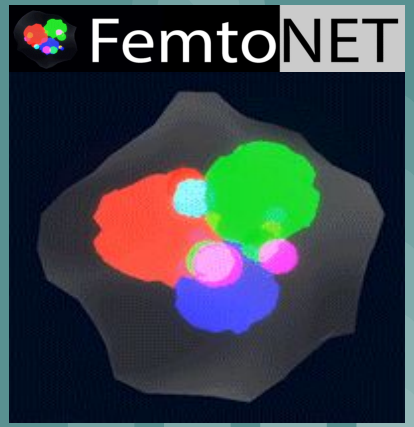
Gluon Spatial Distributions in the Nucleon

B. Kriesten

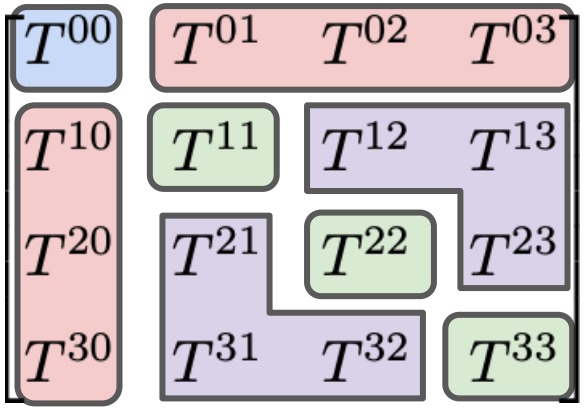


9th APS Group of Hadronic Physics

April 13, 2021

Fundamental Properties of the Nucleon and the QCD EMT

$$T^{\mu\nu}_{QCD} = \frac{1}{4}\,\overline{\psi}\gamma^{(\mu}D^{\nu)}\psi + Tr\left\{F^{\mu\alpha}F^{\nu}_{\alpha} - \frac{1}{2}g^{\mu\nu}F^{2}\right\}$$



Energy Density

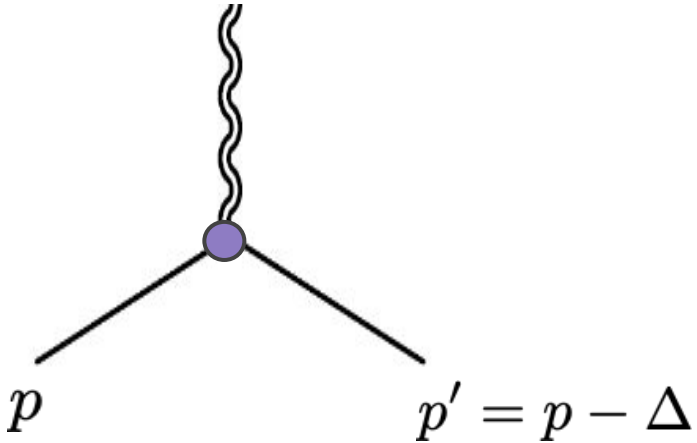
Momentum Density

Pressure Distribution

Shear Forces

Energy Momentum Tensor Form Factors

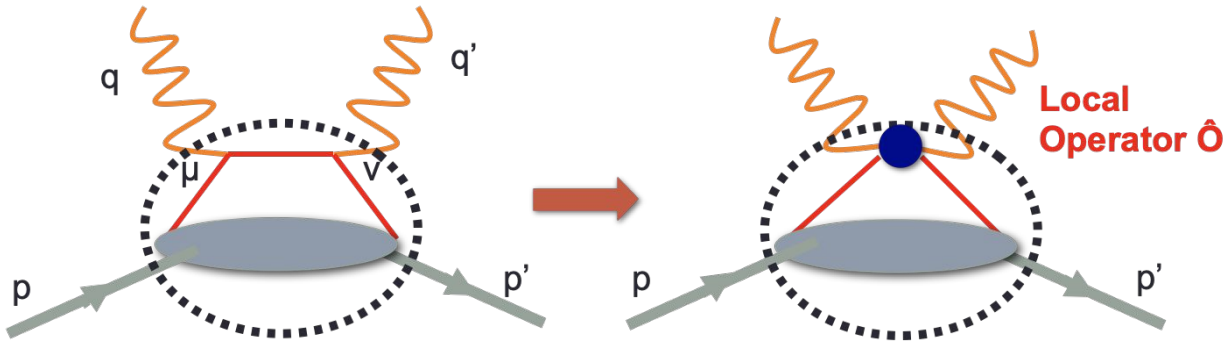
$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \overline{U}(P')[A_{q,g}(\Delta^2)\gamma^{(\mu}\overline{P}^{\nu)} + B_{q,g}(\Delta^2)\overline{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M$$
$$+C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \overline{C}_{q,g}(\Delta^2)g^{\mu\nu}M]U(P)$$



X. Ji **PRL. 78 (1997)**

The matrix elements of the energy momentum tensor can be parameterized by **form factors** describing elastic scattering of a graviton off a proton.

