Coordinate Space Behavior of PDFs and GPDs

Abha Rajan
Old Dominion University

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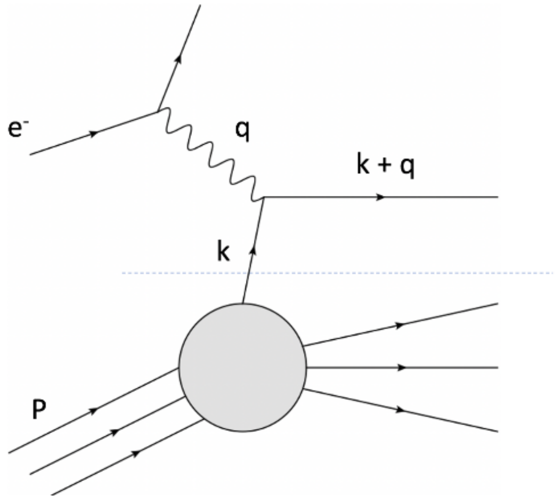
In collaboration with Simonetta Liuti, University of Virginia

Probing hadron structure, Experiment vs Lattice

- Experimental observables are described using momentum degrees of freedom.
- The physical objects measured in a deep inelastic scattering experiment are the structure functions.
- The process is described by the parton model in the limit of Q^2 going to infinity and in terms of coordinate space this amounts to z^2 going to zero – the probed particle is on the lightcone.
- In contrast, lattice QCD observables are calculated in coordinate space and are separated by a space like distance.



$$f(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p \mid \bar{\psi}(0)\,\gamma^+\,\mathcal{U}(0,z^-)\psi\left(z^-\right) \mid p\rangle\big|_{z^+=0,z_T=0}$$

Experiment: momentum space

Fourier transform

Lattice QCD: Coordinate space

The Goal

- Reconstructing PDFs and GPDs using inputs from lattice QCD
  - Local operators → Mellin moments
  - Non local operators → Off the light cone

Outline

- Definitions
- Pseudo PDFs and GPDs in a spectator model
- Reconstructing PDFs and GPDs using Mellin moments
- Conclusions

Definitions

On the light cone

Ioffe time $P \cdot z = \nu$

$$f(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p \mid \bar{\psi}(0)\gamma^+ \mathcal{U}(0,z^-)\psi(z^-) \mid p\rangle\Big|_{z^+=0, z_T=0}$$

Covariant form

$$\langle p|\bar{\psi}(0)\gamma^\alpha \mathcal{U}\psi(z)|p\rangle = M^\alpha((pz), z^2)$$

$$M^\alpha((pz), z^2) = 2p^\alpha \mathcal{M}((pz), z^2) + 2z^\alpha \mathcal{M}_z((pz), z^2)$$

Taking α = + and $z^+ = z_T = 0$,

$$\langle p|\bar{\psi}(0)\gamma^+ \mathcal{U}\psi(z)|p\rangle = 2p^+ \mathcal{M}((pz), 0)$$

X Ji (2013)

Radyushkin (2017)

Lin et al (2014)
Alexandrou et al (2014)

Orginos et al (2017)

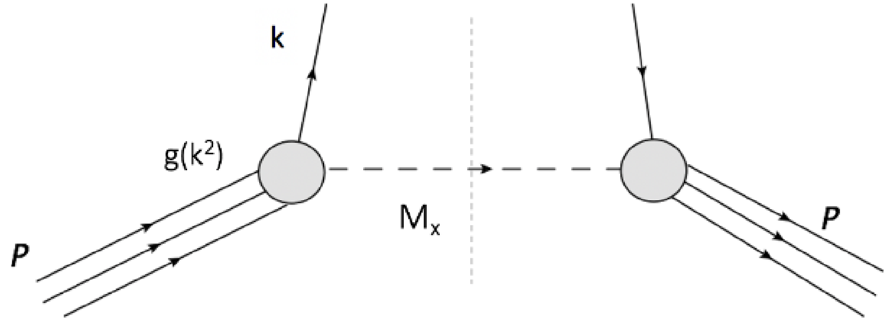
Non local operators on the lattice

- On the lattice one is restricted to taking $z^0 = 0$

$$M^\alpha((pz), z^2) = 2p^\alpha \mathcal{M}((pz), z^2) + 2z^\alpha \mathcal{M}_z((pz), z^2)$$

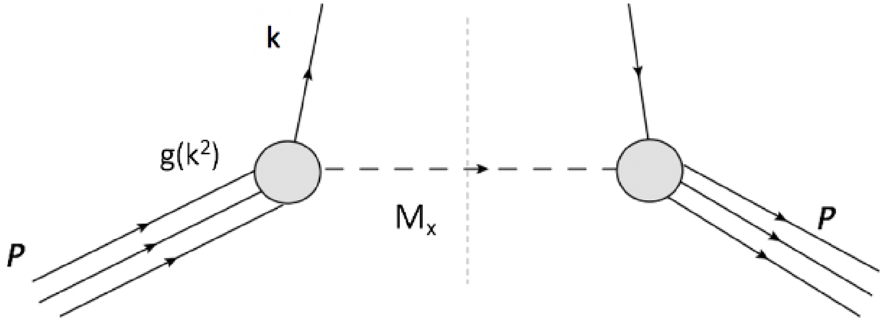
- One way to single out the piece that corresponds to the PDF is to take $\alpha = 0$

PDFs in a spectator model



$$f(x) = \int \frac{d^2k_T\, dk^-}{(2\pi)^4} \frac{(-ig(k^2))^2\, Tr\left[i(\not{k}+m)\gamma^+ i(\not{k}+m)(\not{P}+M)\right]}{((P-k)^2 - M_X^2 + i\epsilon)(k^2 - m^2 + i\epsilon)^2}$$

