Scattering amplitudes using Quantum Computers

Raúl Briceño - http://bit.ly/rbricenoPhD

OLD DOMINION UNIVERSITY

Jefferson Lab

GHP, 2021

Scattering amplitudes using Quantum Computers

Open Access

Role of boundary conditions in quantum computations of scattering observables

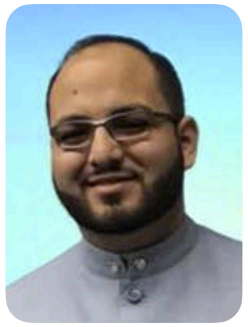
Raúl A. Briceño, Juan V. Guerrero, Maxwell T. Hansen, and Alexandru M. Sturzu
Phys. Rev. D **103**, 014506 – Published 6 January 2021

Article | References | No Citing Articles | PDF | HTML | Export Citation

ABSTRACT

Quantum computing may offer the opportunity to simulate strongly interacting field theories, such as quantum chromodynamics, with physical time evolution. This would give access to Minkowski-signature correlators, in contrast to the Euclidean calculations routinely performed at present. However, as with present-day calculations, quantum computation strategies still require the restriction to a finite system size, including a finite, usually periodic, spatial volume. In this work, we investigate the consequences of this in the extraction of hadronic and Compton-like scattering amplitudes. Using the framework presented in Briceño *et al.* [Phys. Rev. D **101**, 014509 (2020)], we estimate the volume effects for various $1+1$D Minkowski-signature quantities and show that these can be a significant source of systematic uncertainty, even for volumes that are very large by the standards of present-day Euclidean calculations. We then present an improvement strategy, based in the fact that the finite