Connection between Local Operators and GPDs



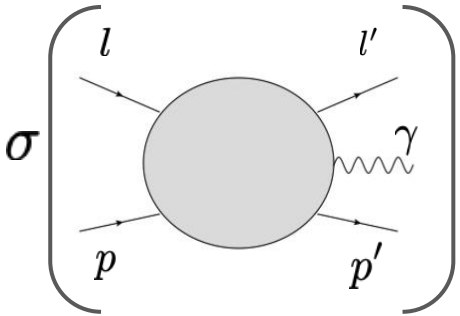
$$\int dx H_g(x,\xi,t) = A_g(t) + \xi^2 C_g(t) \qquad \int dx E_g(x,\xi,t) = B_g(t) - \xi^2 C_g(t)$$

Image credit: **Simonetta Liuti**

Nature of Deeply Virtual Exclusive Processes

A **new era** for understanding and measuring the fundamental properties of nuclei by directly probing the quantum mechanical **phase space distributions** of the quarks and gluons.

Deeply Virtual Compton Scattering



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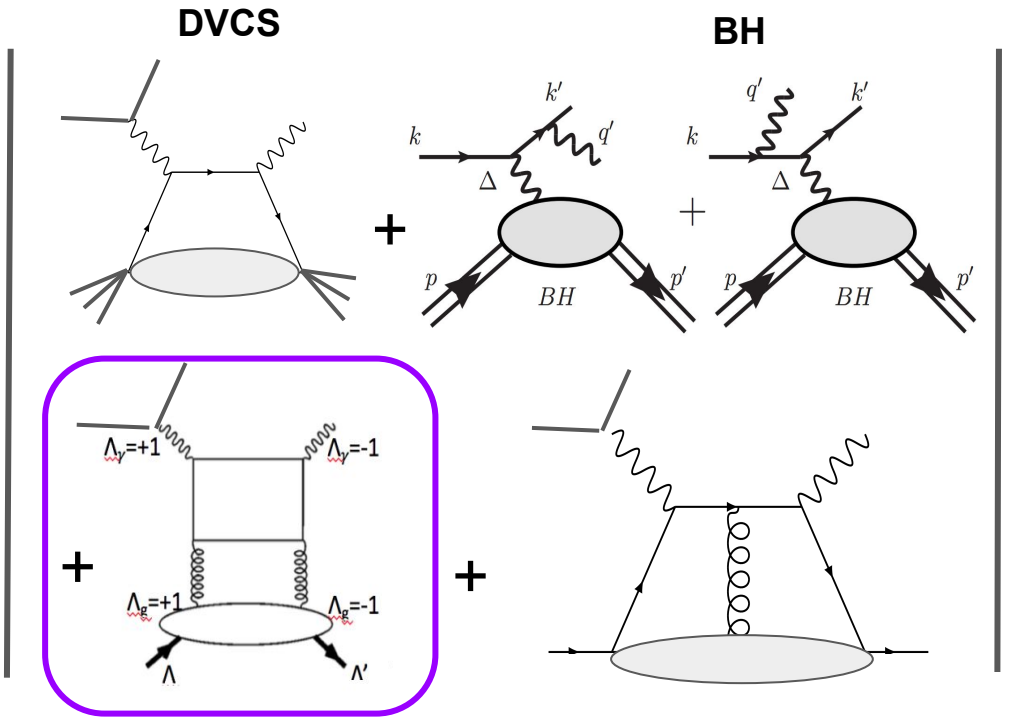
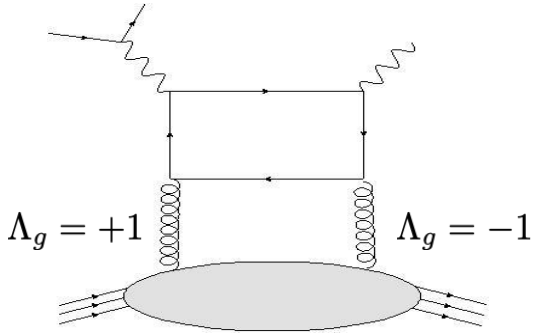


DVCS is known to probe **generalized parton distributions** and is accompanied by various background processes.