Off the light cone PDFs



$$f(k_3, P_3) = \int \frac{d^2k_T\, dk^0}{(2\pi)^4} \frac{(-ig(k^2))^2\, Tr\left[i(\not{k}+m)\gamma^0 i(\not{k}+m)(\not{P}+M)\right]}{((P-k)^2 - M_X^2 + i\epsilon)(k^2 - m^2 + i\epsilon)^2}$$

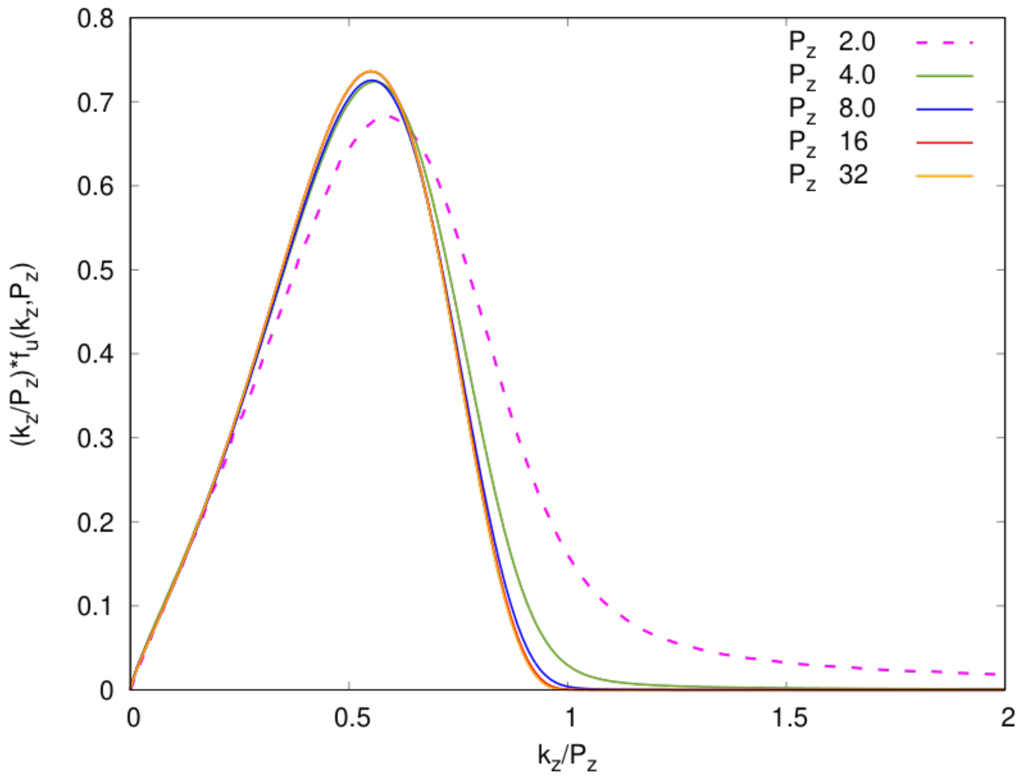
- Quasi PDFs are defined at constant P_z

Gamberg et al (2015)
Bhattacharya et al (2018)