Juan Guerrero
JLab



Alex Sturzu
New College

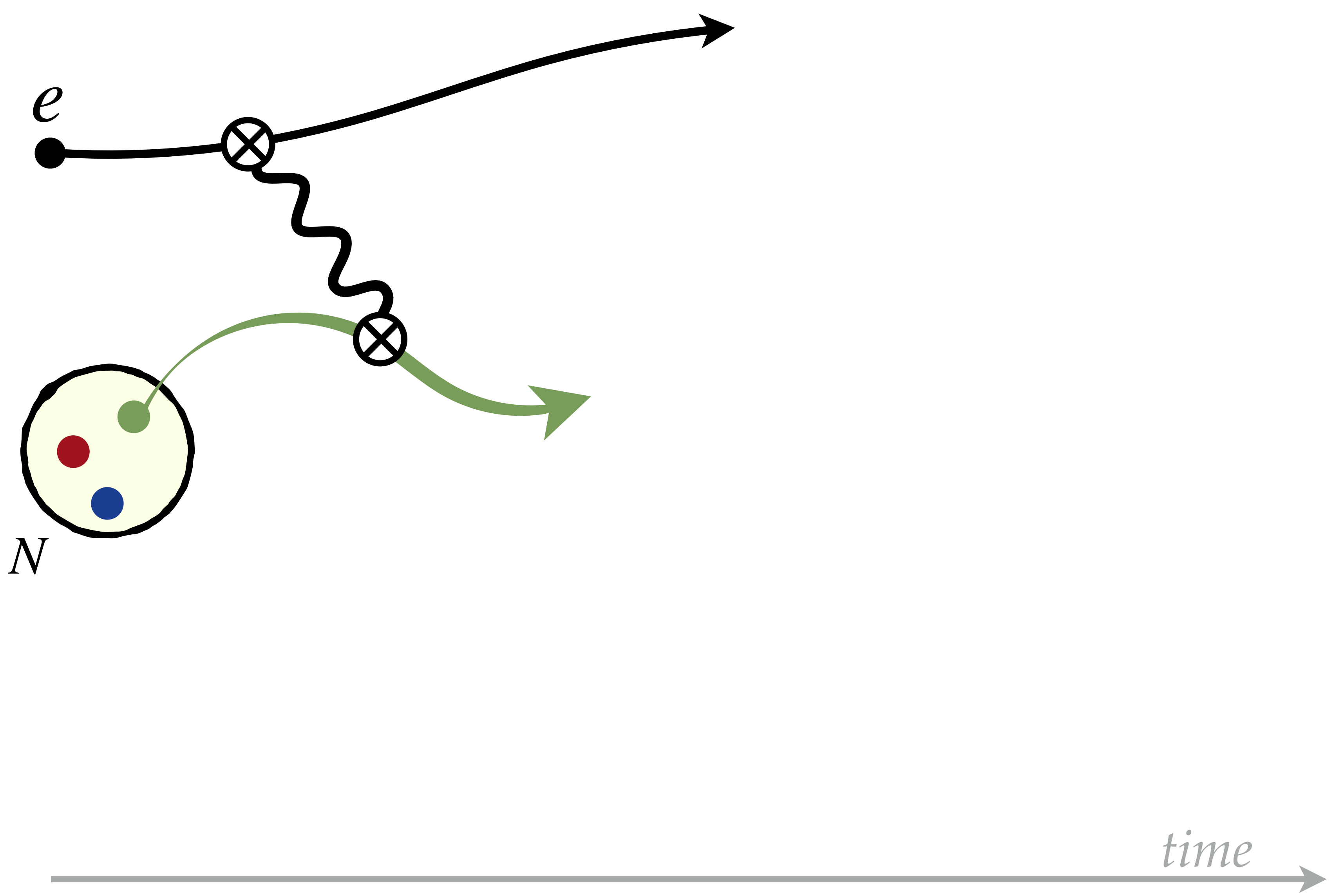
Max Hansen
Edinburgh

Inclusive scattering

- Compton scattering: polarizabilities, PDFs, GPDs,…

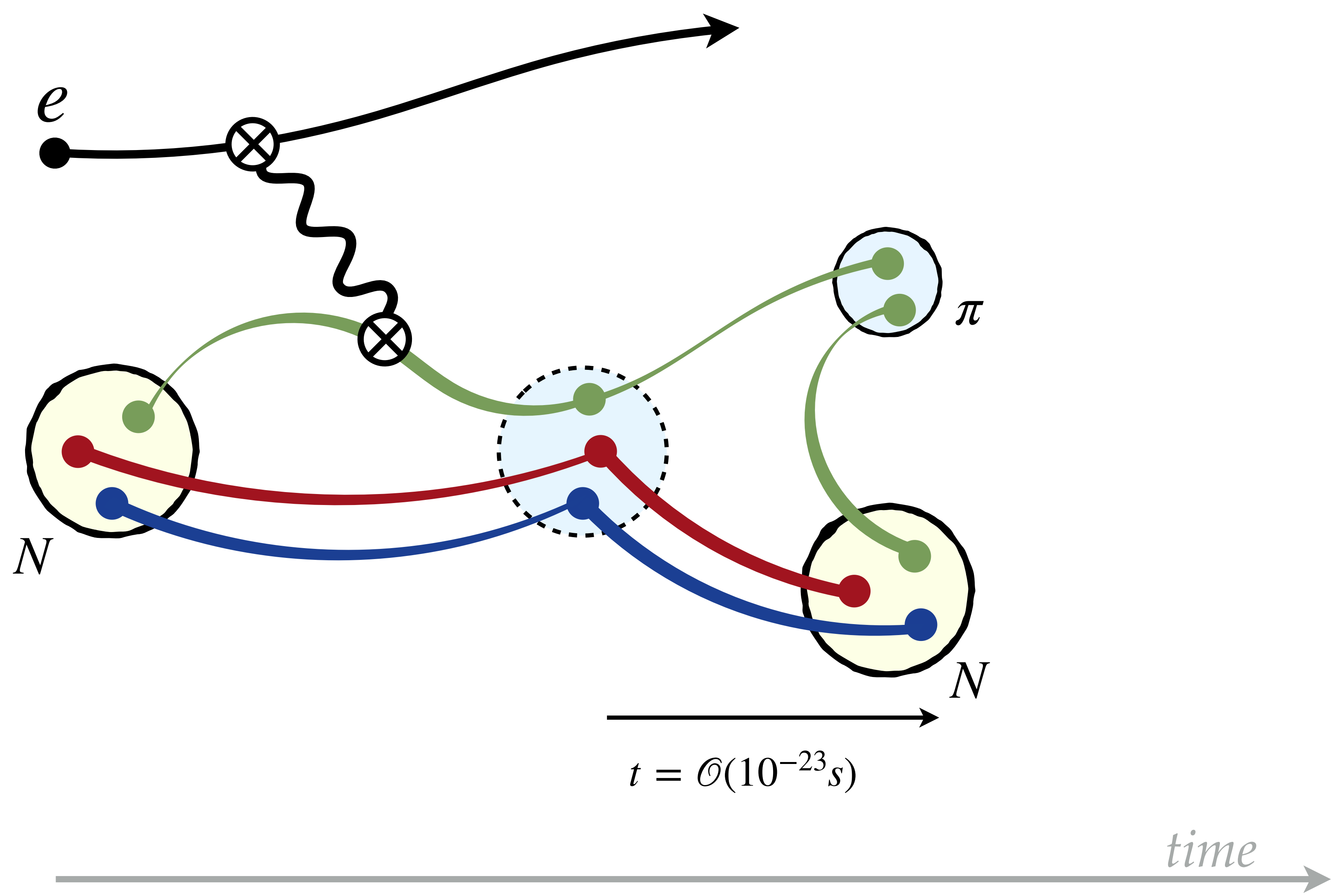
Inclusive scattering

- Compton scattering: polarizabilities, PDFs, GPDs,…



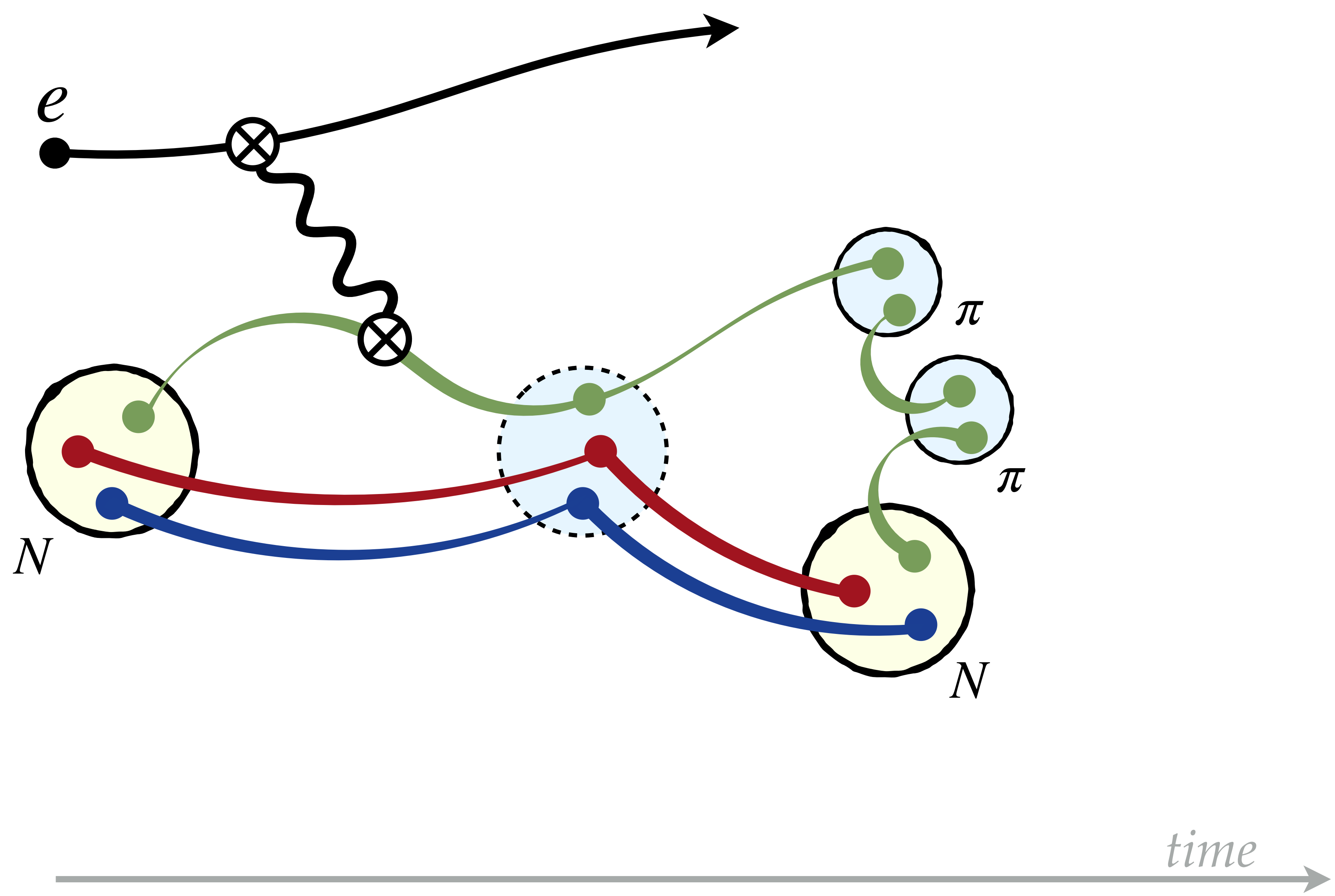
Inclusive scattering

- Compton scattering: polarizabilities, PDFs, GPDs,…



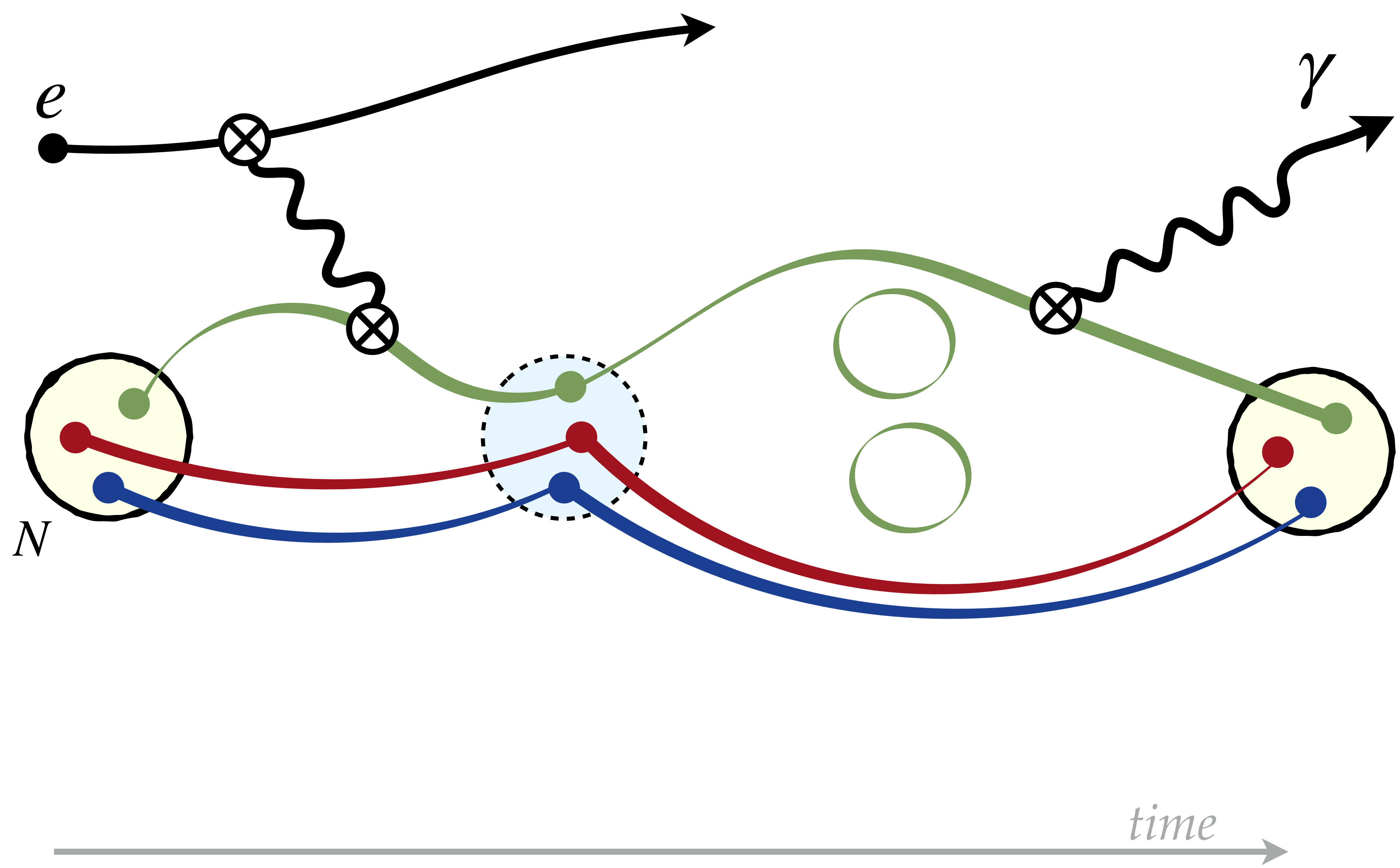
Inclusive scattering

- Compton scattering: polarizabilities, PDFs, GPDs,…



Inclusive scattering

- Compton scattering: polarizabilities, PDFs, GPDs,…

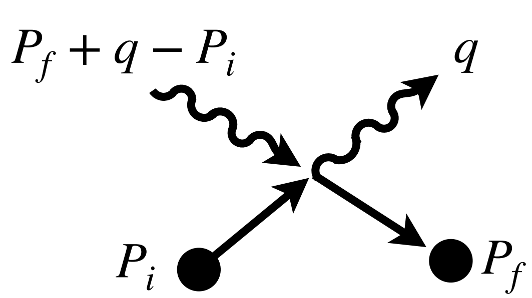


Inclusive scattering

- Compton scattering,
- Inclusive neutrino-nucleus scattering,
- Neutrinoless double beta decay
- …

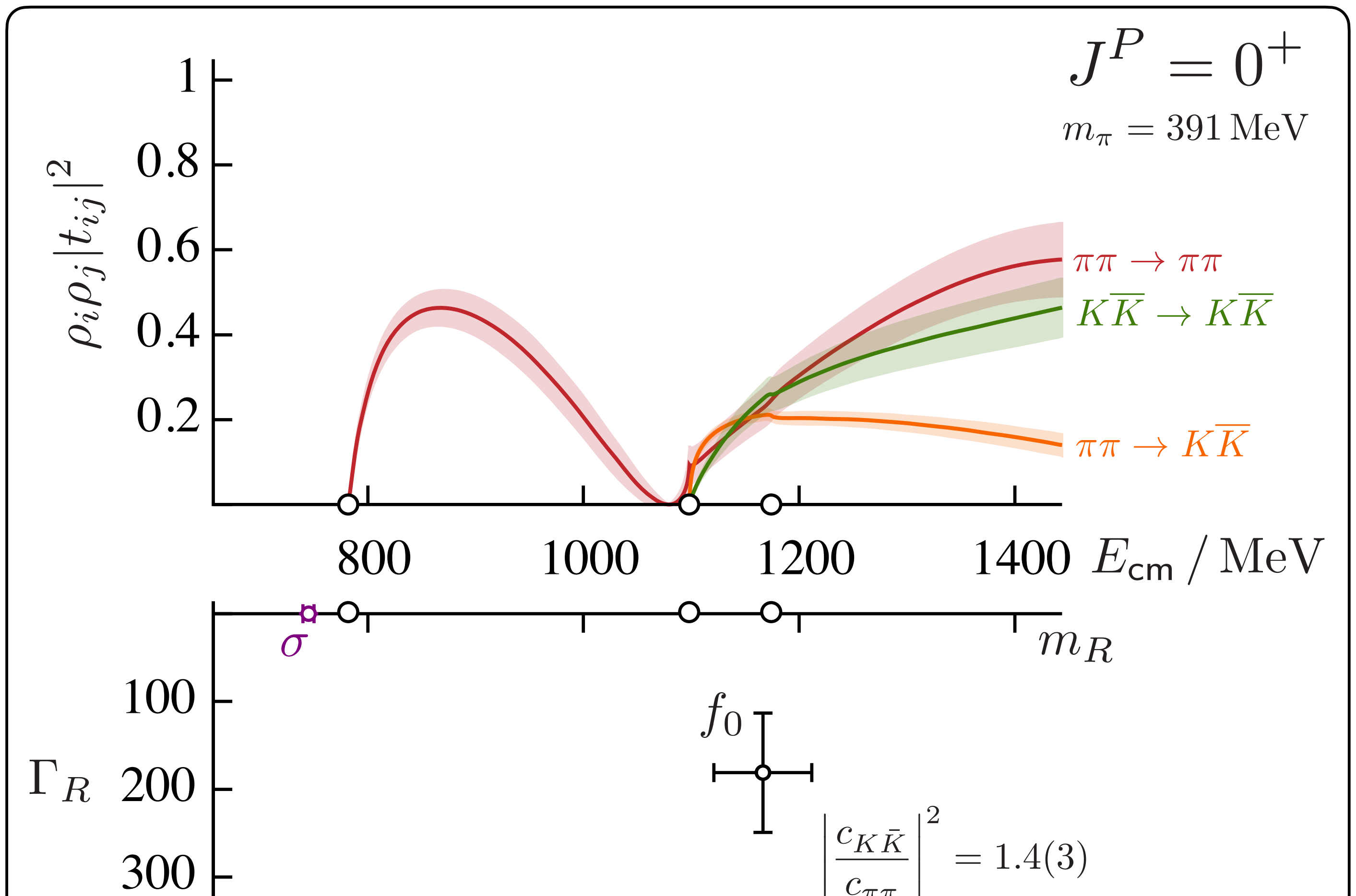
All can be defined as: $$\mathscr{T} \sim \int d^4x\, e^{ix\cdot q} \langle n_f |\, T\left[\mathscr{J}_{2,M}(t)\, \mathscr{J}_1(0)\right] | n_i \rangle_\infty$$

long-range amplitudes

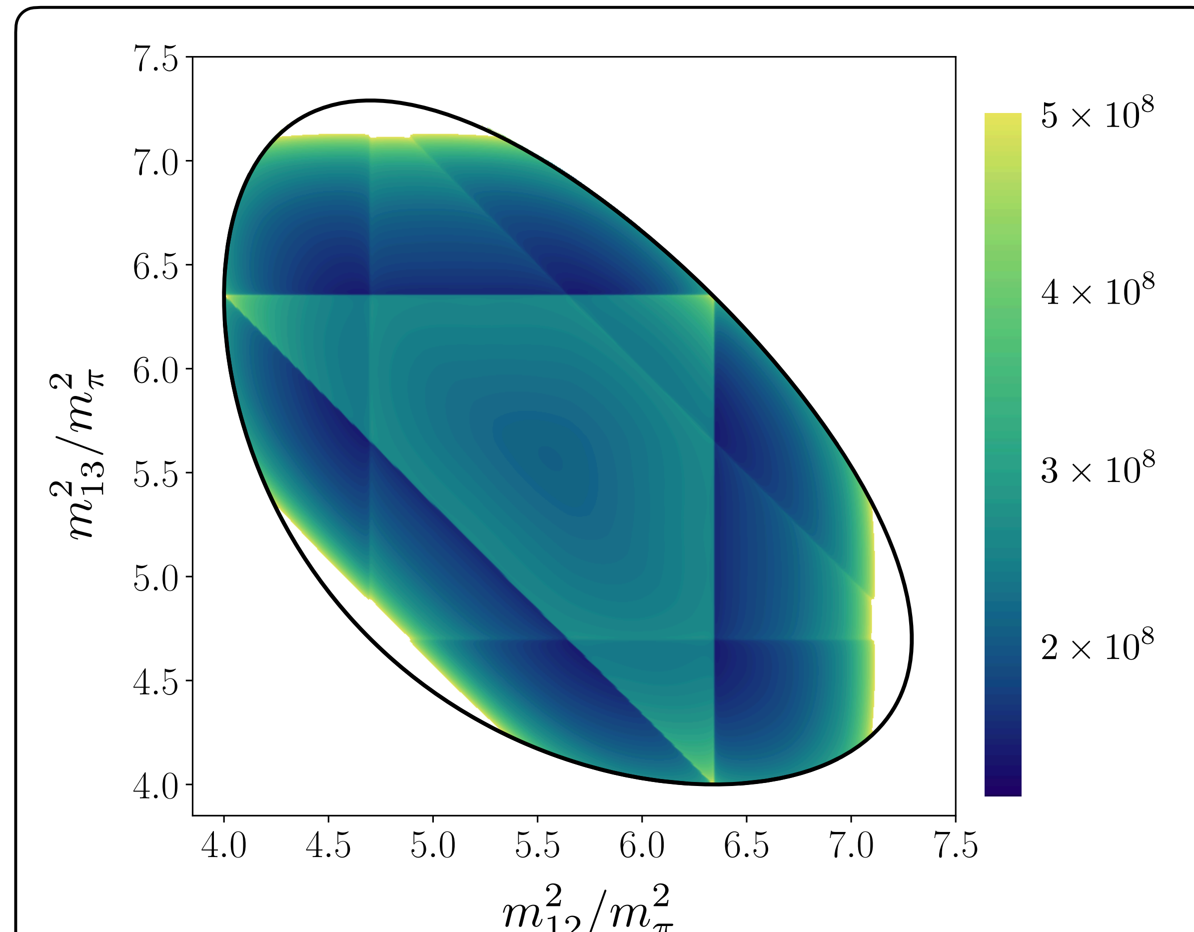


Exclusive vs. inclusive reactions

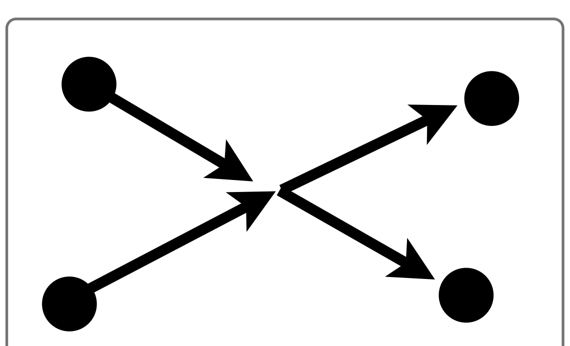
- If exclusive and *interesting*
  - After developing increasingly complex formalism…
  - Lattice QCD **will always win** [see talks by David Wilson, Max Hansen…]

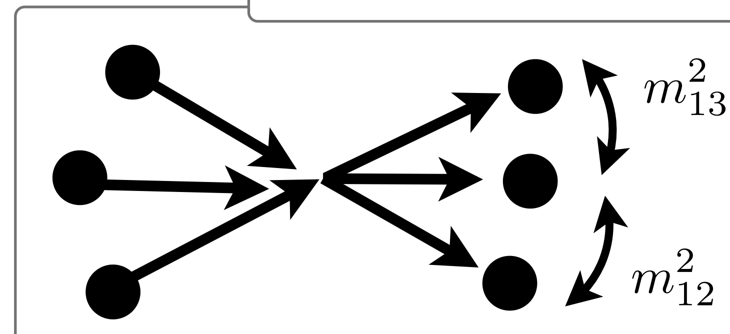


Briceno, Dudek, Edwards, & Wilson (2019)



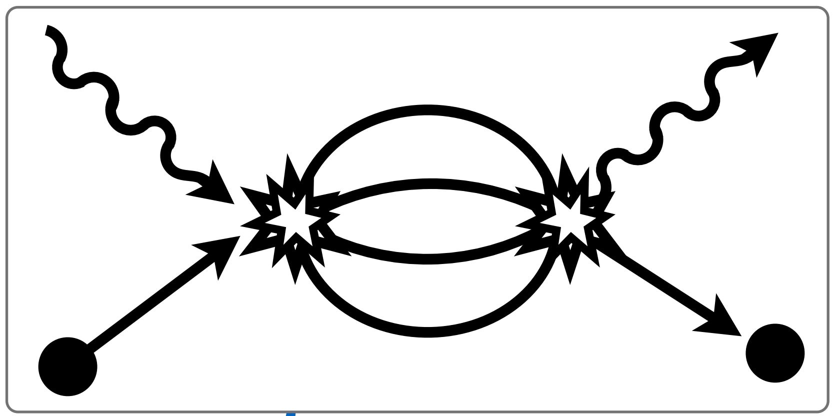
Hansen, Briceno, Edwards, Thomas, & Wilson (2020)





Exclusive vs. inclusive reactions

- If exclusive and *interesting*
  - After developing increasingly complex formalism…
  - Lattice QCD **will always win**
- Inclusive reactions, QC methods *may* be needed and worth investigating.



