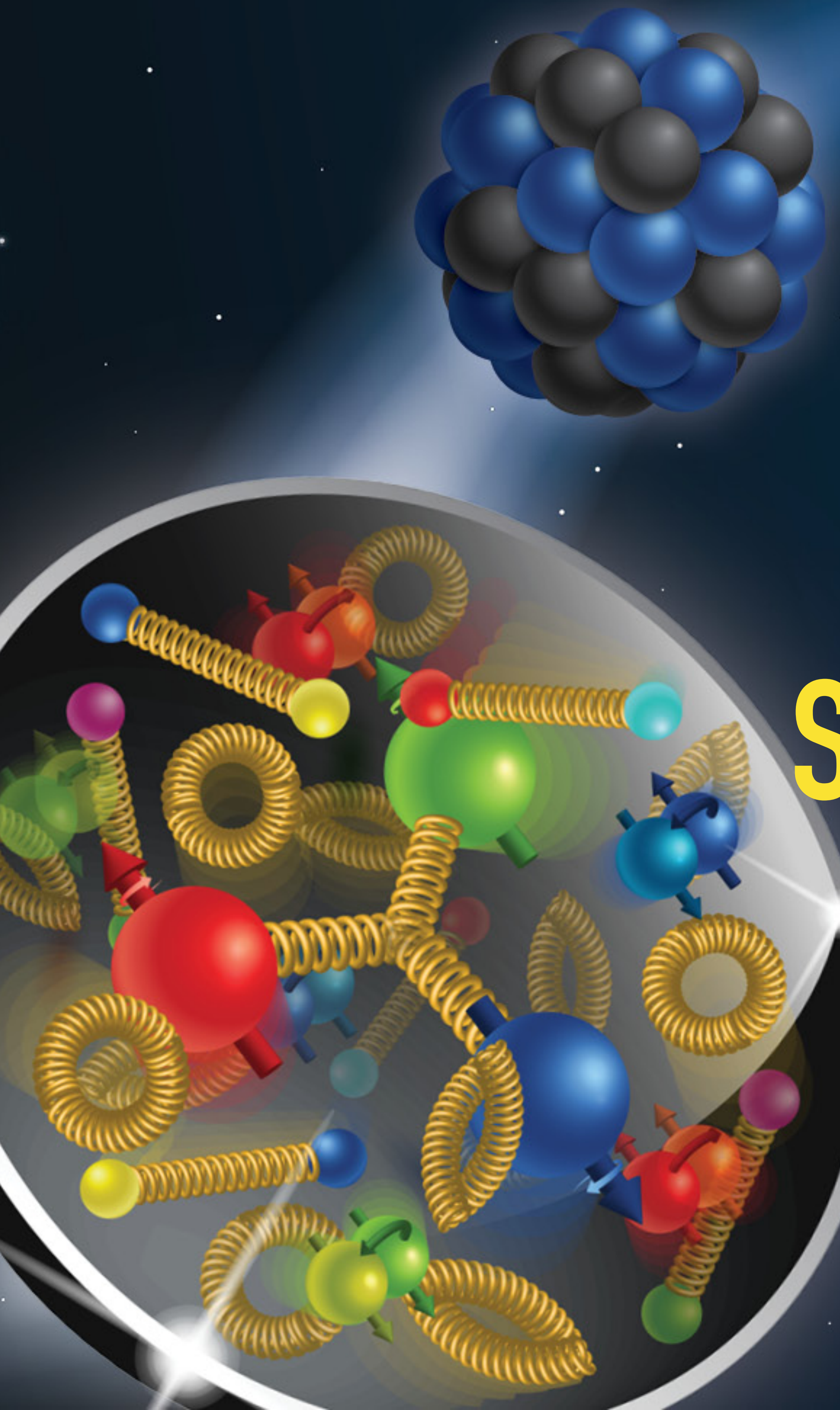
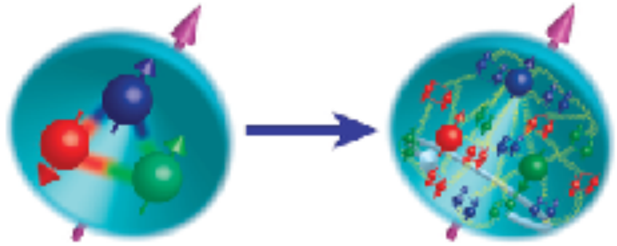


N^{3}LO extraction of the Sivers functions from SIDIS, DY and W$^{\pm}$/Z data

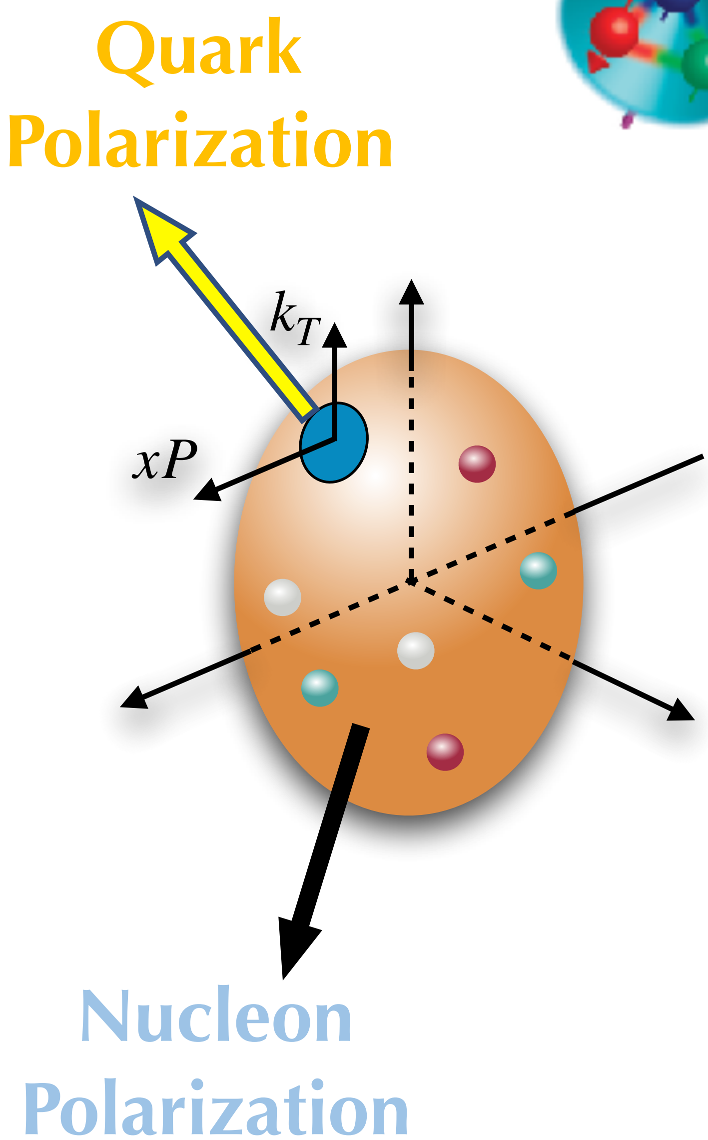
Alexei Prokudin

M. Bury, AP, A. Vladimirov,
PRL 126, 112002 (2021)

Our understanding of hadron evolves: TMDs with Polarization



Nucleon emerges as a strongly interacting, relativistic bound state of quarks and gluons

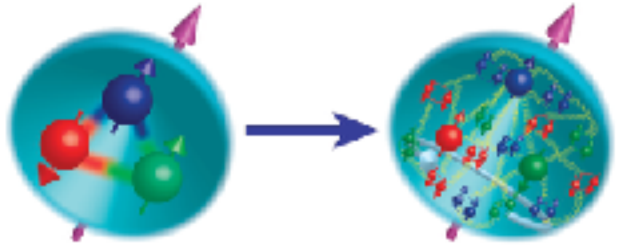


		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$ *Unpolarized*		$h_1^\perp(x,k_T^2)$ - *Boer-Mulders*
	L		$g_1(x,k_T^2)$ - *Helicity*	$h_{1L}^\perp(x,k_T^2)$ - *Kozinian-Mulders,"worm" gear*
	T	$\boxed{f_{1T}^\perp(x,k_T^2) \text{ - Sivers}}$	$g_{1T}(x,k_T^2)$ - *Kozinian-Mulders, "worm" gear*	$h_1(x,k_T^2)$ - *Transversity* $h_{1T}^\perp(x,k_T^2)$ - *Pretzelosity*

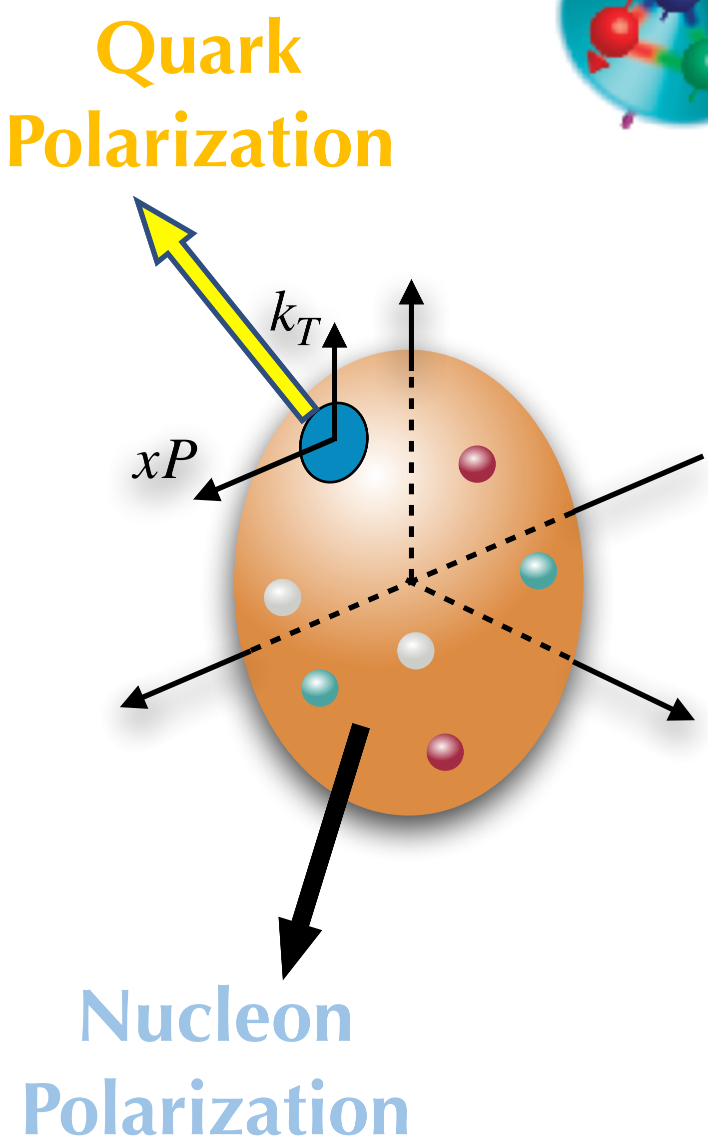
The Sivers function

- Unpolarized quarks in trans. pol. nucleon
- Causes asymmetries in SIDIS and DY
- Changes sign in DY w.r.t. SIDIS

