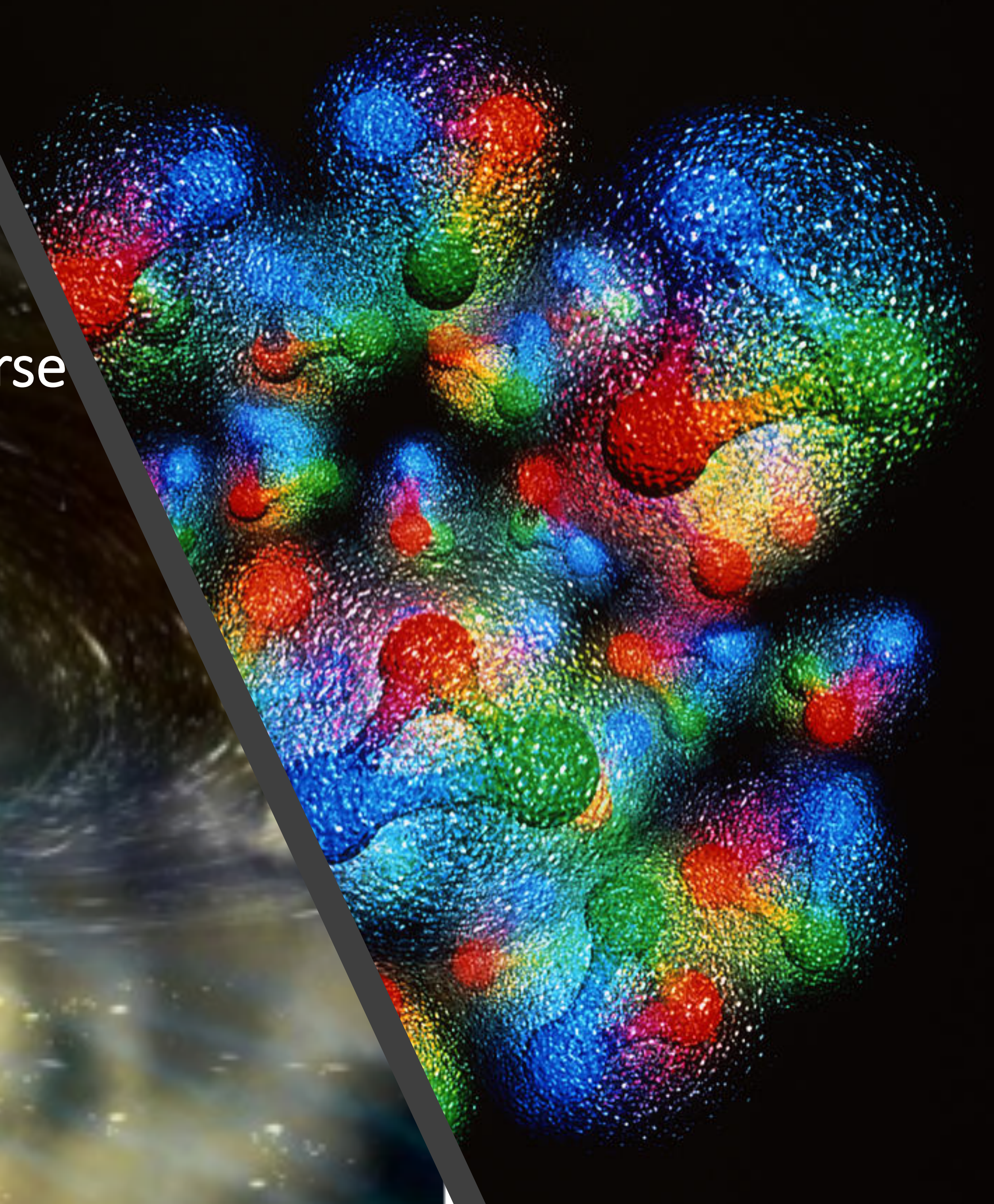
A New Approach to Longitudinal and Transverse Angular Momentum