X. Ji, **PRD. 55 (1997)**

B.Kriesten, S.Liuti, et. al. **PRD. 101 (2020)**

Gluon Transversity Observables in DVCS

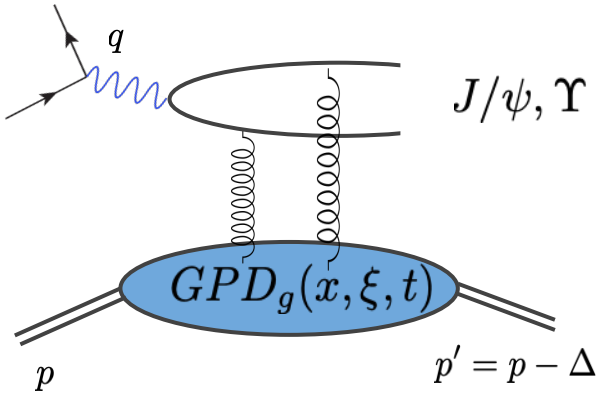


Accessing the **chiral-odd** gluon structure of the nucleon

$$
\begin{aligned}
F_{UU}^{\cos 2\phi} = -2\frac{\alpha_S}{2\pi}\sqrt{1-\xi^2}\frac{t_0-t}{4M^2}\,\Re\mathrm{e}\Bigg[&\sqrt{1-\xi^2}\left(\widetilde{\mathcal{H}}_T^g + (1-\xi)\frac{\mathcal{E}_T^g+\widetilde{\mathcal{E}}_T^g}{2}\right)\left(\mathcal{H}+\widetilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}(\mathcal{E}+\widetilde{\mathcal{E}}\right)^* \\
&+\sqrt{1-\xi^2}\left(\widetilde{\mathcal{H}}_T^g + (1+\xi)\frac{\mathcal{E}_T^g-\widetilde{\mathcal{E}}_T^g}{2}\right)\left(\mathcal{H}-\widetilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}(\mathcal{E}+\widetilde{\mathcal{E}}\right)^* \\
&+\frac{\sqrt{t_0-t}}{2M}\left(\widetilde{\mathcal{H}}_T^g + (1+\xi)\frac{\mathcal{E}_T^g-\widetilde{\mathcal{E}}_T^g}{2}\right)\left(\mathcal{E}+\xi\widetilde{\mathcal{E}}\right)^* \\
&-\sqrt{1-\xi^2}\left(\mathcal{H}_T^g + \frac{t_0-t}{M^2}\widetilde{\mathcal{H}}_T^g - \frac{\xi^2}{1-\xi^2}\mathcal{E}_T^g + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T^g\right)\left(\mathcal{E}-\xi\widetilde{\mathcal{E}}\right)^*\Bigg]
\end{aligned}
$$

Exclusive Measurements of Gluon Distributions

Exclusive electroproduction of vector mesons (such as the J/psi) probe the gluon content of nuclei.



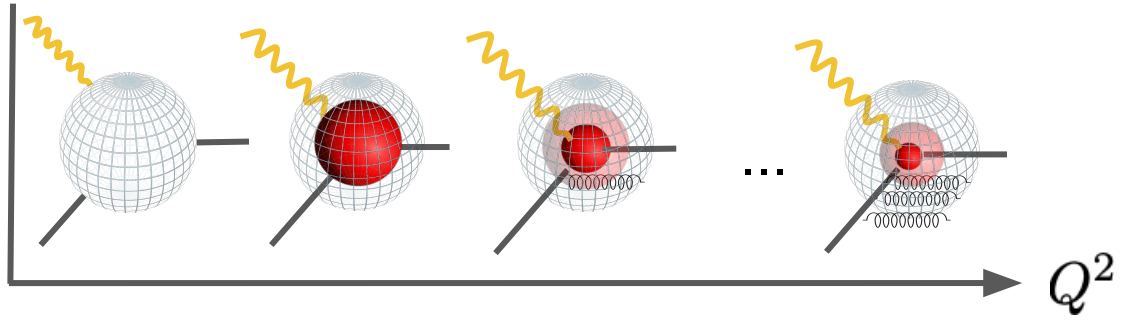
EIC White Paper **arXiv: 1212.1701**

Gluon GPDs enter the vector meson production cross section.
Y. Guo, X. Ji, Y. Liu **arXiv:2103.11506**

$$\frac{d\sigma}{dt} \propto \left(1 - \frac{t}{4M^2}\right) E_2^2 - 2E_2(H_2 + E_2) + (1-\xi^2)(H_2 + E_2)^2$$