Our understanding of hadron evolves: TMDs with Polarization



Nucleon emerges as a strongly interacting, relativistic bound state of quarks and gluons



		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$ *Unpolarized*		$h_1^\perp(x,k_T^2)$ *Boer-Mulders*
	L		$g_1(x,k_T^2)$ *Helicity*	$h_{1L}^\perp(x,k_T^2)$ *Kozinian-Mulders, "worm" gear*
	T	$f_{1T}^\perp(x,k_T^2)$ *Sivers*	$g_{1T}(x,k_T^2)$ *Kozinian-Mulders, "worm" gear*	$h_1(x,k_T^2)$ *Transversity* $h_{1T}^\perp(x,k_T^2)$ *Pretzelosity*

$$\mu^2 \frac{dF(x,b;\mu,\zeta)}{d\mu^2} = \frac{\gamma_F(\mu,\zeta)}{2} F(x,b;\mu,\zeta),$$

$$\zeta \frac{dF(x,b;\mu,\zeta)}{d\zeta} = -\mathcal{D}(b,\mu) F(x,b;\mu,\zeta),$$

Collins-Soper Equations

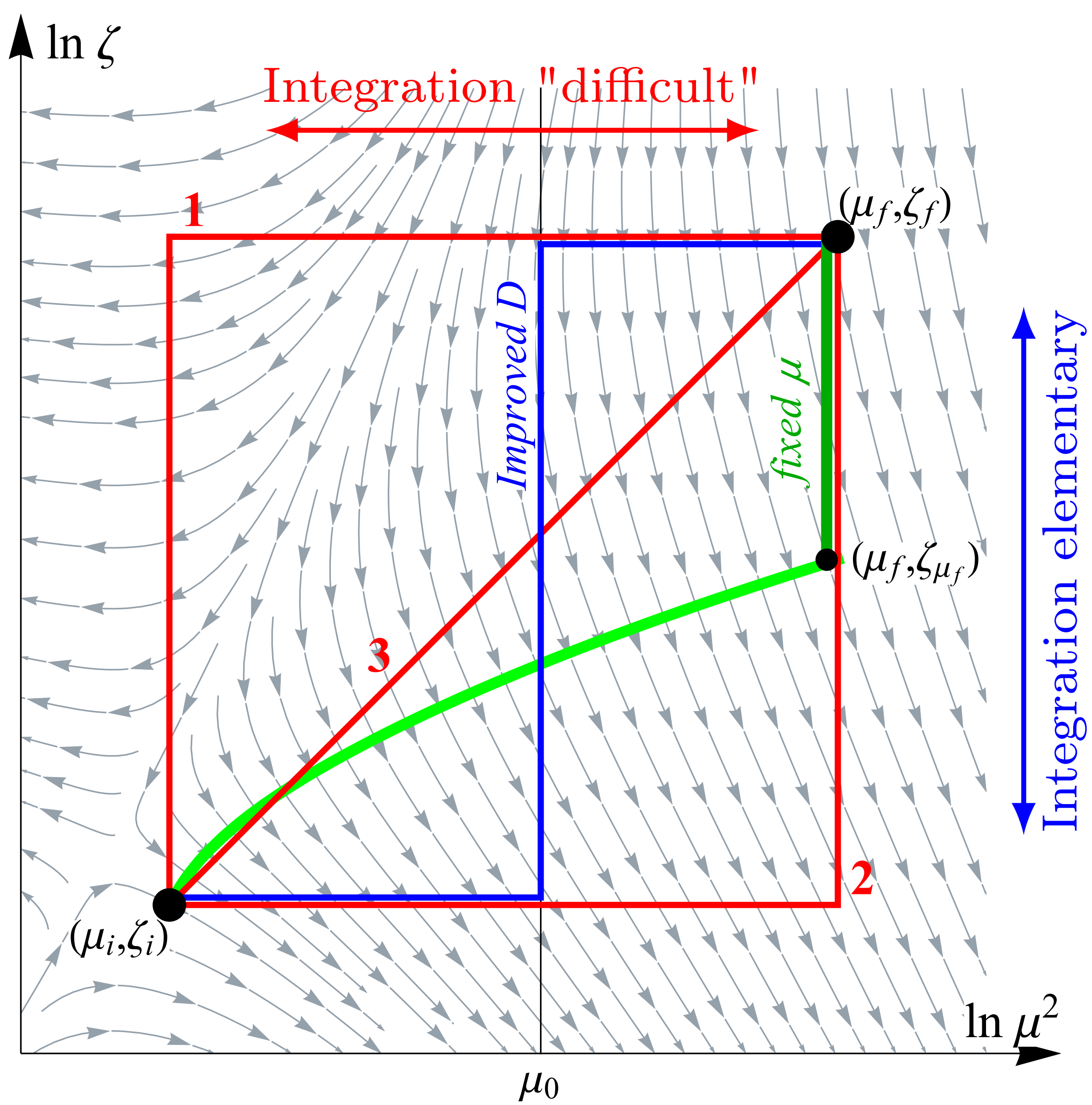
μ = renormalization scale

ζ = Collins-Soper parameter

TMD EVOLUTION CONTAINS NON-PERTURBATIVE COMPONENT

- TMD evolution is a two scale evolution
- Remarkably simple in the zeta-prescription

Scimemi, Vladimirov (18), (20)
Vladimirov (20)



$$F(x,b;\mu,\zeta) = \left(\frac{\zeta}{\zeta_\mu(b)}\right)^{-\mathcal{D}(b,\mu)} F(x,b)$$

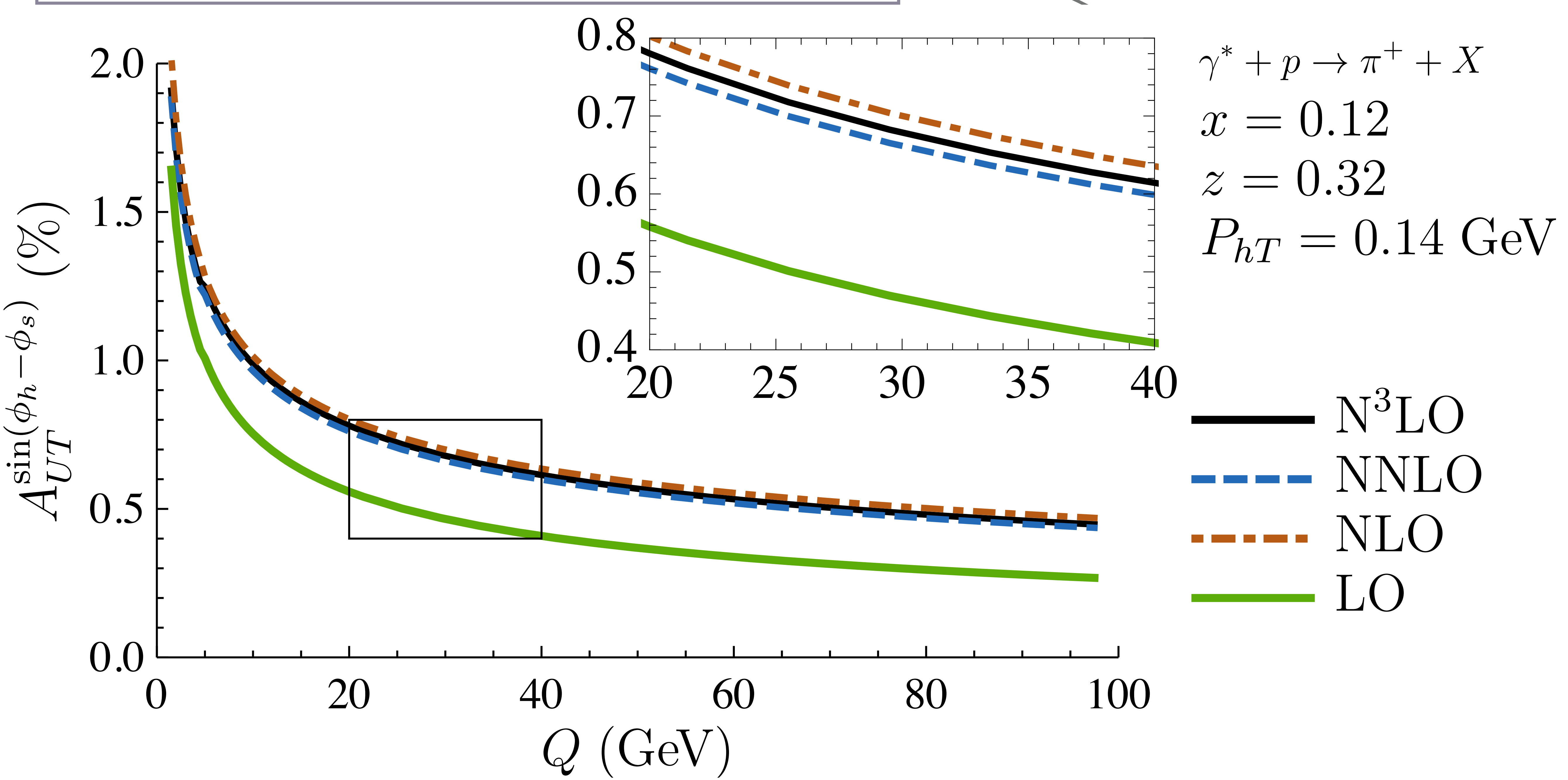
$F(x,b)$ is the “optimal” TMD

$\zeta_\mu(b)$ calculable function

$\mathcal{D}(b,\mu)$ Colins-Soper kernel or rapidity anomalous dimension. Fundamental universal function related to the properties of QCD vacuum