- Pseudo PDFs are defined at constant z^2

Off the light cone PDFs

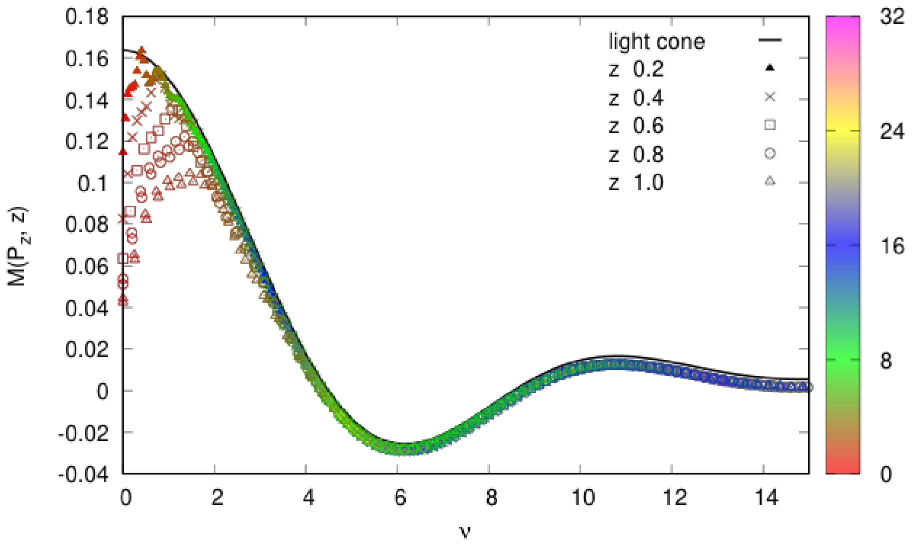
Quasi PDFs - defined at constant P_z



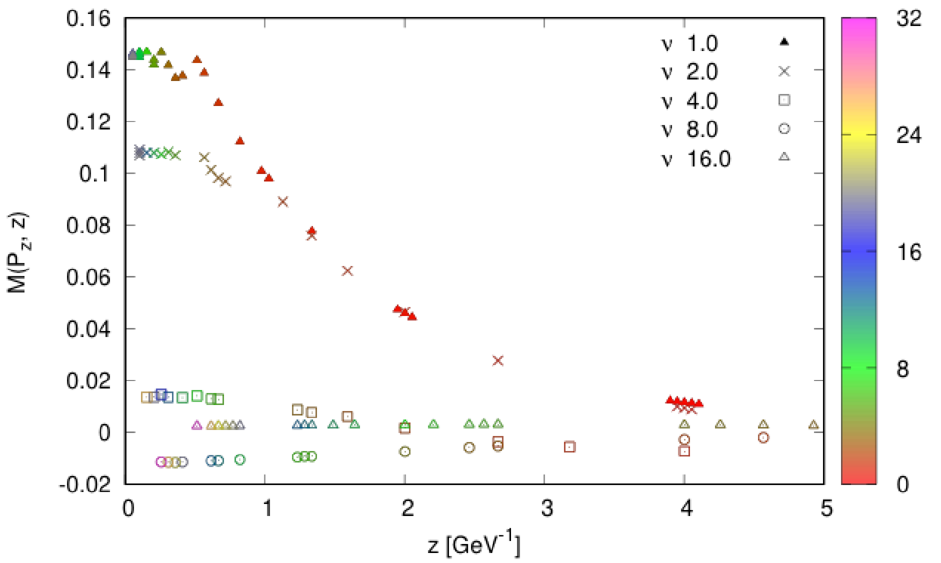
P_z in GeV

Off the light cone PDFs

Points at fixed z. For lower values of Ioffe time, large z contributions come from lower momentum P_z (tagged by color).



Points at fixed Ioffe time. For lower values of Ioffe time, lower momentum P_z (tagged by color) points don't scale.



P_z in GeV, z in GeV^{-1}

Ioffe time $P \cdot z = \nu$

Off the light cone in a rotated frame

Going off the light cone due to transverse spatial separation.

$$\langle p|\bar{\psi}(0)\gamma^\alpha \mathcal{U}\psi(z)|p\rangle = M^\alpha((pz), z^2)$$

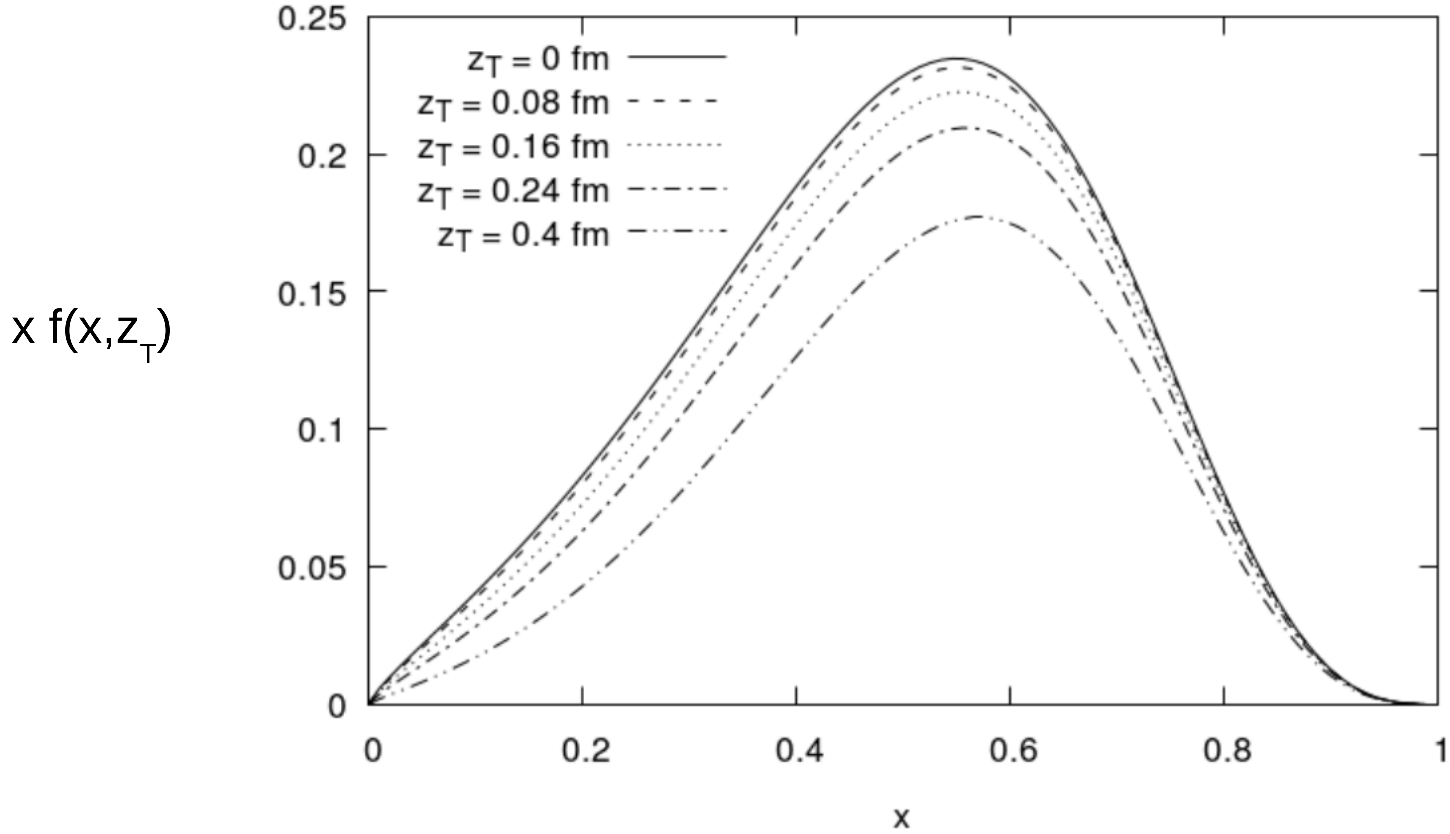
$$M^\alpha((pz), z^2) = 2p^\alpha \mathcal{M}((pz), z^2) + 2z^\alpha \mathcal{M}_z((pz), z^2)$$

If $z^+ = 0$,

$$\langle p|\bar{\psi}(-z/2)\gamma^+ \mathcal{U}\psi(z/2)|p\rangle = 2p^+ \mathcal{M}((pz), z^2) \qquad z = (0, z^-, \mathbf{z_T})$$

The above is essentially the Fourier transform of a Transverse Momentum Dependent Distribution.

