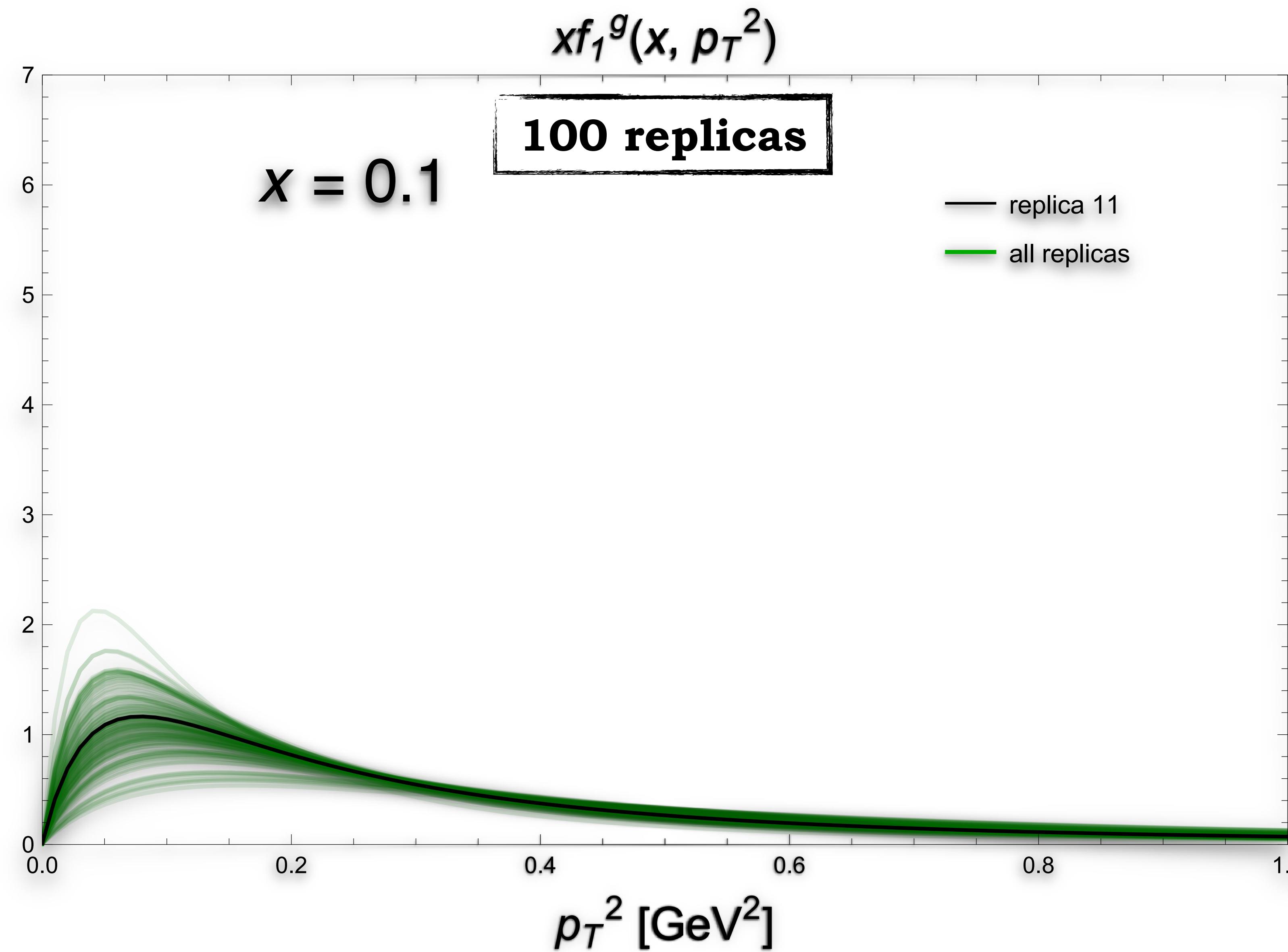
$$\langle S \rangle_g = 0.159(11)$$

Lattice @  $Q_0 = 2$  GeV

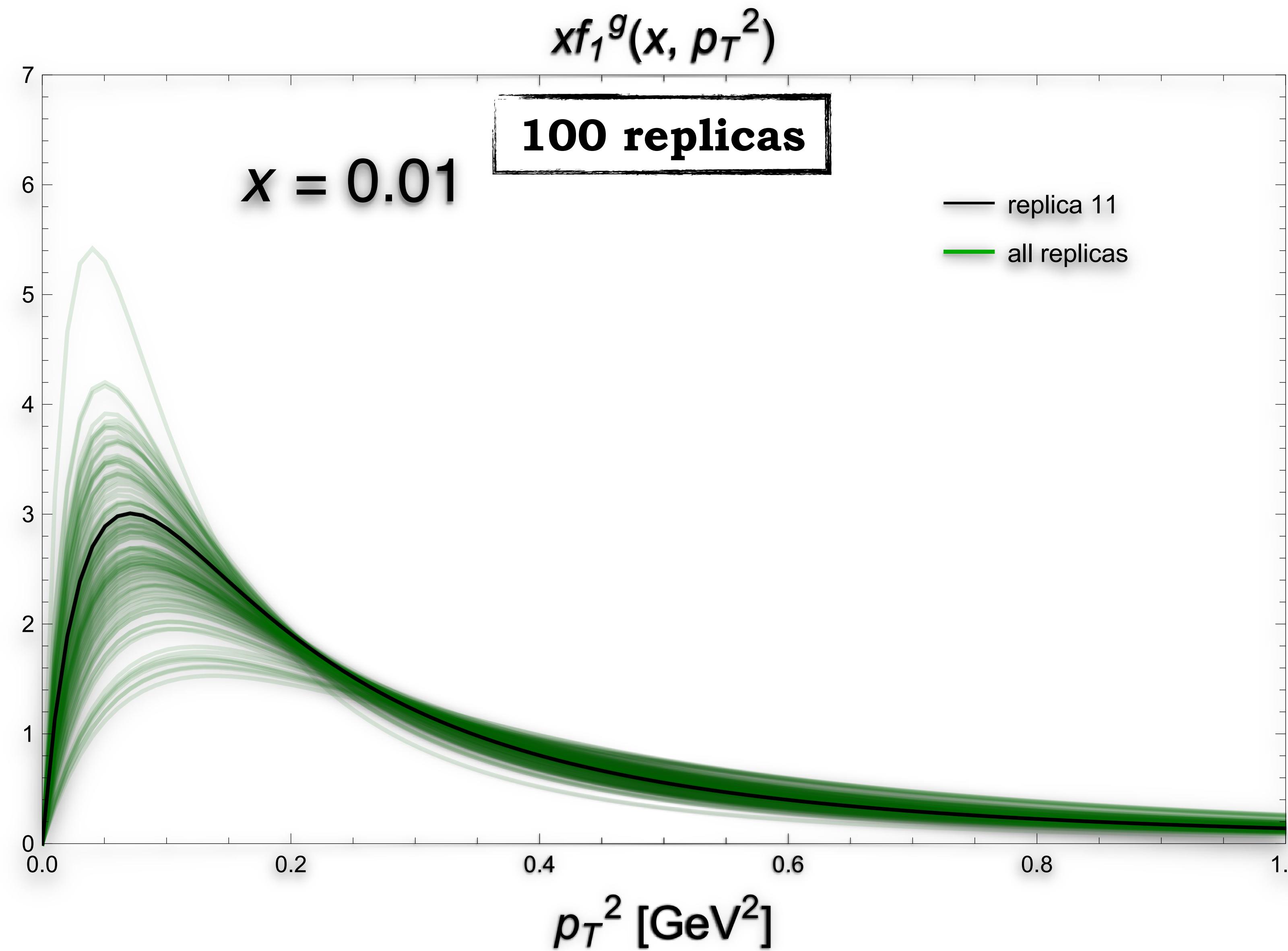
$$\langle x \rangle_g = 0.427(92)$$

$$\langle J \rangle_g = 0.187(46)$$

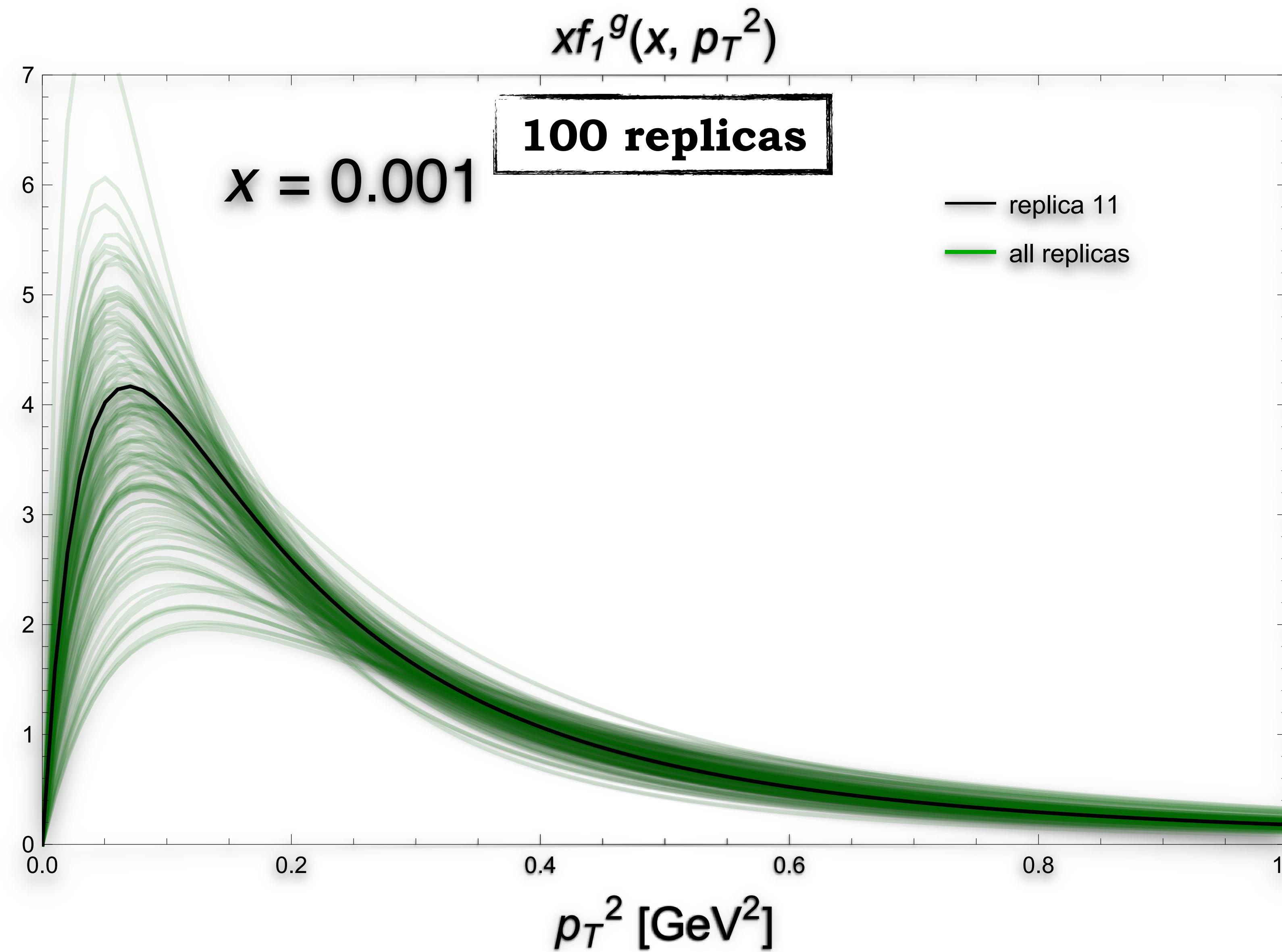
# Unpolarized gluon TMD



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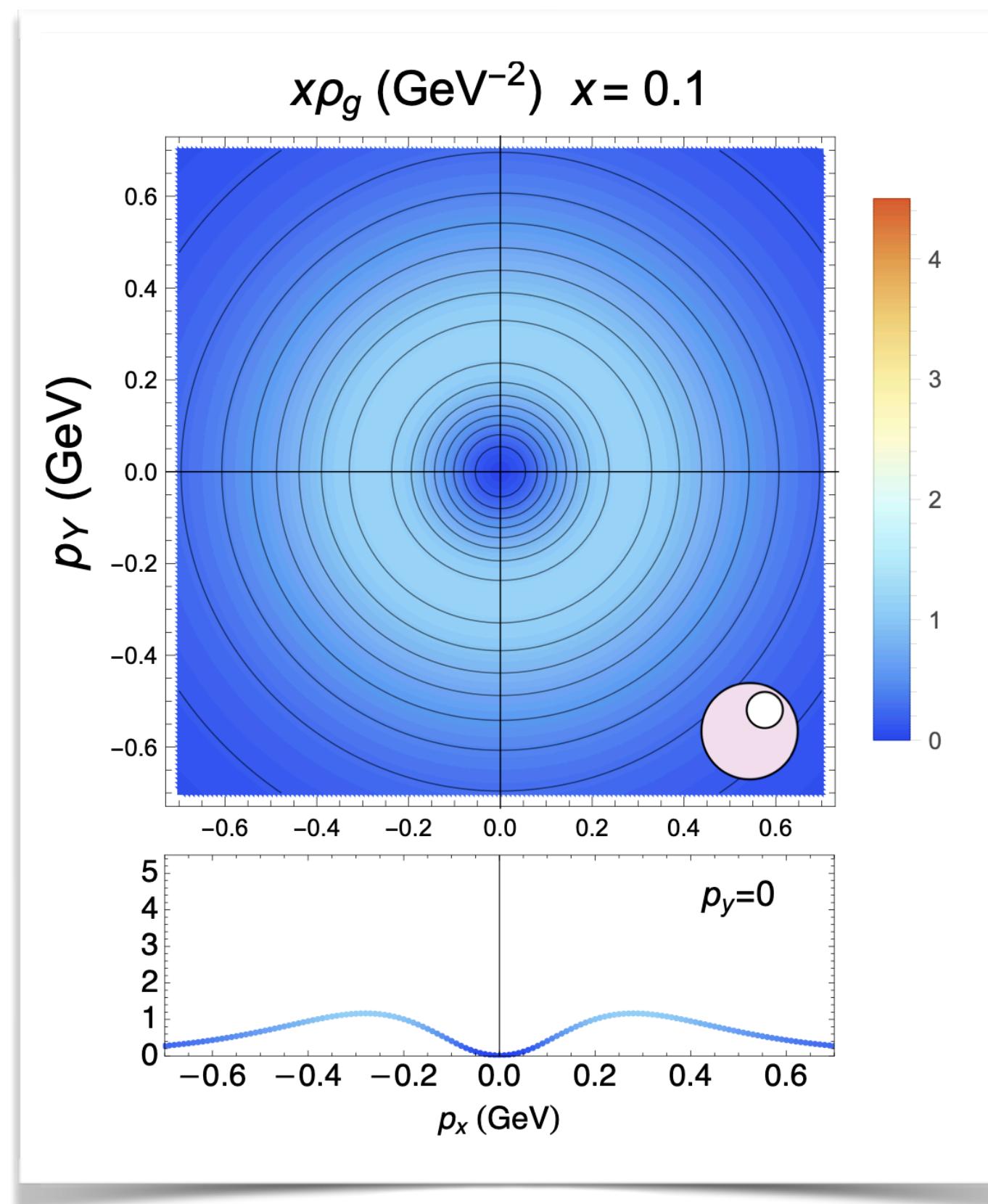