THE SIVERS ASYMMETRY

$$f_{1T;q\leftarrow h}^{\perp}(x,b) = N_q \frac{(1-x)x^{\beta_q}(1+\epsilon_q x)}{n(\beta_q,\epsilon_q)} \exp\left(-\frac{r_0 + x r_1}{\sqrt{1+r_2 x^2 b^2}} b^2\right)$$



DATA SELECTION

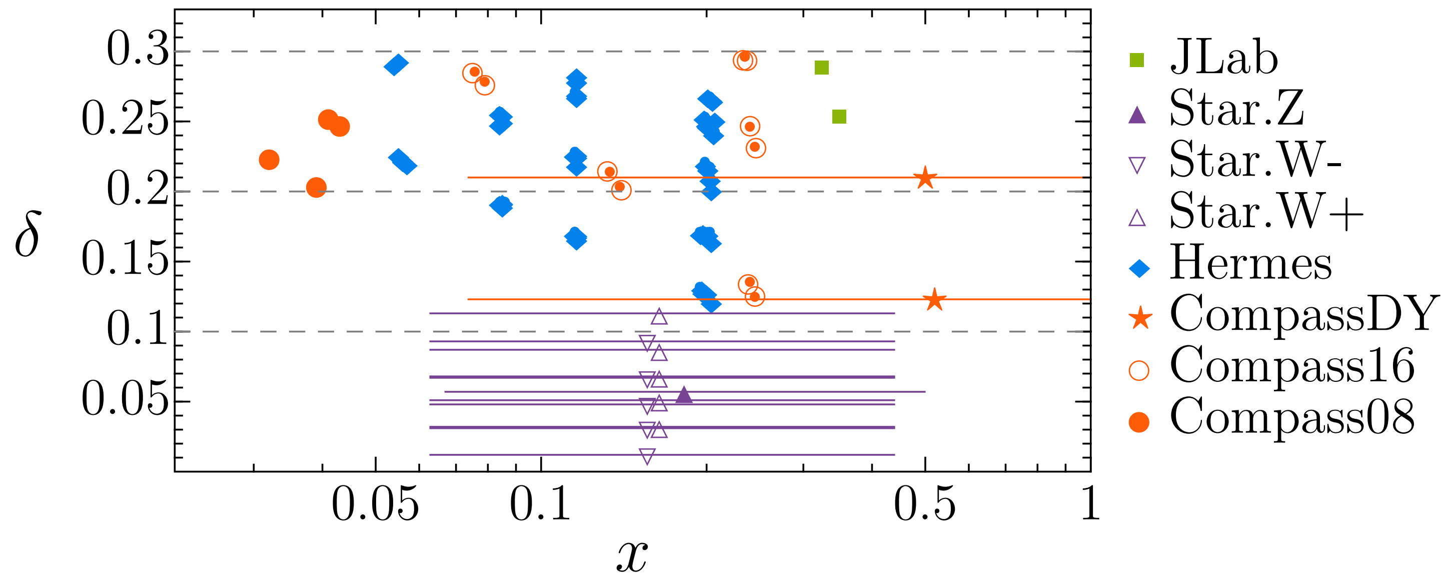
Bury, Prokudin, Vladimirov (2021)

Dataset name	Ref.	Reaction	# Points
Compass08	[36]	$d^\uparrow + \gamma^* \to \pi^+$	1 / 9
		$d^\uparrow + \gamma^* \to \pi^-$	1 / 9
		$d^\uparrow + \gamma^* \to K^+$	1 / 9
		$d^\uparrow + \gamma^* \to K^-$	1 / 9
Compass16	[39]	$p^\uparrow + \gamma^* \to h^+$	5 / 40
		$p^\uparrow + \gamma^* \to h^-$	5 / 40
Hermes	[35]	$p^\uparrow + \gamma^* \to \pi^+$	11 / 64
		$p^\uparrow + \gamma^* \to \pi^-$	11 / 64
		$p^\uparrow + \gamma^* \to K^+$	12 / 64
		$p^\uparrow + \gamma^* \to K^-$	12 / 64
JLab	[41, 42]	${}^3He^\uparrow + \gamma^* \to \pi^+$	1 / 4
		${}^3He^\uparrow + \gamma^* \to \pi^-$	1 / 4
		${}^3He^\uparrow + \gamma^* \to K^+$	1 / 4
		${}^3He^\uparrow + \gamma^* \to K^-$	0 / 4
SIDIS total			63
CompassDY	[40]	$\pi^- + d^\uparrow \to \gamma^*$	2 / 3
Star.W+	[43]	$p^\uparrow + p \to W^+$	5 / 5
Star.W-		$p^\uparrow + p \to W^-$	5 / 5
Star.Z		$p^\uparrow + p \to \gamma^*/Z$	1 / 1
DY total			13
Total			76

- Only P_T dependence used to avoid double counting
- Data selection compatible with TMD factorization requirement

$$\delta = \frac{|P_{hT}|}{zQ}(\text{in SIDIS}), \qquad \delta = \frac{|q_T|}{Q}(\text{in DY}).$$

$$\langle Q \rangle > 2\ \text{GeV} \qquad \text{and} \qquad \delta < 0.3$$



FIT RESULTS

Bury, Prokudin, Vladimirov (2021)

- Replica method using Artemide framework
- Errors both from the data and the uncertainty due to unpolarized TMD

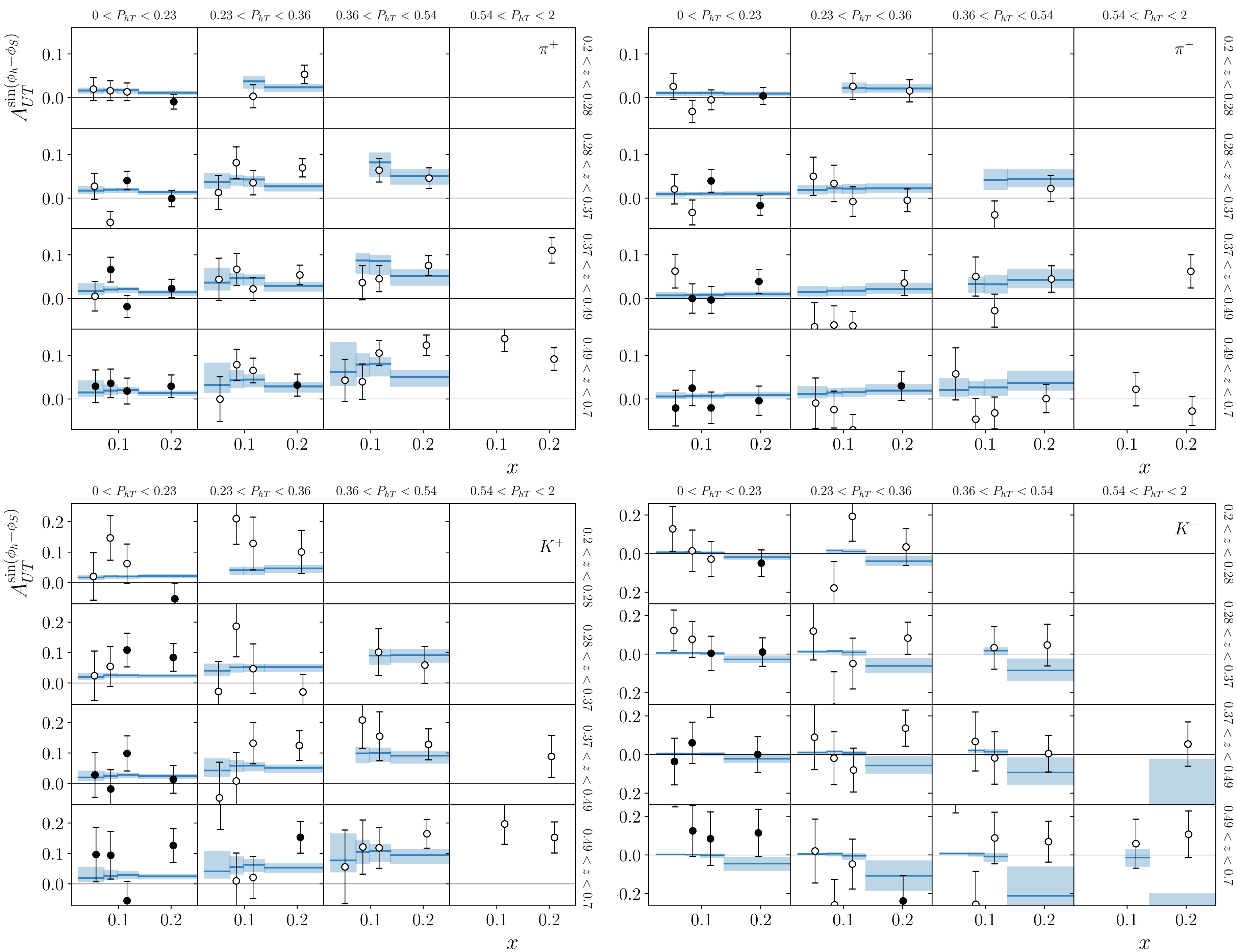
Name	χ^2/N_{pt}[SIDIS]	χ^2/N_{pt}[DY]	χ^2/N_{pt}[total]
SIDIS at N^{3}LO	$0.87^{+0.13}_{+0.03}$	$1.23^{+0.50}_{-0.24}$ no fit	$0.93^{+0.16}_{+0.01}$
SIDIS+DY at N^{3}LO	$0.88^{+0.15}_{+0.04}$	$0.90^{+0.31}_{+0.00}$	$0.88^{+0.15}_{+0.05}$