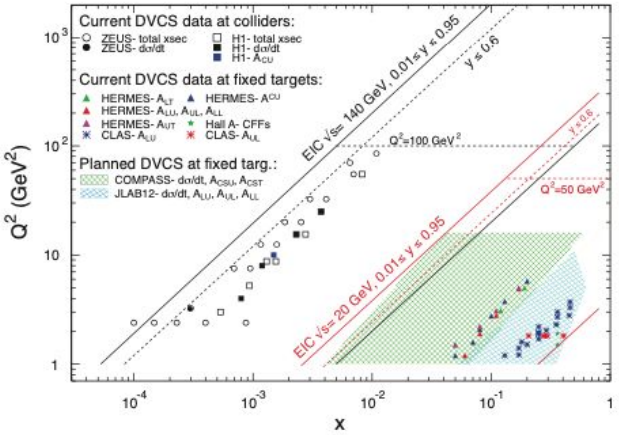
$$E_2 = \int_0^1 dx E_g(x,\xi,t) \quad H_2 = \int_0^1 dx H_g(x,\xi,t)$$

Gluon distributions through scaling violations



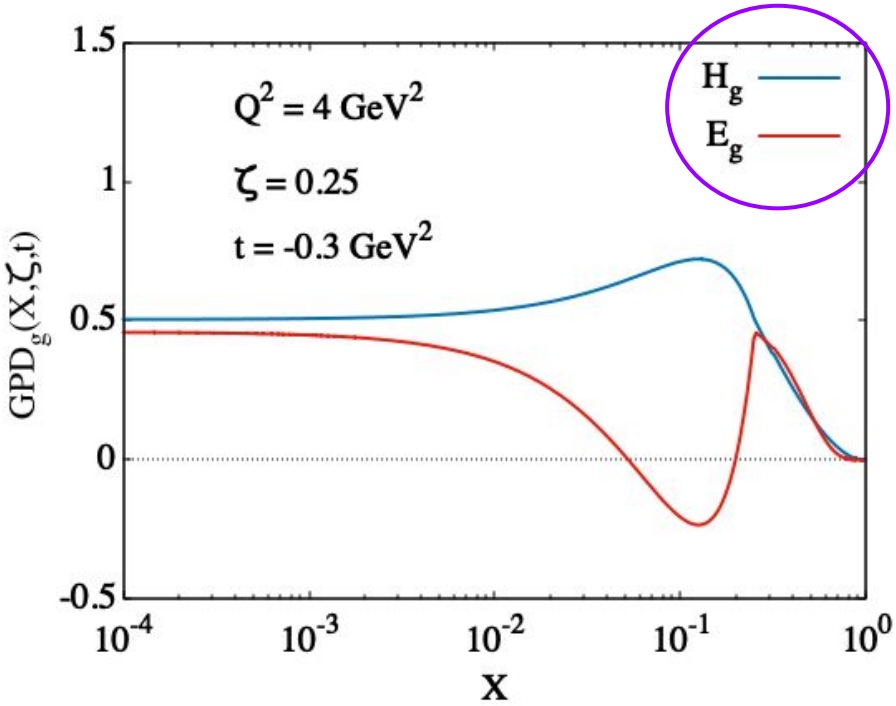
$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_S(Q^2)}{2\pi}\left[P_{QQ}\otimes F_2 + 2e^2 P_{QG}\otimes xg(x)\right]$$

A **lever arm in Q2** hopefully allows us to use perturbative evolution to extract the gluon distribution through **scaling violations**.



EIC Yellow Report **arXiv: 2103.05419**
EIC White Paper **arXiv: 1212.1701**

Flexible Gluon GPD Model

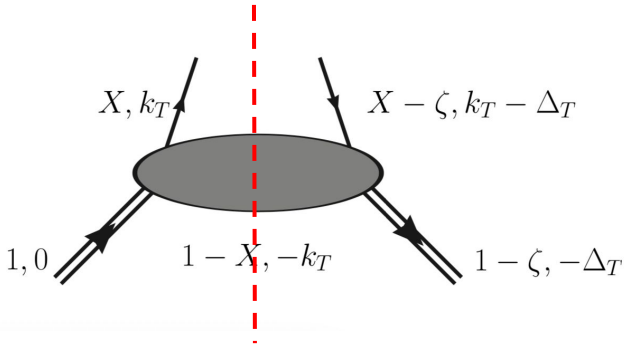


Allows us to calculate **gluon angular momentum** observables as they appear in experiment.