# Unpolarized gluon TMD



# 3D tomography: the gluon content in the proton

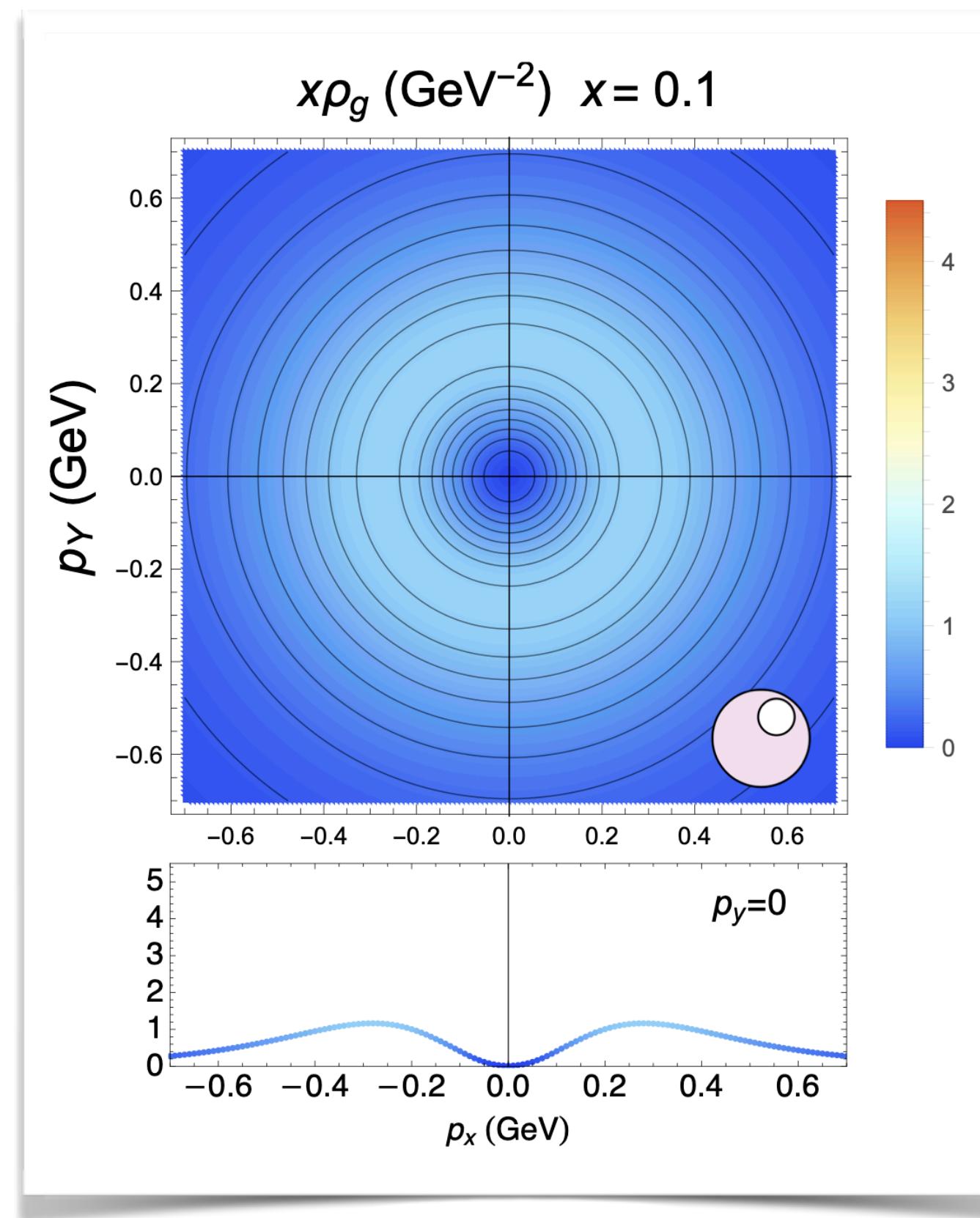
unpolarized TMD



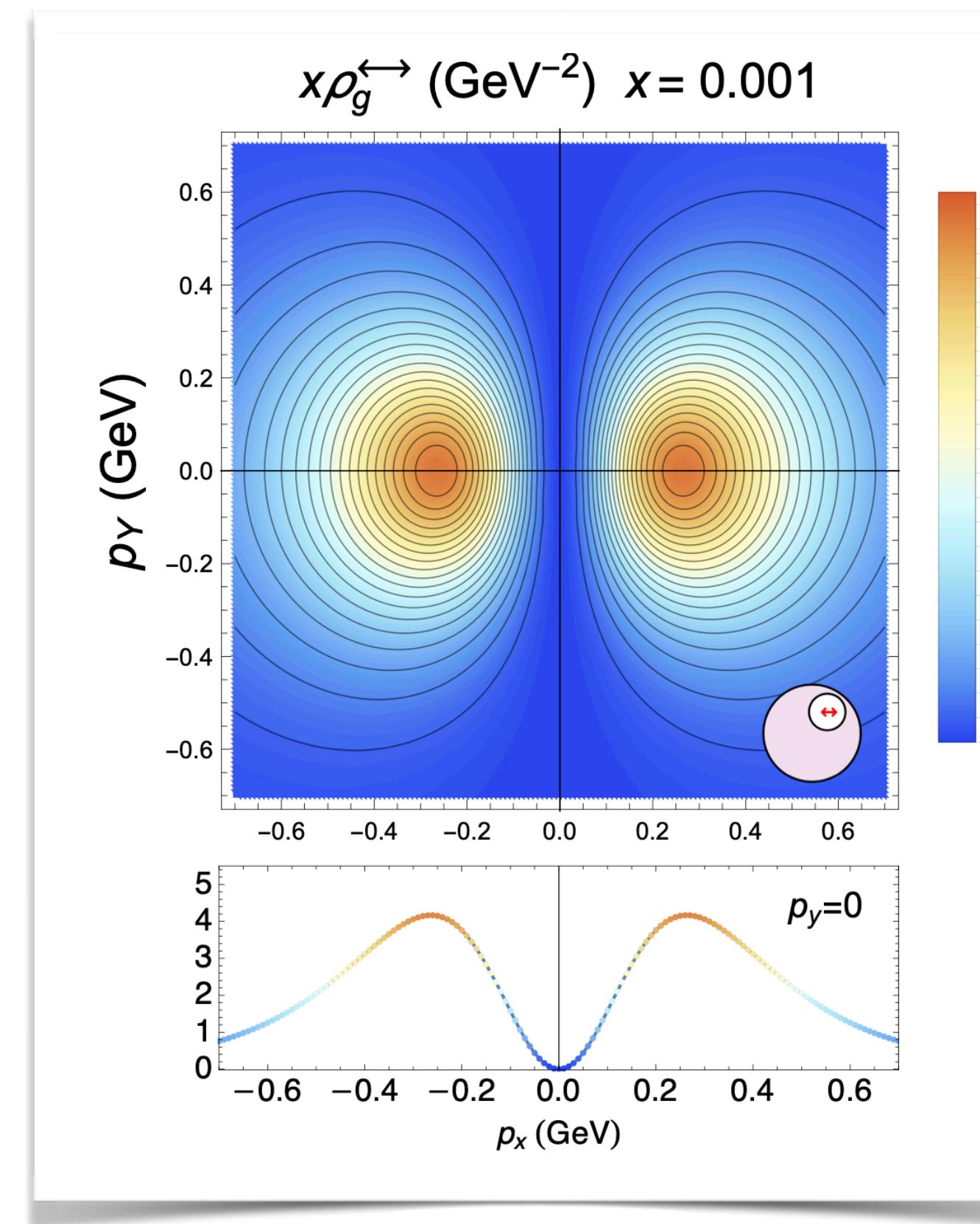
[A. Bacchetta, F.G.C., M. Radici, P. Taels, *Eur. Phys. J. C* **80** (2020) no.8 [[arXiv:2005.02288](https://arxiv.org/abs/2005.02288)]]