Infinite-volume reactions

- complex functions
- kinematic singularities

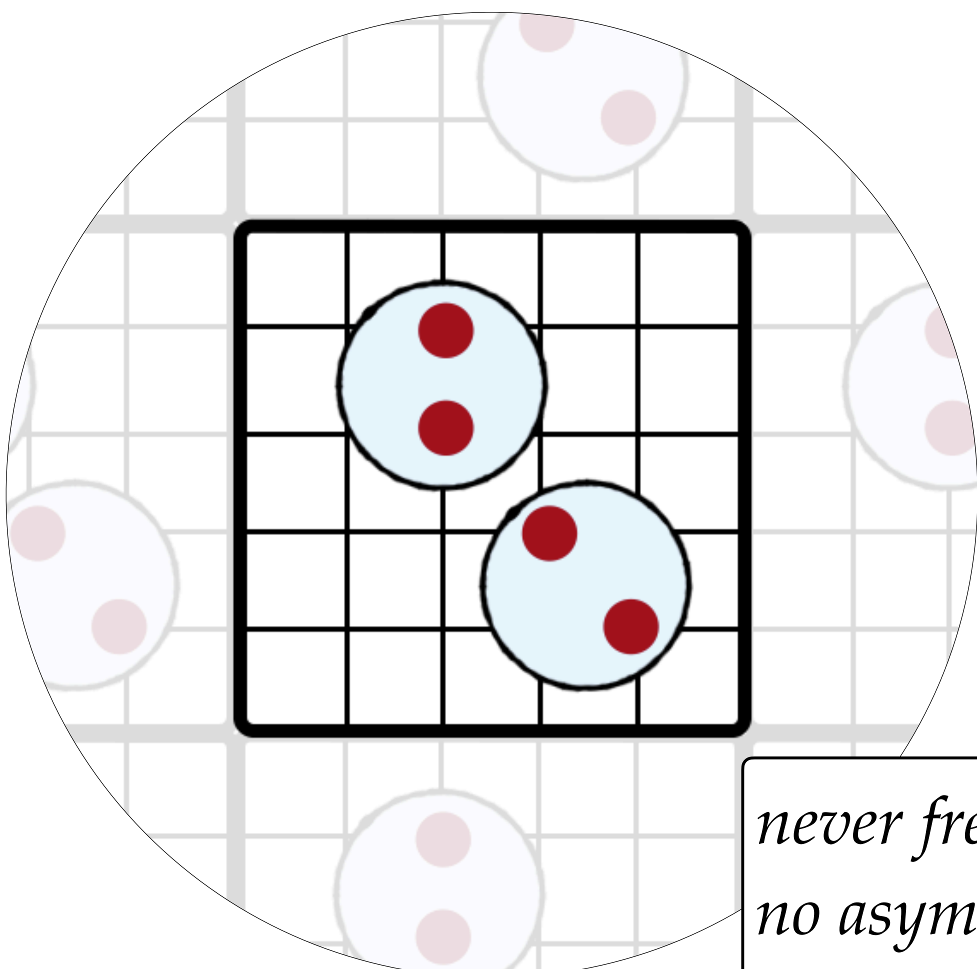
Finite-volume quantities

- real functions
- power-law finite-volume errors

For "large" volumes, shouldn't these be simply related? 🤔 . . . 🤷‍♂️

Real-time QFT calculations

- lattice spacing
- finite volume: L
- quark masses: $m_q \to m_q^{\text{phys}}$



never free
no asymptotic states
no scattering

Long-range processes in a finite volume

BBDHS formalism for finite-volume long range matrix elements:

$$\boxed{\mathcal{T}_L = \mathcal{T} - \mathcal{H}(s,Q^2)\,\frac{1}{F^{-1}(P,L)+\mathcal{M}(s)}\,\mathcal{H}'(s,Q^2_{if})}$$

Alessandro Baroni
LANL Postdoc

Zohreh Davoudi
Maryland

Max Hansen
Edinburgh

Matthias Schindler
South Carolina

implication for 3+1
Euclidean lattice QCD

see talk by Raúl Briceño
on Friday 🤓

Juan Guerrero
JLab

Alex Sturzu
New College

Max Hansen
Edinburgh

implication for 1+1
real-time correlators

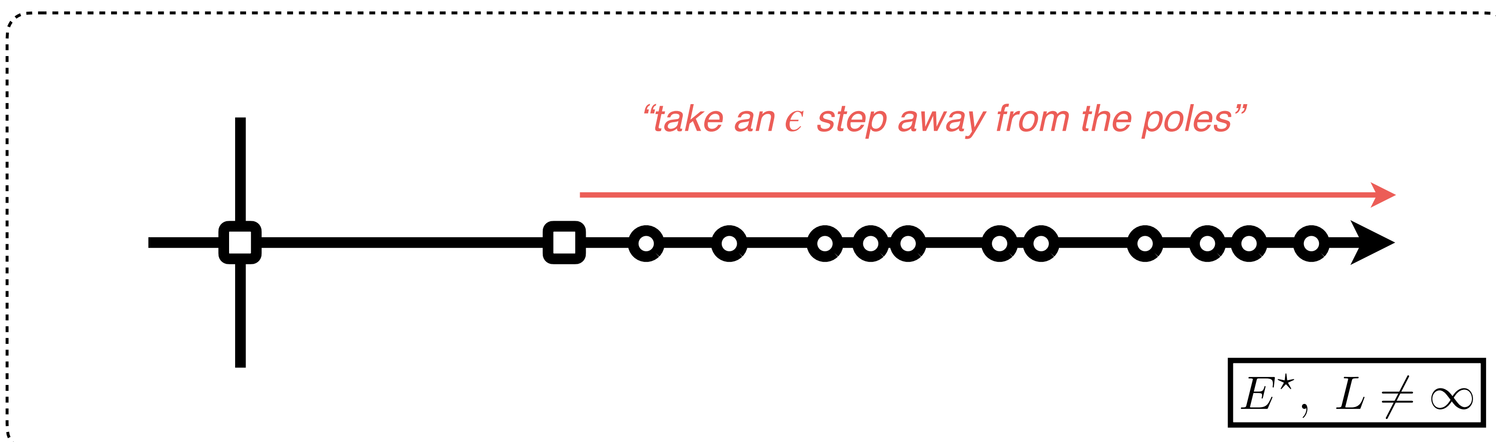
Three silver bullets

$$\boxed{\mathcal{T}_L = \mathcal{T} - \mathcal{H}(s, Q^2)\, \frac{1}{F^{-1}(P, L) + \mathcal{M}(s)}\, \mathcal{H}'(s, Q^2_{if})}$$

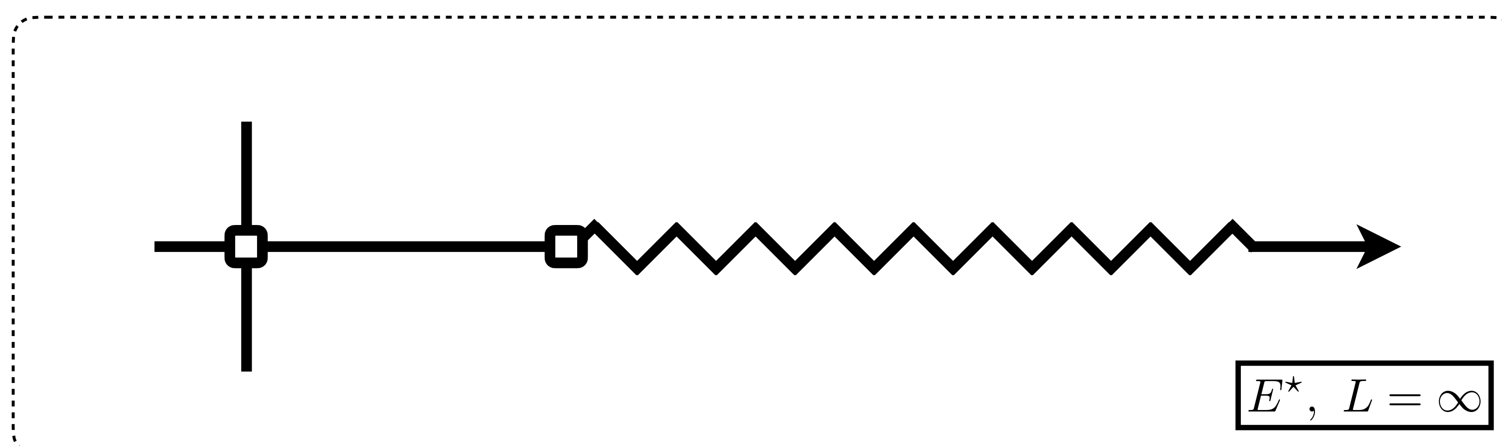
- [ ] Introduce an $i\epsilon$ by hand

$$\mathcal{T}_L(\epsilon) \sim \int_{-\infty}^{\infty} d\tau\, e^{iq_0 t - \epsilon|t|}\, \underline{\langle n_f |\, T[\mathcal{J}_2(t)\, \mathcal{J}_1(0)]\, | n_i \rangle_L}$$

this is what would be accessed via a quantum computer or a real-time calculation



For large enough L and small enough ϵ



Three silver bullets

$$\boxed{\mathcal{T}_L = \mathcal{T} - \mathcal{H}(s, Q^2)\,\frac{1}{F^{-1}(P,L) + \mathcal{M}(s)}\,\mathcal{H}'(s, Q^2_{if})}$$

- Introduce an $i\epsilon$ by hand

$$\mathscr{T}_L(\epsilon) \sim \int_{-\infty}^{\infty} d\tau\, e^{iq_0 t - \epsilon |t|}\, \langle n_f |\, T[\mathscr{J}_2(t)\, \mathscr{J}_1(0)]\, | n_i \rangle_L$$

- Exploit symmetry:
  - The physical amplitudes depend on a finite number of Lorentz scalars.
  - Boost average