- Unbiased parametrization
- No tension between SIDIS and DY data — universality
- Good convergence of the fit for all data sets

N3LO EXTRACTION OF THE SIVERS FUNCTION

Bury, Prokudin, Vladimirov (2021)

HERMES 20 3D binning description

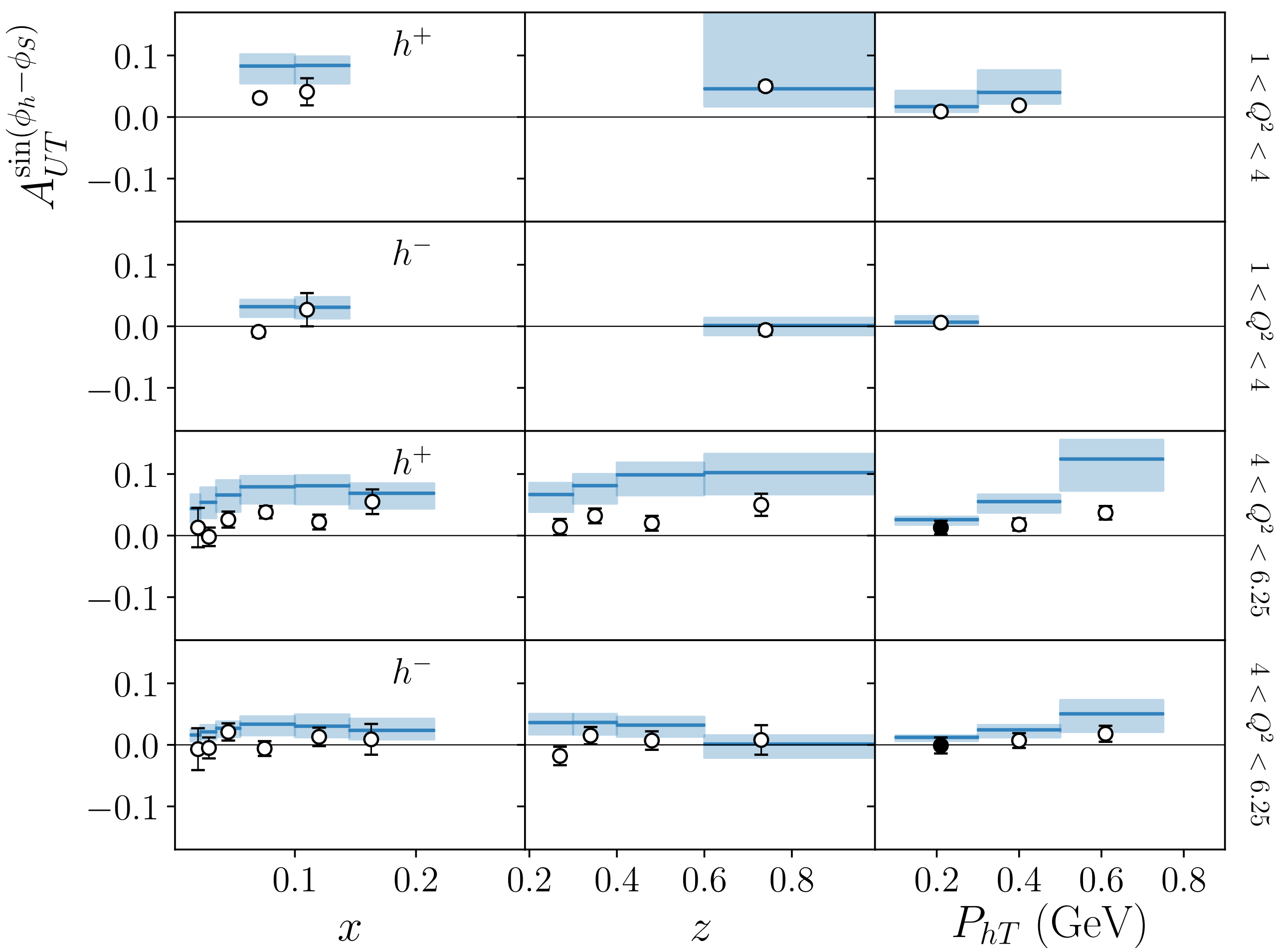


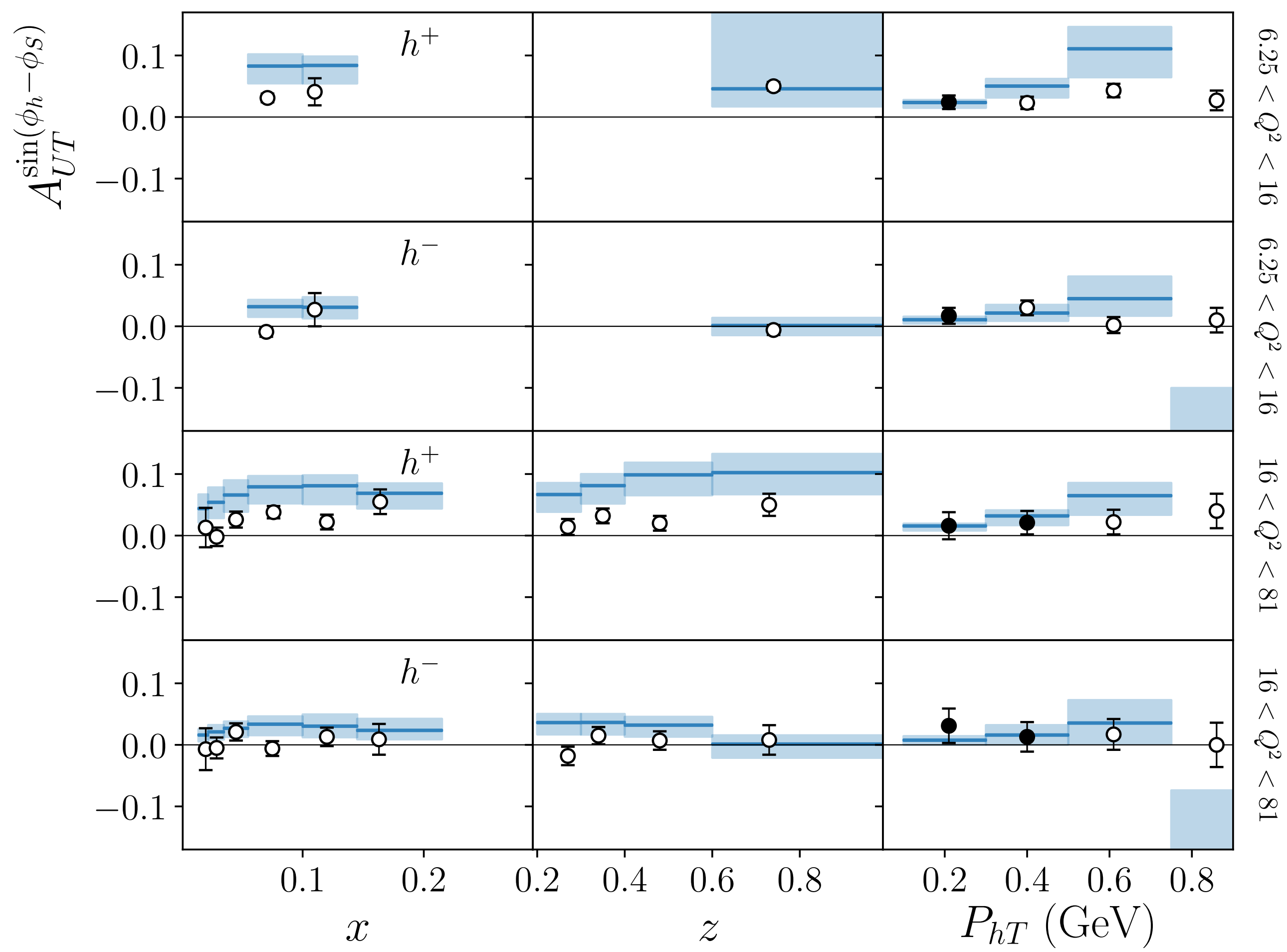
Filled in points used in the fit

N3LO EXTRACTION OF THE SIVERS FUNCTION

Bury, Prokudin, Vladimirov (2021)

