$N^3LO$ extraction of the Sivers functions from SIDIS, DY and $W^{\pm}/Z$ data

Alexei Prokudin

M. Bury, AP, A. Vladimirov, PRL 126, 112002 (2021)
Our understanding of hadron evolves: TMDs with Polarization

Nucleon emerges as a strongly interacting, relativistic bound state of quarks and gluons

The Sivers function

### Quark Polarization

<table>
<thead>
<tr>
<th>Nucleon Polarization</th>
<th>Quark Polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unpolarized (U)</td>
<td>$f_1(x, k_T^2)$</td>
</tr>
<tr>
<td>Longitudinally Polarized (L)</td>
<td>$g_1(x, k_T^2)$</td>
</tr>
<tr>
<td>Transversely Polarized (T)</td>
<td>$h_T^\perp(x, k_T^2)$</td>
</tr>
</tbody>
</table>

- **Unpolarized quarks in trans. pol. nucleon**
- **Causes asymmetries in SIDIS and DY**
- **Changes sign in DY w.r.t. SIDIS**
Our understanding of hadron evolves: TMDs with Polarization

Nucleon emerges as a strongly interacting, relativistic bound state of quarks and gluons

Quark Polarization

Unpolarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T)
---|---|---
**U** | $f_1(x, k_T^2)$ | $h_1^\perp(x, k_T^2)$
Unpolarized |  | Boer-Mulders

**L** | $g_1(x, k_T^2)$ | $h_{1L}^\perp(x, k_T^2)$
Helicity |  | Kozinian-Mulders, “worm” gear

**T** | $f_{1T}^\perp(x, k_T^2)$ | $g_{1T}(x, k_T^2)$ | $h_1(x, k_T^2)$
Sivers | Kozinian-Mulders, “worm” gear | Transversity | Pretzelosity

Collins-Soper Equations

\[ \frac{\mu^2 dF(x, b; \mu, \zeta)}{d\mu^2} = \frac{\gamma F(\mu, \zeta)}{2} F(x, b; \mu, \zeta), \]

\[ \zeta \frac{dF(x, b; \mu, \zeta)}{d\zeta} = -\mathcal{D}(b, \mu) F(x, b; \mu, \zeta), \]

\( \mu \) = renormalization scale

\( \zeta \) = Collins-Soper parameter
TMD evolution contains non-perturbative component

- TMD evolution is a two-scale evolution
- Remarkably simple in the zeta-prescription

\[ F(x, b; \mu, \zeta) = \left( \frac{\zeta}{\zeta_\mu(b)} \right)^{-D(b, \mu)} F(x, b) \]

- \( F(x, b) \) is the “optimal” TMD
- \( \zeta_\mu(b) \) is a calculable function
- \( D(b, \mu) \) is the Collins-Soper kernel or rapidity anomalous dimension. Fundamental universal function related to the properties of QCD vacuum
THE SIVERS ASYMMETRY

\[ f_{1T,q\rightarrow h}^+(x,b) = N_q \frac{(1-x)x^\beta_q(1+\epsilon_q x)}{n(\beta_q,\epsilon_q)} \exp \left( -\frac{r_0 + x r_1}{\sqrt{1 + r_2 x^2 b^2}} b^2 \right) \]

\[ \gamma^* + p \rightarrow \pi^+ + X \]

\[ x = 0.12 \]

\[ z = 0.32 \]

\[ P_{hT} = 0.14 \text{ GeV} \]

- N^3LO
- NNLO
- NLO
- LO
DATA SELECTION

Only $P_T$ dependence used to avoid double counting

Data selection compatible with TMD factorization requirement

$$\delta = \frac{|P_{hT}|}{zQ} \text{ (in SIDIS)}, \quad \delta = \frac{|q_T|}{Q} \text{ (in DY)}.$$ 