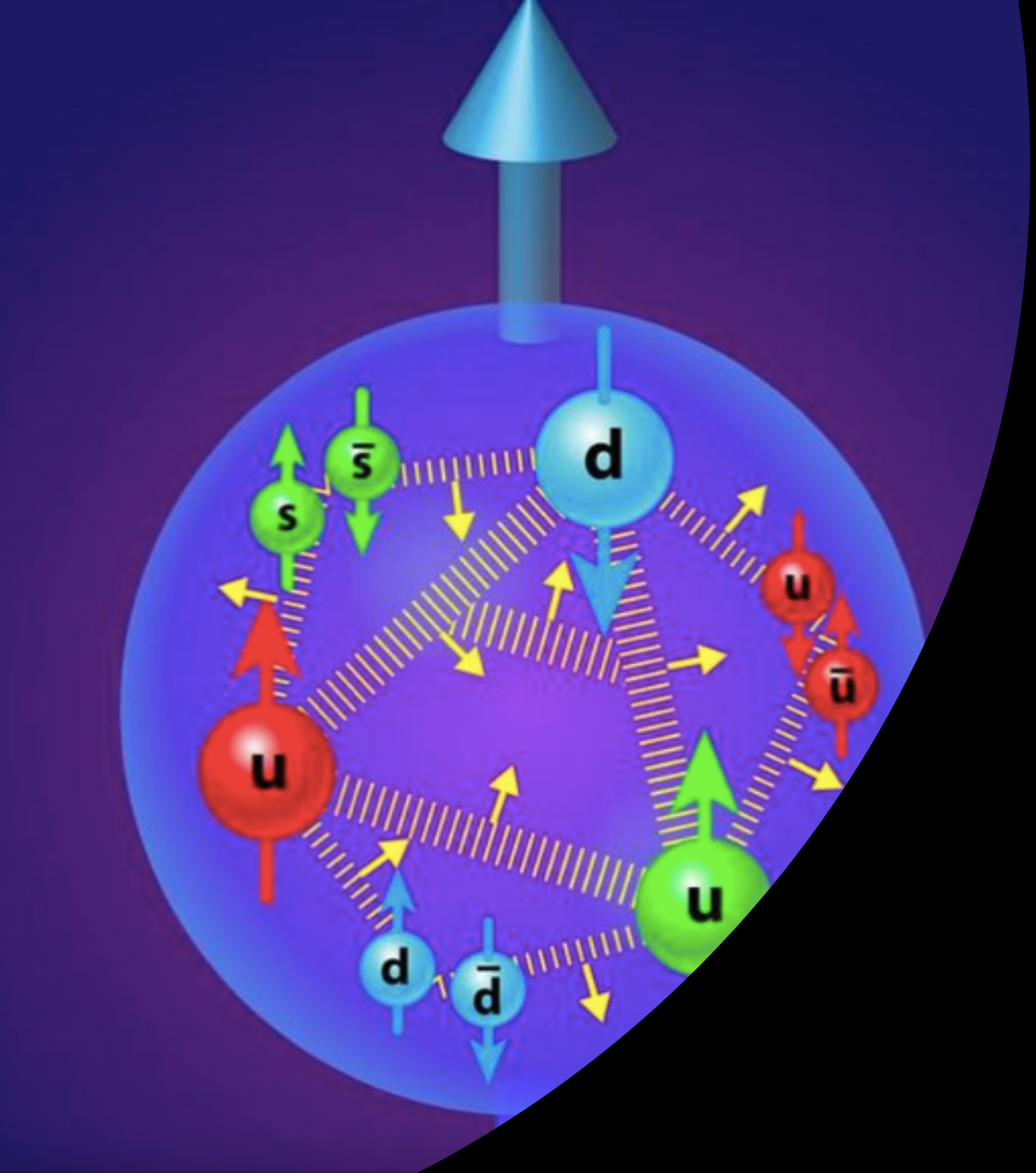
SIMONETTA LIUTI

UNIVERSITY OF VIRGINIA

APS GHP

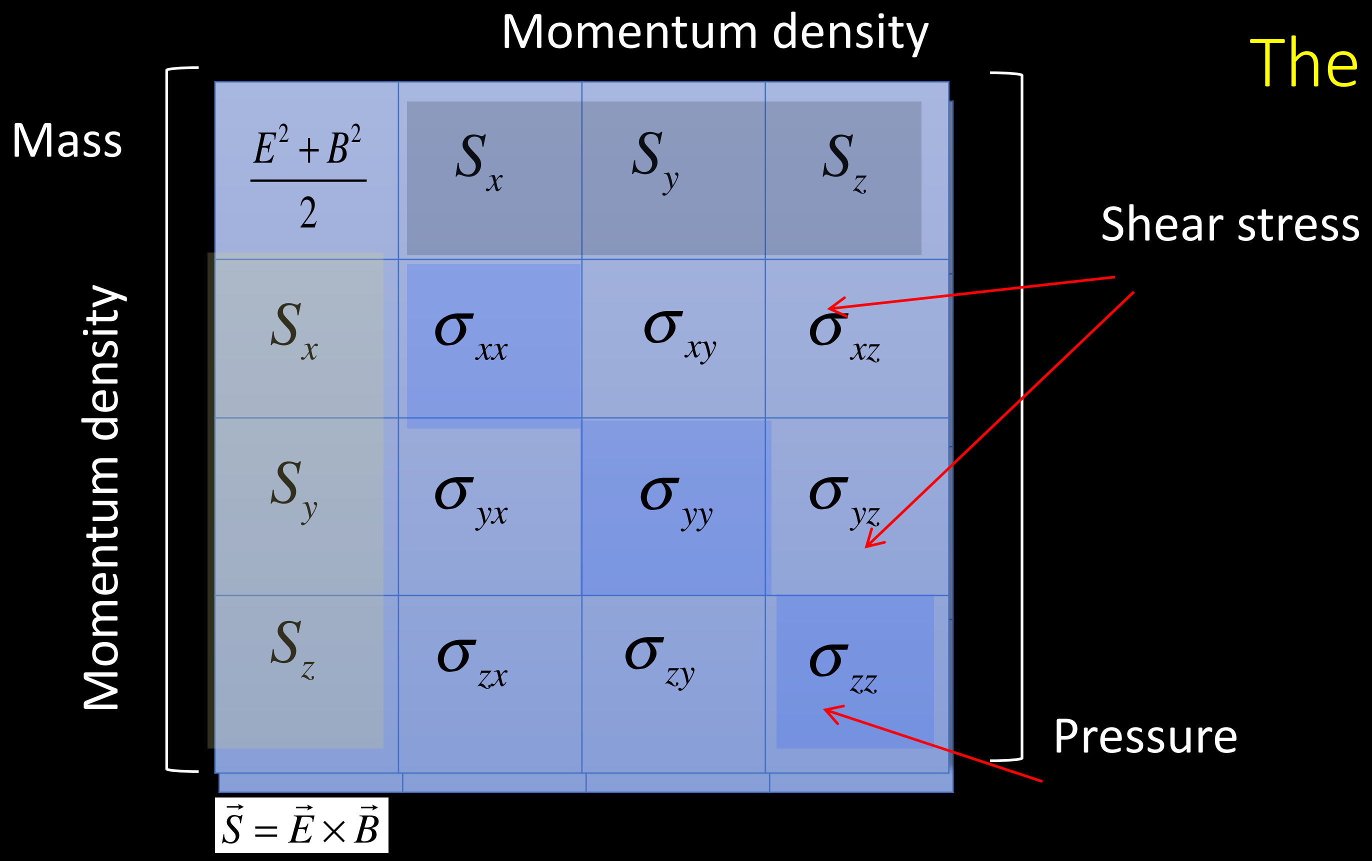
April 13, 2021

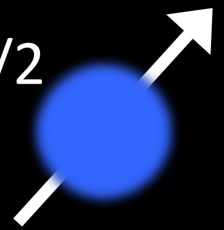




How does the proton/neutron get its mass and spin and how do we test this dynamics?