COMPASS SIDIS data

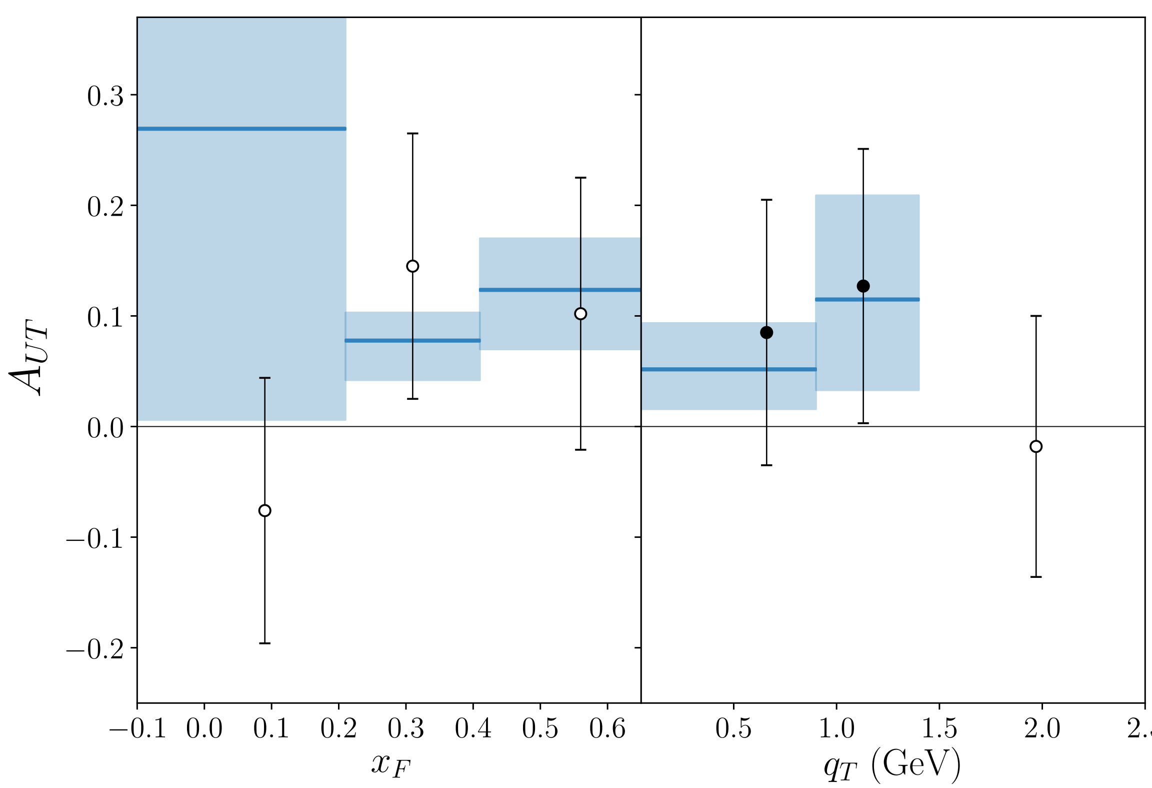




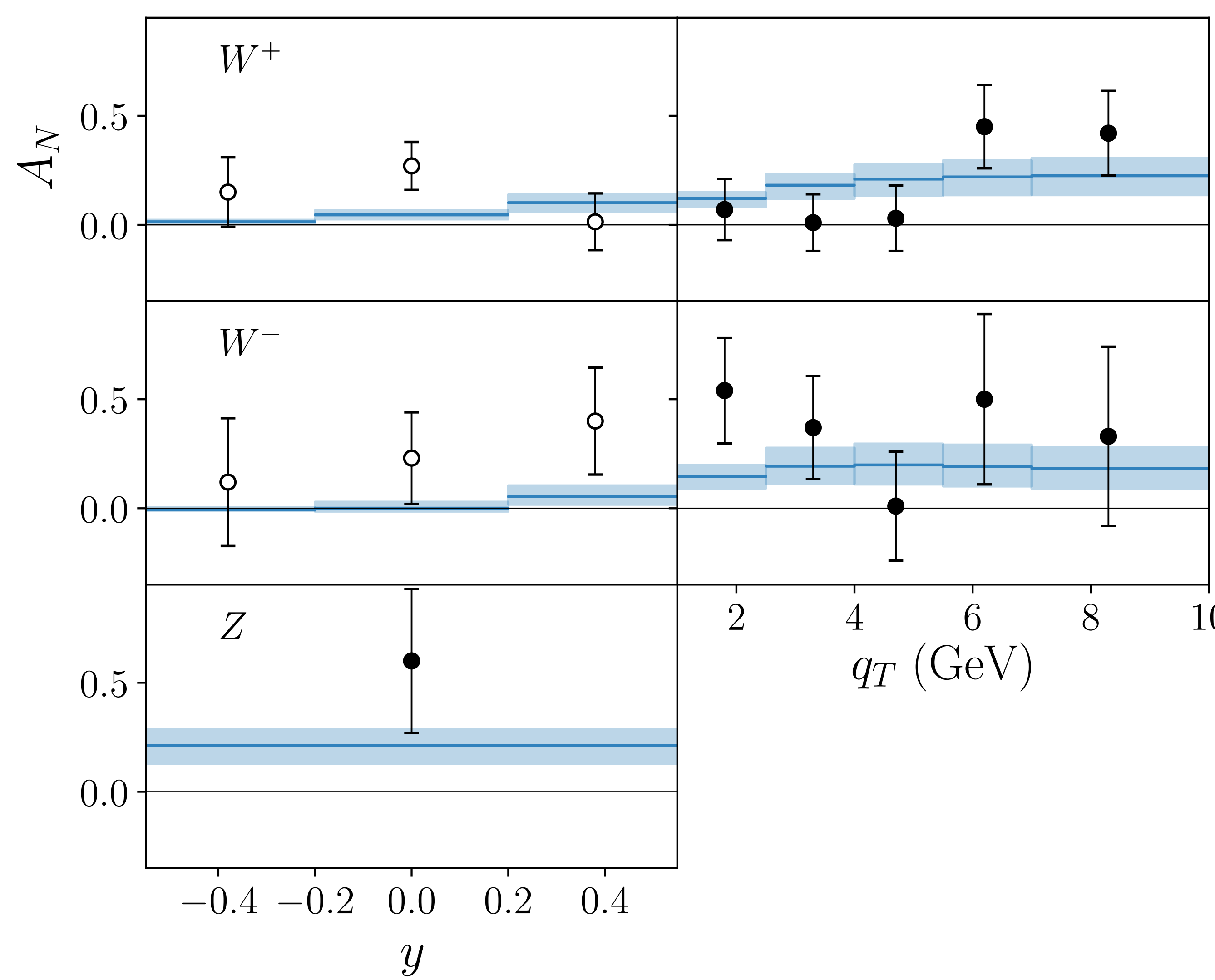
N3LO EXTRACTION OF THE SIVERS FUNCTION

Bury, Prokudin, Vladimirov (2021)