Off the light cone in a rotated frame



Off forward off the light cone

Going off forward introduces another vector Δ = p' - p

$$\langle p'|\bar{\psi}(-z/2)\gamma^\alpha \mathcal{U}\psi(z/2)|p\rangle = M^\alpha((\Delta z),(pz),z^2)$$

$$M^\alpha((\Delta z),(pz),z^2) = 2p^\alpha \mathcal{M}((pz),z^2) + 2z^\alpha \mathcal{M}_z((\Delta z),(pz),z^2) + 2\Delta^\alpha \mathcal{M}_\Delta((\Delta z),(pz),z^2)$$

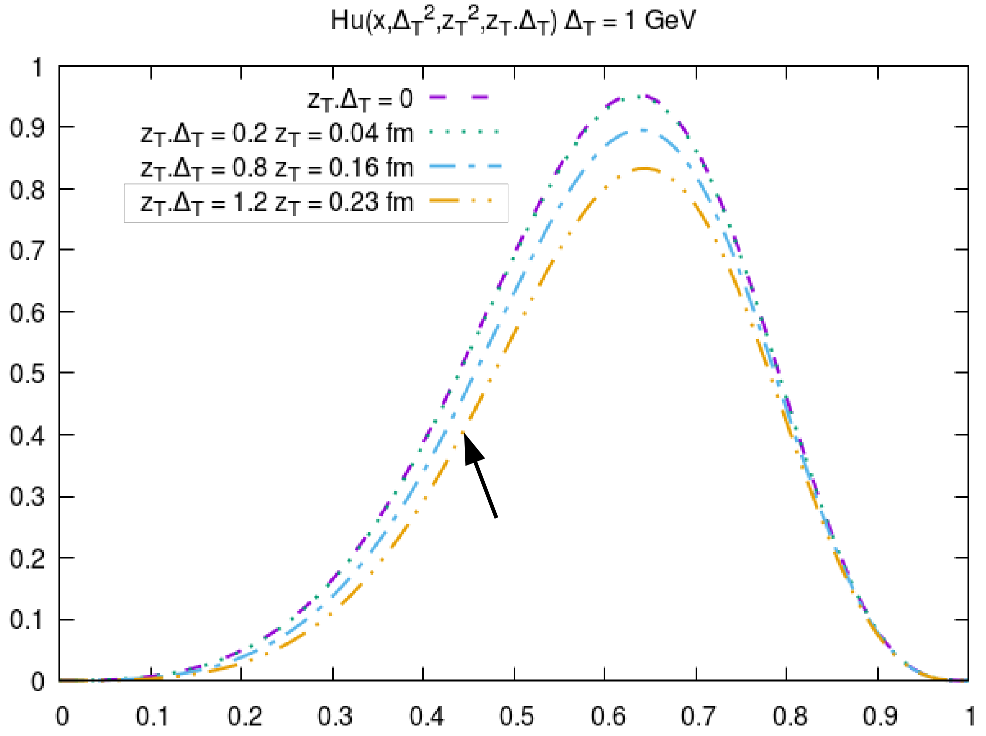
$$(\Delta z) = \Delta^+ z^- + \Delta^- z^+ - \mathbf{\Delta_T . z_T}$$

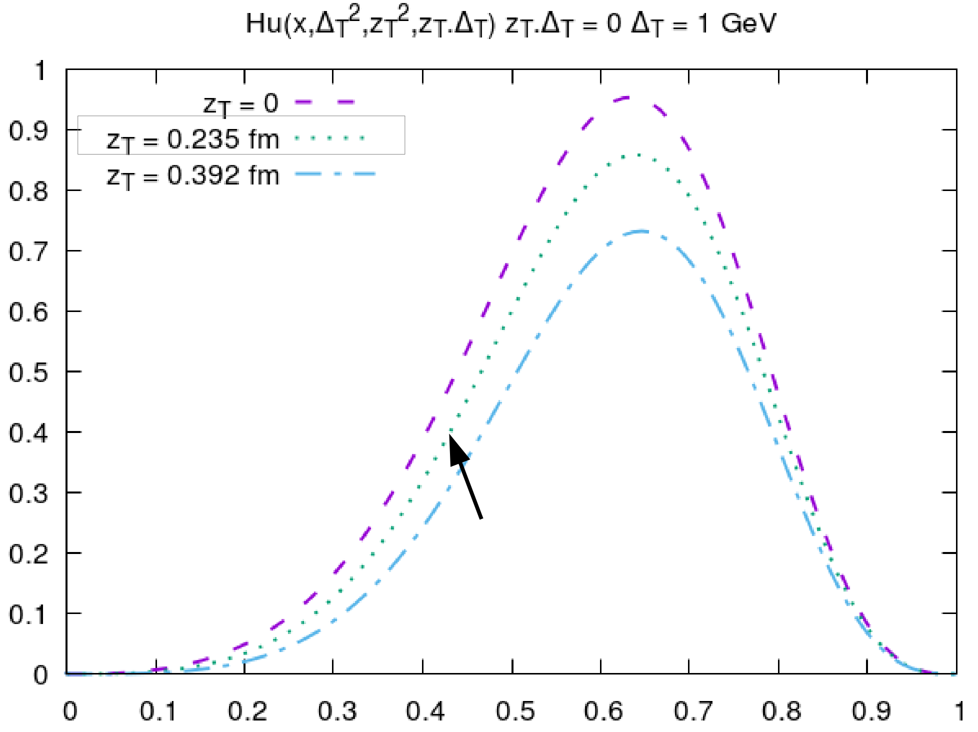
For an on the light cone GPD, for zero skewness or Δ^+ = 0, $(\Delta z) = 0$ irrespective of $\mathbf{\Delta_T}$

For an off the light cone GPD in a rotated frame, if skewness is zero , $(\Delta z) = -\mathbf{\Delta_T . z_T}$

Effectively a mixing of skewness and transverse momentum transfer.