The QCD Energy Momentum Tensor





(X. Ji, 1997)

$$\langle p',\Lambda \mid T^{\mu\nu} \mid p,\Lambda\rangle = A(t)\bar{U}(p',\Lambda')[\gamma^{\mu}P^{\nu}+\gamma^{\nu}P^{\mu}]U(p,\Lambda) + B(t)\bar{U}(p',\Lambda')i\,\frac{\sigma^{\mu(\nu}\Delta^{\nu)}}{2M}U(p,\Lambda)$$
$$+C(t)[\Delta^{2}g^{\mu\nu}-\Delta^{\mu\nu}]\bar{U}(p',\Lambda')U(p,\Lambda)+\widetilde{C}(t)g^{\mu\nu}\bar{U}(p',\Lambda')U(p,\Lambda)$$

off-forward

q and g not separately conserved

$$P=\frac{p+p'}{2}$$
$$\Delta = p'-p = q-q'$$
$$t=(p-p')^2=\Delta^2$$

Direct calculation of EMT form factors

Donoghue et al. PLB529 (2002),

C. Corianò et al. PRD(2018),

A. Freese , QCD Evolution 2019

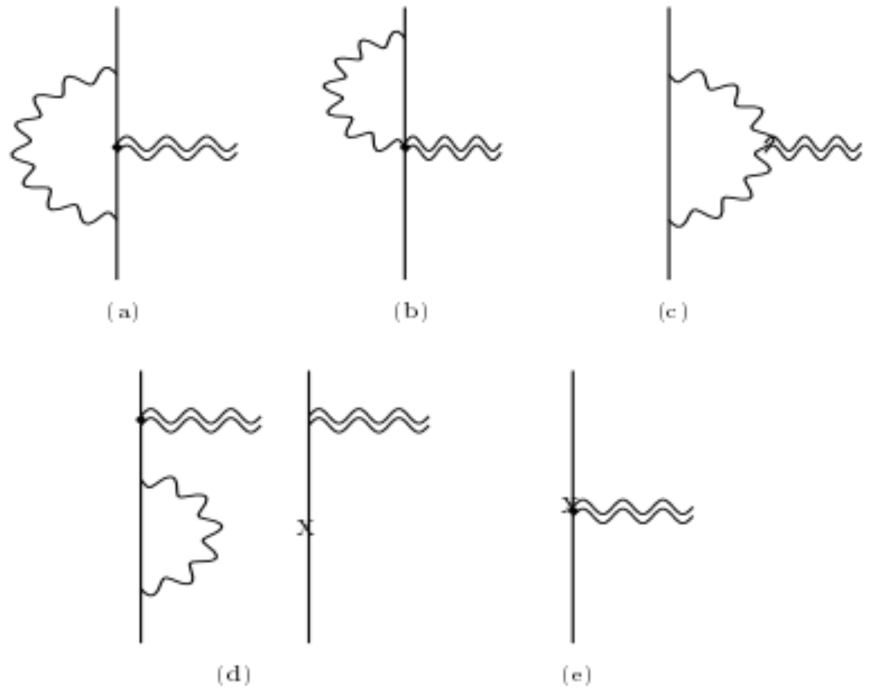
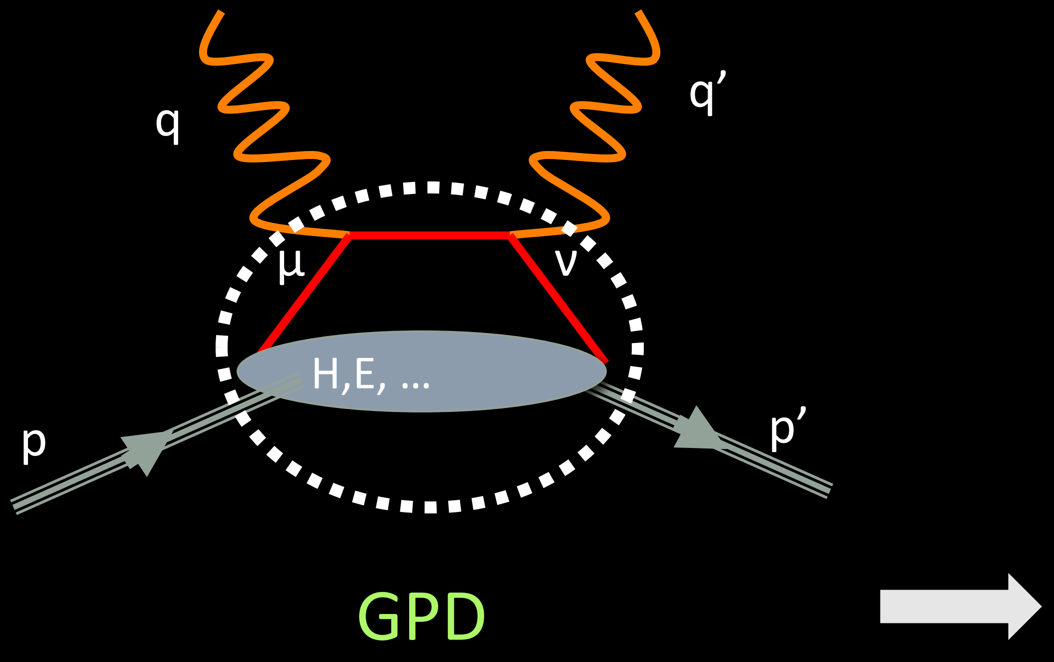
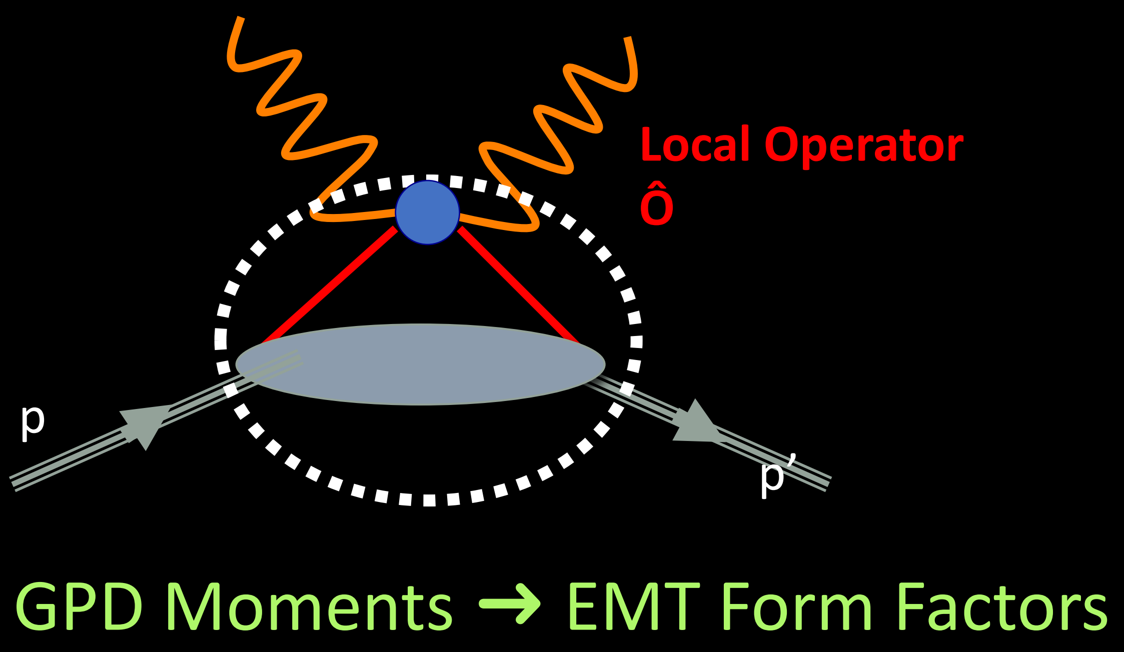


Figure 2: Feynman diagrams for spin 1/2 radiative corrections to $T_{\mu\nu}$.