B. Kriesten, P. Velie, E. Yeats, F.Y. Lopez, S. Liuti **arXiv:2101.01826**

General Framework

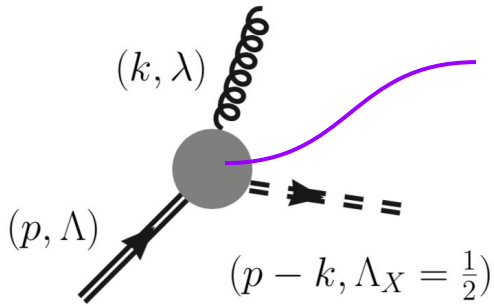
Helicity Amplitude



$$A_{\Lambda'\lambda'_g,\Lambda\lambda_g} = \int \frac{d^2k_\perp}{1-X} \sum_{\Lambda_X} \phi^{*\Lambda_X}_{\lambda'_g\Lambda'}(k',p')\,\phi^{\Lambda_X}_{\lambda_g\Lambda}(k,p)$$

$$\phi^{\Lambda_X}_{\lambda_g\Lambda}(k,p) = \Gamma(k)\,\frac{\bar{U}_{\Lambda_X}(p-k)\,U_\Lambda(p)}{k^2-m_g^2}\,\epsilon^*_{\lambda_g}(k)$$

Dipole Form Factor



$$\Gamma(k) \approx g\,\frac{k^2-m_g^2}{(k^2-M_\Lambda^2)^2}$$

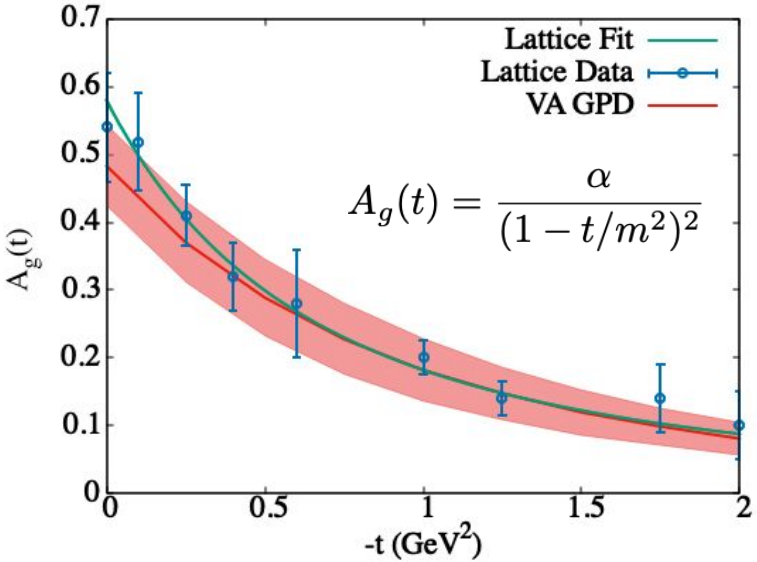
$$H^g(X,\zeta,t) = \begin{cases} H^{M_\Lambda}_{M_X}(X,\zeta,t)\,R^{\alpha,\alpha'}_p(X,t), & \zeta \le X \le 1 \\ a_g X^2 - a_g\,\zeta X + H(\zeta,t), & 0 \le X < \zeta \\ 0 & -1+\zeta \le X < 0 \end{cases}$$

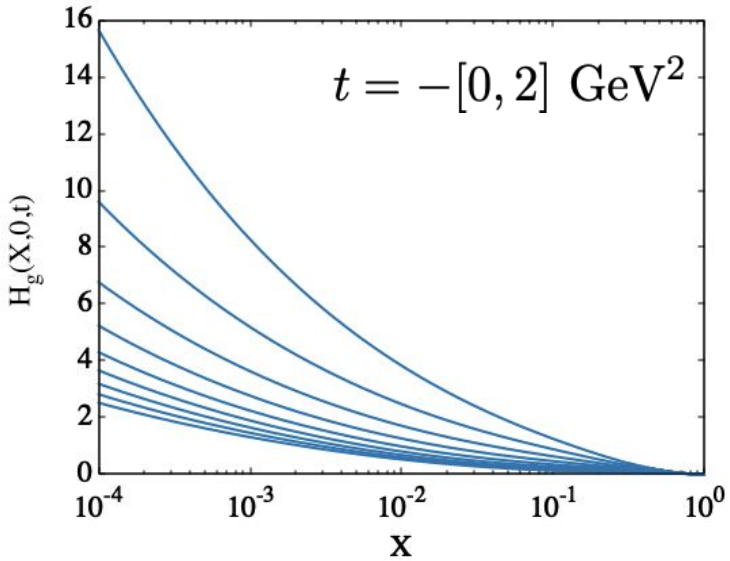
- 7 Fitted parameters and the initial scale for perturbative evolution.