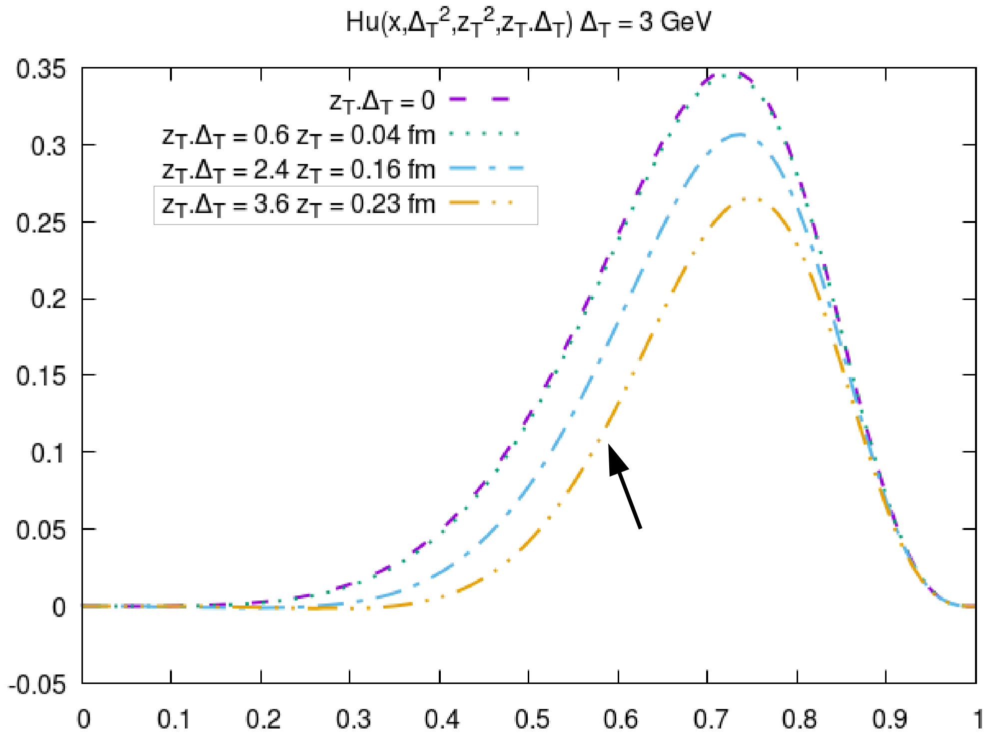
Off forward off the light cone

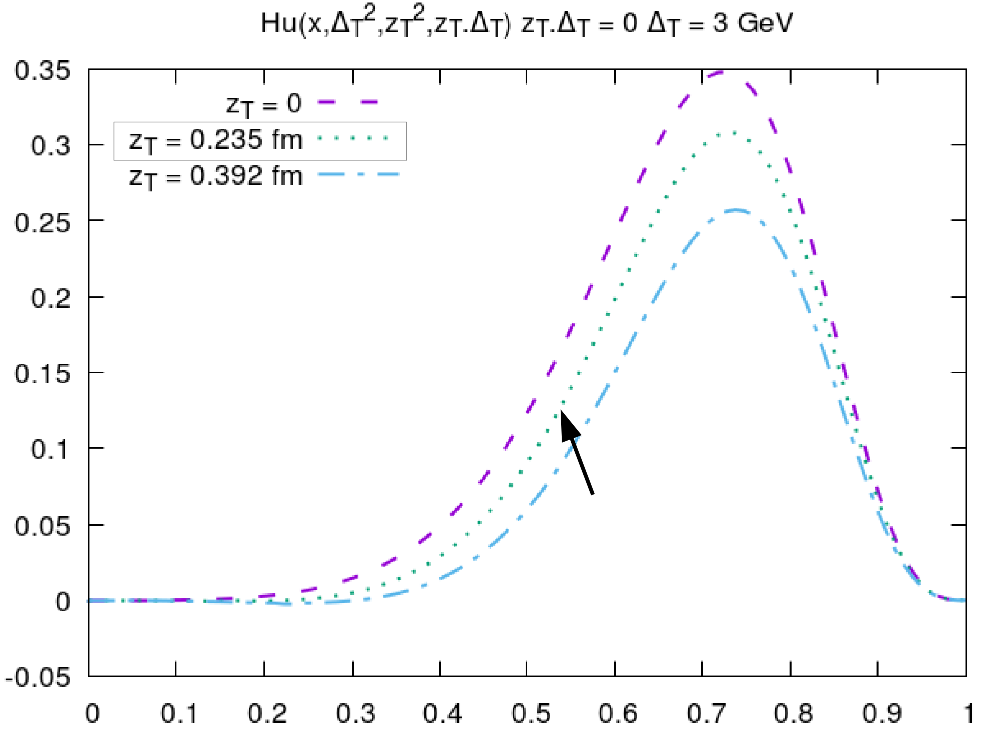




Off forward off the light cone

The effect is higher for higher momentum transfer.





Reconstructing PDFs using lattice QCD moments

- Reconstruct the PDF in Ioffe time space using Mellin moments and Regge behavior.

- Inverse Fourier transform to obtain actual PDF.