EMT matrix elements from Generalized Parton Distributions Moments



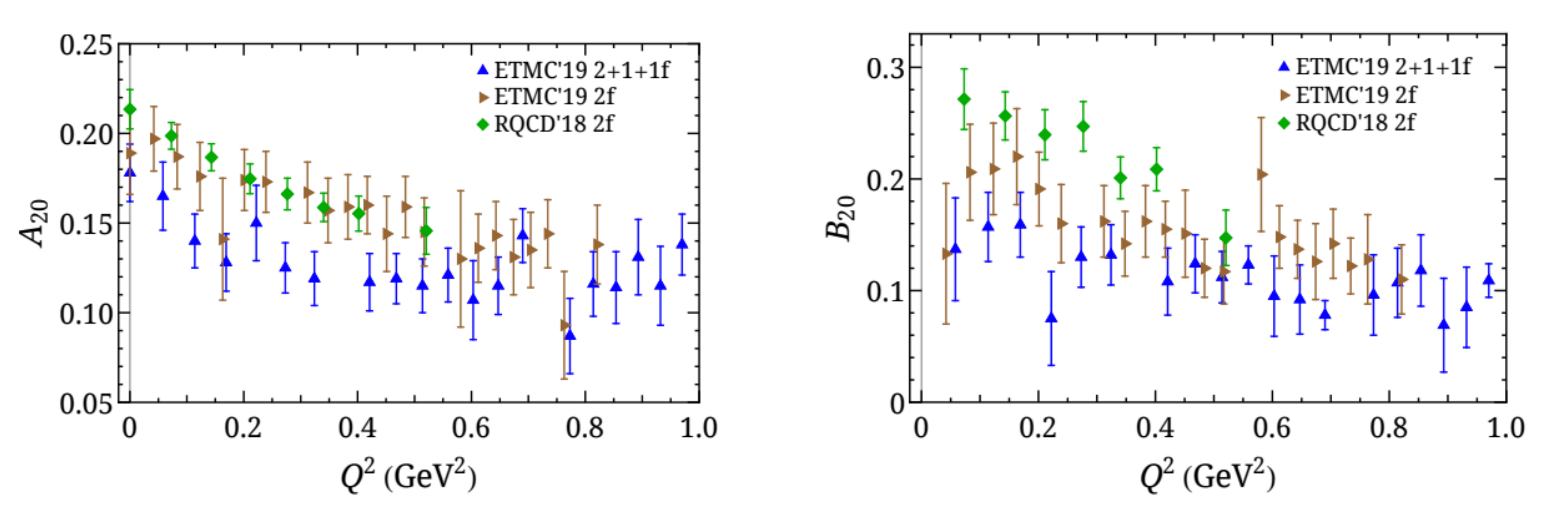


- Large momentum transfer $Q^2 >> M^2$ ➔ "deep"
- Large Invariant Mass $W^2 >> M^2$ ➔ equivalent to an "inelastic" process