$$\langle Q \rangle > 2 \text{ GeV} \quad \text{and} \quad \delta < 0.3$$

\[\begin{array}{|c|c|c|c|}
\hline
\text{Dataset name} & \text{Ref.} & \text{Reaction} & \text{\# Points} \\
\hline
\text{Compass08} & [36] & d^\uparrow + \gamma^* \to \pi^+ & 1/9 \\
 & & d^\uparrow + \gamma^* \to \pi^- & 1/9 \\
 & & d^\uparrow + \gamma^* \to K^+ & 1/9 \\
 & & d^\uparrow + \gamma^* \to K^- & 1/9 \\
\hline
\text{Compass16} & [39] & p^\uparrow + \gamma^* \to h^+ & 5/40 \\
 & & p^\uparrow + \gamma^* \to h^- & 5/40 \\
\hline
\text{Hermes} & [35] & p^\uparrow + \gamma^* \to \pi^+ & 11/64 \\
 & & p^\uparrow + \gamma^* \to \pi^- & 11/64 \\
 & & p^\uparrow + \gamma^* \to K^+ & 12/64 \\
 & & p^\uparrow + \gamma^* \to K^- & 12/64 \\
\hline
\text{JLab} & [41, 42] & ^3He^\uparrow + \gamma^* \to \pi^+ & 1/4 \\
 & & ^3He^\uparrow + \gamma^* \to \pi^- & 1/4 \\
 & & ^3He^\uparrow + \gamma^* \to K^+ & 1/4 \\
 & & ^3He^\uparrow + \gamma^* \to K^- & 0/4 \\
\hline
\text{SIDIS total} & & & 63 \\
\hline
\text{CompassDY} & [40] & \pi^- + d^\uparrow \to \gamma^* & 2/3 \\
\hline
\text{Star.W+} & [43] & p^\uparrow + p \to W^+ & 5/5 \\
\text{Star.W-} & & p^\uparrow + p \to W^- & 5/5 \\
\text{Star.Z} & & p^\uparrow + p \to \gamma^*/Z & 1/1 \\
\hline
\text{DY total} & & & 13 \\
\hline
\text{Total} & & & 76 \\
\hline
\end{array}\]
FIT RESULTS

- Replica method using Artemide framework
- Errors both from the data and the uncertainty due to unpolarized TMD

<table>
<thead>
<tr>
<th>Name</th>
<th>$\chi^2/N_{pt}[\text{SIDIS}]$</th>
<th>$\chi^2/N_{pt}[\text{DY}]$</th>
<th>$\chi^2/N_{pt}[\text{total}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIDIS at N$^3$LO</td>
<td>$0.87^{+0.13}_{-0.03}$</td>
<td>$1.23^{+0.50}_{-0.24} \text{ no fit}$</td>
<td>$0.93^{+0.16}_{-0.01}$</td>
</tr>
<tr>
<td>SIDIS+DY at N$^3$LO</td>
<td>$0.88^{+0.15}_{-0.04}$</td>
<td>$0.90^{+0.31}_{-0.00}$</td>
<td>$0.88^{+0.15}_{-0.05}$</td>
</tr>
</tbody>
</table>

- Unbiased parametrization
- No tension between SIDIS and DY data — universality
- Good convergence of the fit for all data sets
N3LO EXTRACTION OF THE SIVERS FUNCTION

HERMES 20 3D binning description

Filled in points used in the fit
N3LO EXTRACTION OF THE SIVERS FUNCTION

COMPASS SIDIS data

\[ A_{UT} \sin(\phi - \phi_s) \]

\[ h^+ \]

\[ h^- \]

\[ P_{hT} \text{ (GeV)} \]

\[ x \]

\[ z \]
**N3LO EXTRACTION OF THE SIVERS FUNCTION**

Pion induced Drell-Yan, COMPASS

W/Z production, STAR

Bury, Prokudin, Vladimirov (2021)
Sivers function in the momentum space

Bury, Prokudin, Vladimirov (2021)