Pion induced Drell-Yan, COMPASS

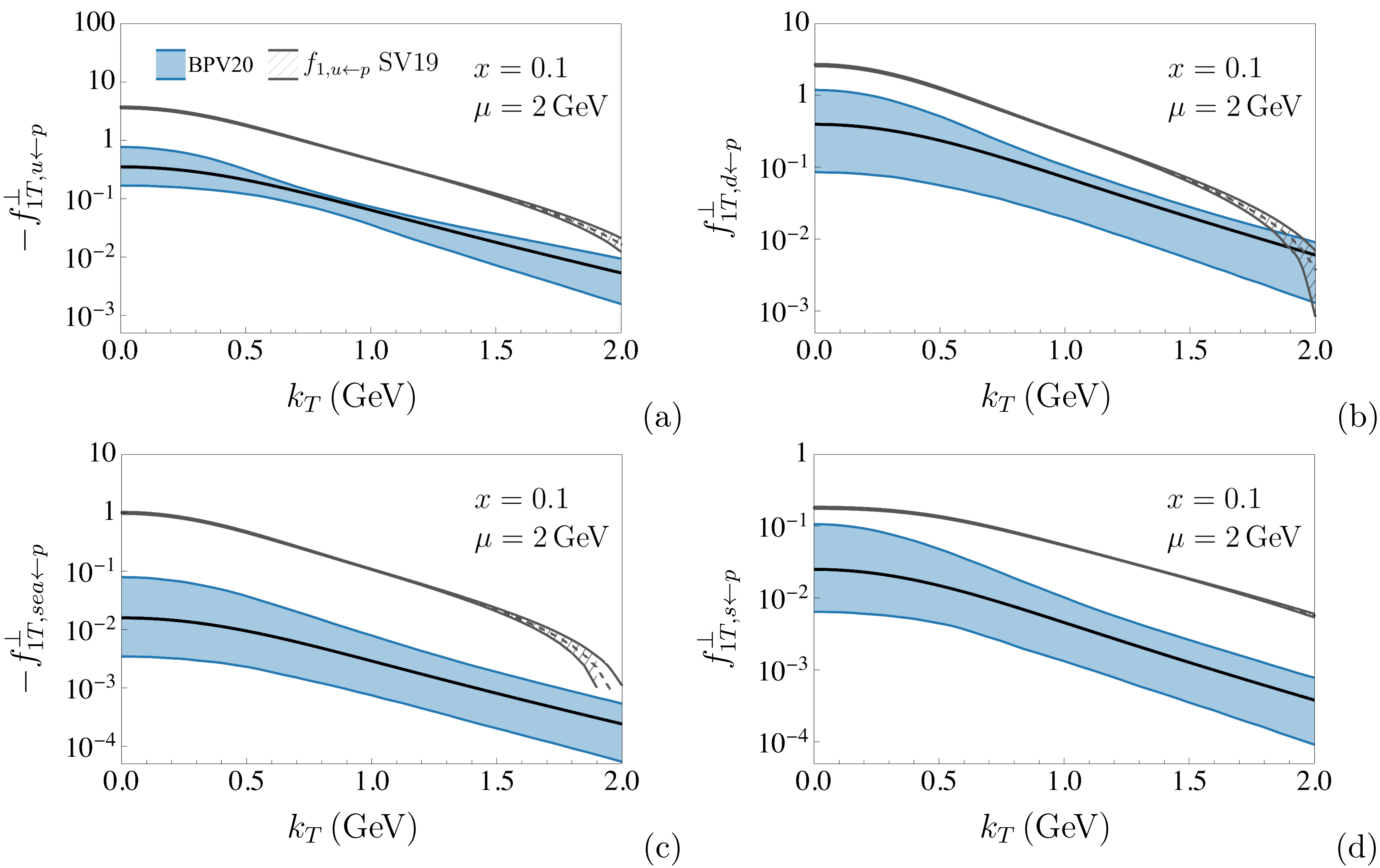


W/Z production, STAR



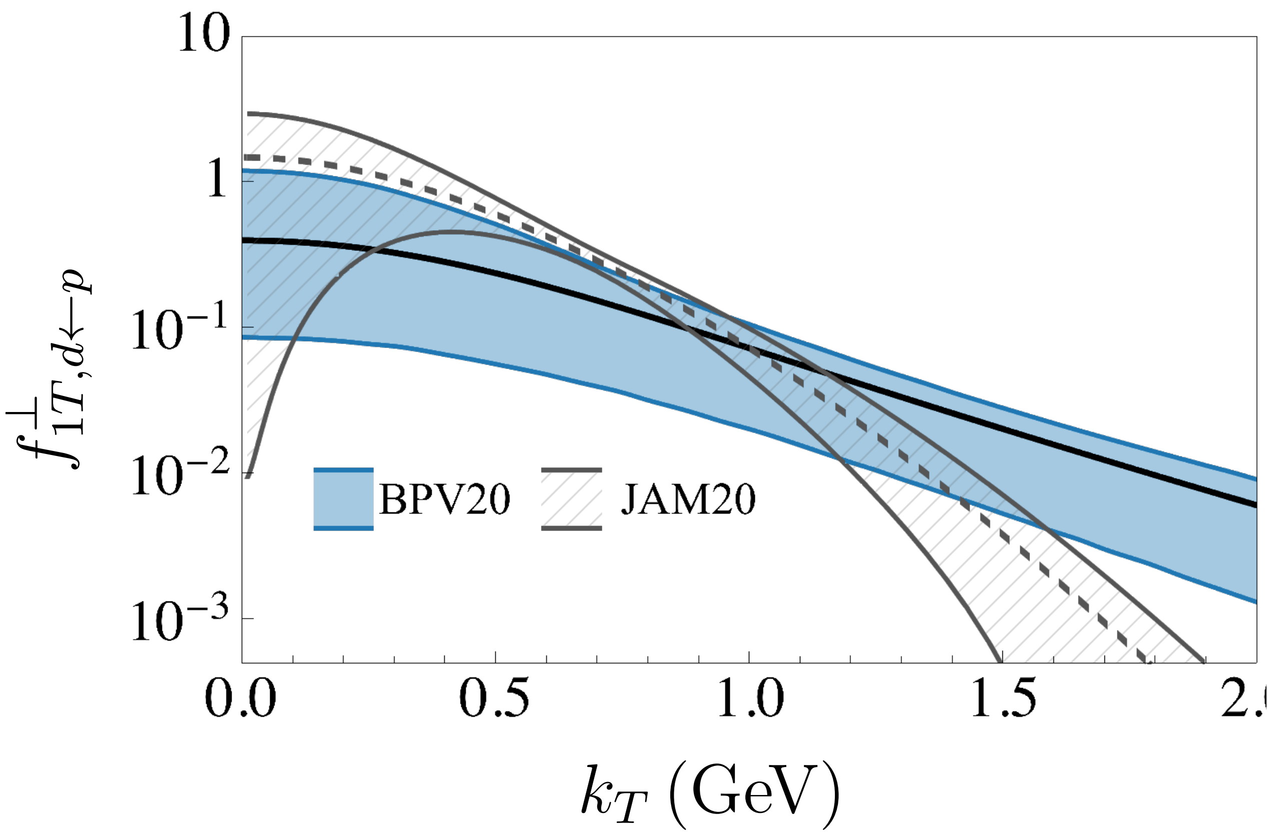
SIVERS FUNCTION IN THE MOMENTUM SPACE

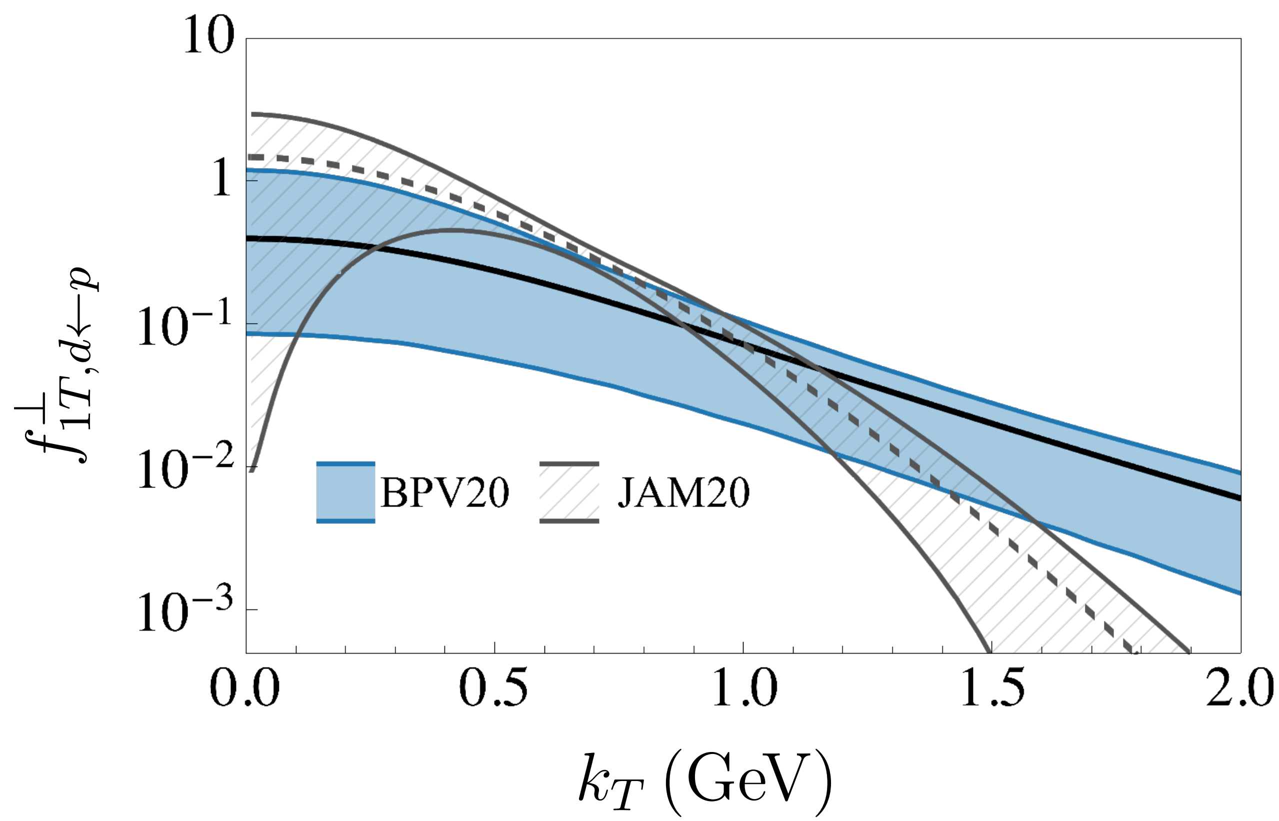
Bury, Prokudin, Vladimirov (2021)



SIVERS FUNCTION IN THE MOMENTUM SPACE

Bury, Prokudin, Vladimirov (2021)



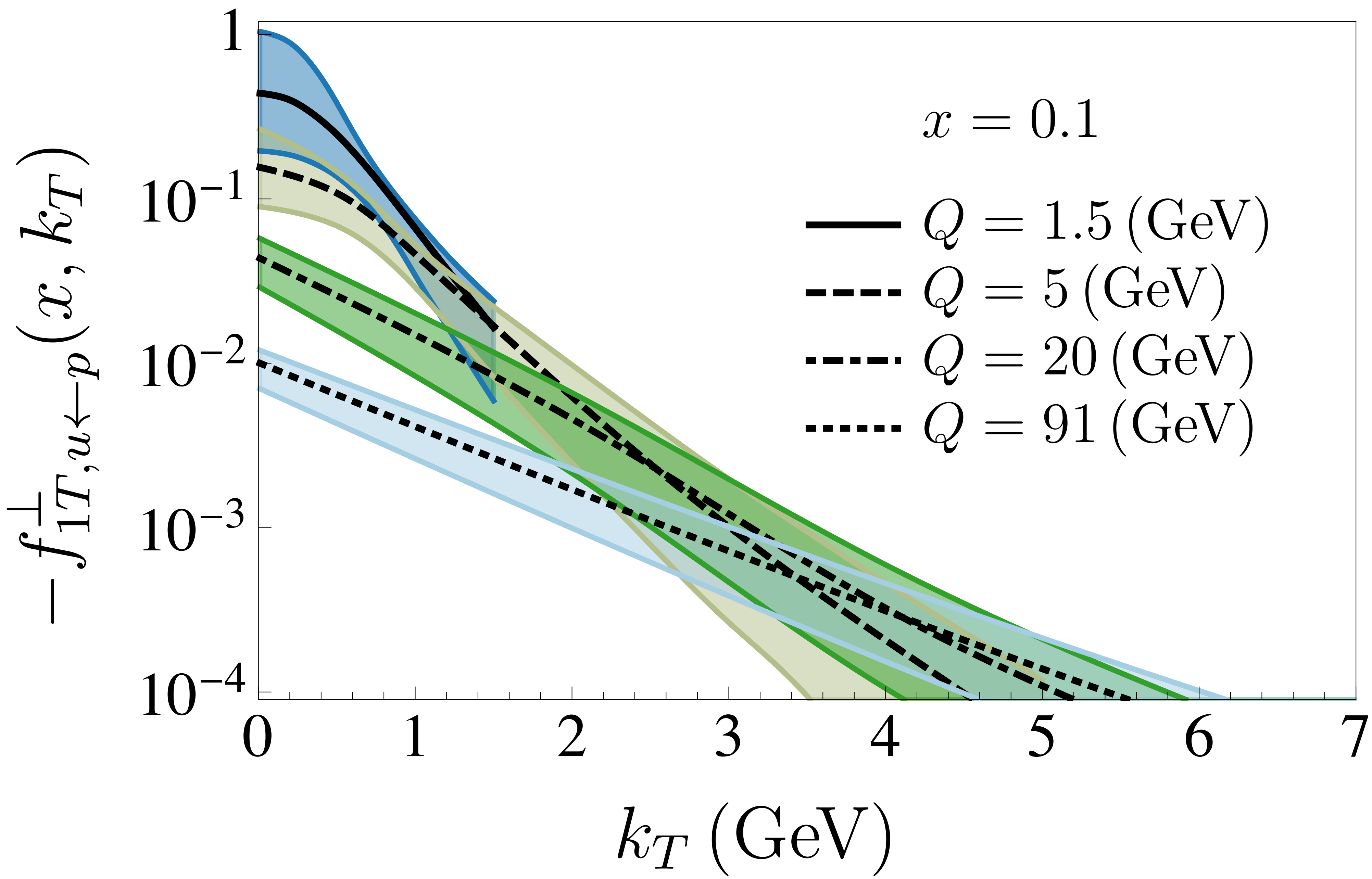


- Comparison to Jam20 (LO) analysis

Jam20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)