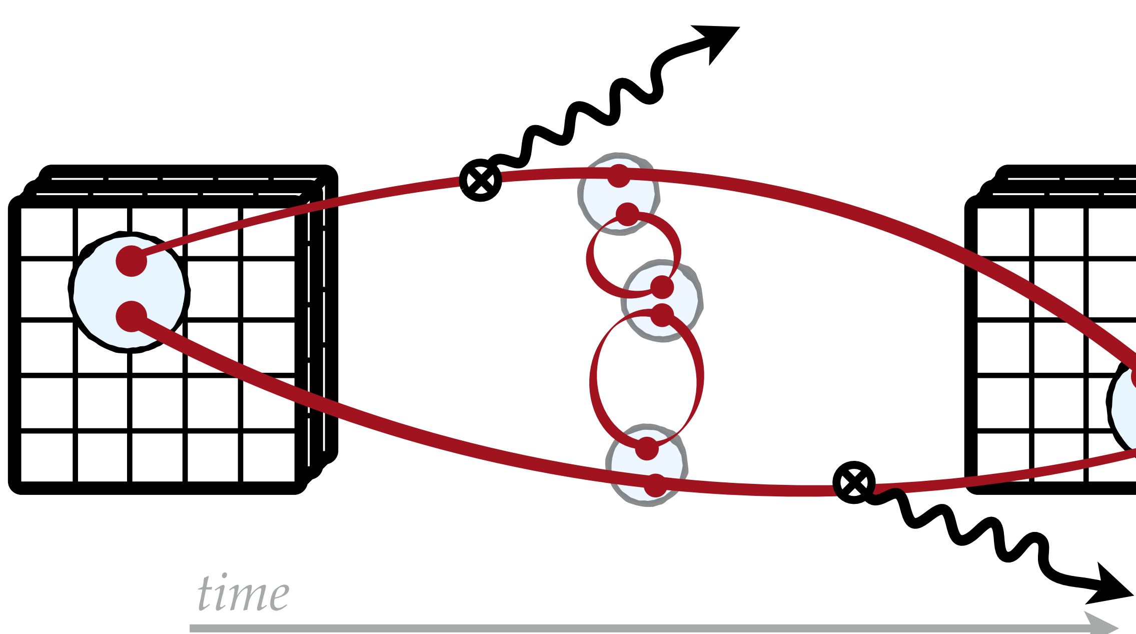
- Binning / wave packets

Toy model investigation *no quantum simulations were performed*

In principle

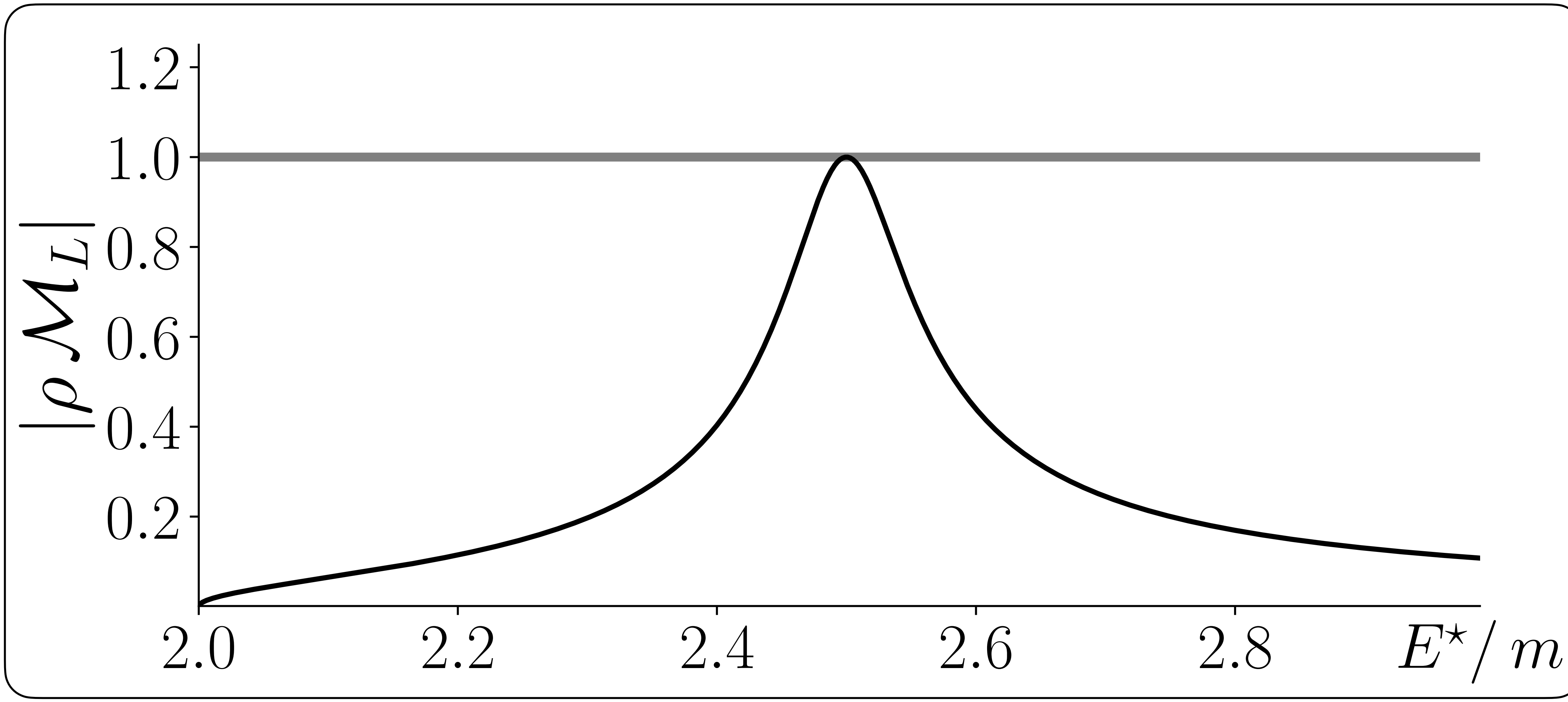
$$\mathscr{M} = \lim_{\epsilon \to 0} \lim_{L \to \infty} \mathscr{M}_L(\epsilon) \qquad\qquad \mathscr{T} = \lim_{\epsilon \to 0} \lim_{L \to \infty} \mathscr{T}_L(\epsilon)$$

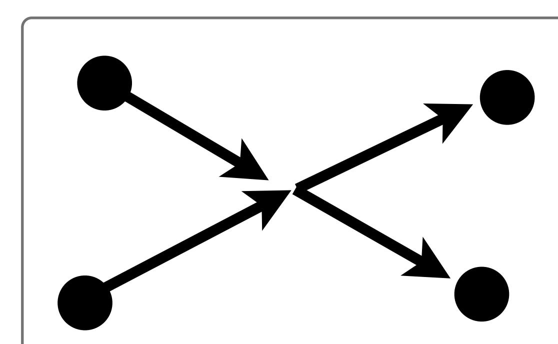
- ☐ Do we really need to take the $L \to \infty$ limit?
- ☐ How quickly do we recover the asymptotic behavior?
- ☐ Does this depend on the dynamics of the system?



Purely "hadronic" amplitude

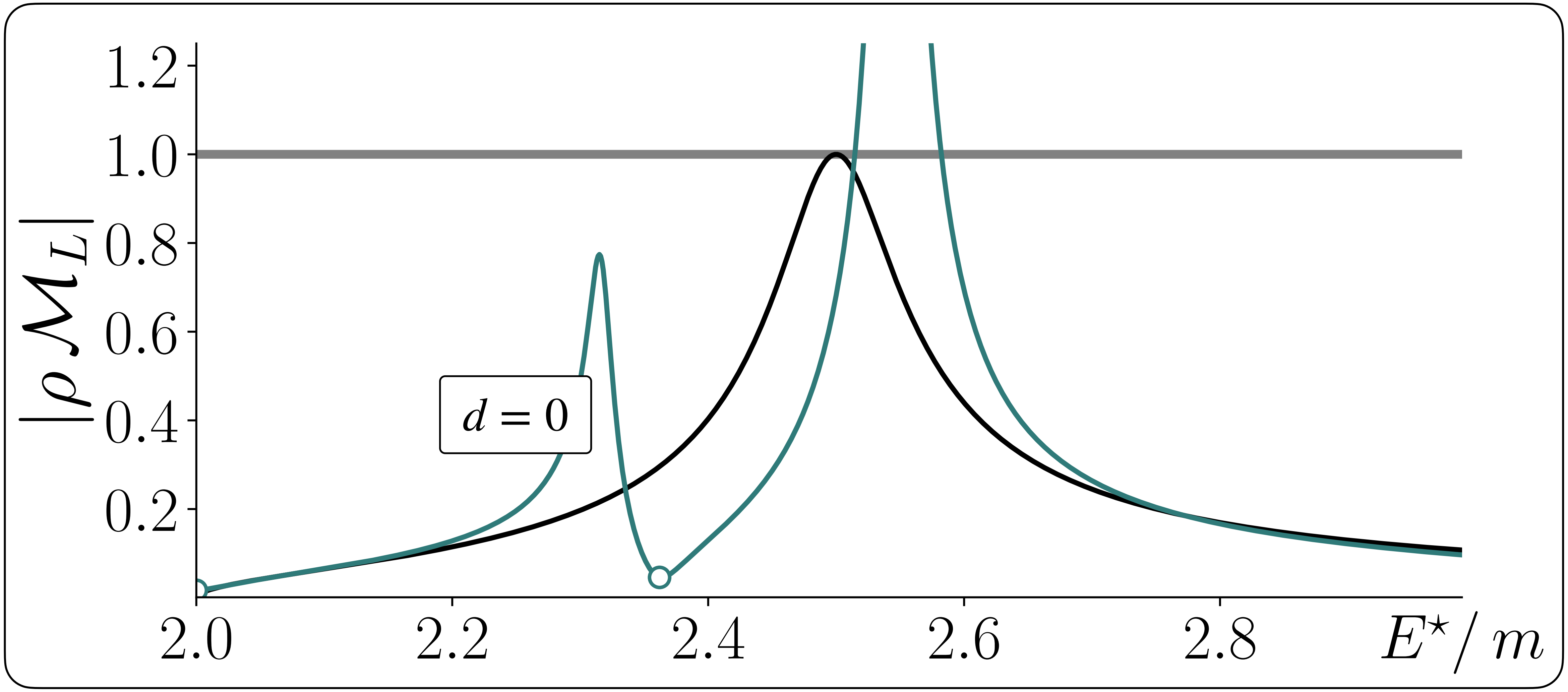
In a finite-volume, the spectrum and all observables depend on the total momentum $P = 2\pi d/L$, where d is discrete.

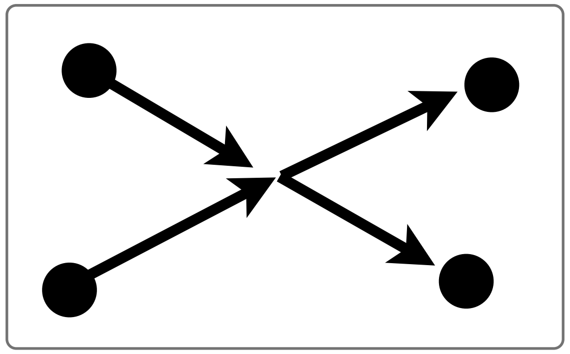




Purely “hadronic” amplitude

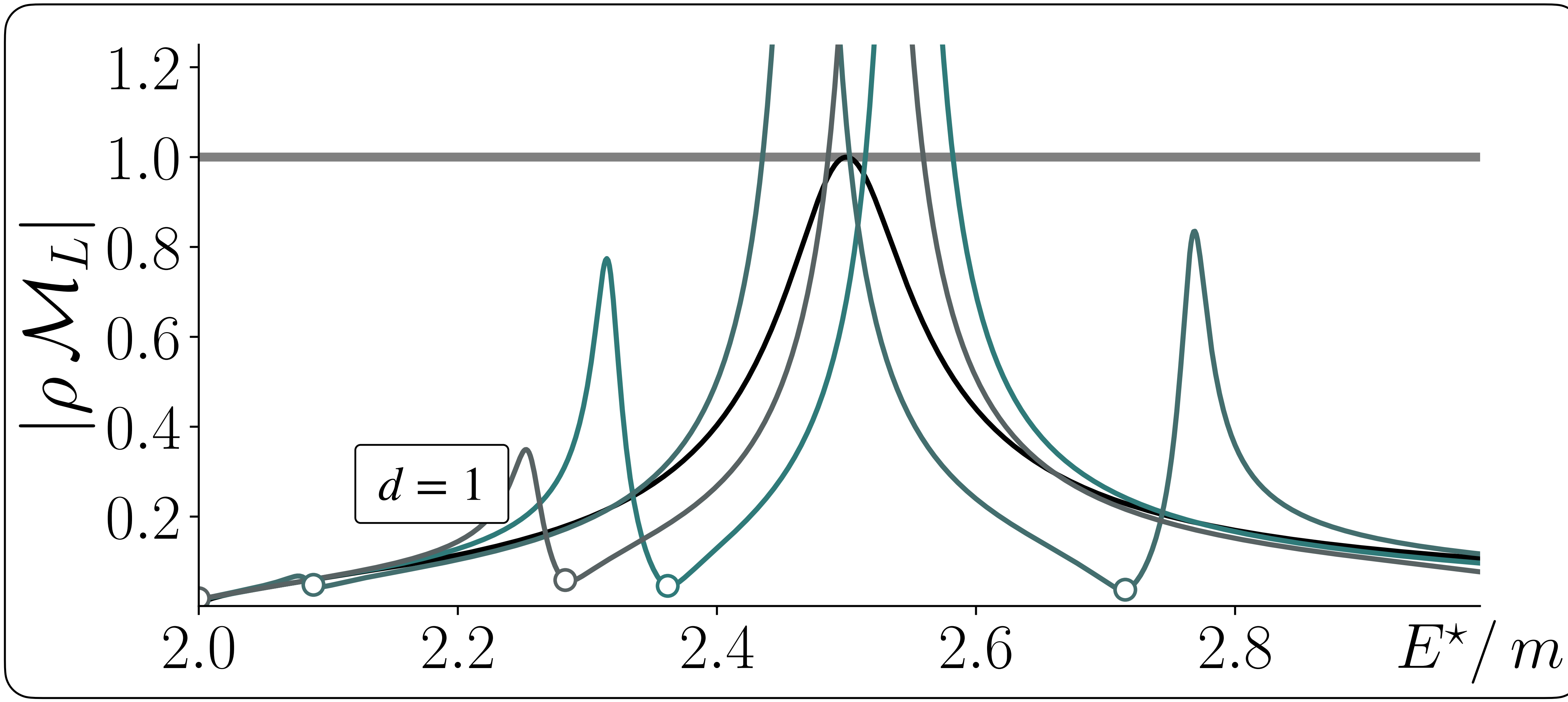
In a finite-volume, the spectrum and all observables depend on the total momentum $\mathrm{P} = 2\pi d/L$, where d is discrete.

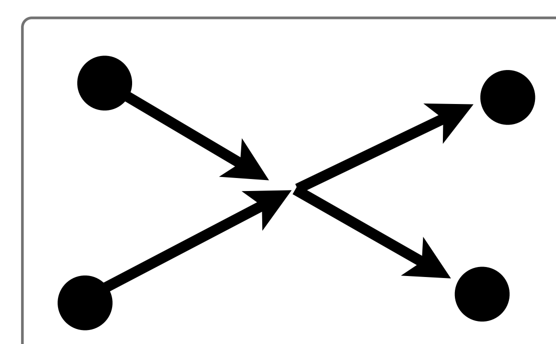




Purely "hadronic" amplitude

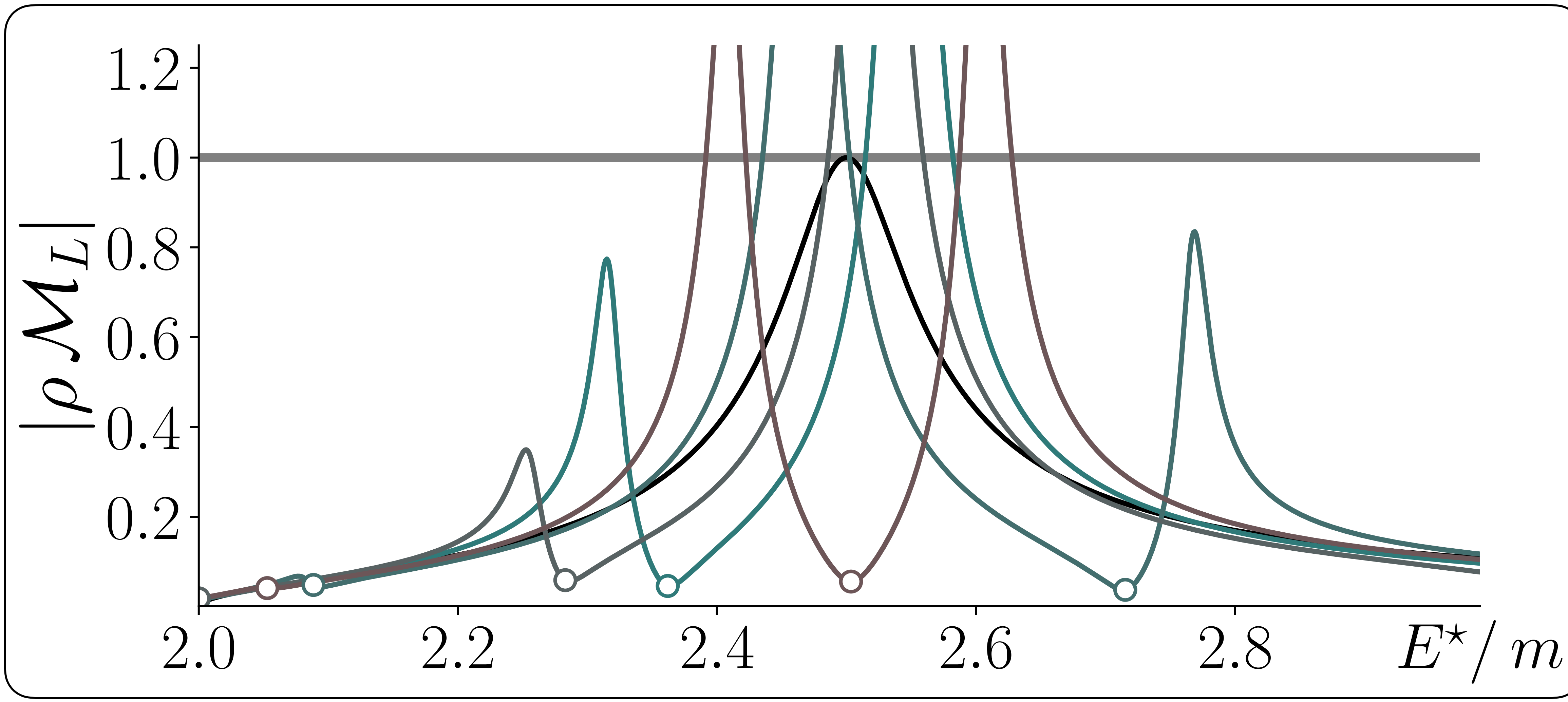
In a finite-volume, the spectrum and all observables depend on the total momentum $\mathrm{P} = 2\pi d/L$, where d is discrete.