Quark Parametrization **PRD. 84 (2010), PRC 88 (2013)**

Gluon GPD H(X,0,t)

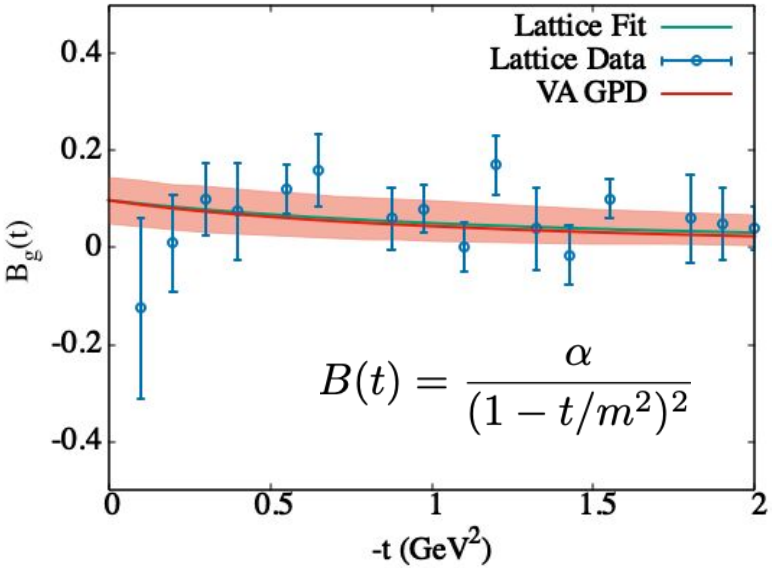
Dipole fit of Lattice QCD calculated moments (P.E. Shanahan, W. Detmold **PRD 99, (2019)**) allows us to fit the the gluon GPD t-dependence.

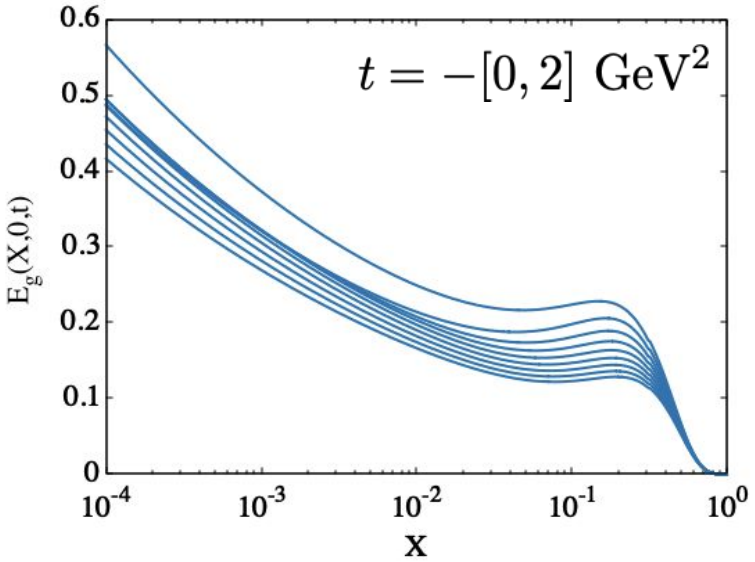




Gluon GPD E(X,0,t)