# 3D tomography: the gluon content in the proton

unpolarized TMD



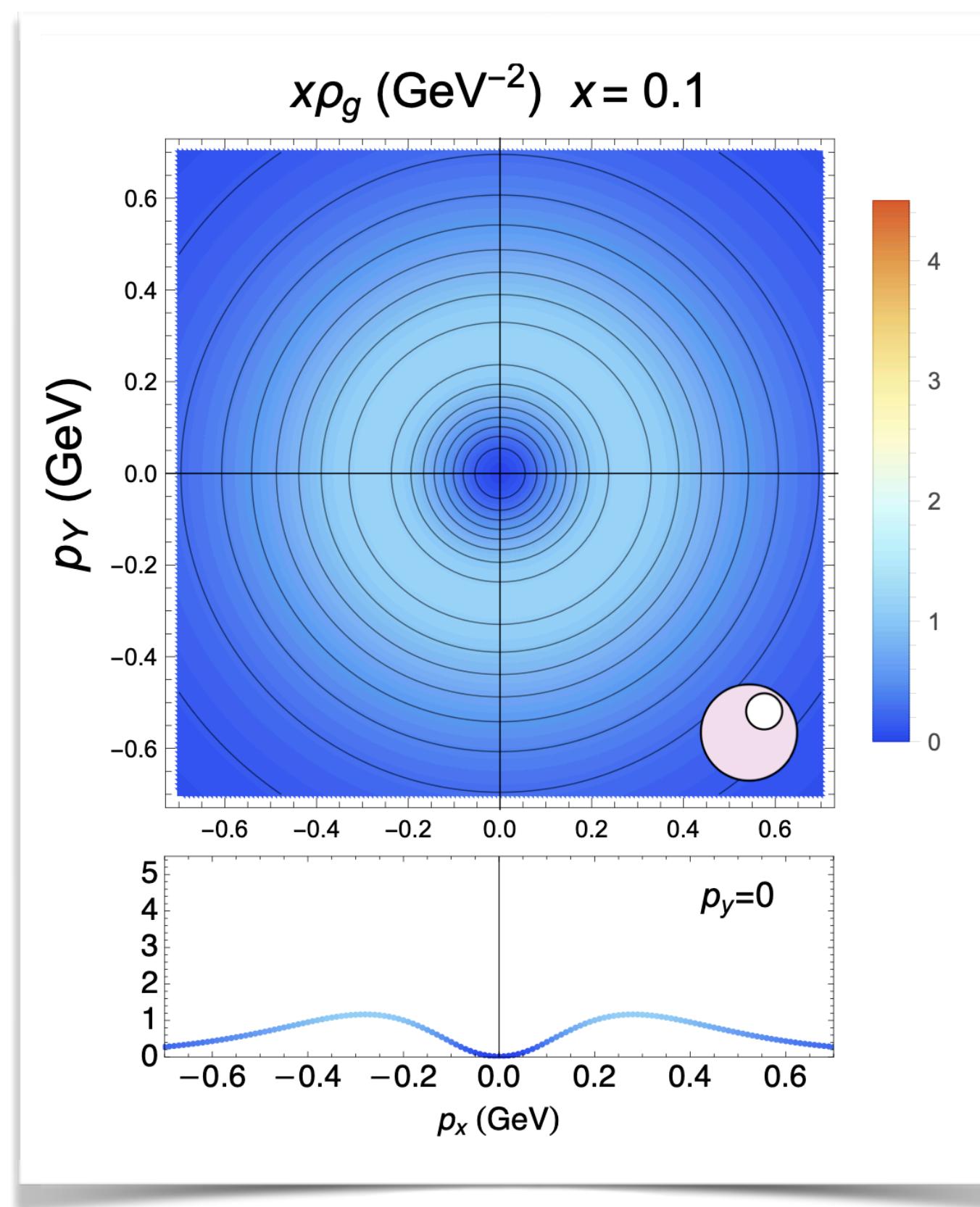
Boer-Mulders



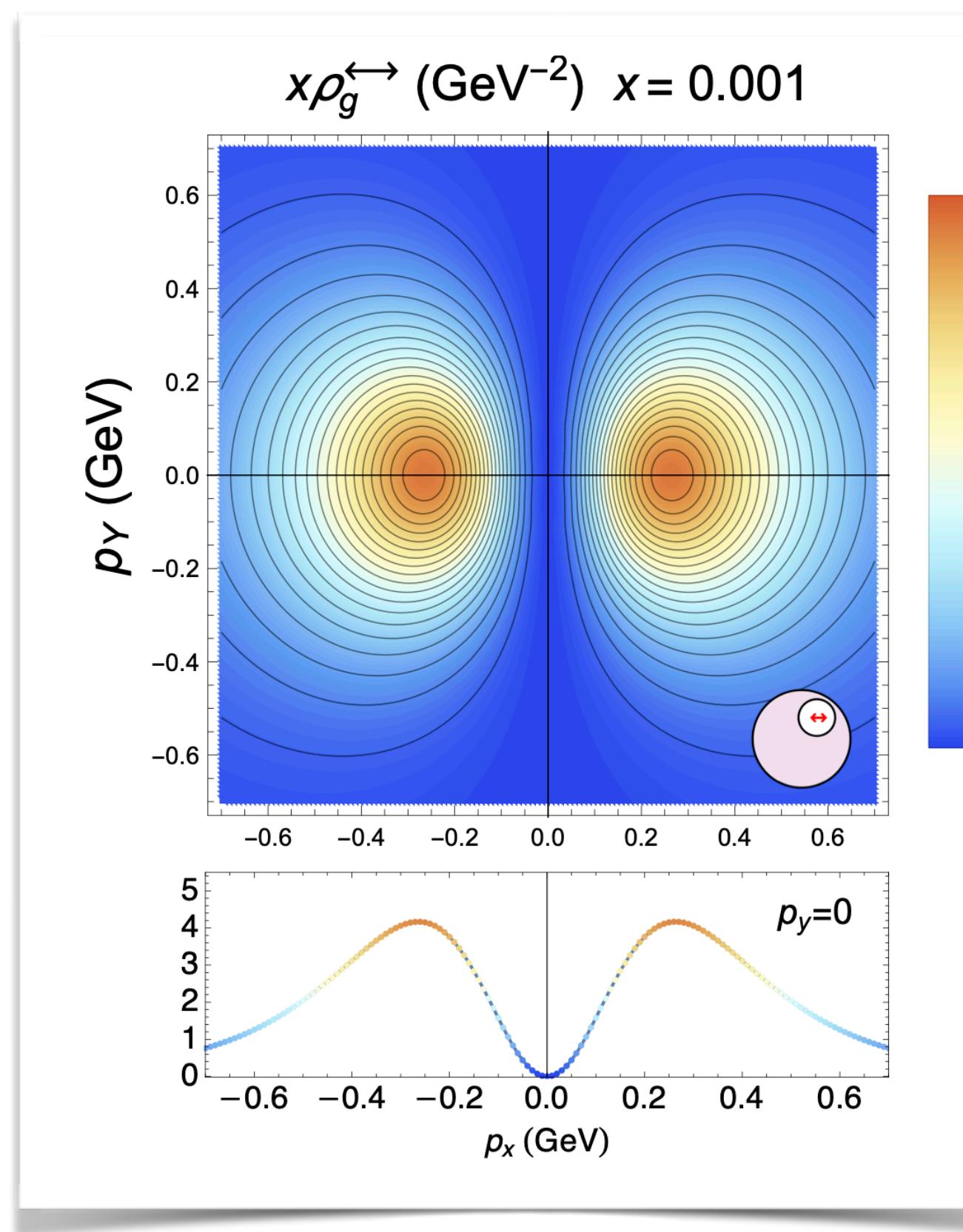
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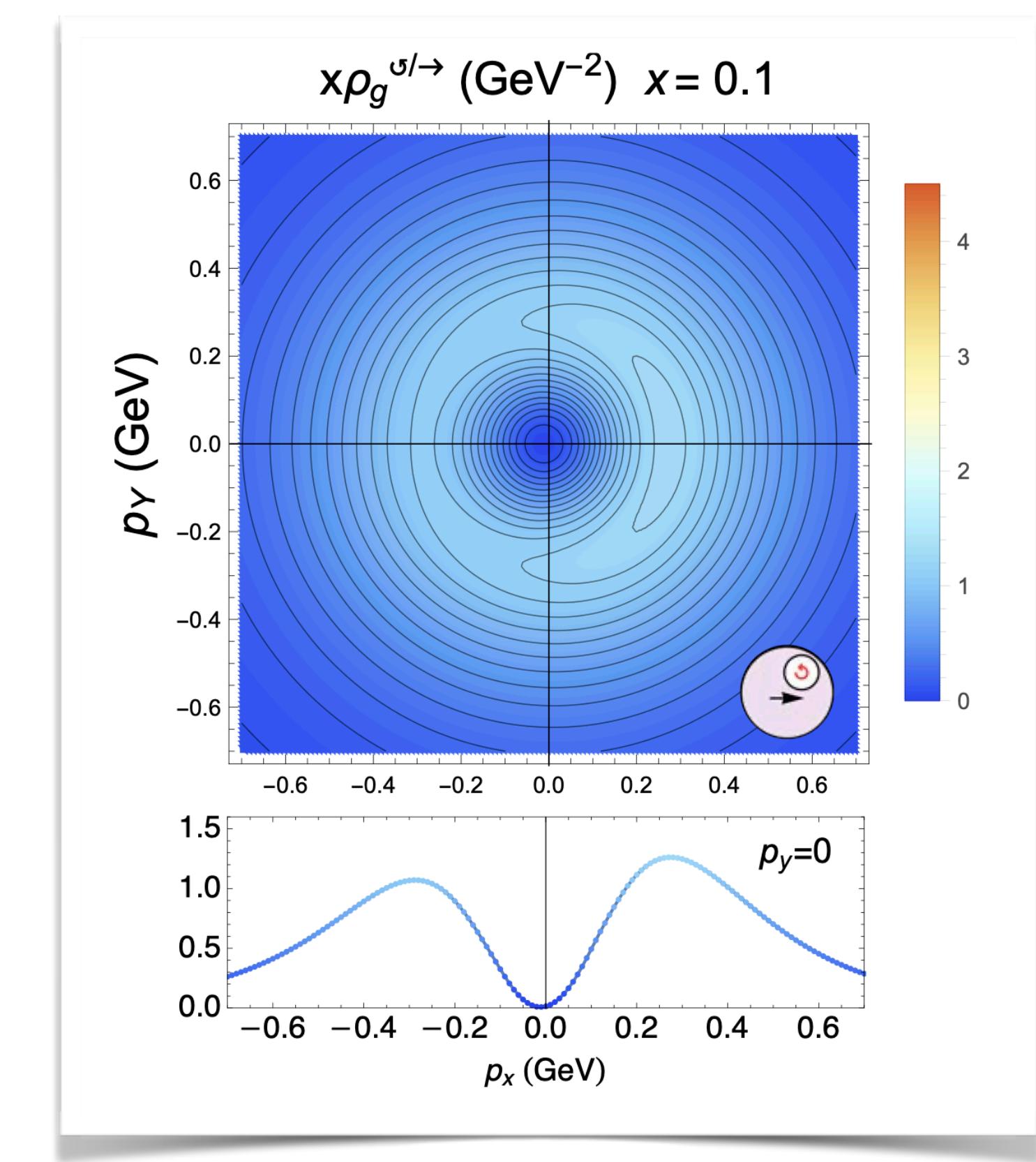
unpolarized TMD



Boer-Mulders



worm-gear



[A. Bacchetta, F.G.C., M. Radici, P. Taels, *Eur. Phys. J. C* **80** (2020) no.8 [[arXiv:2005.02288](https://arxiv.org/abs/2005.02288)]]

**...towards twist-2  
T-odd gluon TMDs**

# T-odd gluon TMDs at twist-2

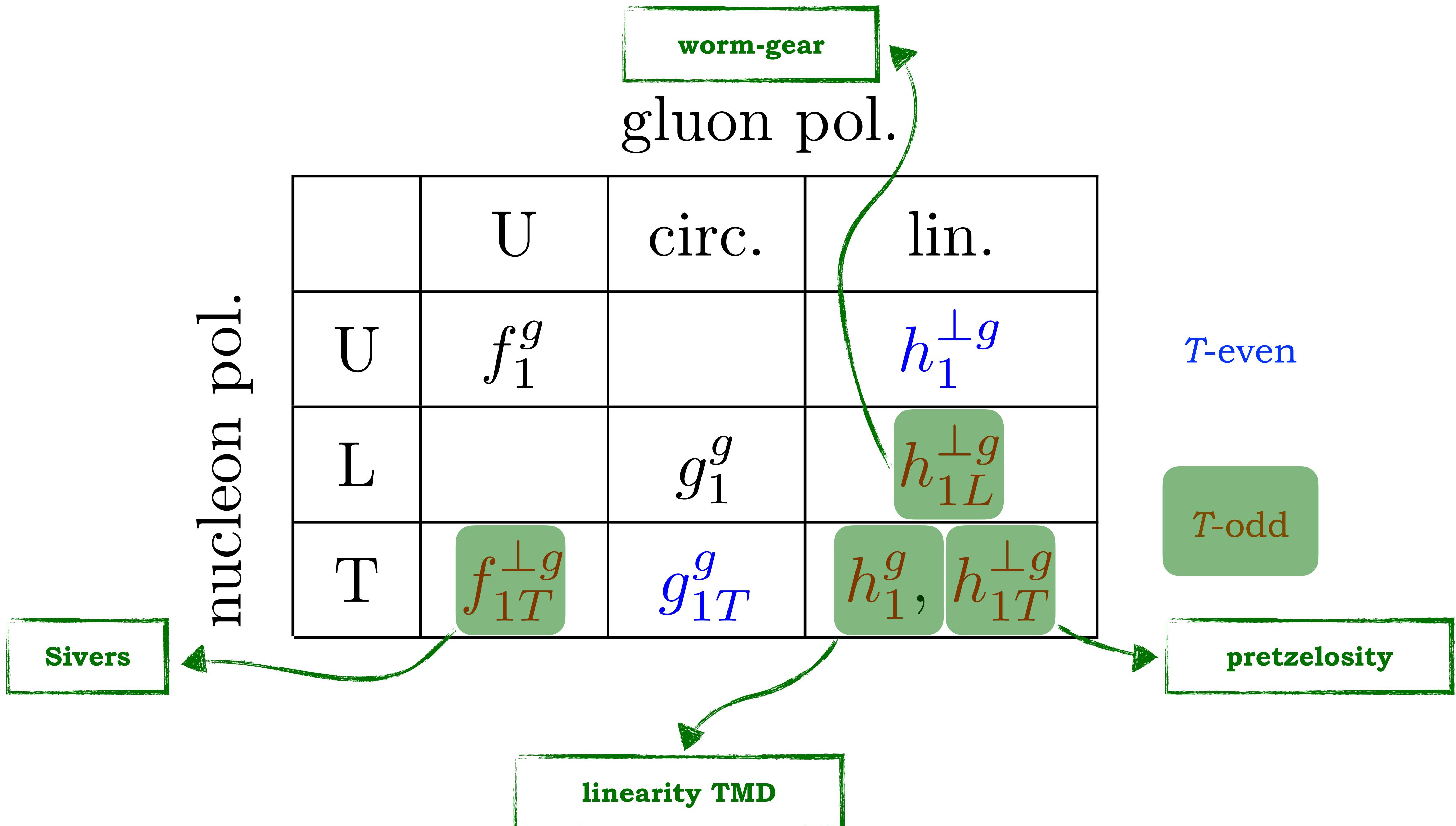
gluon pol.

	U	circ.	lin.
U	$f_1^g$		$h_1^{\perp g}$
L		$g_1^g$	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_1^g, h_{1T}^{\perp g}$

*T-even*

*T-odd*

# T-odd gluon TMDs at twist-2

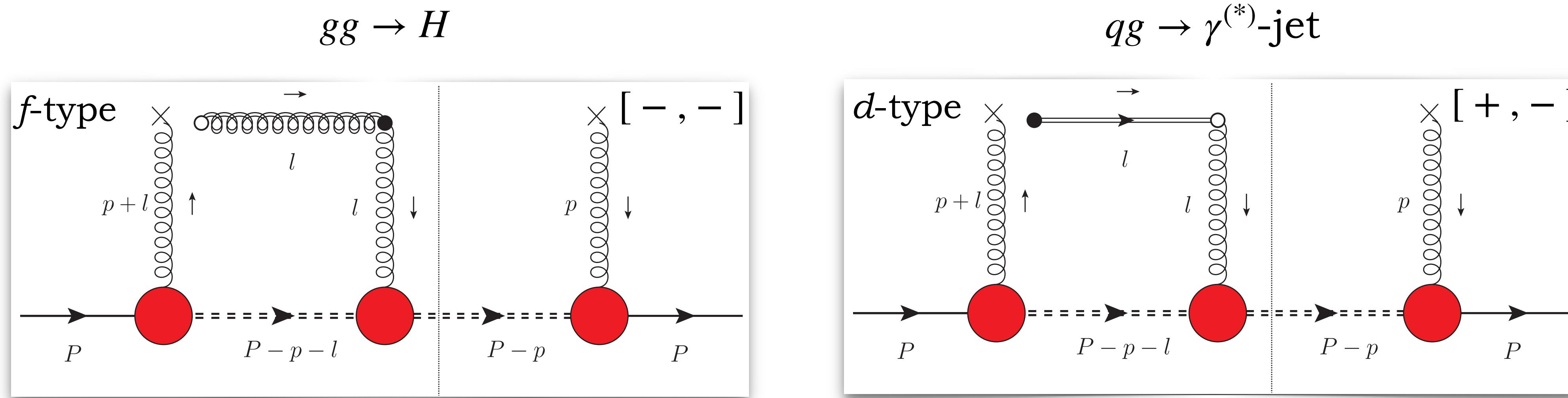


# **T-odd gluon TMDs in a spectator model**

- No residual gluon-spectator interaction at tree level
- *Interference with one-gluon exchange (eikonal)*

# T-odd gluon TMDs in a spectator model

- No residual gluon-spectator interaction at tree level
- Interference with one-gluon exchange (eikonal)*



- Leading-twist one-gluon-exchange of the gauge-link operator
- Sensitivity to WW/DP structures
- Calculation of **Sivers** function *underway!*

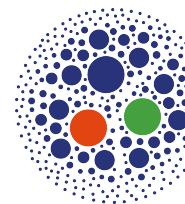
# Closing statements

- Systematic calculation of all twist-2  $T$ -even gluon TMDs
- Spectral mass to catch small- and large- $x$  effects
- Simultaneous fit** of  $f_1$  and  $g_1$  PDFs via **replica method**

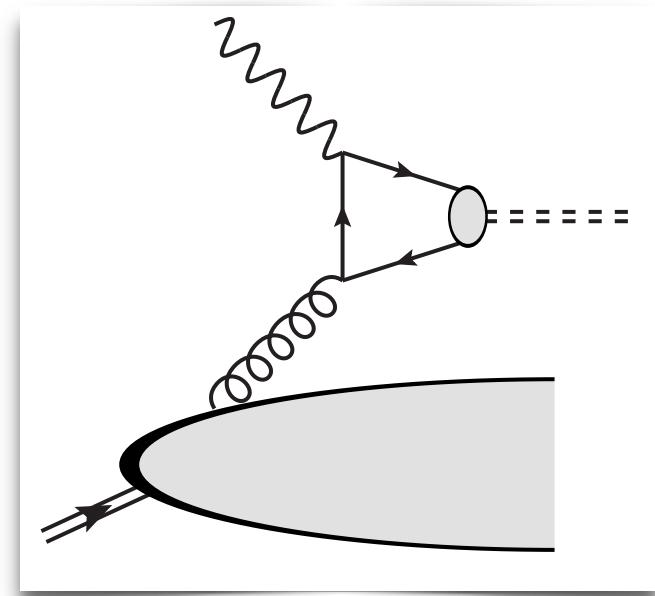
# Closing statements

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- Twist-2  $T$ -odd gluon TMDs (**Sivers**, etc.) soon available!
- Pheno: **spin asymmetries**, **pseudodata** and **impact studies**
- Evolution: extension to quark TMDs in the same framework
- Explorative studies on gauge-link sensitivity and factorization
- Studies on GPD and small- $x$  UGD sectors