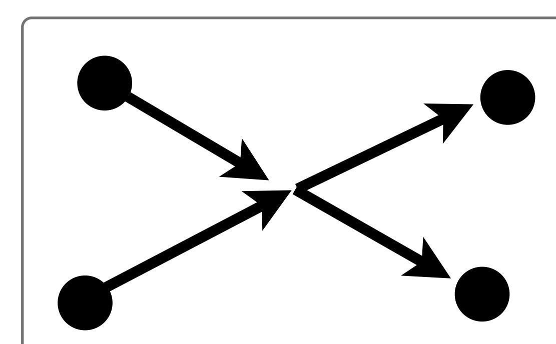




Purely "hadronic" amplitude

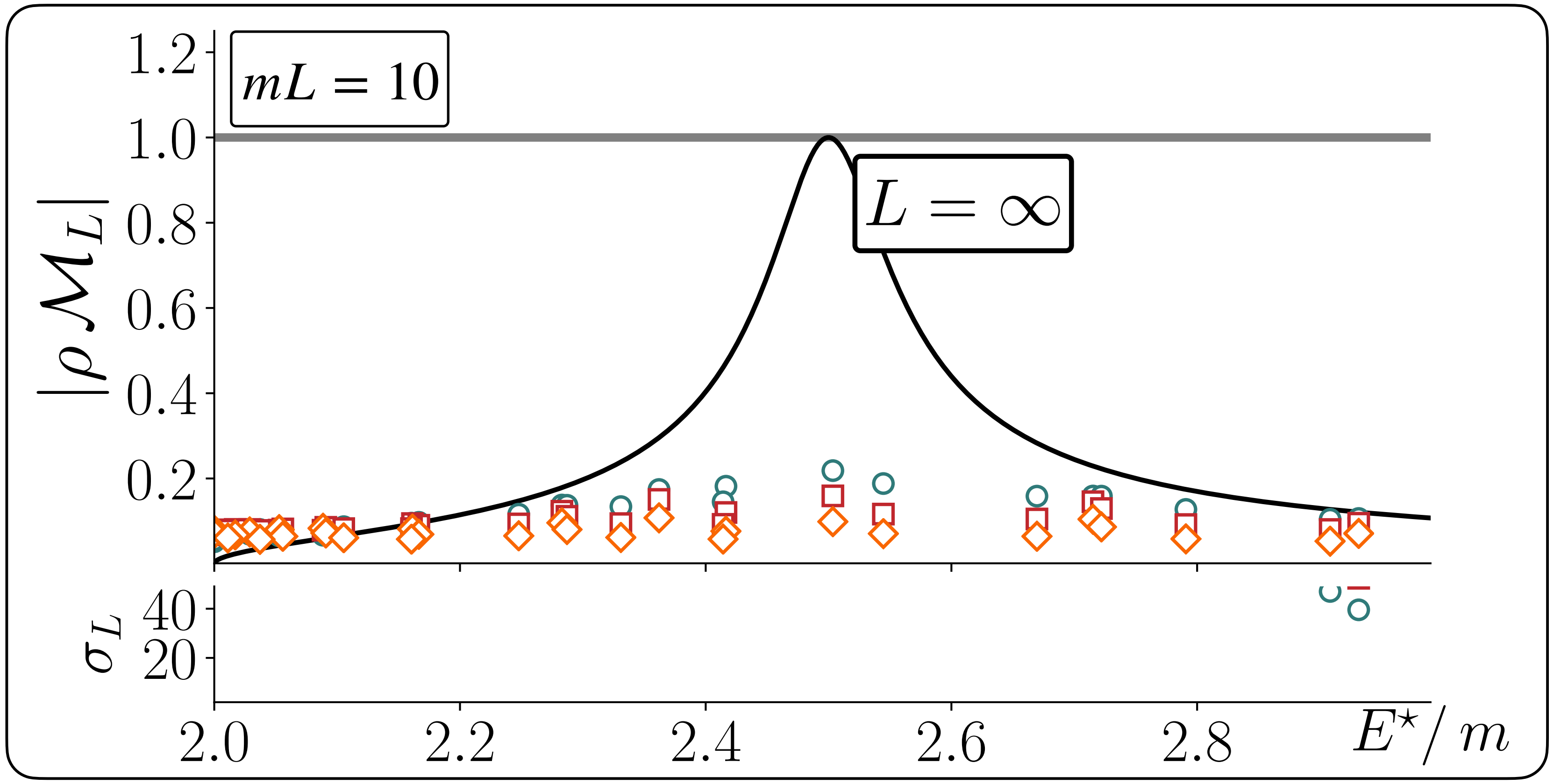
In a finite-volume, the spectrum and all observables depend on the total momentum $\mathrm{P} = 2\pi d/L$, where d is discrete.