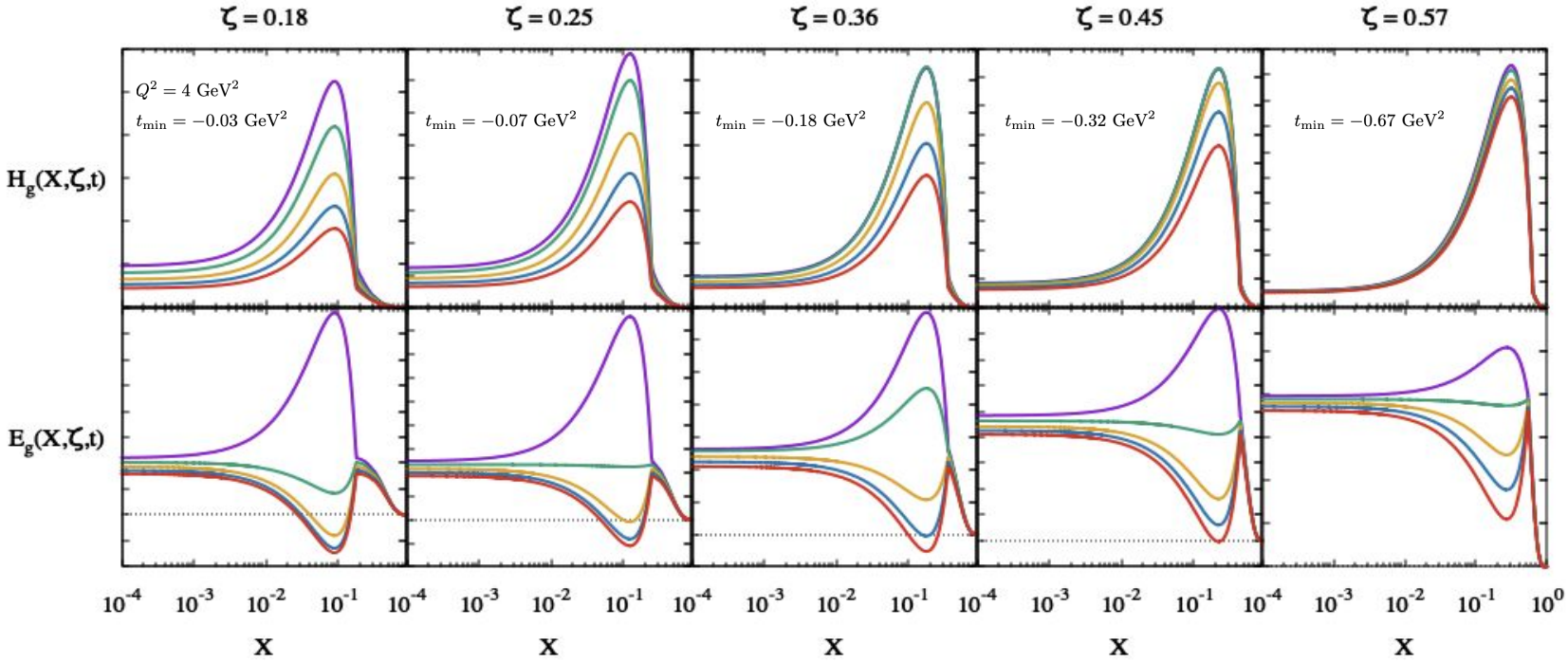
Similarly, we use a dipole fit of Lattice QCD calculated moments (P.E. Shanahan, W. Detmold **PRD 99, (2019)**) to fit the gluon GPD t-dependence, but there is **much flexibility in this fit**.





Full Gluon GPD

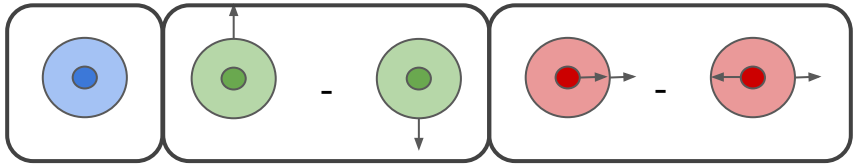
Allows us to calculate **gluon observables** as they appear in experimental cross sections.



Momentum Space to Transverse Position Space

Probability density of finding a quark at transverse position b from the center of momentum as a function of quark and proton polarization.

$$\rho^q_{\Lambda\lambda}(\mathbf{b}) = H_q(\mathbf{b}^2) + \frac{b^i}{M}\epsilon_{ij}S_T^j\frac{\partial}{\partial b}E_q(\mathbf{b}^2) + \Lambda\lambda\widetilde{H}_q(\mathbf{b}^2)$$