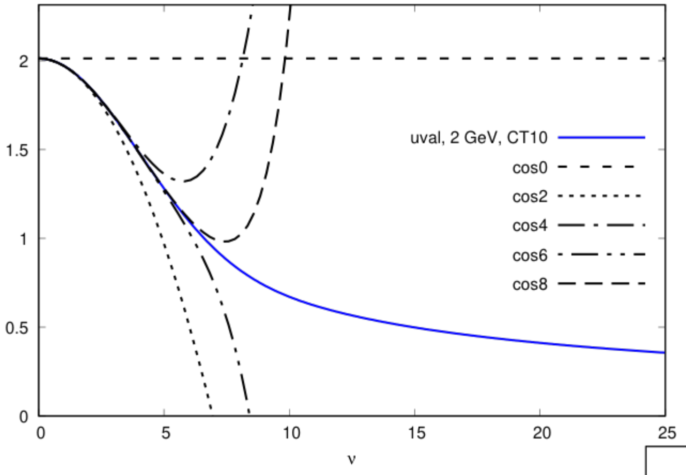
$$Im\,\mathcal{M}(\nu) = \int_0^1 dx f(x) sin(x\nu) = M_1\nu - \frac{1}{3!}M_3\nu^3 + \dots$$

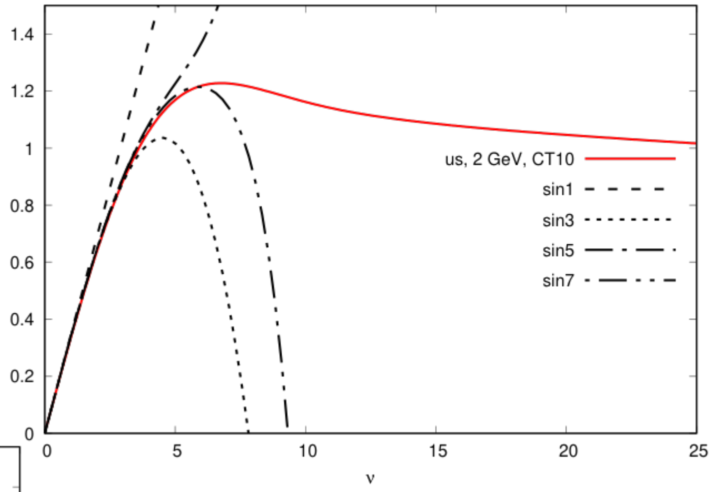
$$Re\,\mathcal{M}(\nu) = \int_0^1 dx f(x) cos(x\nu) = M_0 - \frac{1}{2!}M_2\nu^2 + \dots.$$

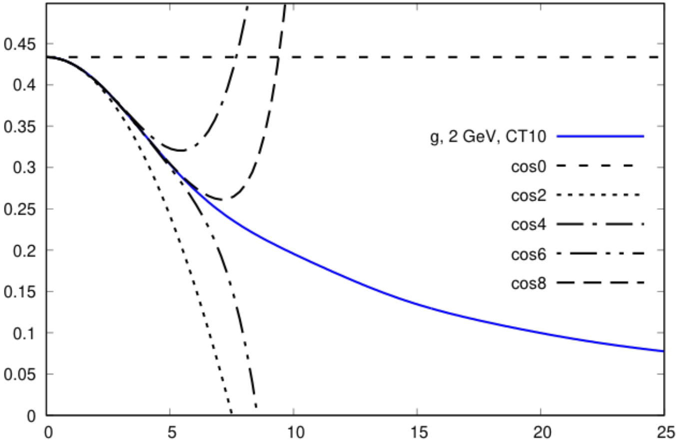
$$x^{-\alpha} \to \nu^{-\alpha+1}$$

Fourier Transform

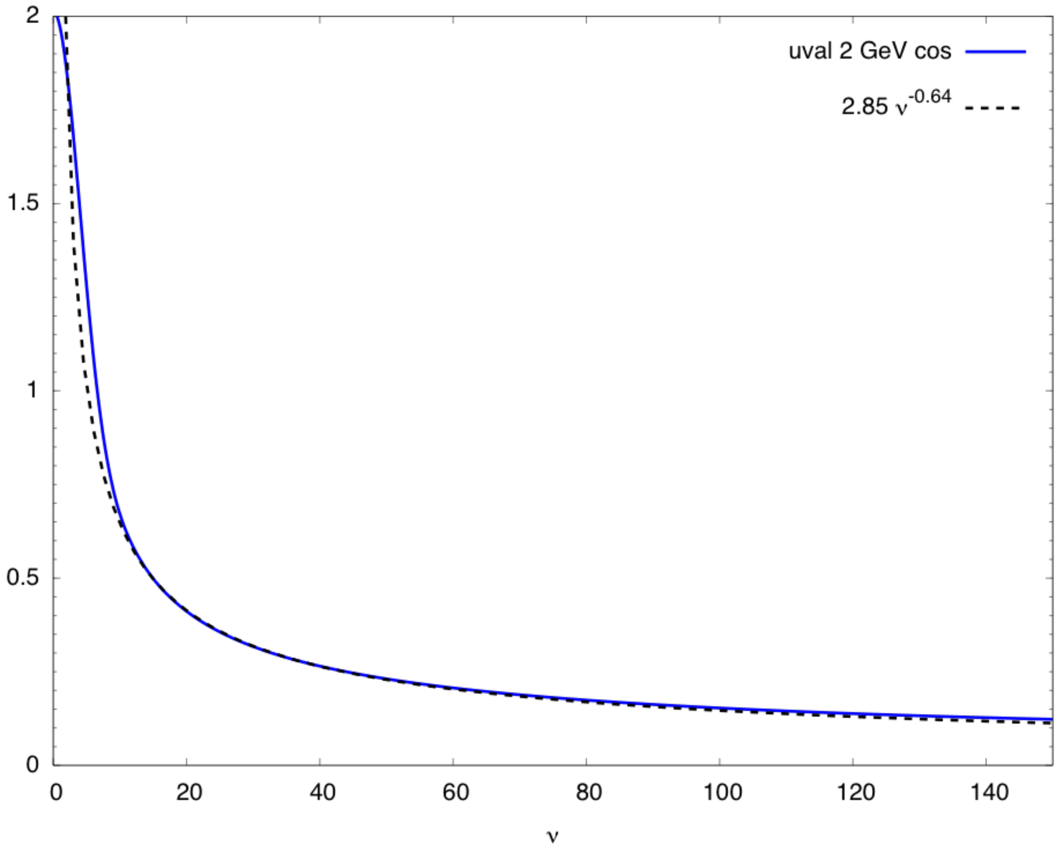
Reconstructing PDFs using lattice QCD moments