Purely "hadronic" amplitude

(ϵ, L) - dependence



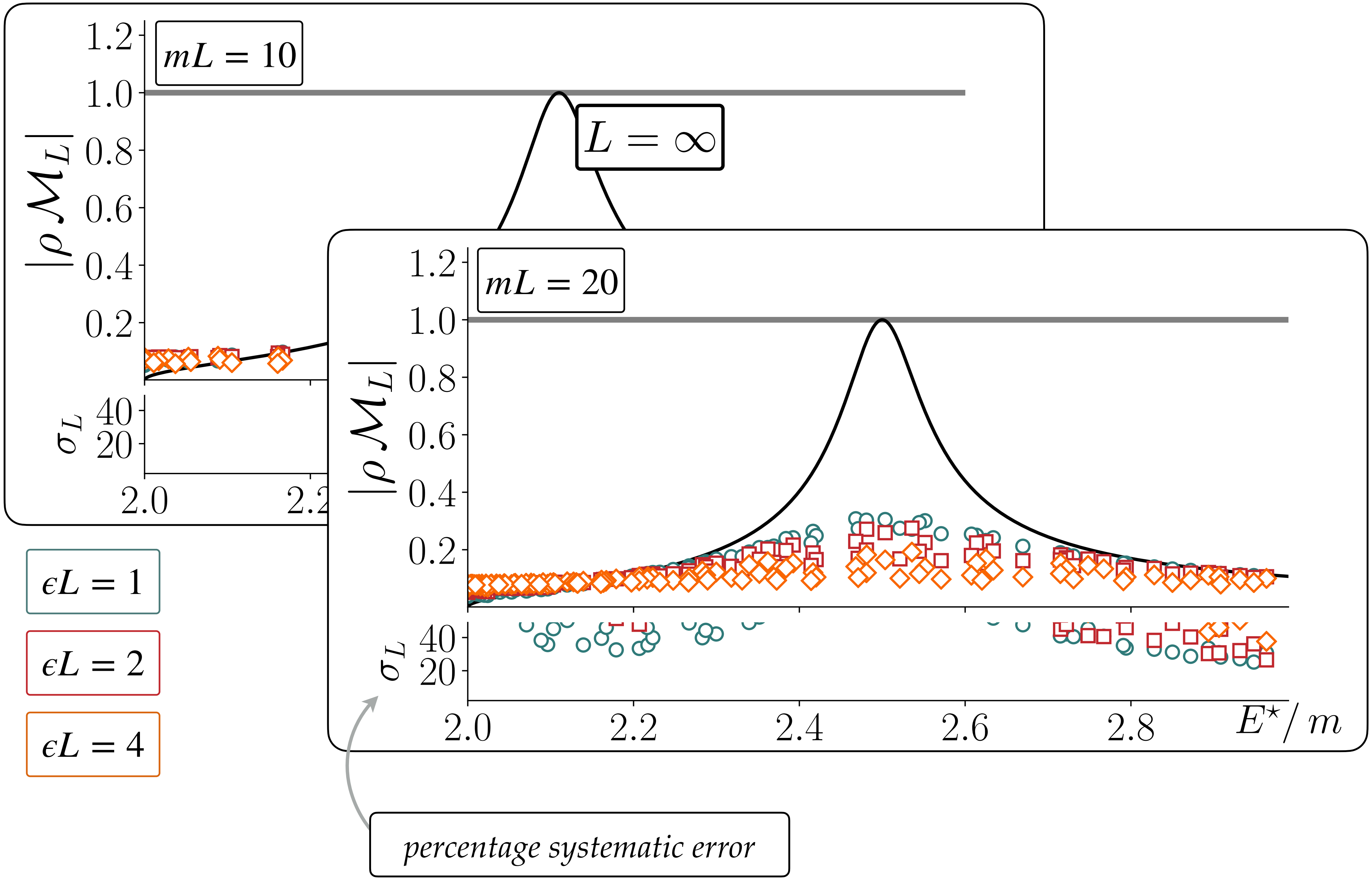
$\epsilon L = 1$

$\epsilon L = 2$

$\epsilon L = 4$

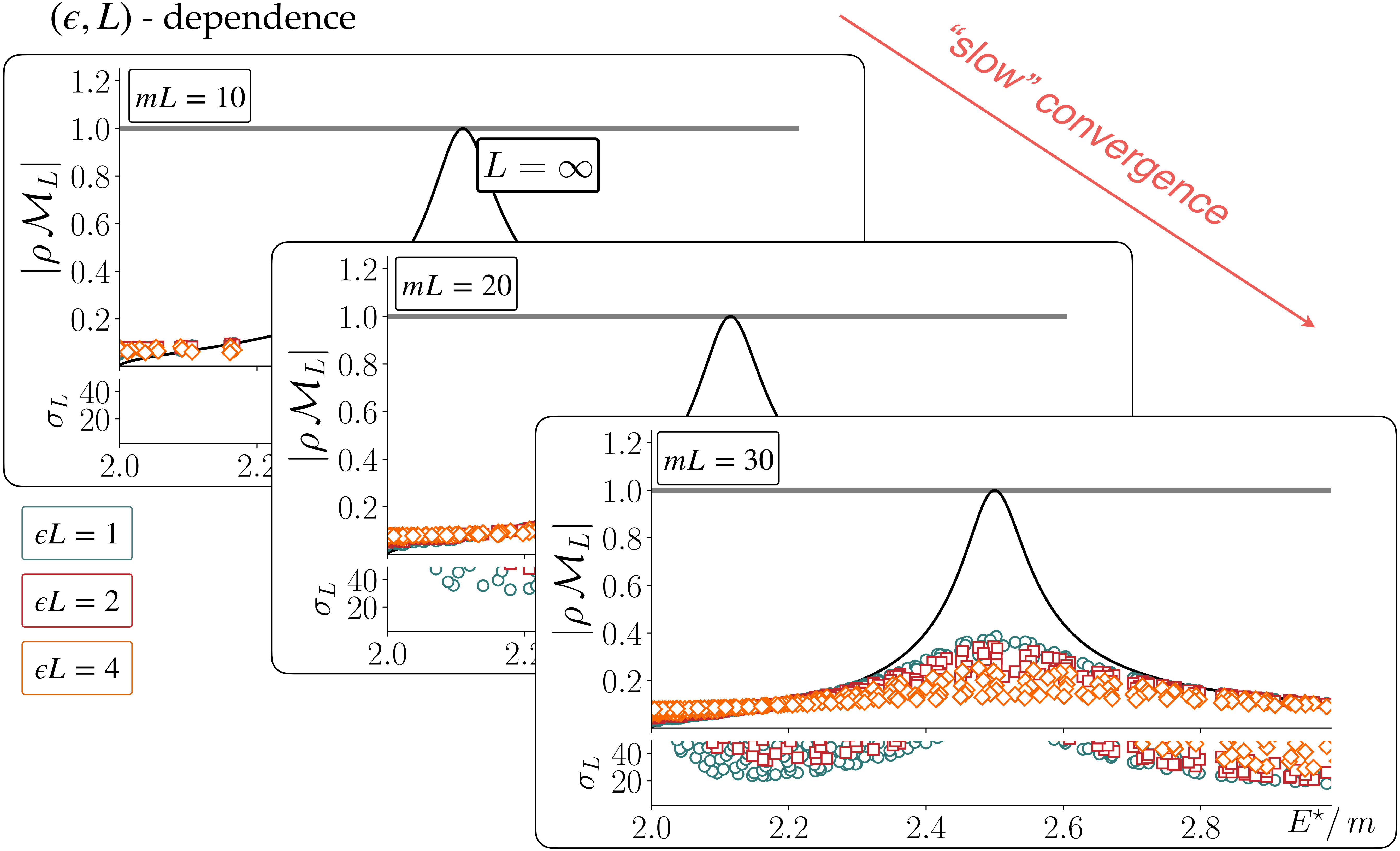
Purely "hadronic" amplitude

(ϵ, L) - dependence

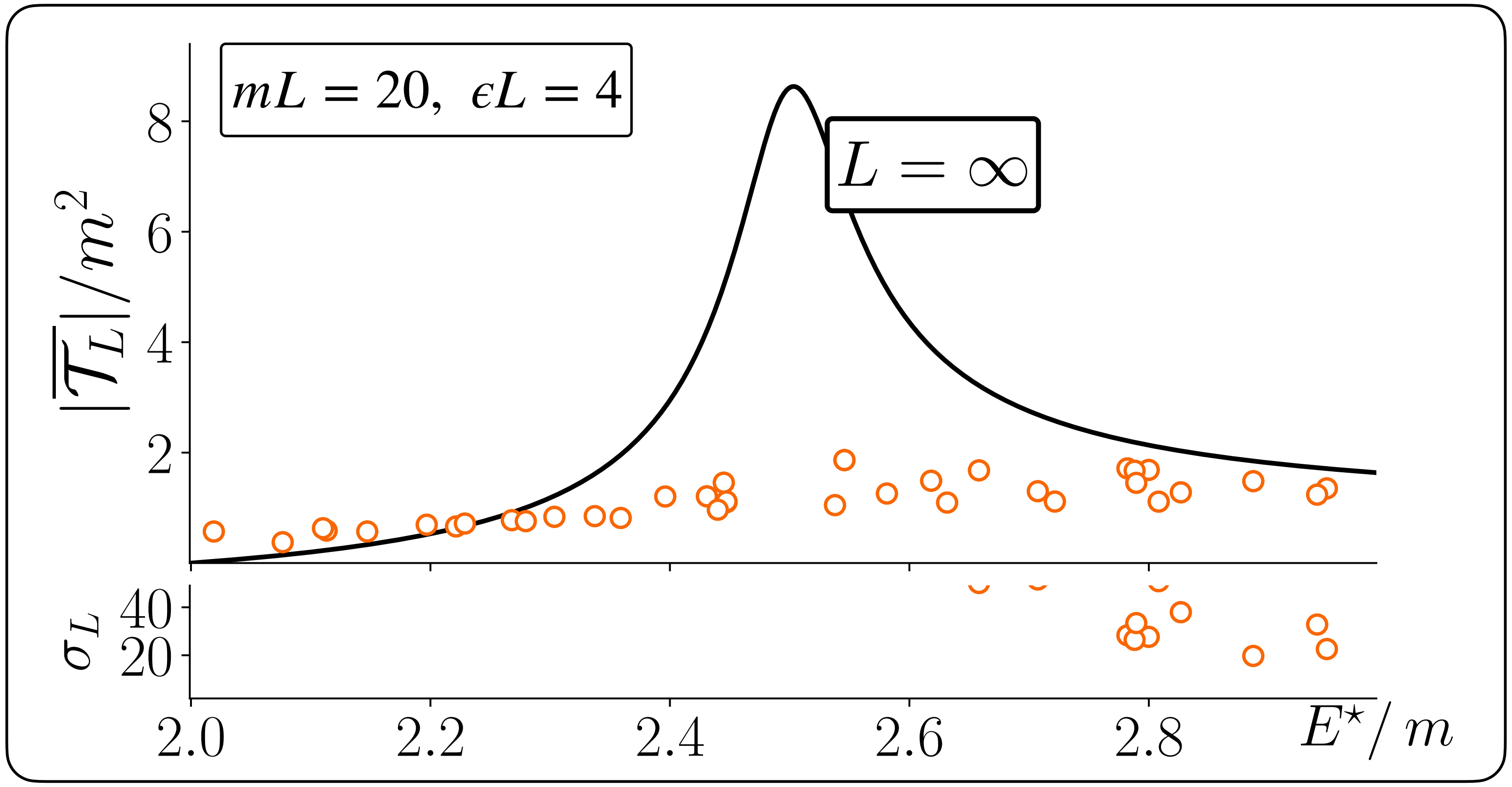


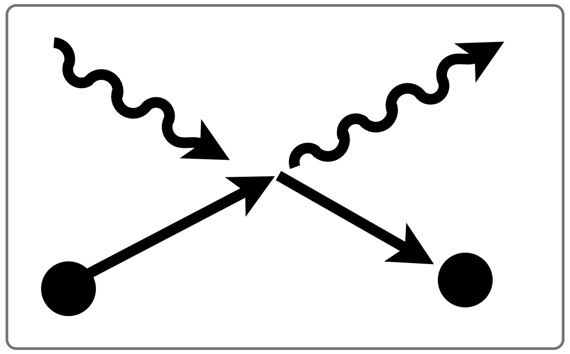
Purely "hadronic" amplitude

(ϵ, L) - dependence



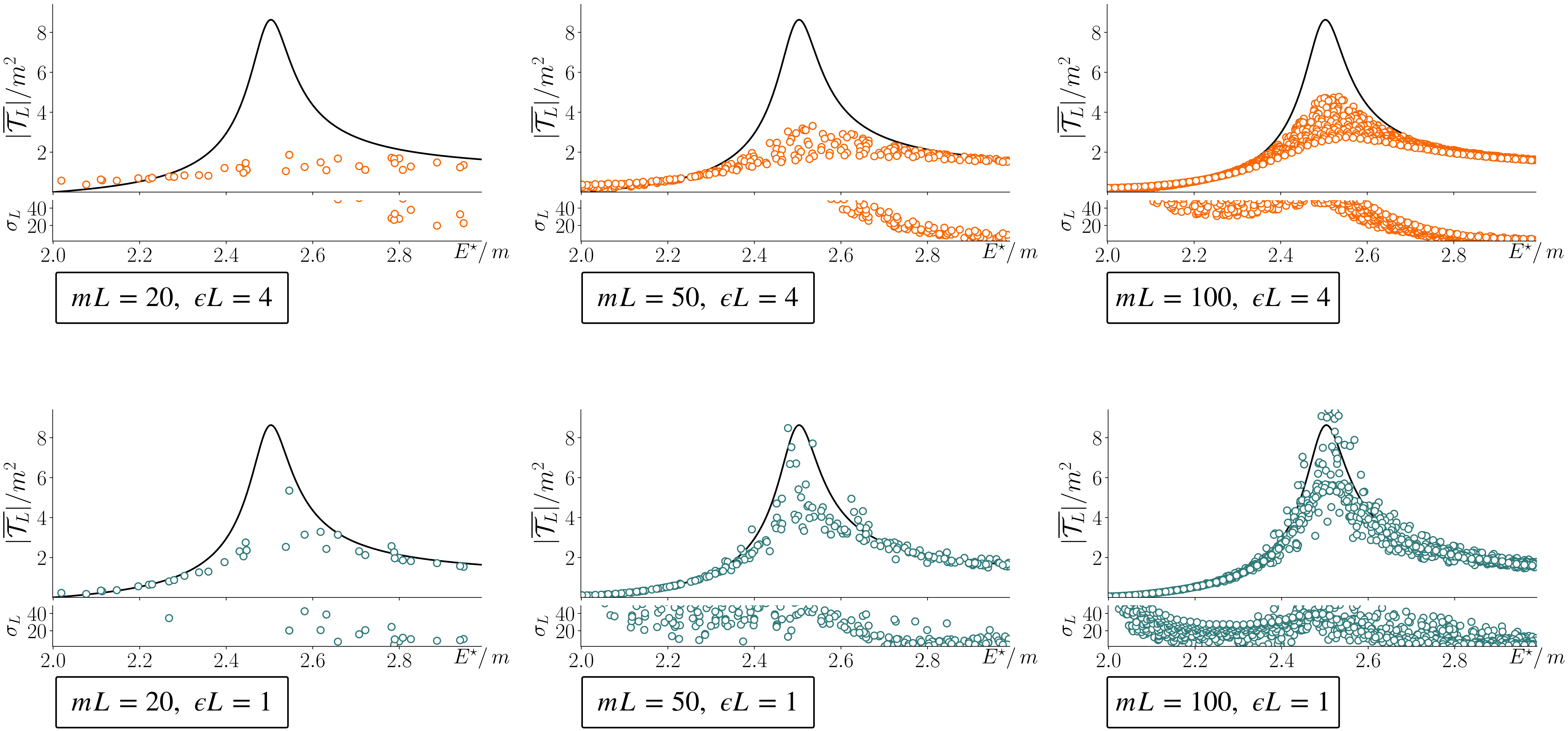
Compton-like amplitudes





$$\boxed{\mathcal{T}_L = \mathcal{T} - \mathcal{H}(s, Q^2)\, \frac{1}{F^{-1}(P,L) + \mathcal{M}(s)}\, \mathcal{H}'(s, Q^2_{if})}$$

Compton-like amplitudes

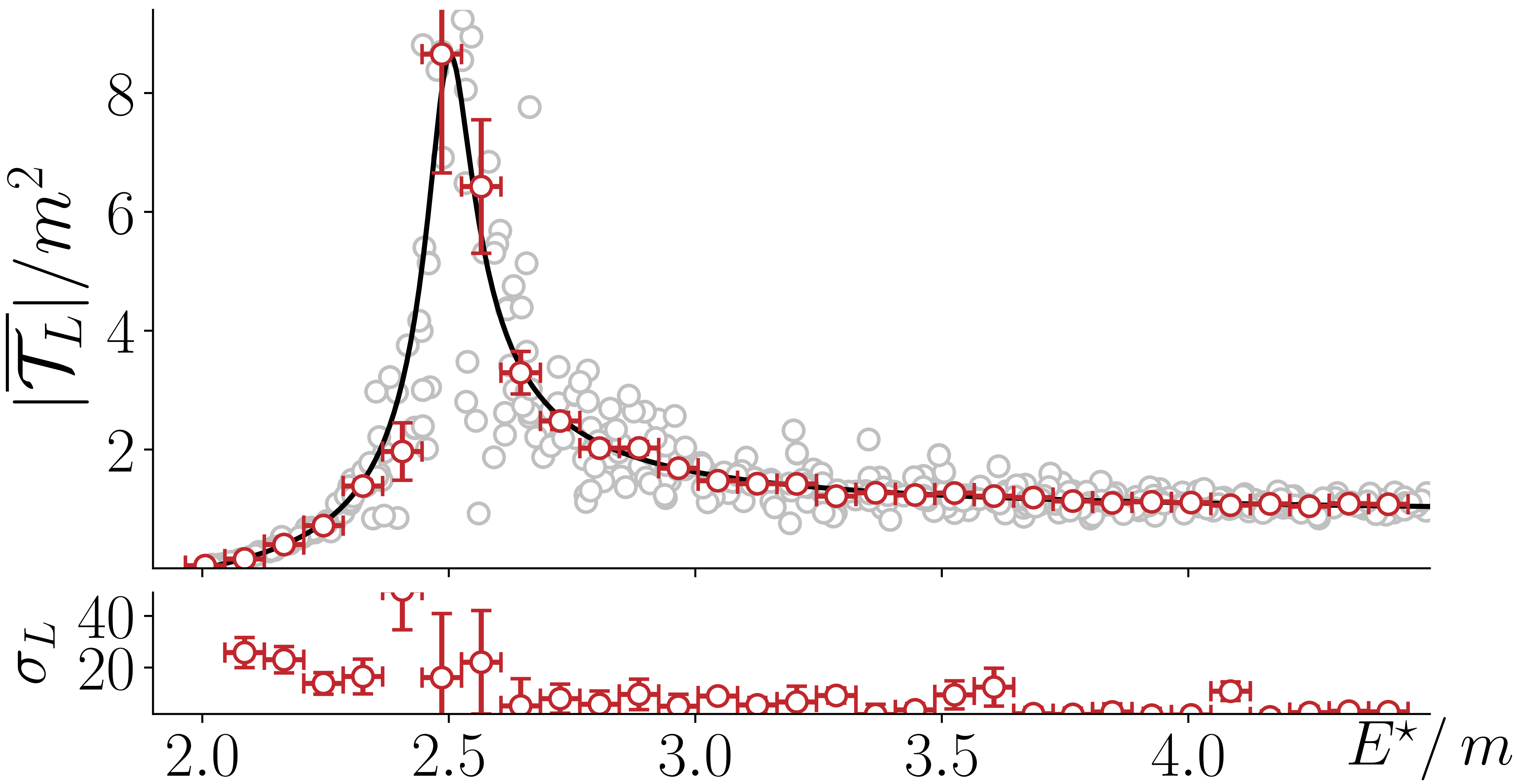


$$\boxed{\mathcal{T}_L = \mathcal{T} - \mathcal{H}(s, Q^2)\, \frac{1}{F^{-1}(P, L) + \mathcal{M}(s)}\, \mathcal{H}'(s, Q^2_{if})}$$