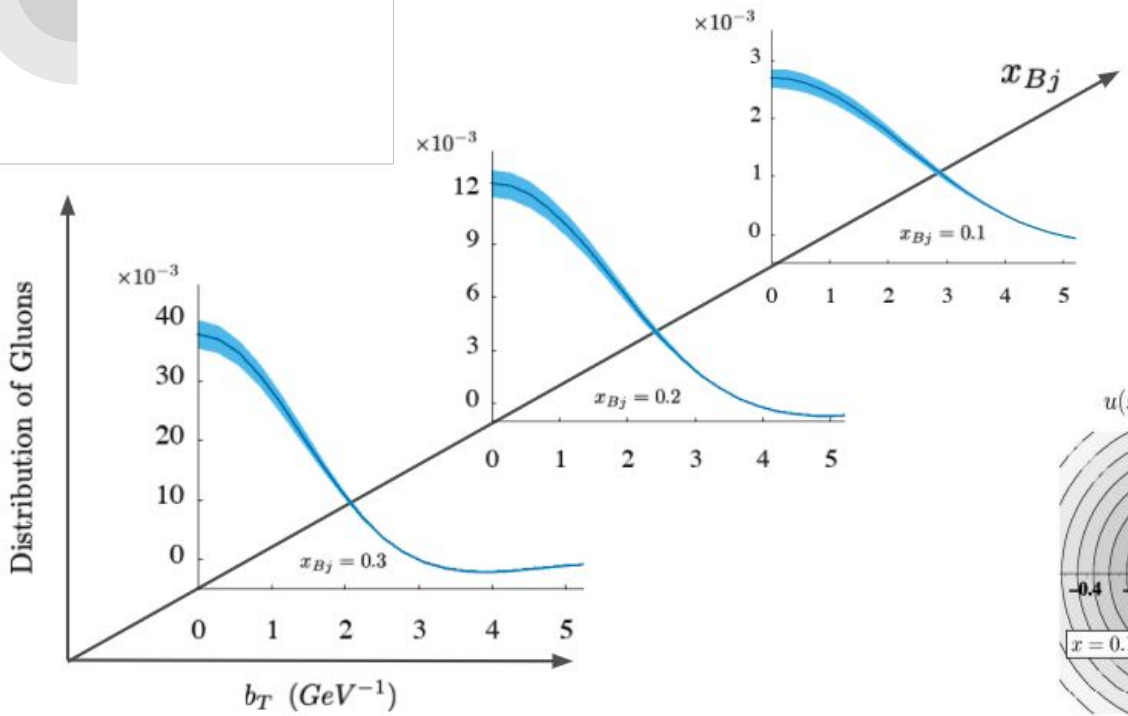
$$\rho^q_{\Uparrow}(x,b_T) = \int \frac{d^2\Delta_T}{(2\pi)^2}\left[H(x,0,t) - \frac{1}{2M}E(x,0,t)\frac{\partial}{\partial b_y}\right]e^{-i\Delta_T\cdot b_T}$$

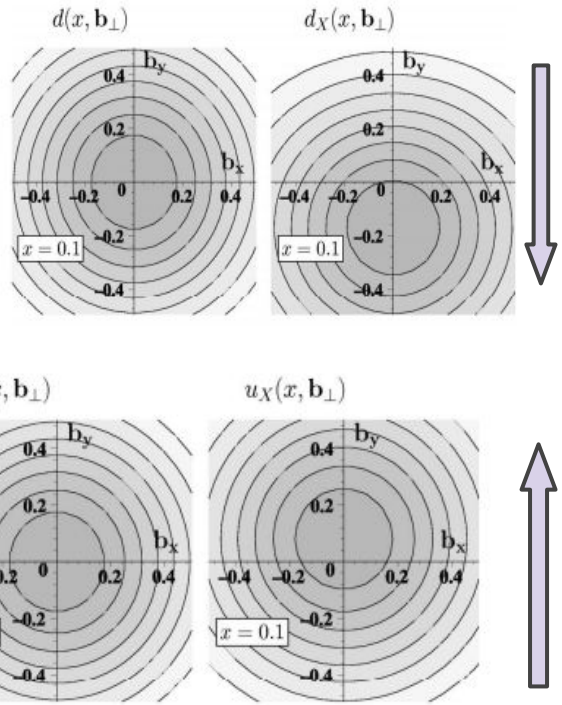
M. Burkardt **PRD. 62 (2000)**
M. Burkardt ***Int.J.Mod.Phys.A* 18 (2003)**

Transverse polarization shifts the unpolarized distribution proportional to the GPD E.

2+1 Dimensional Imaging in Impact Parameter Space

M. Burkardt **PRD. 66 (2002)**





Gluon GPDs: B. Kriesten, P. Velie, E. Yeats, F.Y. Lopez, S. Liuti **arXiv:2101.01826**
Fourier Transforms: A. Rajan, B. Kriesten, S. Liuti **(in progress)**

Conclusions

- Gluon spatial distributions are the key to understanding the **fundamental properties of the nucleon** (spin, mass, pressure, etc.).

- An EIC will explore **new kinematic regimes** for femtography using a variety of exclusive processes to place constraints on the quark and gluon distributions.

- Model calculations are necessary for predicting the **size of gluon observables** particularly at an EIC.