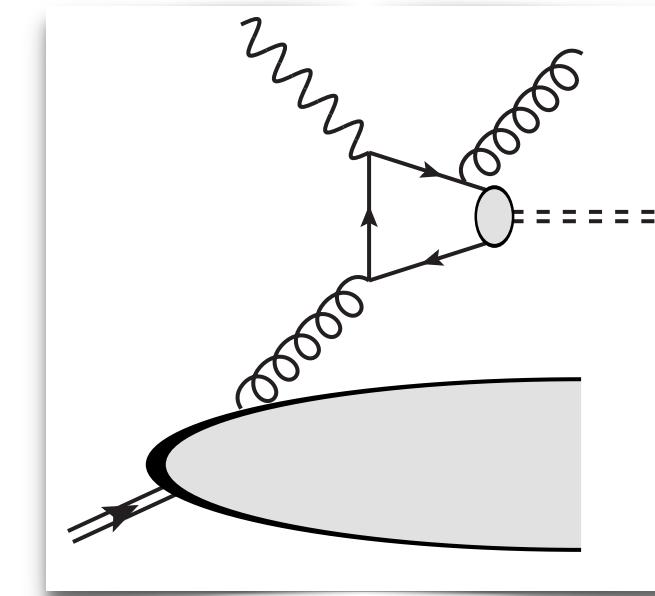
# Gluon TMDs and quarkonia



**SIDIS**



CO



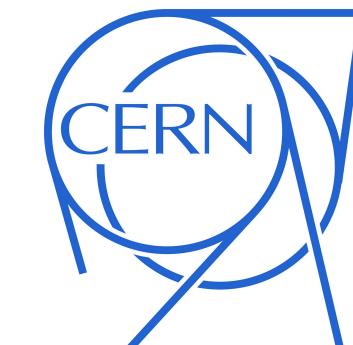
CO + CS



EIC Yellow Report Document

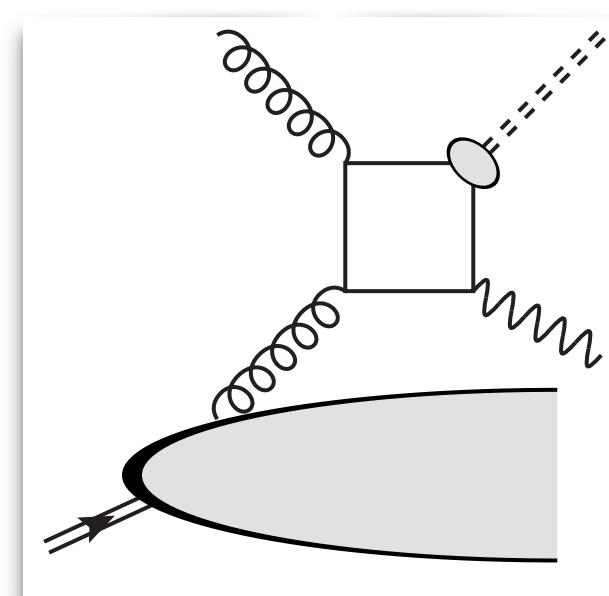
🔗 (EICUG website)

🔗 [EICUG [arXiv:2103.05419]]



On the physics potential to study the gluon content of proton and deuteron at NICA SPD

🔗 [NICA Collaboration [arXiv:2011.15005]]



CO + CS

Perspectives for quarkonium studies at the high-luminosity LHC

🔗 (QAT 2021 Workshop)

🔗 [Quarkonia As Tools Collaboration [arXiv:2012.14161]]

**Backup  
slides**

# Assumptions of the model



## Spectator-system spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectral-mass function

spectator-model TMD

∅ [Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X(M_X; \{X^{(\text{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\}) = \mu^{2a} \left[ \frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

low- $x$  (high- $\mu^2$ ) tail  $\propto (a - b)$

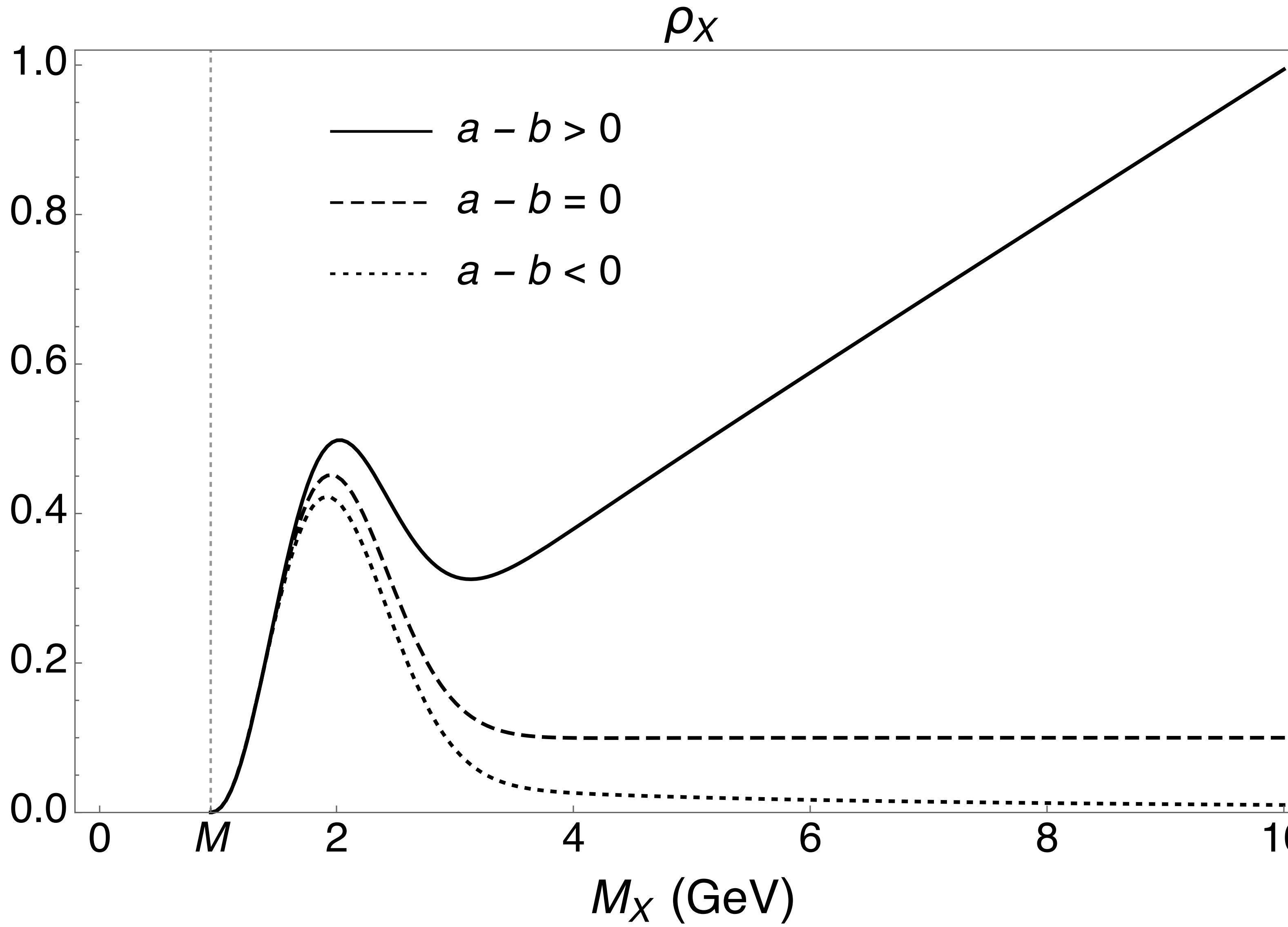
$q\bar{q}$  contributions energetically available at large  $M_X$