Toy model results

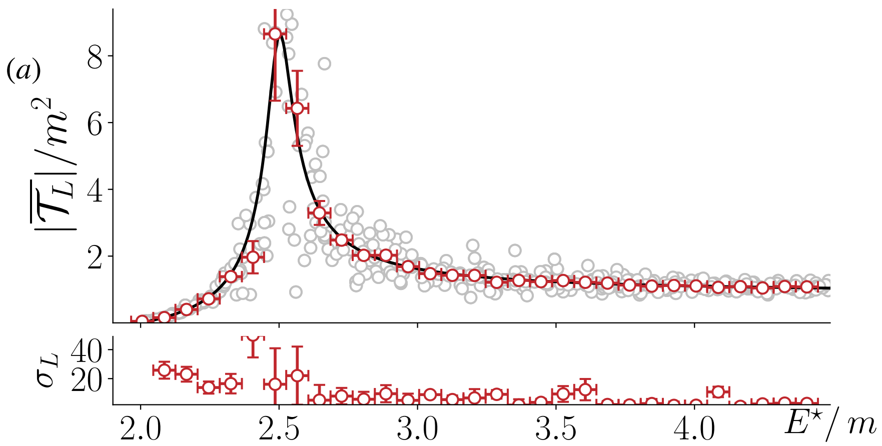
By averaging over $mL = [20,25,30]$ boost with $d \leq mL$,

and binning in energy and virtualities…

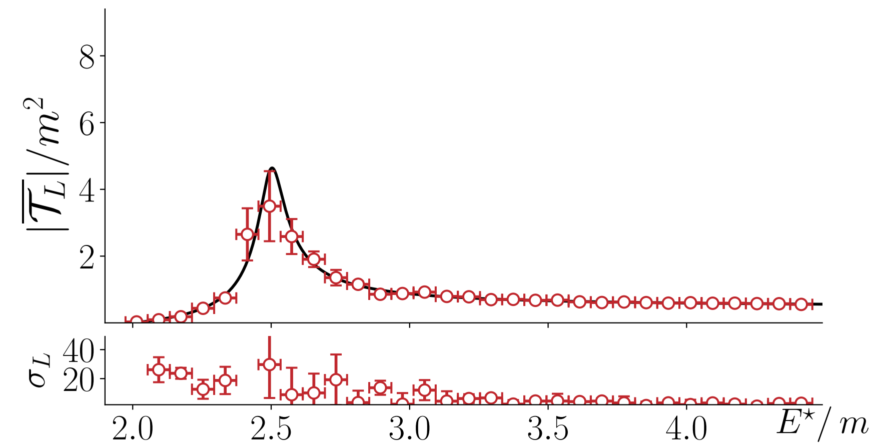


Toy model results

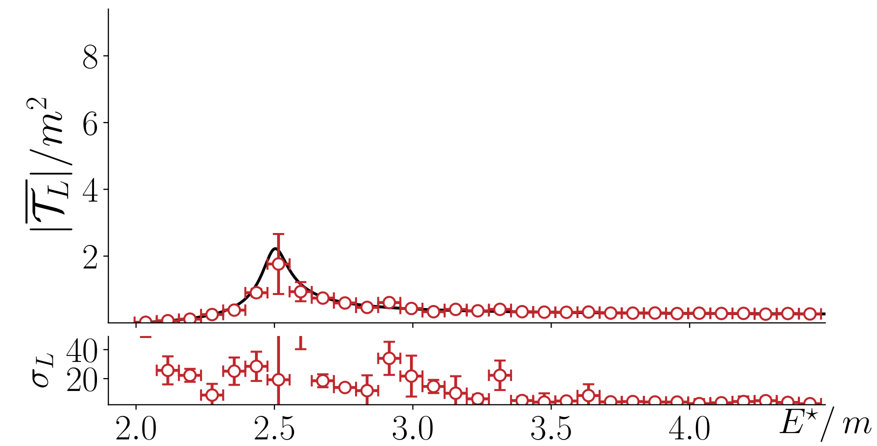
By averaging and binning…



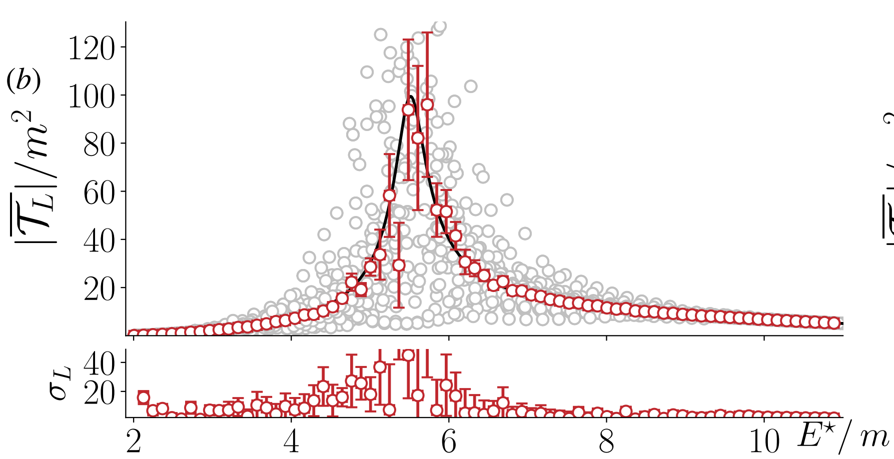
$Model\ 1,\ \overline{Q^2} = 2\, m^2$



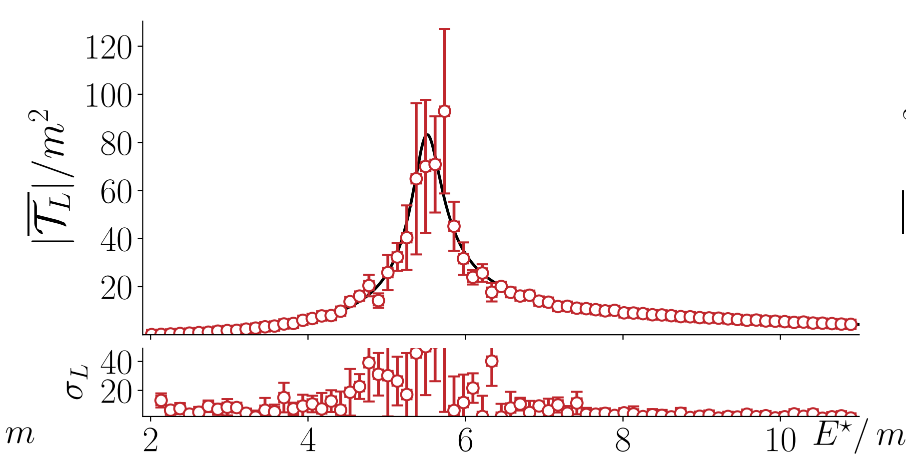
$Model\ 1,\ \overline{Q^2} = 5\, m^2$



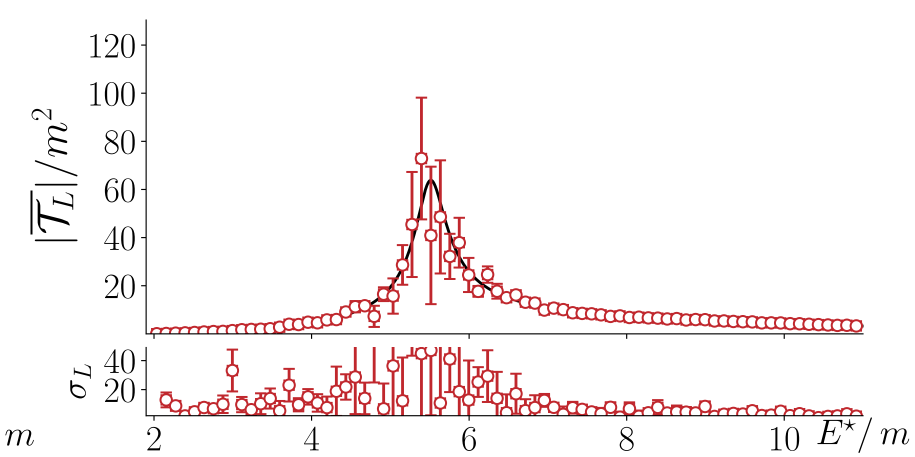
$Model\ 1,\ \overline{Q^2} = 10\, m^2$



$Model\ 2,\ \overline{Q^2} = 2\, m^2$



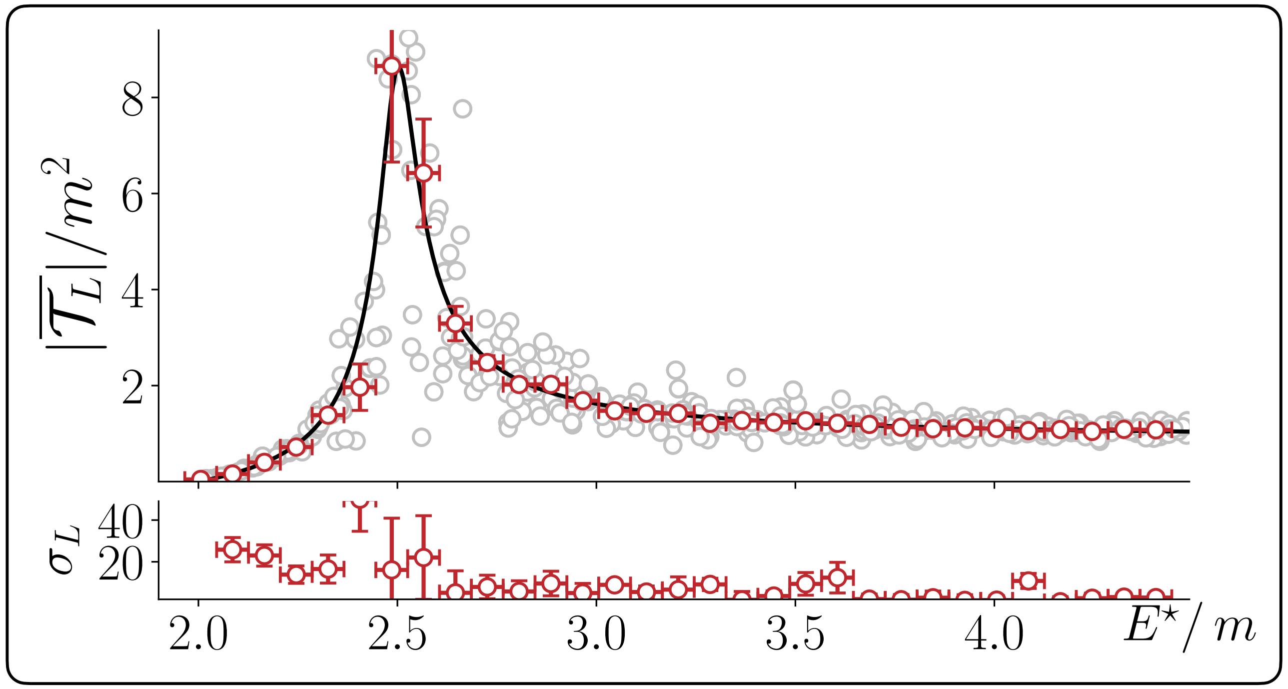
$Model\ 2,\ \overline{Q^2} = 5\, m^2$

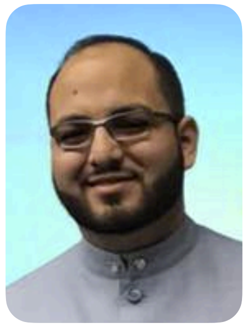


$Model\ 2,\ \overline{Q^2} = 10\, m^2$

Summary and Outlook

- ☑ Inclusive scattering observables are in principle accessible
- ☑ No sophisticated formalism is needed
  - ☑ existing formalism serves as diagnostic tool
- ☐ Optimal choices of smearing/binning/boosting?
- ☐ Inelastic processes?
- ☐ Does life get harder or easier in 3 + 1D?
- ☐ Test on toy theory [quantum simulation vs. lattice]





Juan Guerrero
JLab

Alex Sturzu
New College

Max Hansen
Edinburgh

Spectra

- Infinite-volume Hamiltonian

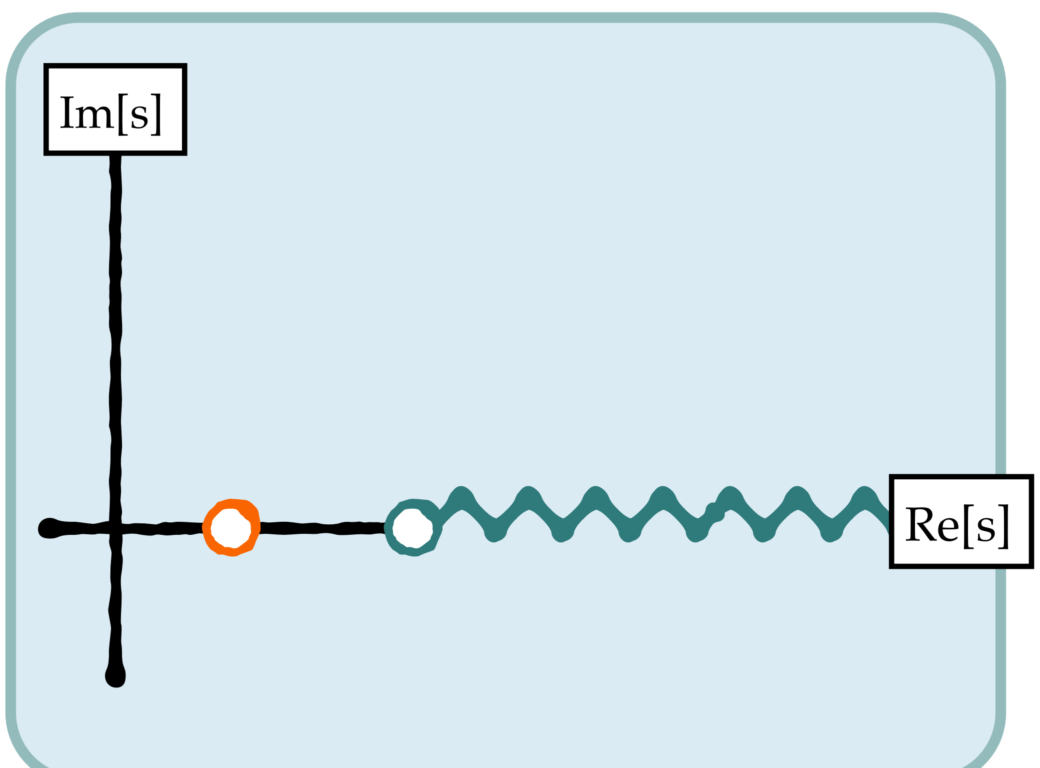
$$\hat{H}_\infty = \int_\infty d^3x \mathscr{H}$$

- Asymptotic states satisfy:

$$\hat{H}_{\infty,0} | p_1, p_2, \cdots p_n \rangle_0 = | p_1, p_2, \cdots p_n \rangle_0 \sum_{i=0}^{n} \sqrt{p_i^2 + m^2}$$

This is what the interaction picture buys us. At asymptotically large separations in space/time, we have nearly free states — which are the only ones we can generally define. 🤓

- No gap between states [continuum of states]