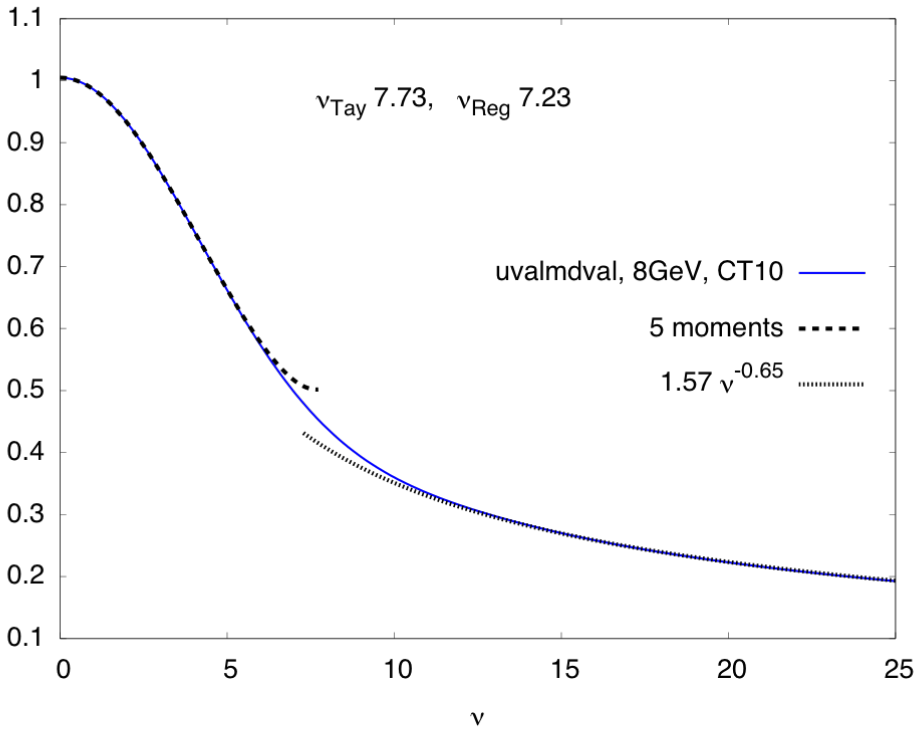


Reconstructing PDFs using lattice QCD moments



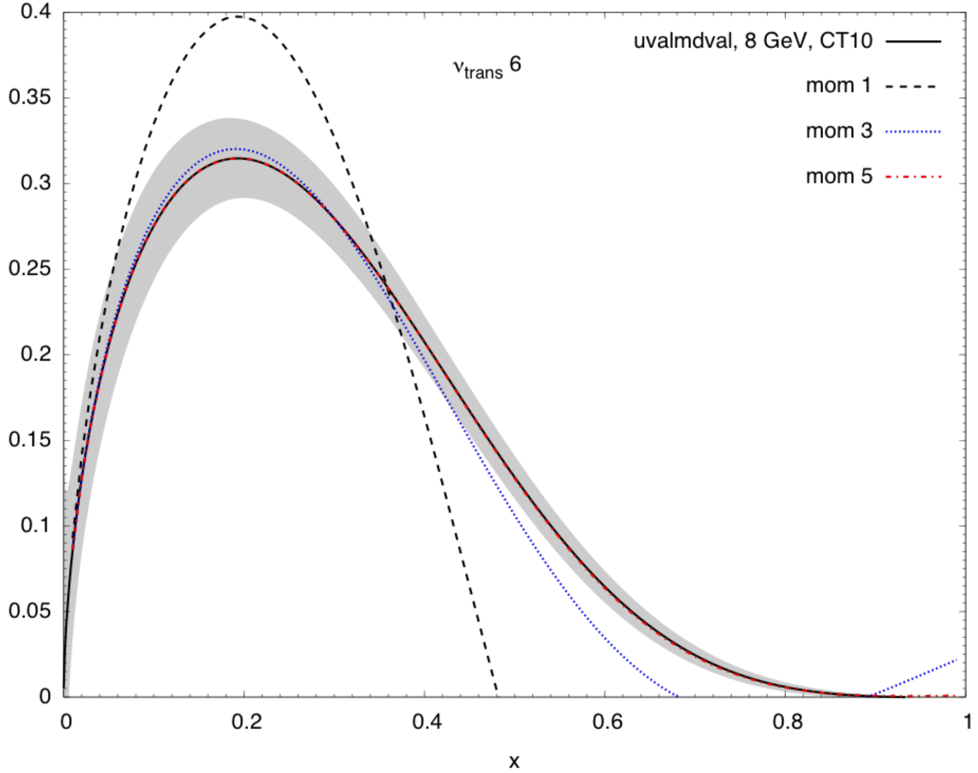
Large Ioffe time described mostly by Regge behavior.