$$\mu^2 = M_X^2 - M^2$$

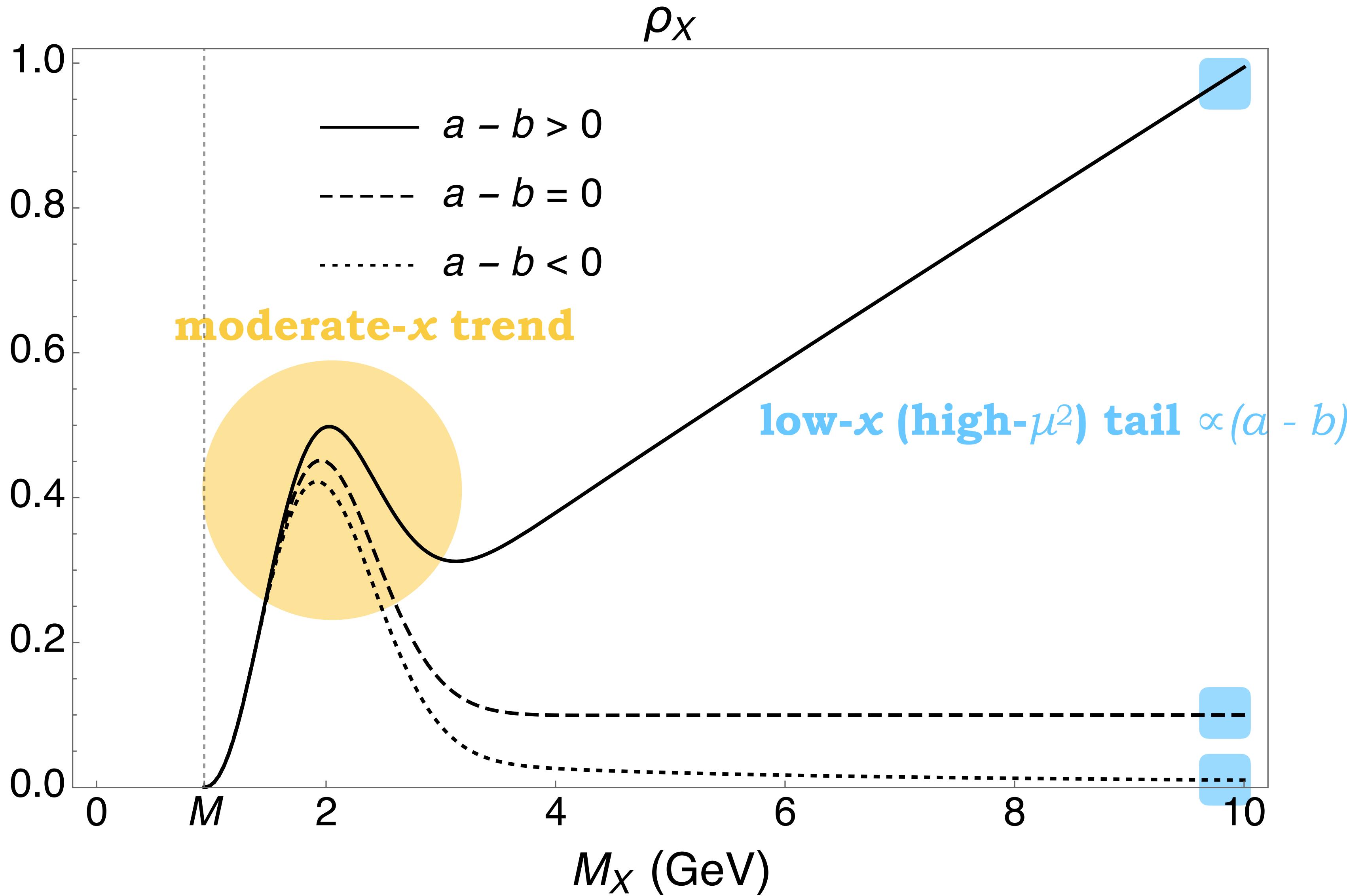
moderate- $x$  trend

pure tri-quark contribution at low  $M_X$

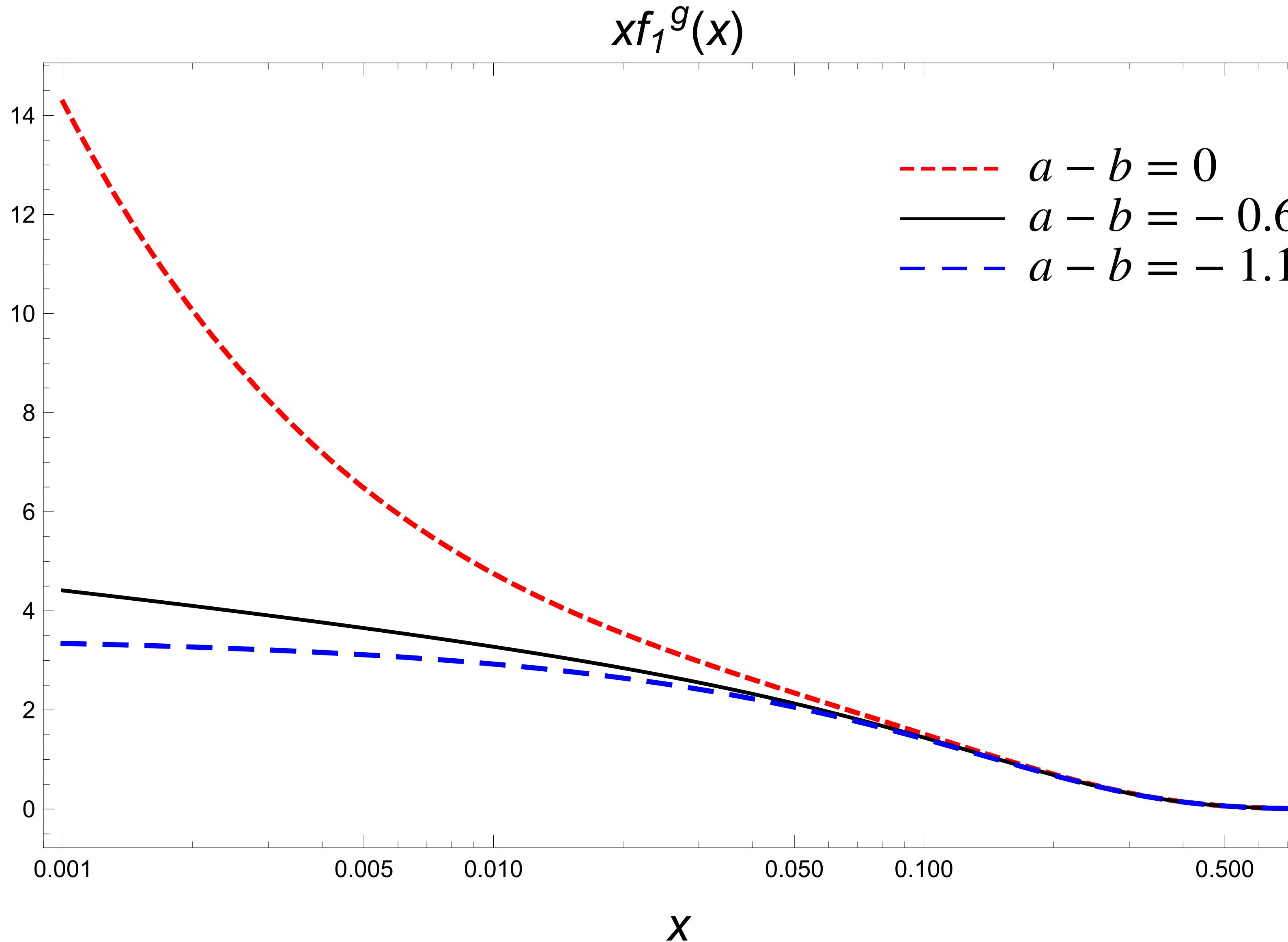
# Spectral function vs $(a - b)$



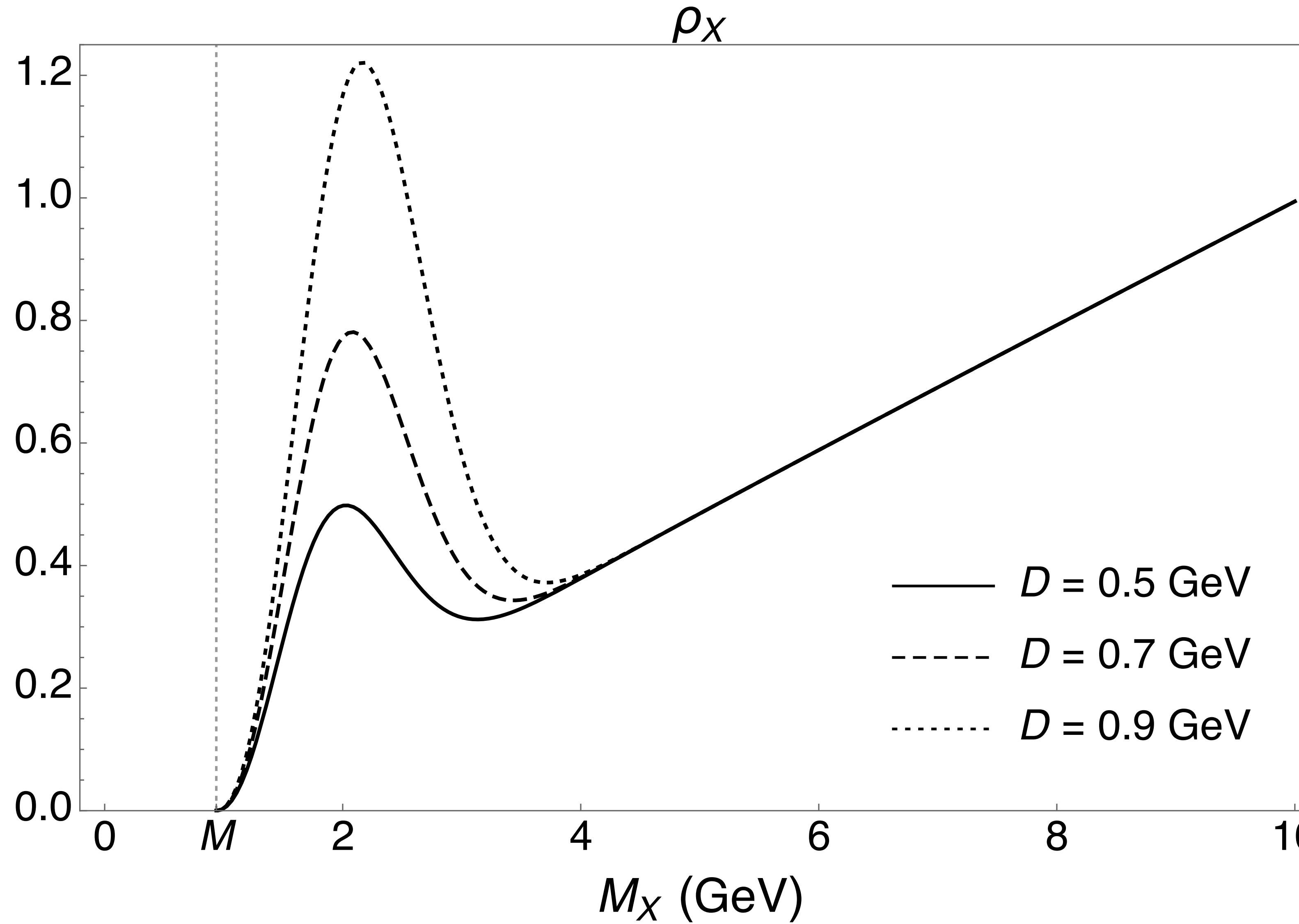
# Spectral function vs $(a - b)$



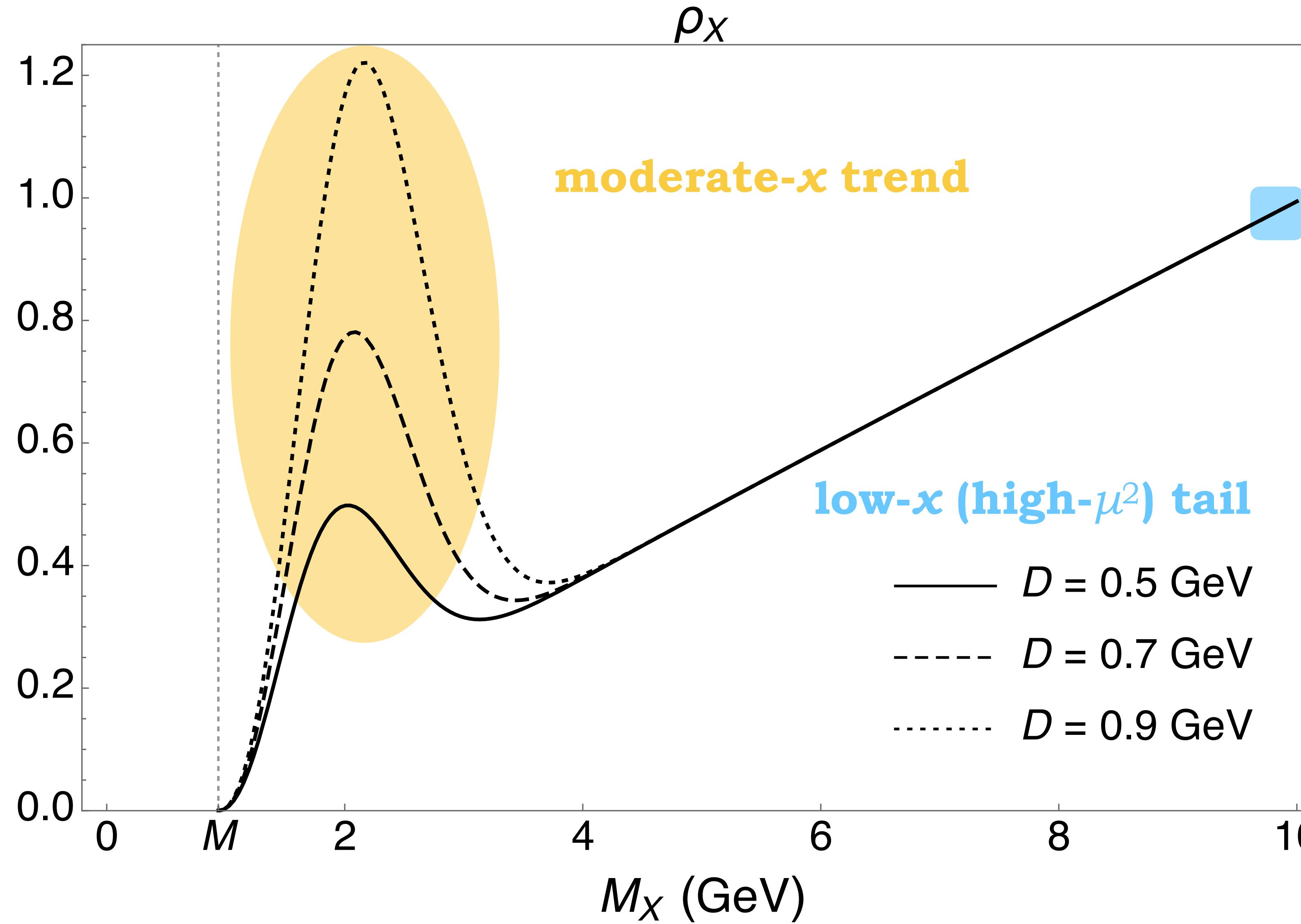