N3LO EXTRACTION OF THE SIVERS FUNCTION

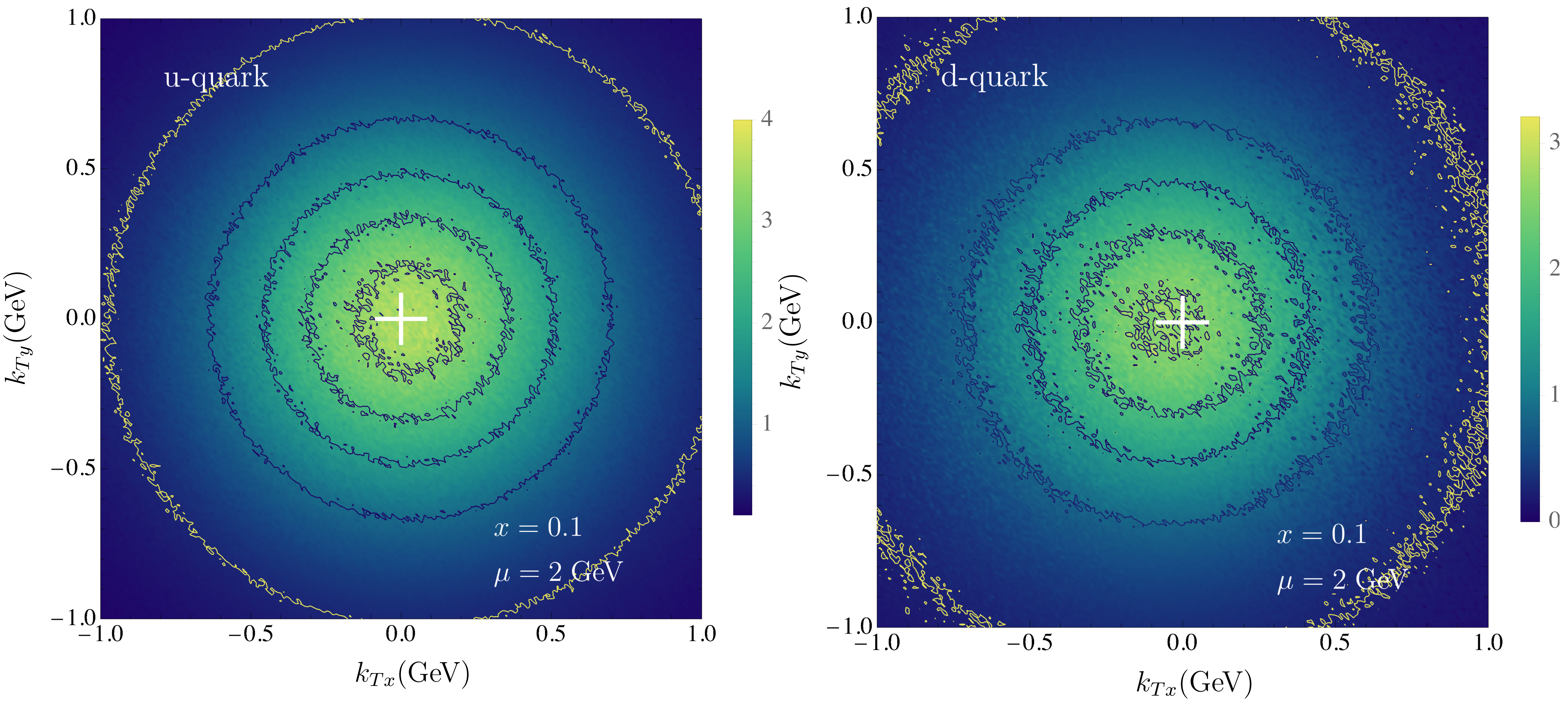
Bury, Prokudin, Vladimirov (2021)



NUCLEON TOMOGRAPHY

Bury, Prokudin, Vladimirov (2021)

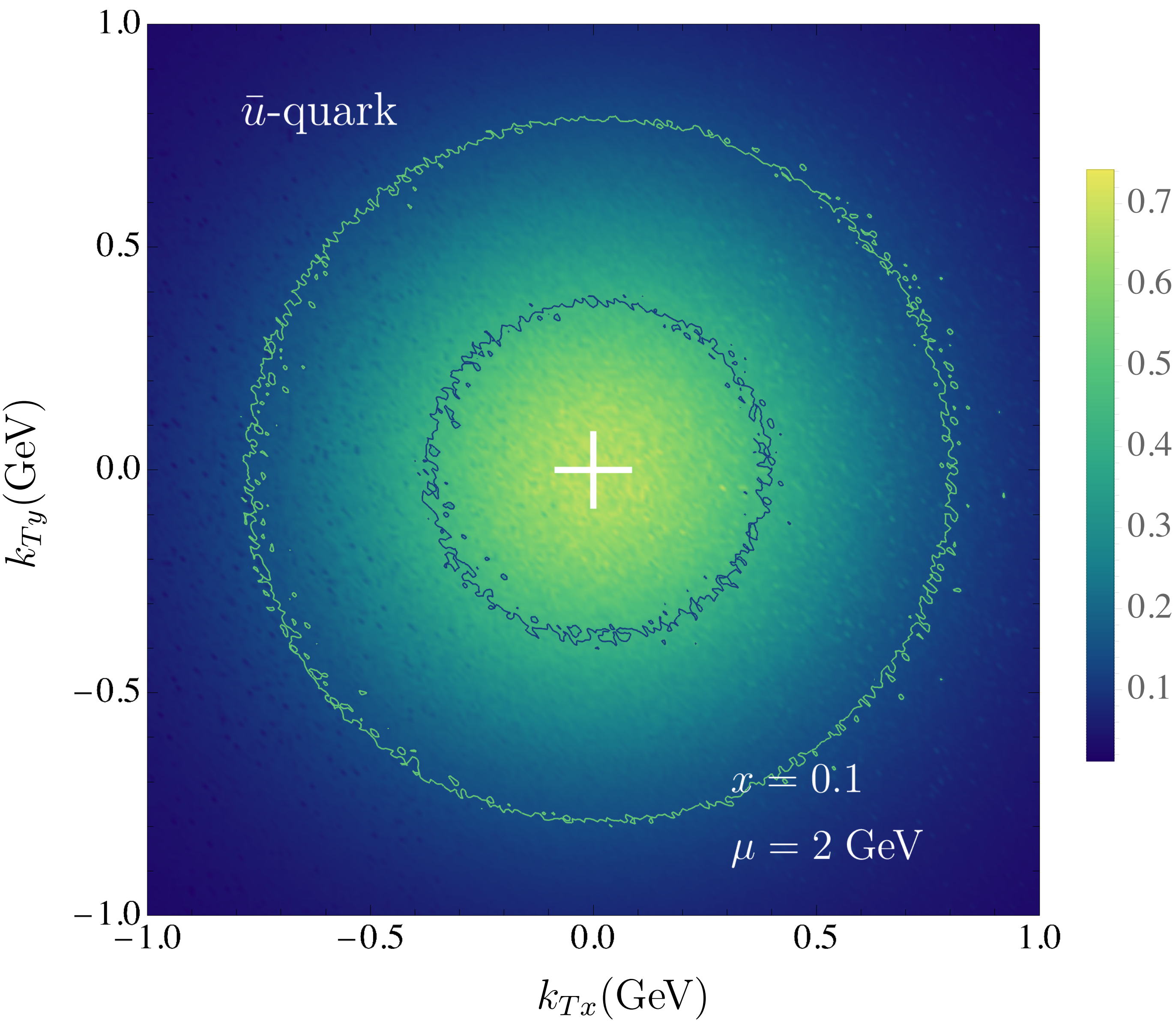
$$\rho_{1;q\leftarrow h^{\uparrow}}(x, \boldsymbol{k}_T, \boldsymbol{S}_T, \mu) = f_{1;q\leftarrow h}(x, k_T; \mu, \mu^2) - \frac{k_{Tx}}{M} f^{\perp}_{1T;q\leftarrow h}(x, k_T; \mu, \mu^2)$$