Reconstructing PDFs using lattice QCD moments

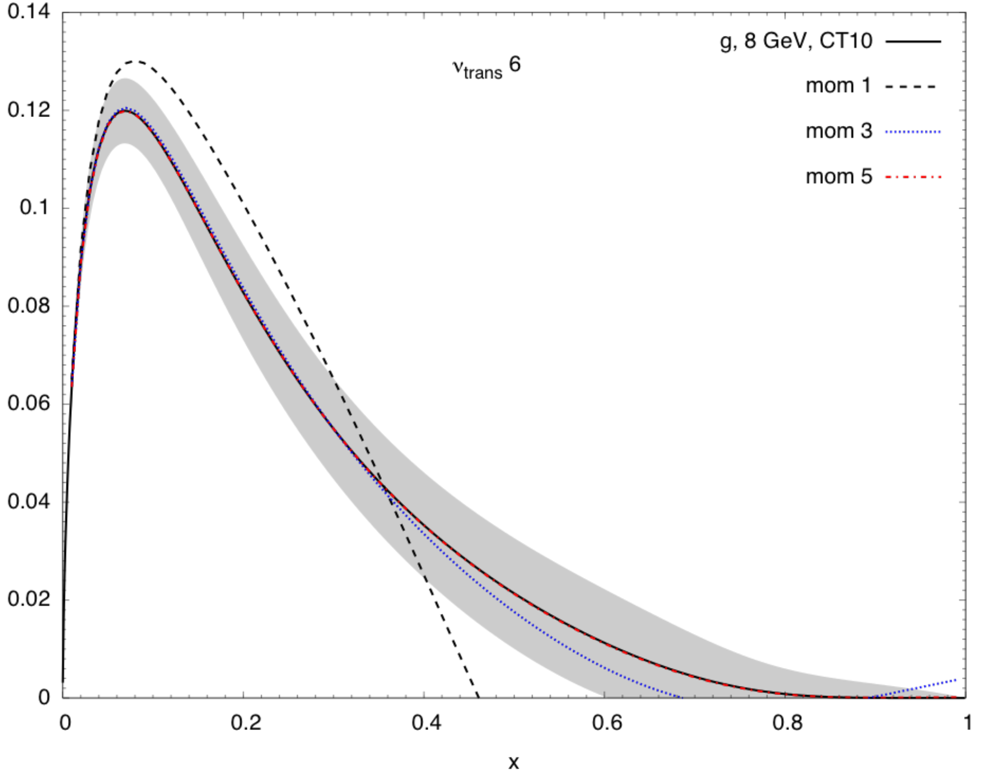


Reconstructing PDFs using lattice QCD moments

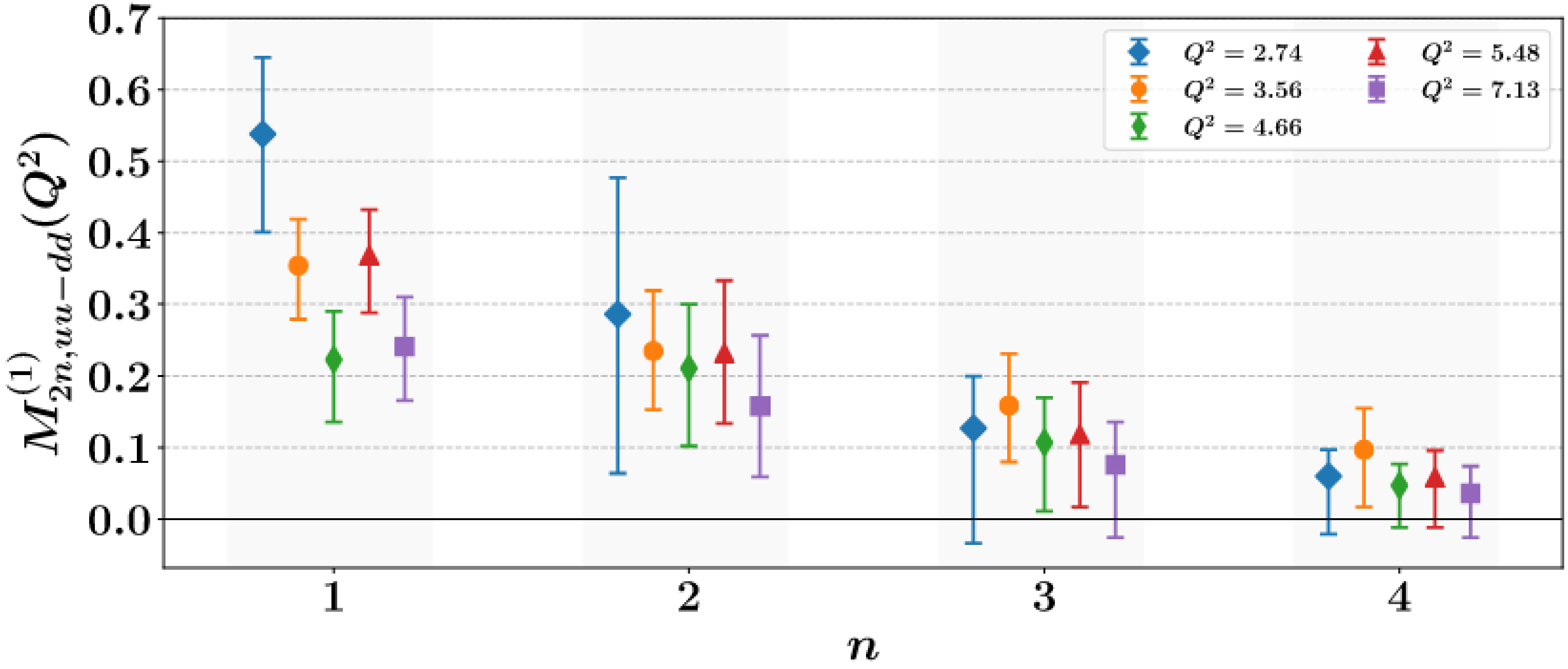
$u_{val} - d_{val}$



gluons



State of the art in lattice calculations of Mellin moments



Can et al, QCDSF / UKQCD / CSSM, Phys. Rev. D 102 (2020)

Summary

- Both local and non local operators calculated on the lattice carry a wealth of information
- With more Mellin moments one covers more region in Ioffe time space.
- In the case of pseudo PDFs, this is achieved by boosting to higher and higher momentum.
- Limited range in Ioffe time makes it tricky to perform an inverse Fourier transform.
- Interesting effects of skewness and transverse momentum transfer for off forward distributions off the light cone.