# $xf_1$ collinear PDF vs $(a - b)$



# Spectral function vs $D$

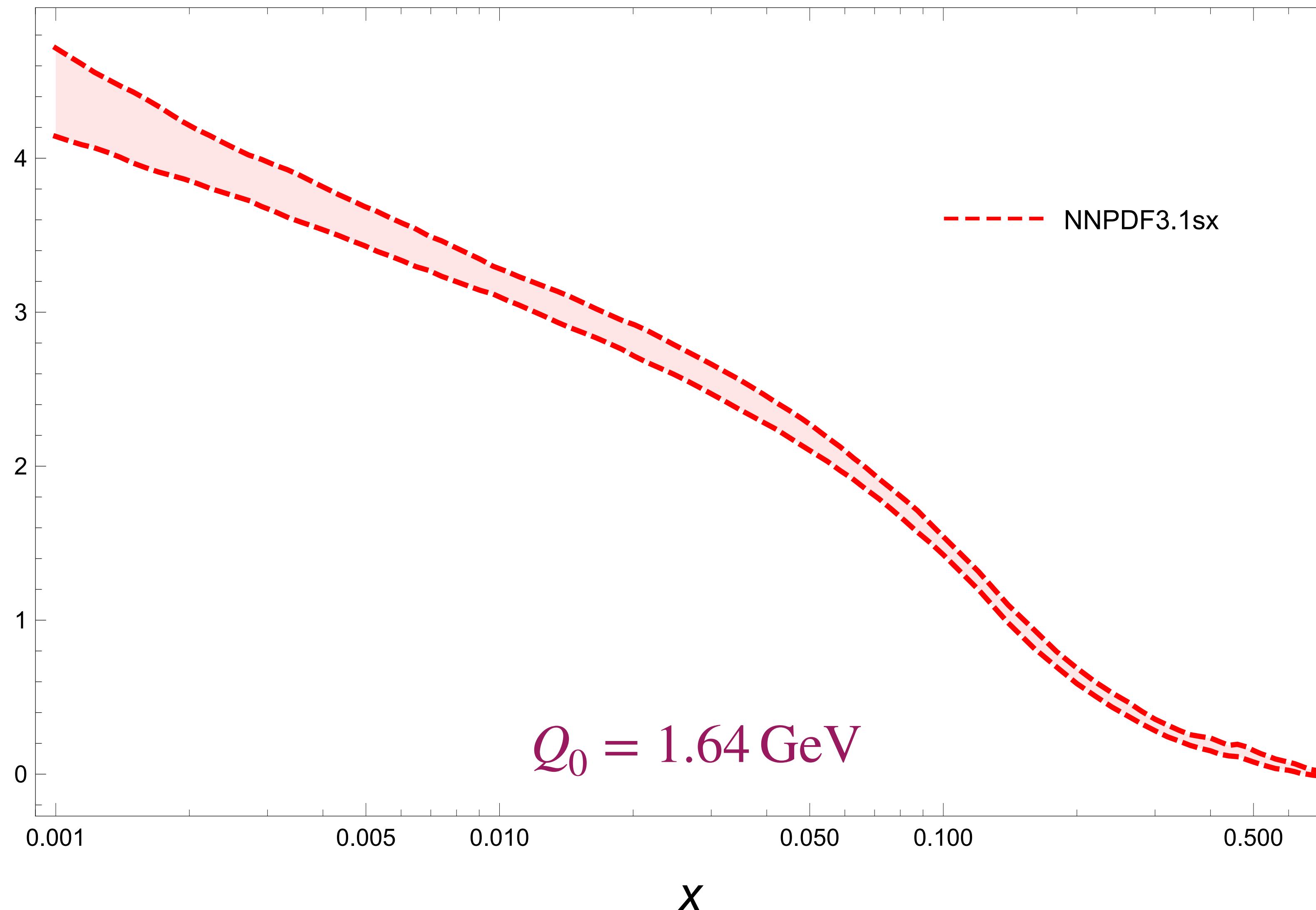


# Spectral function vs $D$

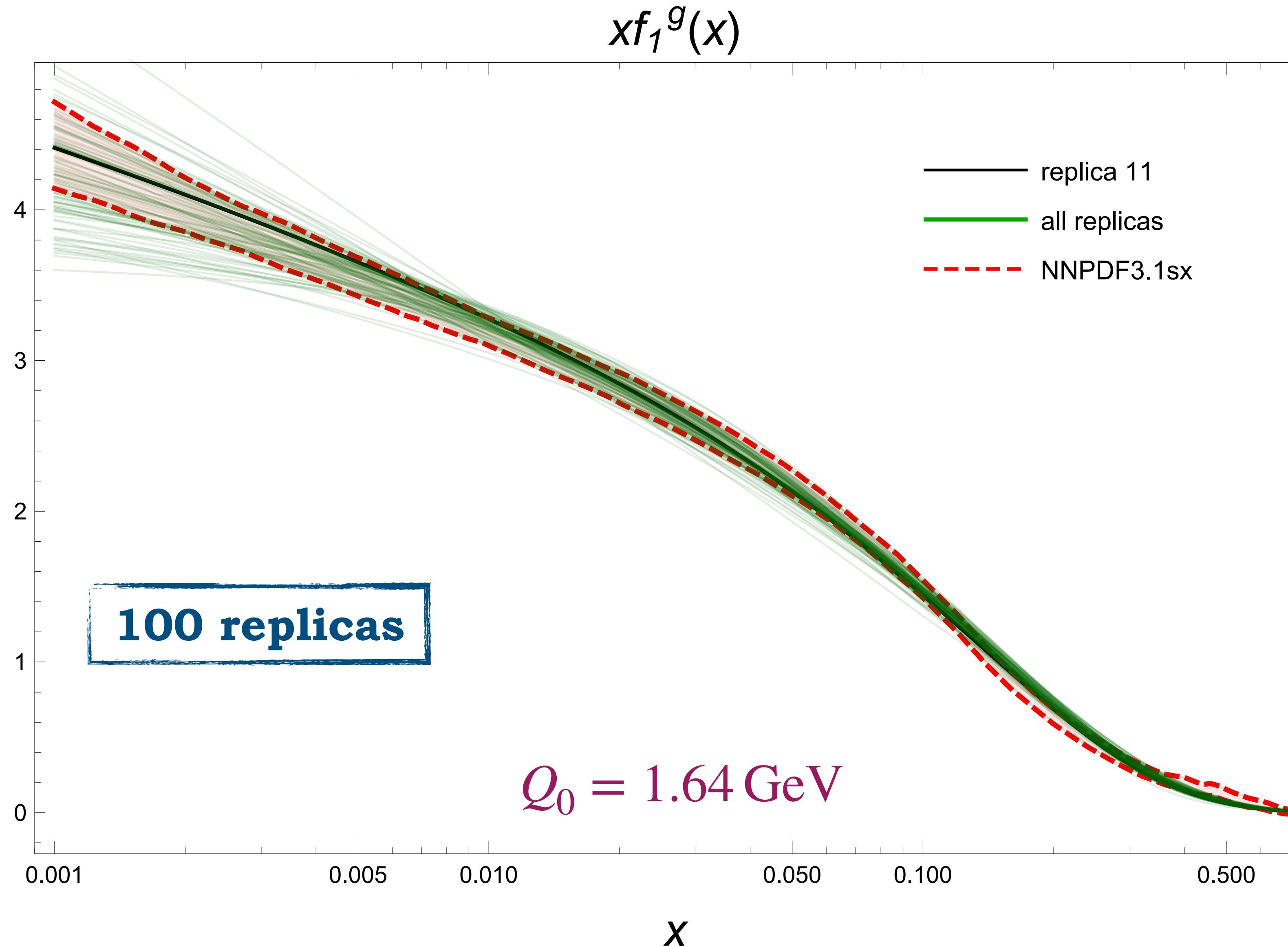


# Unpolarized gluon PDF

$xf_1^g(x)$

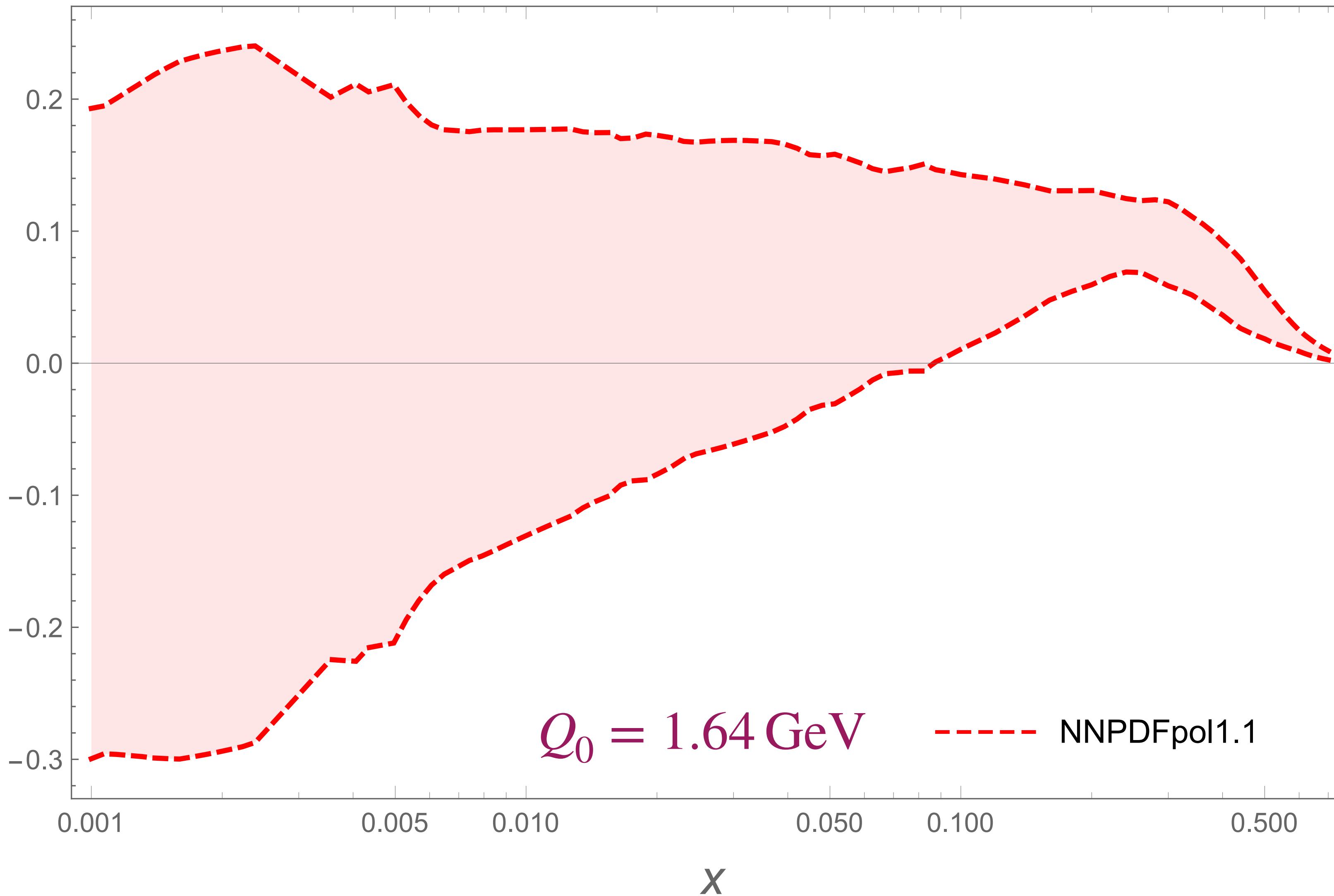


# Unpolarized gluon PDF



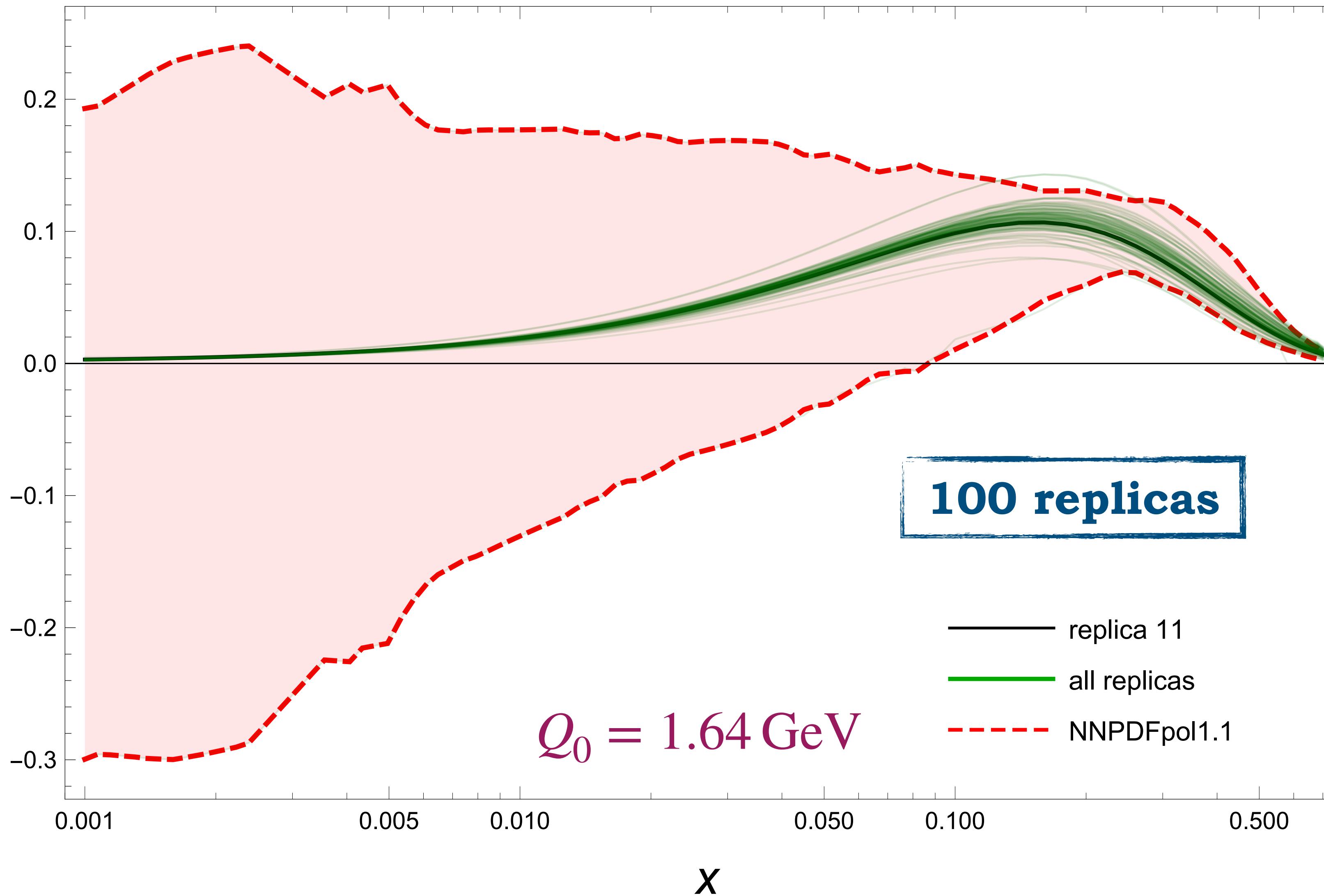
# Helicity gluon PDF

$xg_1^g(x)$