Figure 14. Sivers function in the momentum space (black solid line) for $u$, $d$, sea, and $s$ quarks at $x=0$ and $\mu=2$ GeV. The blue band is the 68% CI. The gray dashed line is the unpolarized TMD PDF extracted in SV19 shown for the comparison (for $u$ and sea-quark the Sivers function is multiplied by $f_{1u,p}$). The extracted Sivers function is shown in Fig. 14. The Fourier transformation, Eq. (4.4), effectively inverses the ranges of variables. Therefore, a large uncertainty at large-$b$ (given by parameters $r_0$, $r_1$, $r_2$) transforms to a large uncertainty at small-$k_T$. For comparison, we also show the values and uncertainties of the unpolarized TMD PDFs extracted in SV19 fit. We observe that the Sivers function's typical size is about 4-5 times as small as the corresponding unpolarized distribution. Figure 14 shows the functions at $x=0$, for the value of $x$ of the data used in our fit $x \approx 0.01-0.25$ profiles are similar.

Figure 15. Sivers function in the momentum space for $u$ quark at $x=0$ as a function of $k_T$ (GeV). The bands are the 68% CI. The calculations are performed at four different values of $Q$. 

\[ -f_{1T,u,p} \]
SIVERS FUNCTION IN THE MOMENTUM SPACE

Comparison to Jam20 (LO) analysis

Jam20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)
The extracted Sivers function is shown in Fig. 14. The Fourier transformation, Eq. (4.4), effectively inverses the ranges of variables. Therefore, a large uncertainty at large-$b$ (given by parameters $r_0$, $r_1$, $r_2$) transforms to a small uncertainty at small-$k_T$. For comparison, we also show the values and uncertainties of the unpolarized TMD PDFs extracted in SV19 fit. We observe that the Sivers function’s typical size is about 4-5 times as small as the corresponding unpolarized distribution. Figure 14 shows the functions at $x = 0.1$, for the value of $x$ of the data used in our fit $x \llgeq 0.01$. Profiles are similar.
\[
\rho_{1; q \leftarrow h^\uparrow}(x, k_T, S_T, \mu) = f_{1; q \leftarrow h}(x, k_T; \mu, \mu^2) - \frac{kTx}{M} f_{1T; q \leftarrow h}(x, k_T; \mu, \mu^2)
\]
\[
\rho_{1; q \leftarrow h}^{\uparrow}(x, k_T, S_T, \mu) = f_{1; q \leftarrow h}(x, k_T; \mu, \mu^2) - \frac{k_T x}{M} f_{1T; q \leftarrow h}(x, k_T; \mu, \mu^2)
\]
The Qiu–Sterman Matrix Element

Invert the formula for Operator Product Expansion of Sivers via the QS functions

\[ T_q(-x, 0, x; \mu_b) = -\frac{1}{\pi} \left( 1 + C_F \frac{\alpha_s(\mu_b) \pi^2}{4\pi} \right) f_{1T;q\rightarrow h}^{\perp}(x, b) \]

\[ -\frac{\alpha_s(\mu_b)}{4\pi^2} \int_{x}^{1} \frac{dy}{y} \left[ \frac{\bar{y}}{N_c} f_{1T;q\rightarrow h}^{\perp}(x/y, b) + \frac{3y^2 \bar{y}}{2x} G^{(+)} \left( \frac{-x}{y}, 0, \frac{x}{y}; \mu_b \right) \right] + O(\alpha_s^2, b^2) \]

\[ \mu_b = \frac{2e^{-\gamma_E}}{b} \]

We choose \( b = 0.11 \text{ (GeV}^{-1}) \), \( \mu_b = 10 \text{ (GeV)} \)

and estimate gluon contribution \( G^{(+)} = \pm(|T_u| + |T_d|) \)
We have extracted Sivers function from the first global fit of SIDIS, pion-induced Drell-Yan and $W^\pm/Z$-bozon production experimental data at N3LO precision. Conservative data cuts are used to ensure validity of factorization and unbiased parametrization. Good agreement between SIDIS and DY data in an analysis with TMD evolution is achieved for the first time. The Qiu-Sterman functions are extracted in a model independent way. Our results set a new benchmark and the standard of precision for studies of TMD polarized functions.
SIGN CHANGE

Large contribution from antiquark Sievers functions to DY makes it possible to describe data without the sign change

\[ f_{1T}^{+ \text{sea}} \rightarrow -f_{1T}^{+ \text{sea}} \]