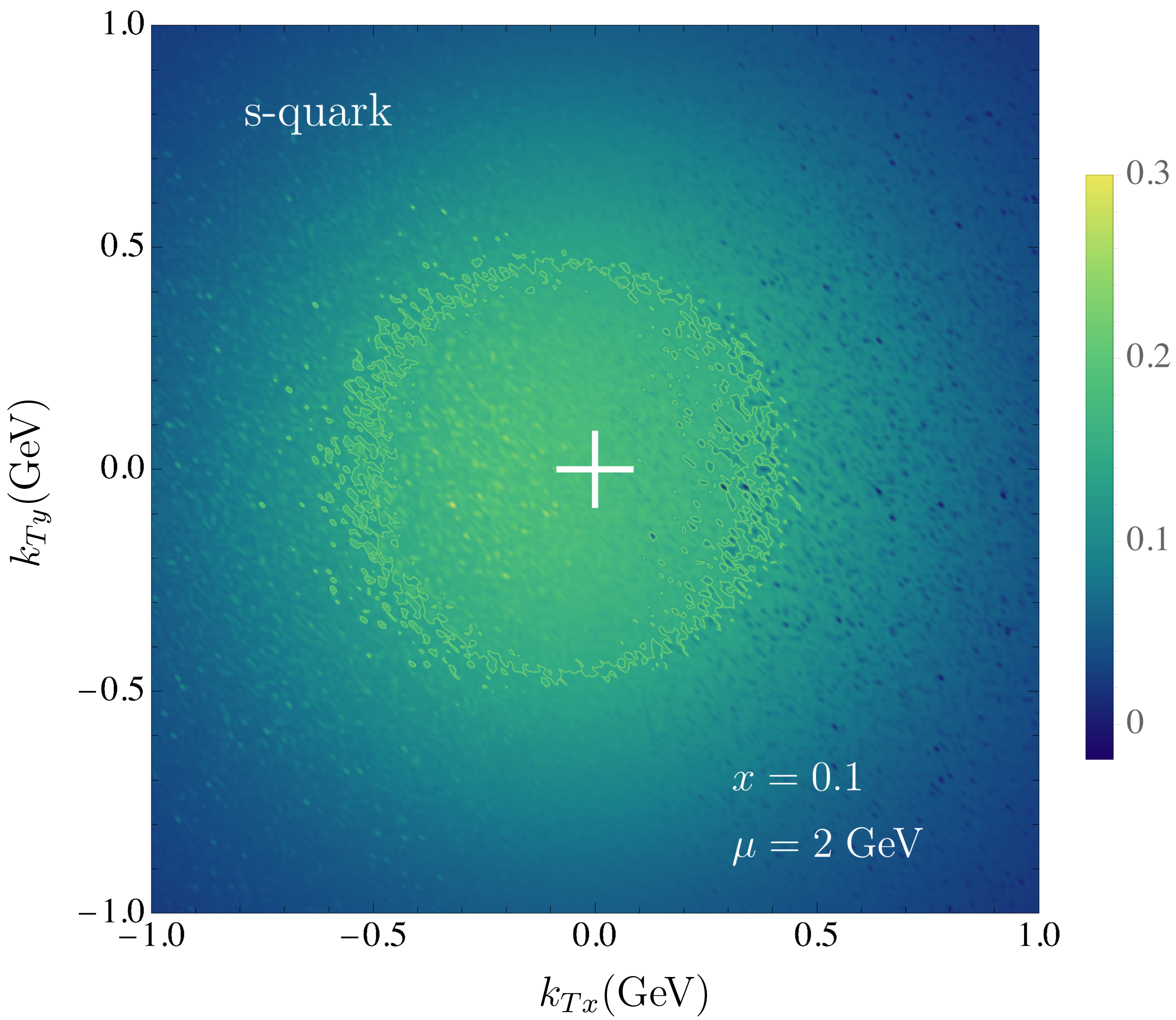


NUCLEON TOMOGRAPHY

Bury, Prokudin, Vladimirov (2021)

$$\rho_{1;q\leftarrow h^{\uparrow}}(x,\boldsymbol{k}_T,\boldsymbol{S}_T,\mu) = f_{1;q\leftarrow h}(x,k_T;\mu,\mu^2) - \frac{k_{Tx}}{M} f^{\perp}_{1T;q\leftarrow h}(x,k_T;\mu,\mu^2)$$





THE QIU-STERMAN MATRIX ELEMENT

Bury, Prokudin, Vladimirov (2020)

- Invert the formula for Operator Product Expansion of Sivers via the QS functions

Scimemi, Tarasov, Vladimirov (19)

$$T_q(-x,0,x;\mu_b) = -\frac{1}{\pi}\left(1 + C_F\frac{\alpha_s(\mu_b)}{4\pi}\frac{\pi^2}{6}\right) f_{1T;q\leftarrow h}^{\perp}(x,b)$$
$$-\frac{\alpha_s(\mu_b)}{4\pi^2}\int_x^1 \frac{dy}{y}\left[\frac{\bar{y}}{N_c} f_{1T;q\leftarrow h}^{\perp}\left(\frac{x}{y},b\right) + \frac{3y^2\bar{y}}{2x} G^{(+)}\left(-\frac{x}{y},0,\frac{x}{y};\mu_b\right)\right] + \mathcal{O}(a_s^2,b^2)$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b}$$

We choose $b = 0.11\ (\mathrm{GeV}^{-1}),\quad \mu_{\mathrm{b}} = 10\ (\mathrm{GeV})$

and estimate gluon contribution $G^{(+)} = \pm(|T_u| + |T_d|)$

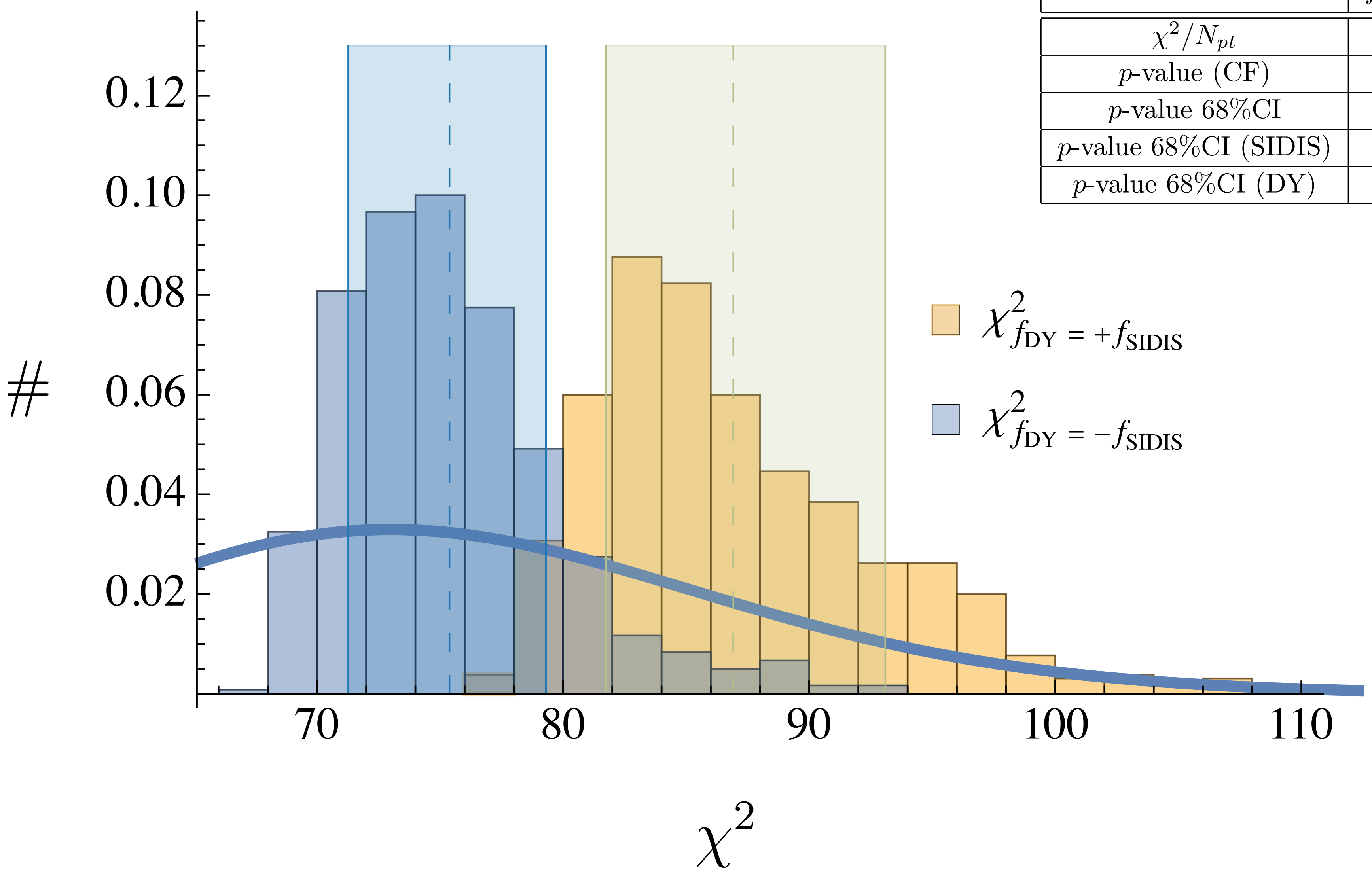
CONCLUSIONS

- We have extracted Sivers function from the first global fit of SIDIS, pion-induced Drell-Yan and W±/Z-bozon production experimental data at N3LO precision
- Conservative data cuts are used to ensure validity of factorization and unbiased parametrization
- Good agreement between SIDIS and DY data in an analysis with TMD evolution is achieved for the first time
- The Qiu-Sterman functions are extracted in a model independent way
- Our results set a new benchmark and the standard of precision for studies of TMD polarized functions

BACKUP SLIDES

SIGN CHANGE

Bury, Prokudin, Vladimirov (2020)



	$f^{\perp}_{1T\,[DY]} = -f^{\perp}_{1T\,[SIDIS]}$	$f^{\perp}_{1T\,[DY]} = +f^{\perp}_{1T\,[SIDIS]}$
χ^2/N_{pt}	$0.88^{+0.16}_{+0.06}$	$1.00^{+0.22}_{+0.08}$
p-value (CF)	0.74	0.44
p-value 68%CI	[0.60, 0.34]	[0.28, 0.08]
p-value 68%CI (SIDIS)	[0.67, 0.42]	[0.53, 0.11]
p-value 68%CI (DY)	[0.56, 0.17]	[0.68, 0.02]

Large contribution from antiquark Sievers functions to DY makes it possible to describe data without the sign change

$$f_{1T}^{\perp sea} \to -f_{1T}^{\perp sea}$$