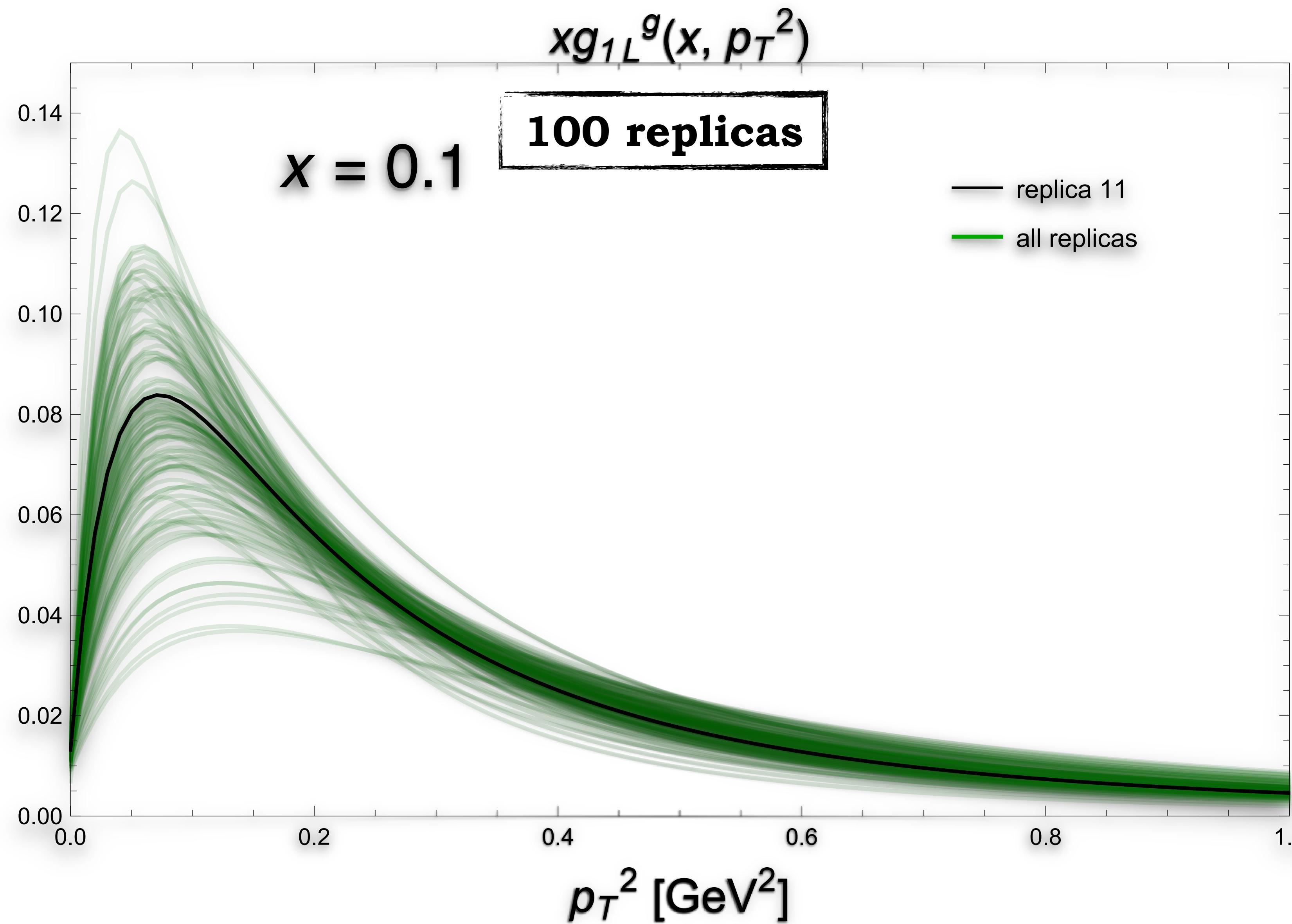


# Helicity gluon PDF

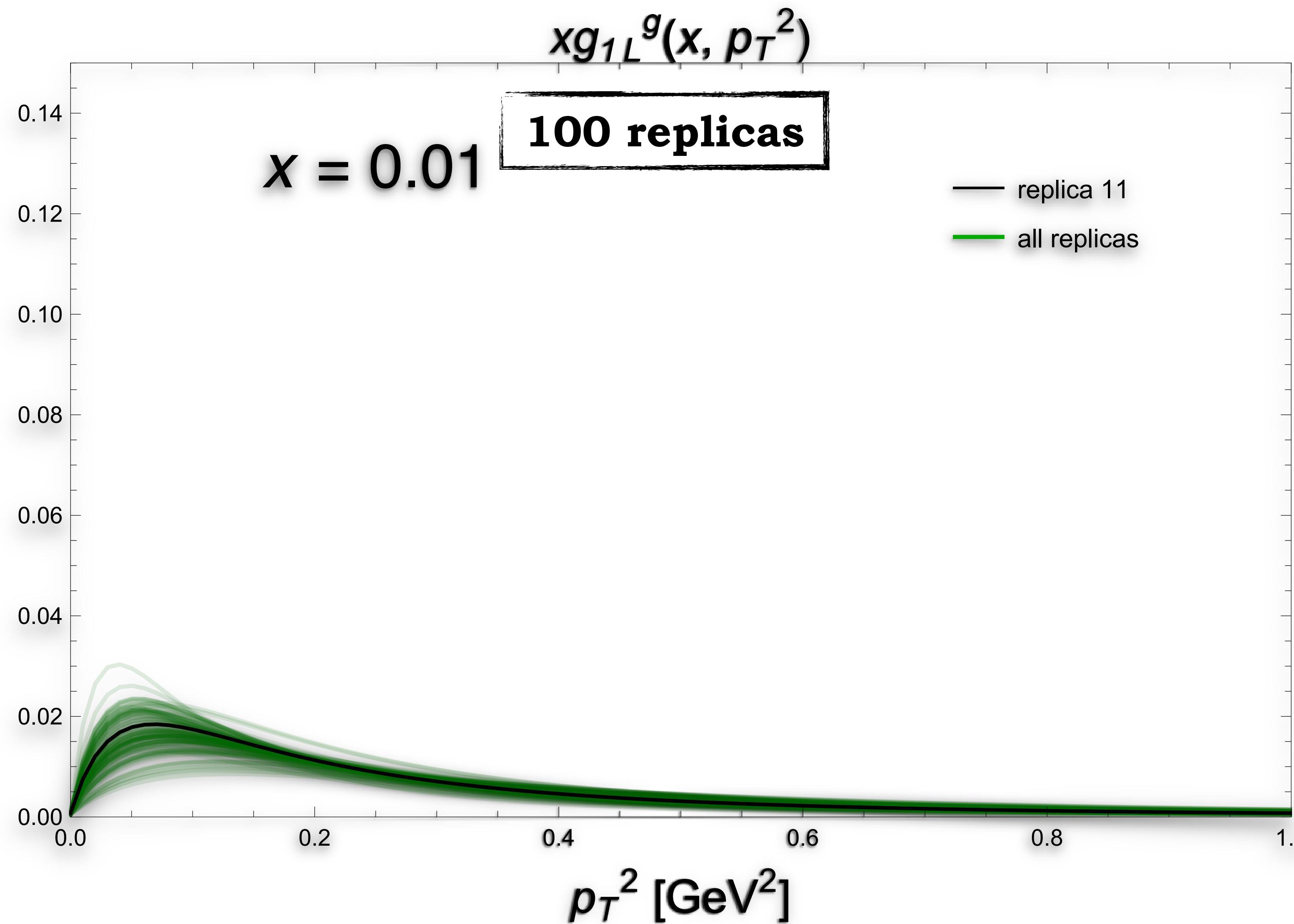
$xg_1^g(x)$



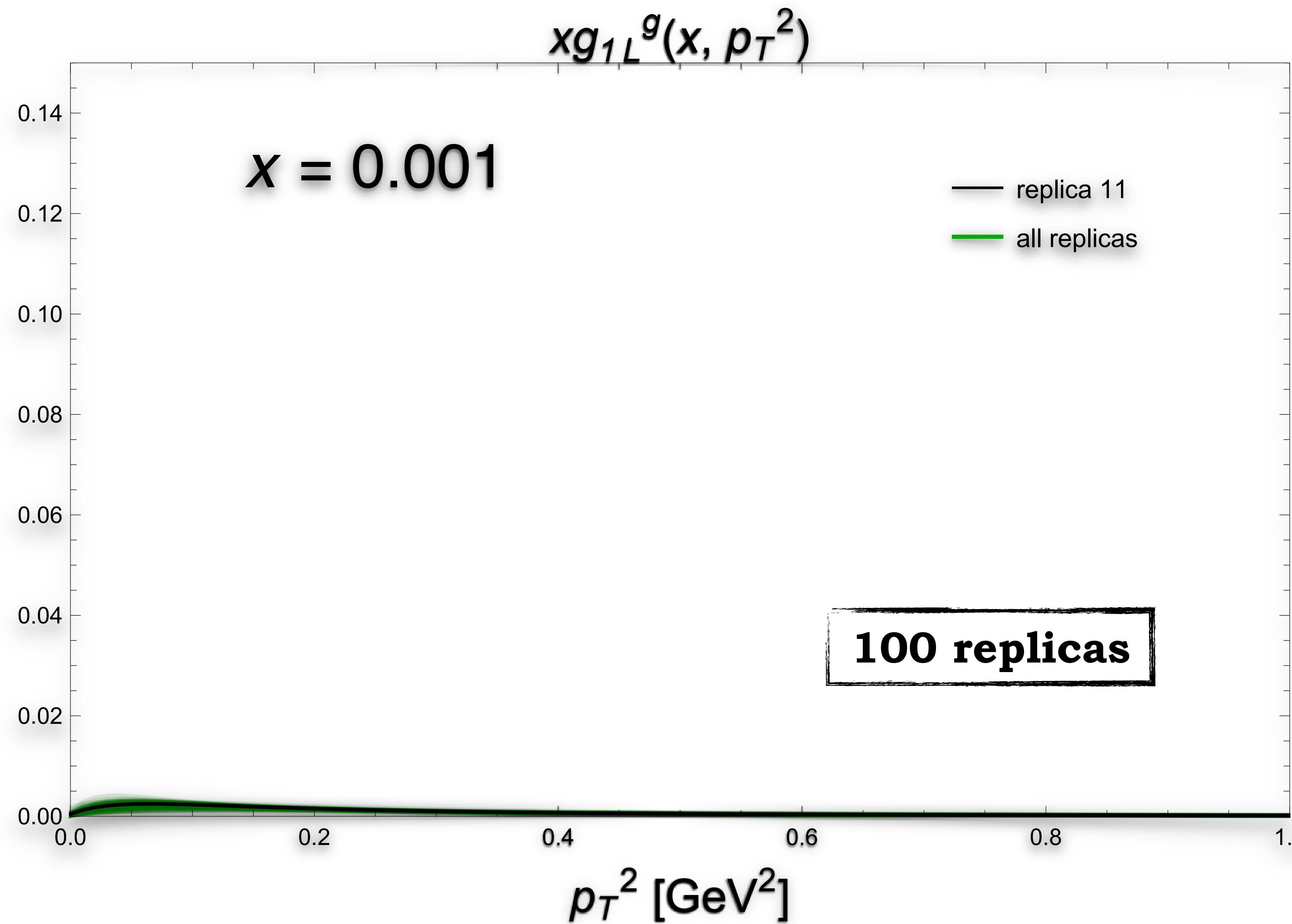
# Helicity gluon TMD



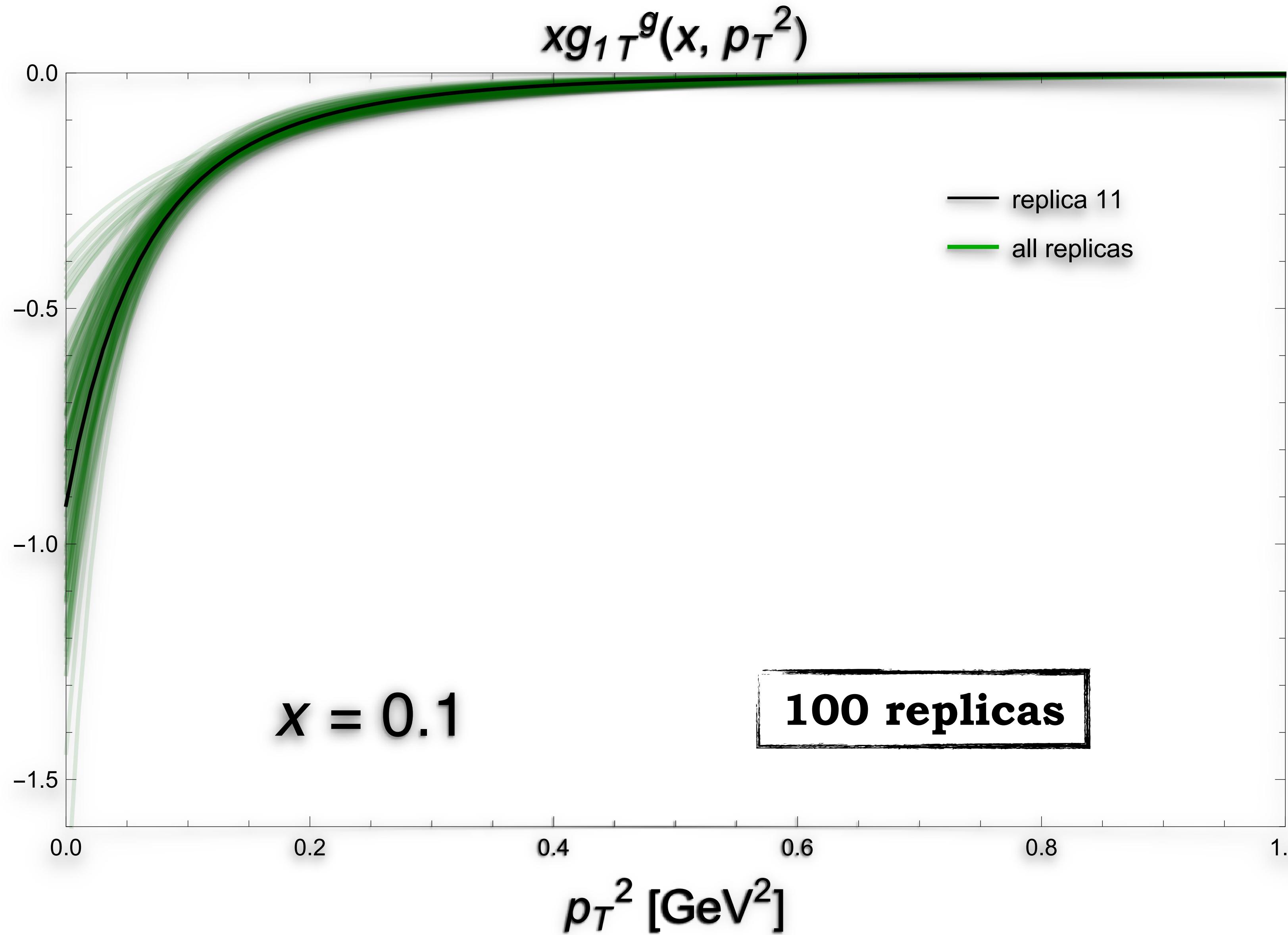
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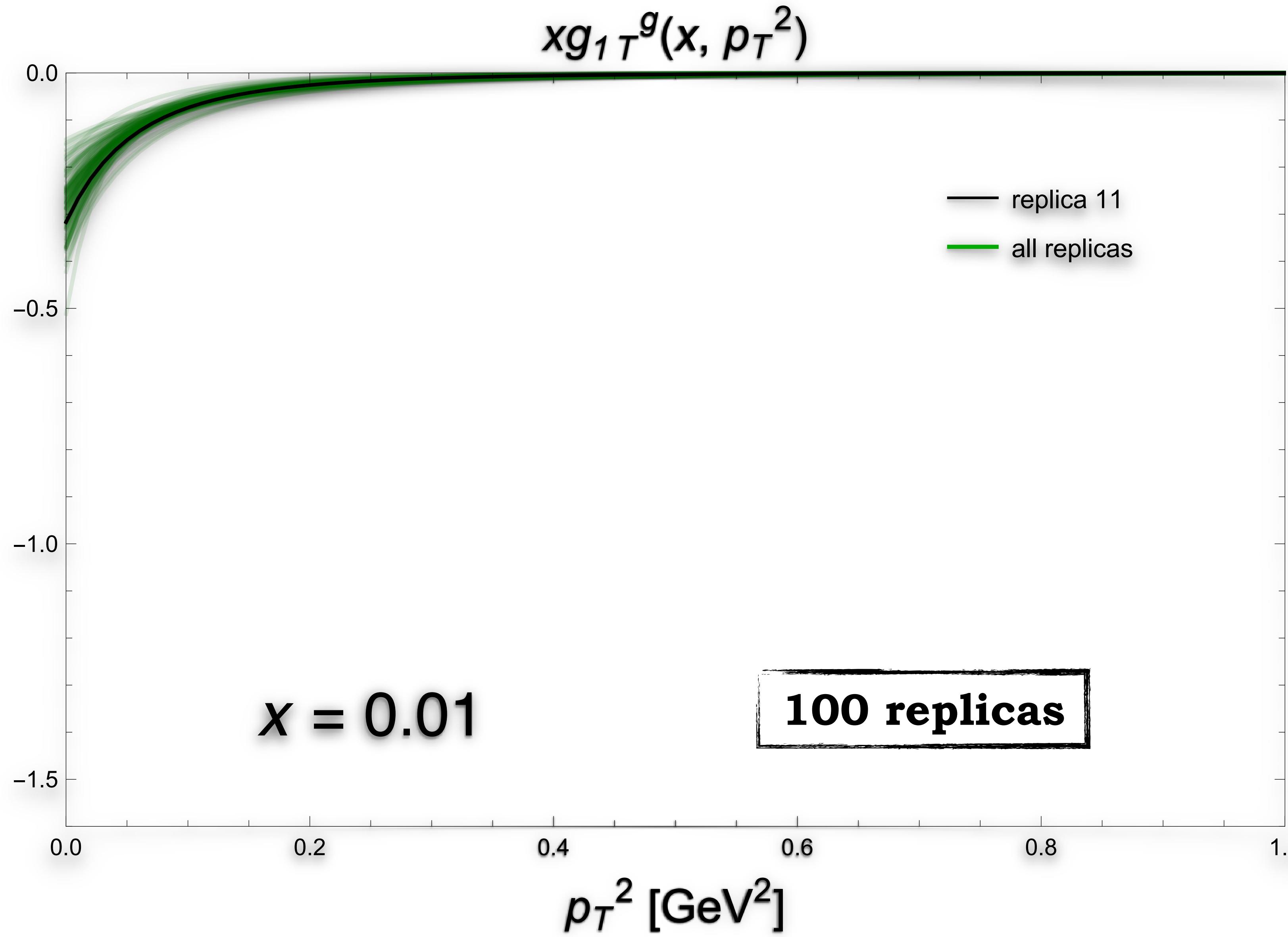
# Helicity gluon TMD