Spectra

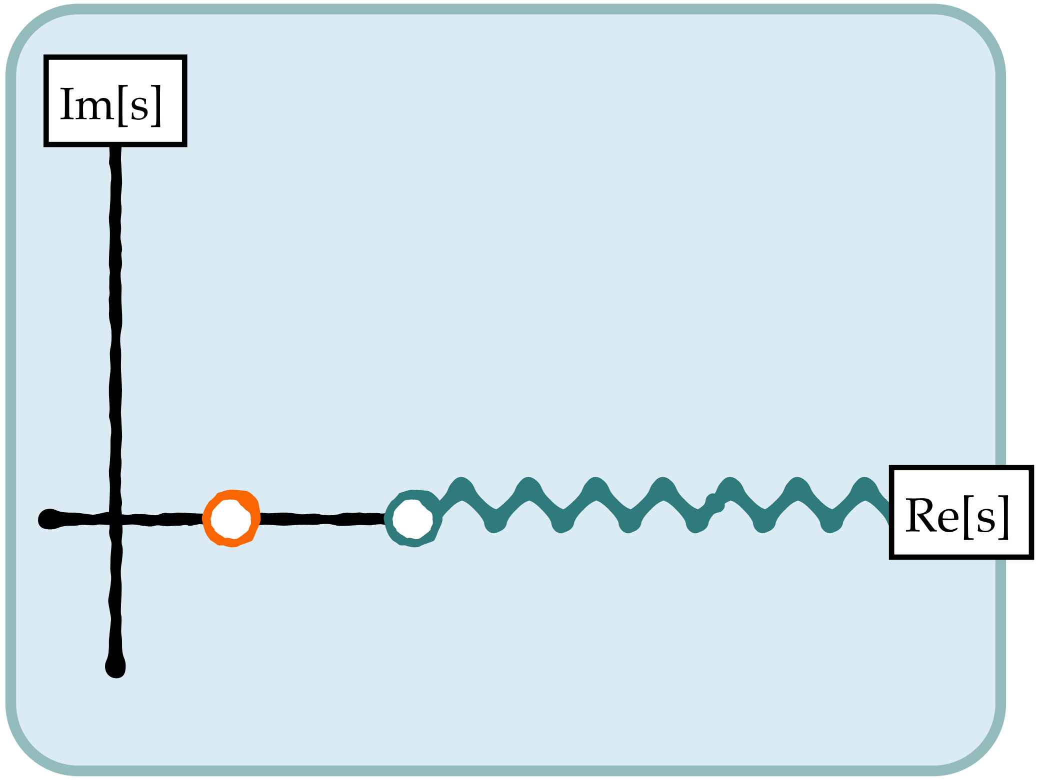
- Infinite-volume Hamiltonian

$$\hat{H}_\infty = \int_\infty d^3x\mathscr{H}$$

- Asymptotic states satisfy:

$$\hat{H}_{\infty,0}|p_1, p_2, \cdots p_n\rangle_0 = |p_1, p_2, \cdots p_n\rangle_0 \sum_{i=0}^{n} \sqrt{p_i^2 + m^2}$$

- No gap between states [continuum of states]



- Finite-volume Hamiltonian

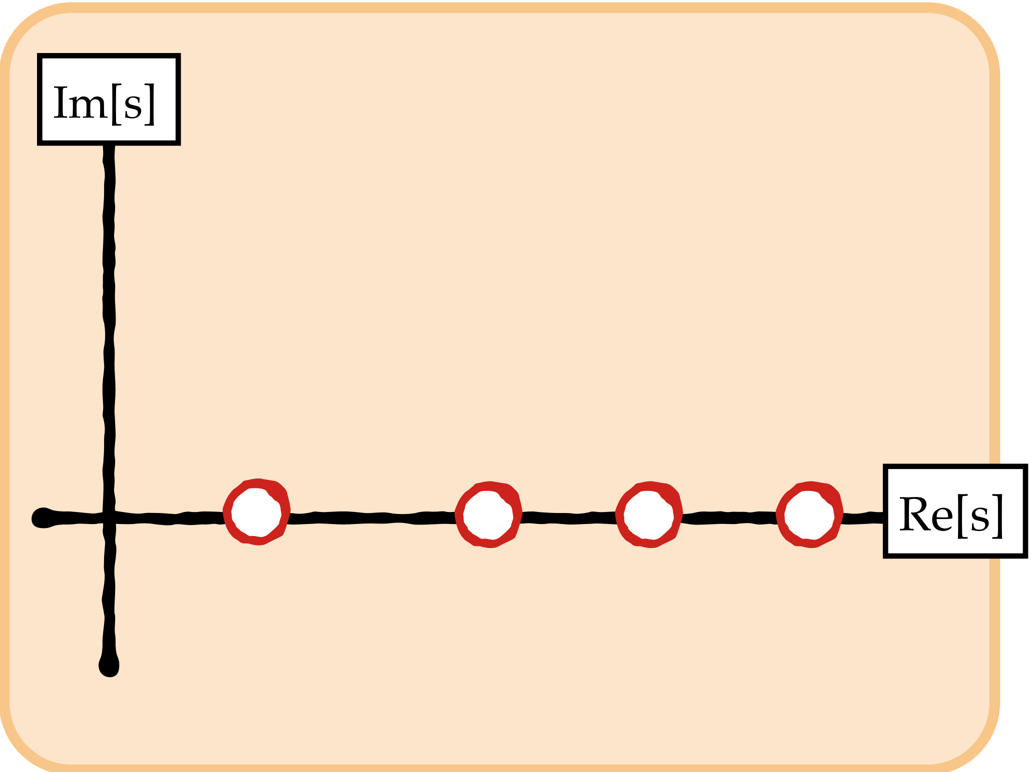
$$\hat{H}_L = \int_{V=L^3} d^3x\mathscr{H}$$

- No asymptotic states:

$$\hat{H}_L|n\rangle_L = |n\rangle_L E_n$$

- Intrinsic gap between states:

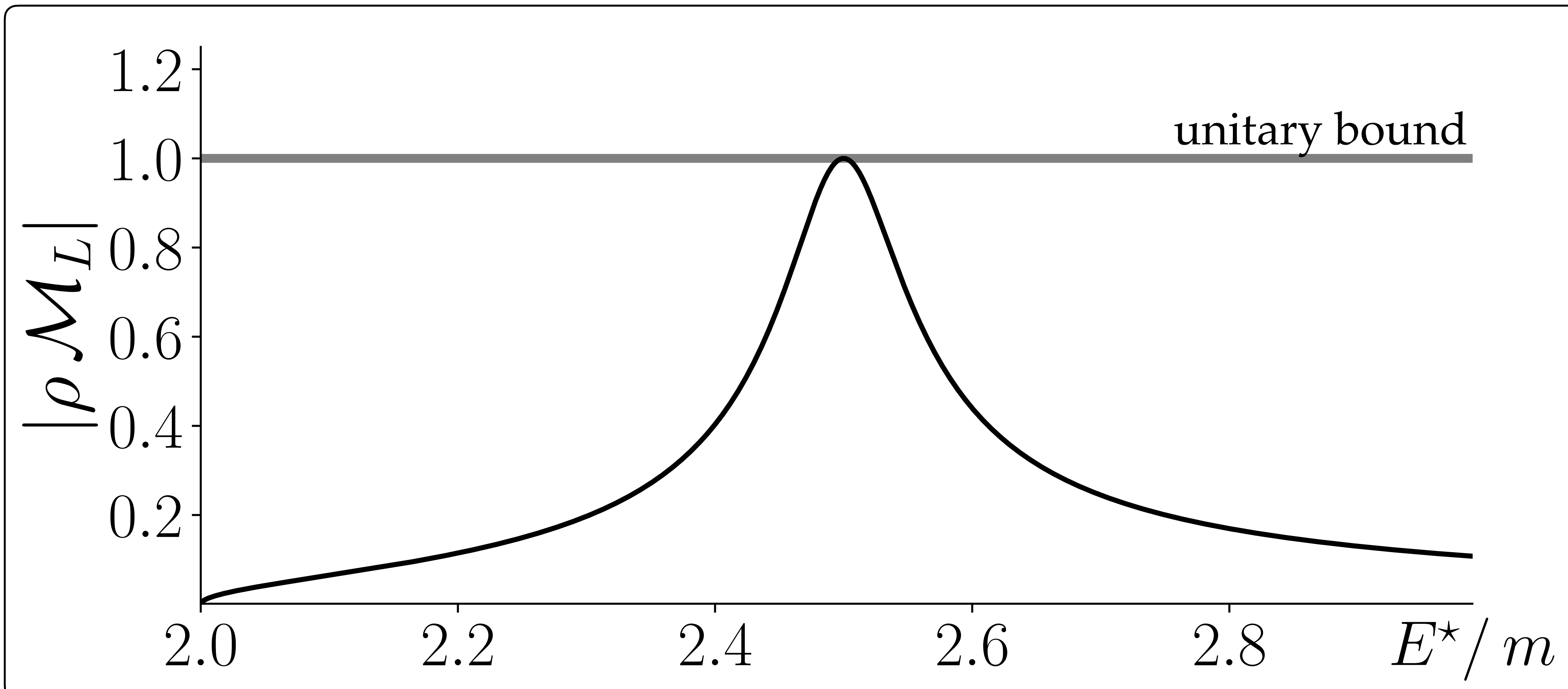
$$E_{n+1}(L) - E_n(L) \sim \frac{1}{L^\#}$$



Toy model *no quantum simulations were performed*

Scattering amplitude parametrization

$$\mathcal{M} = \frac{1}{\mathcal{K}^{-1} - i\rho}, \qquad \mathcal{K}(s) = m^2 q^2 \frac{g^2}{m_R^2 - s}$$



$E^\star = \sqrt{s}$

Toy model *no quantum simulations were performed*

Scattering amplitude parametrization

$$\mathcal{M} = \frac{1}{\mathcal{K}^{-1} - i\rho}, \qquad \mathcal{K}(s) = m^2 q^2 \frac{g^2}{m_R^2 - s}$$

All-order perturbation theory implies that the Compton-like amplitudes satisfy

$$\mathcal{T}(s, Q^2, Q^2_{if}) = w_2(s, Q^2, Q^2_{if}) + \mathcal{A}_{12}(s, Q^2)\mathcal{M}(s)\mathcal{A}'_{21}(s, Q^2_{if})$$

real and "smooth" for space-like virtualities

A perfectly reasonable parametrization:

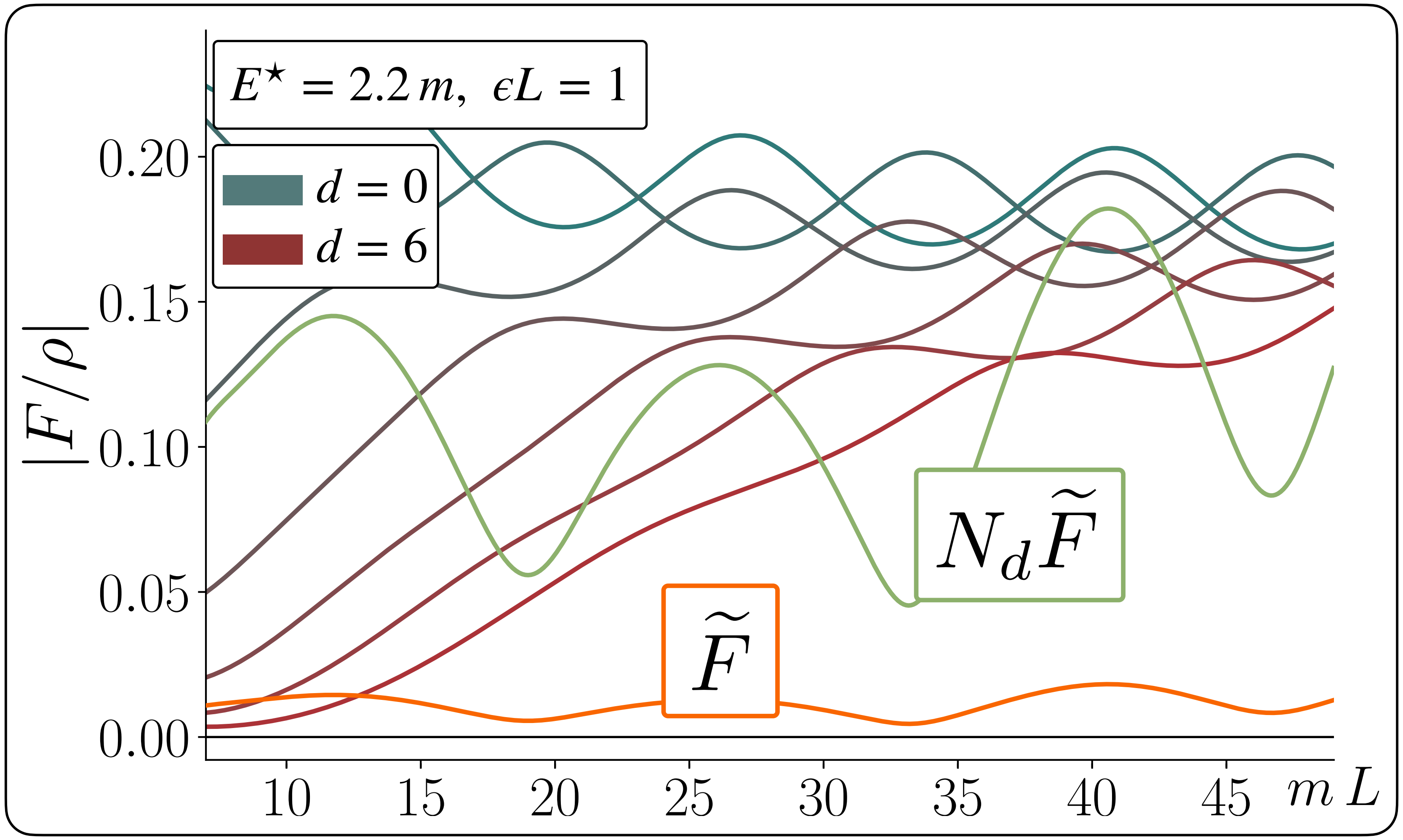
$$w_2(s, Q^2, Q^2_{if}) = 0, \qquad \mathcal{A}_{12}(s, Q^2) = \mathcal{A}'_{21}(s, Q^2) = \frac{1}{1 + Q^2/M^2}$$

Boost averaging

The volume effects are encoded in $F(P, L)$, which is not a Lorentz scalar.

Asymptotic behavior $F \sim e^{-L\epsilon\alpha_0}(-1)^{\boldsymbol{d}}$

Averaging over different boosts, volume effects are expected to reduce.



$$\boxed{\mathcal{T}_L = \mathcal{T} - \mathcal{H}(s, Q^2)\,\frac{1}{F^{-1}(P, L) + \mathcal{M}(s)}\,\mathcal{H}'(s, Q_{if}^2)}$$