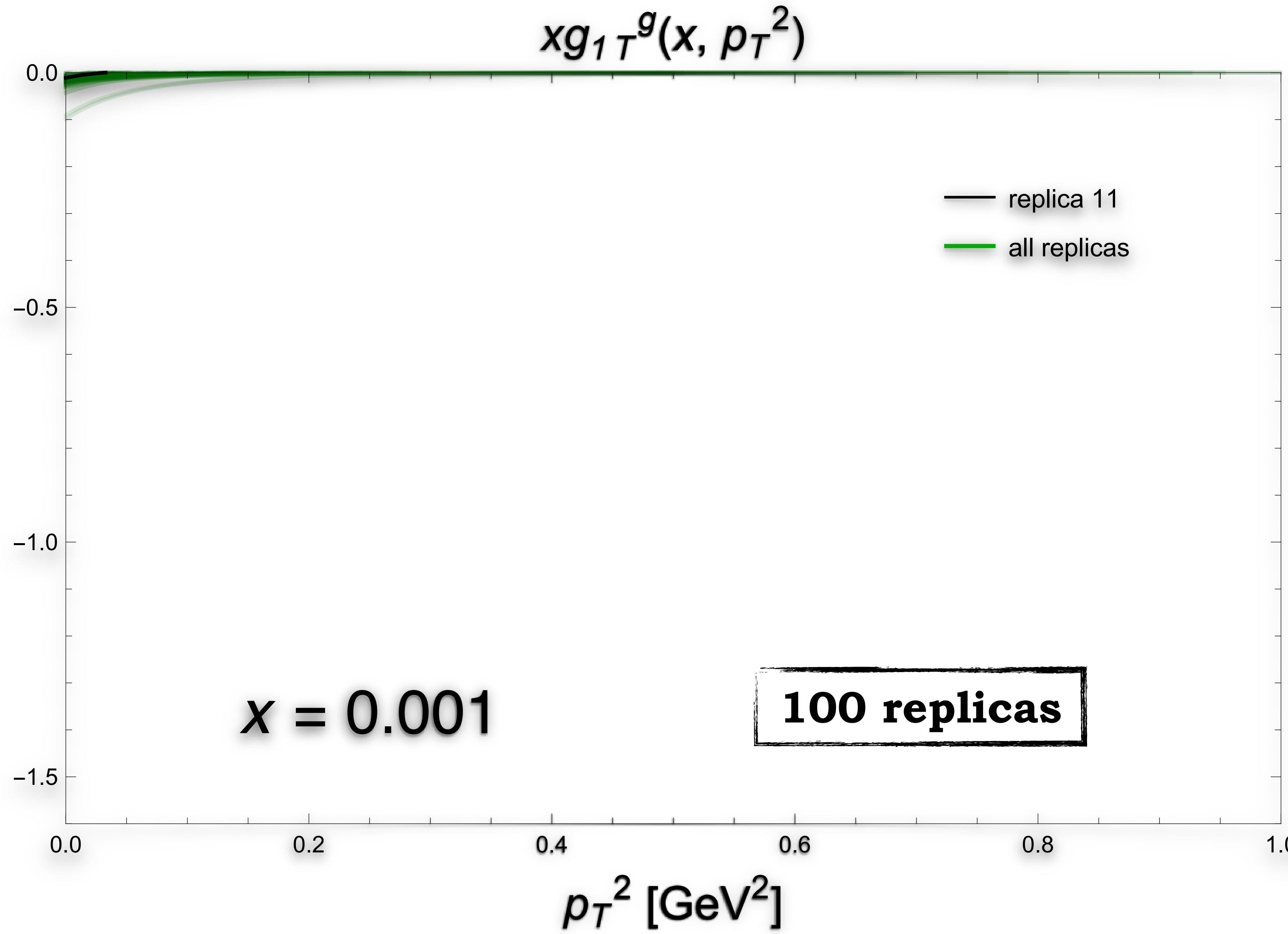
# Worm-gear gluon TMD



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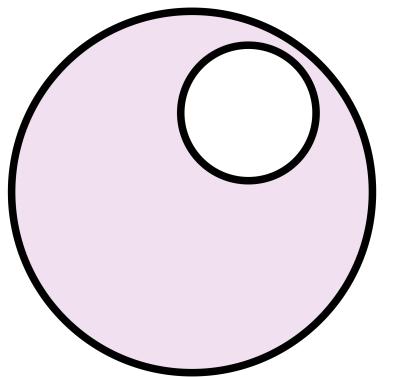
# Worm-gear gluon TMD



# Bottom line

- \* Each TMD shows a distinctive  $x$ - and  $p_T$ -behavior
- \* Data on gluon TMDs will exclude many replicas and constrain parameters not yet so well constrained by collinear PDFs
- \* Simultaneous fit on two distinct PDFs provides with *corroborating evidence* of reliability of our model
- \* Standard CSS  $\mathcal{O} \mathcal{O}$  evolution can be turned on

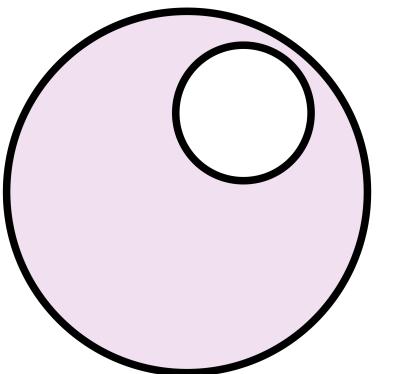
# $\rho$ -densities



**Unpolarized [u/u]**

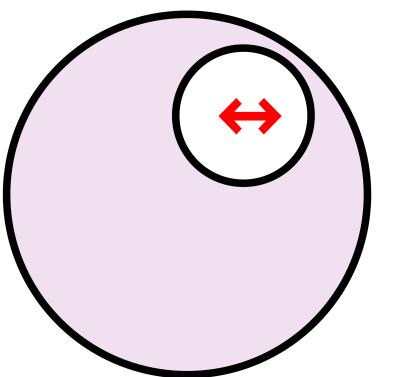
$$f_1(x, p_x, p_y)$$

# $\rho$ -densities



**Unpolarized [u/u]**

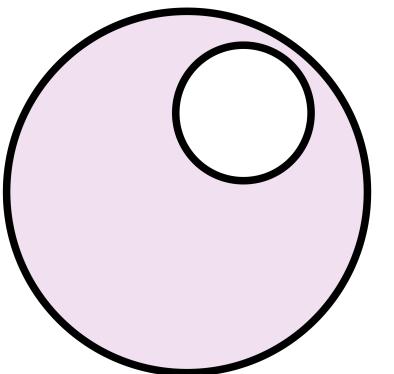
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**Boer-Mulders [↔/u]**

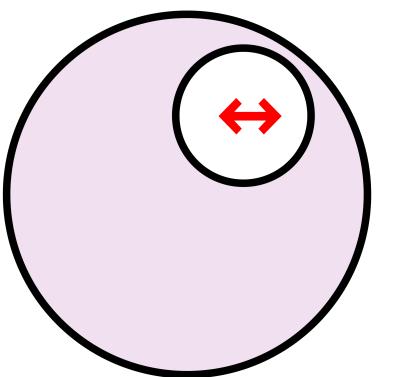
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

# $\rho$ -densities



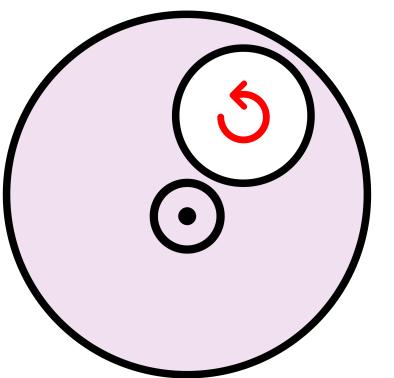
**Unpolarized** [u/u]

$$f_1(x, p_x, p_y)$$



**Boer-Mulders** [ $\leftrightarrow/u$ ]

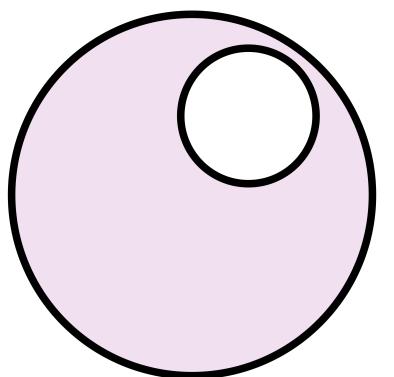
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



**Helicity** [ $\cup/+$ ]

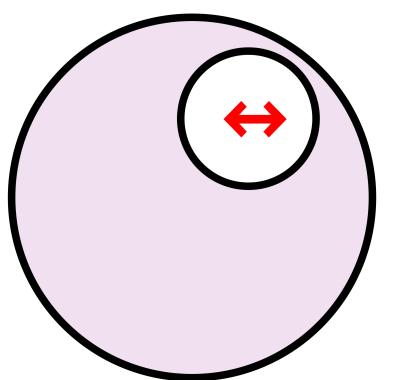
$$\frac{1}{2} \left[ f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$

# $\rho$ -densities



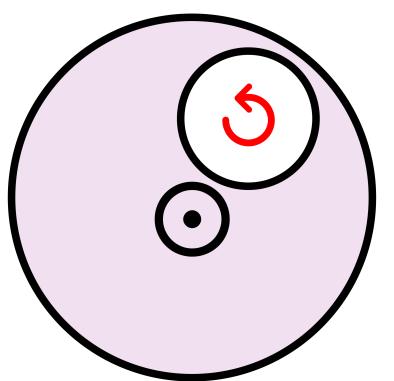
**Unpolarized** [u/u]

$$f_1(x, p_x, p_y)$$



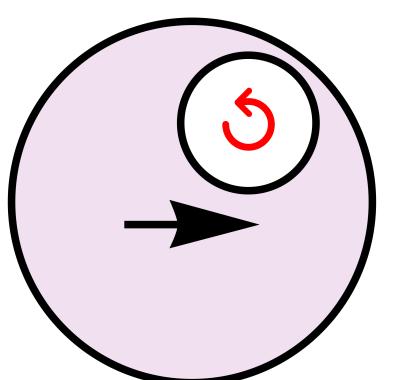
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**Helicity** [ $\cup/+$ ]

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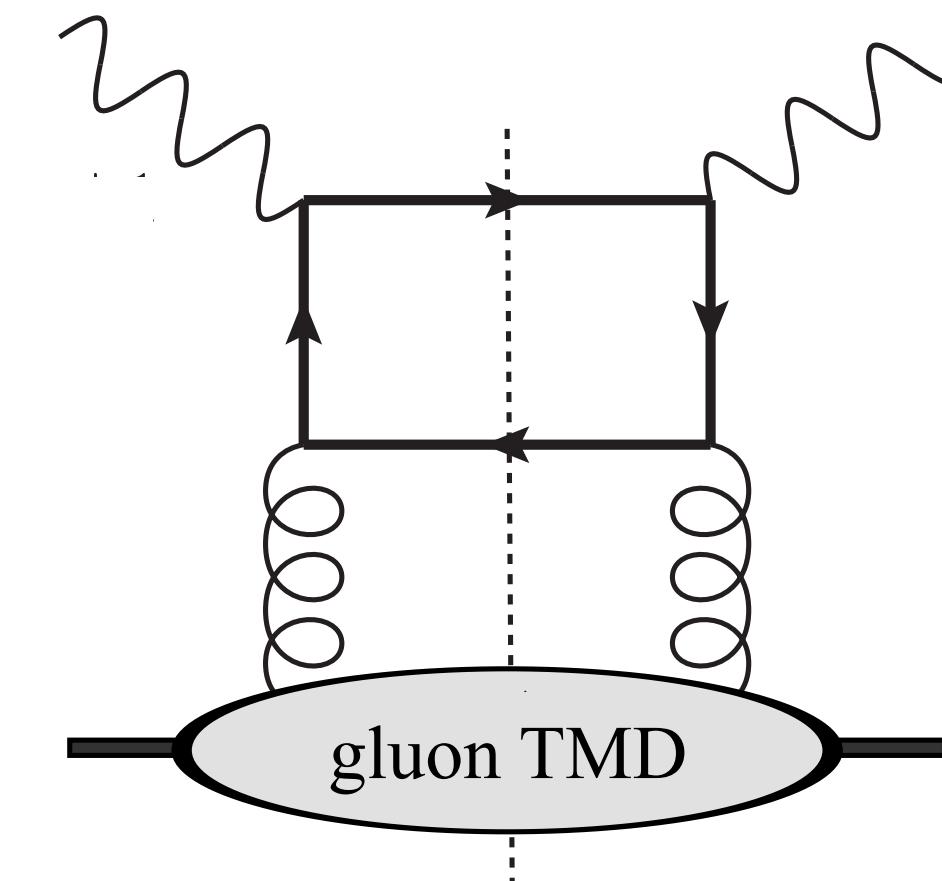
**Worm-gear** [ $\cup/\rightarrow$ ]

$$f_1(x, p_x, p_y) - \frac{p_x}{M} g_{1T}(x, p_x, p_y)$$

# T-odd gluon TMDs and semi-inclusive reactions

1. Almost back-to-back di-jet production
2. Open-charm (heavy-light meson) states
3. Almost back-to-back  $J/\Psi$ -plus-jet production
4. Inclusive  $J/\Psi$  production at low  $p_T$

$$ep \rightarrow e + \text{jet} + \text{jet} + X$$



# T-odd gluon TMDs and semi-inclusive reactions

1. Almost back-to-back di-jet production
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3. Almost back-to-back  $J/\Psi$ -plus-jet production
4. Inclusive  $J/\Psi$  production at low  $p_T$

- \* Gluon-induced processes
- \* **Spin-asymmetry** studies feasible
- \* Small- and **large-** $x$  physics supported

$$ep \rightarrow e + \text{jet} + \text{jet} + X$$