Physical interpretation of EMT form factors

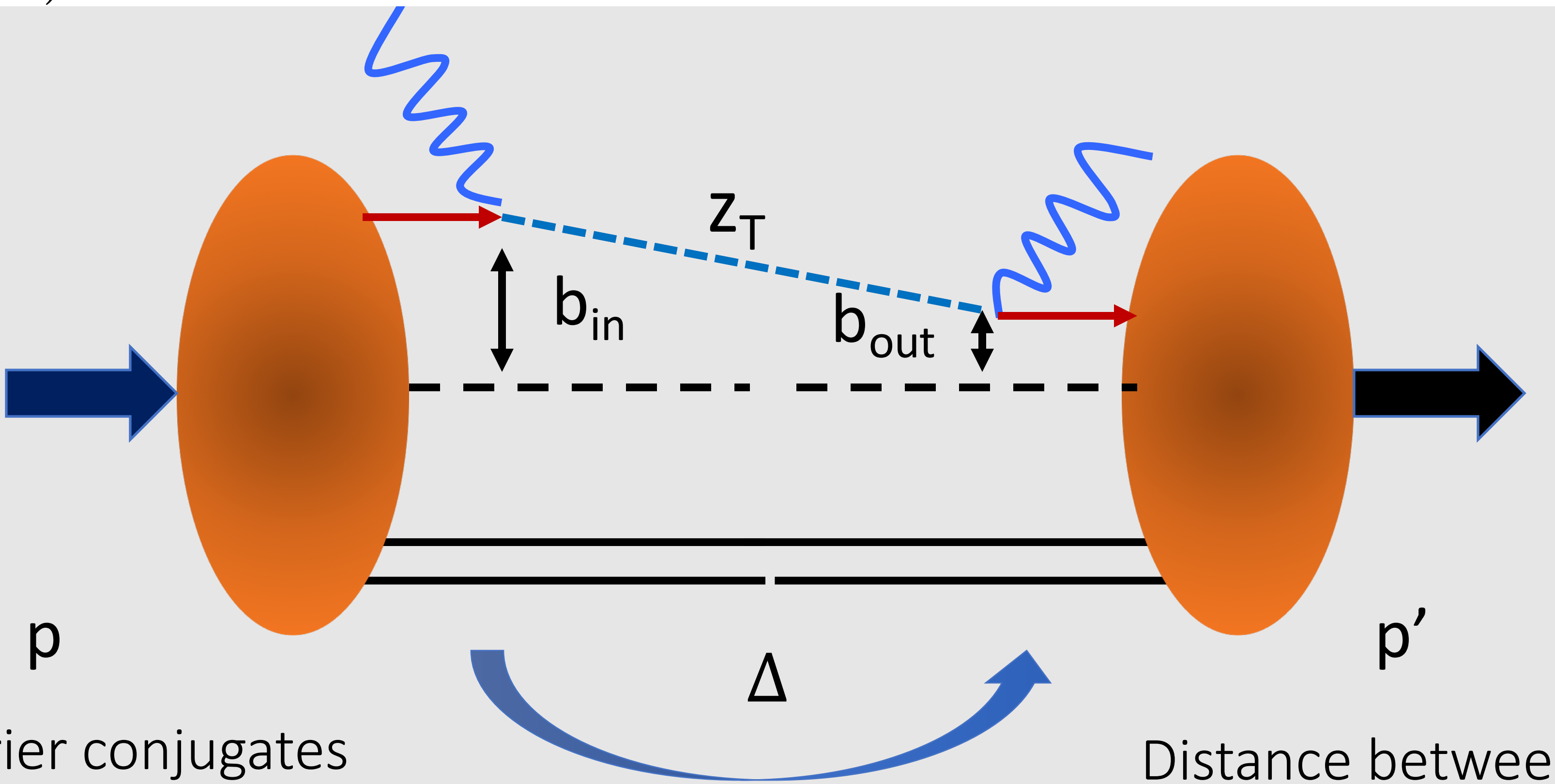
$\frac{1}{2}\left(A_q + B_q\right) = J_q = \frac{1}{2}\left(A_{20} + B_{20}\right)$	$J_q^i = \int d^3 r \epsilon^{ijk} r_j T_{0k}$ Angular Momentum
$A_q = \langle x_q \rangle = A_{20}$	Momentum
$C_q = \text{Internal Forces} = C_{20}$	$\int d^3 r \left(r^i r^j - \delta^{ij} r^2\right) T_{ij}$ Pressure

u-d



arXiv 2006.08636

$$\mathcal{W}^{\mathcal{U}} = \frac{1}{2}\int \frac{d^2\boldsymbol{\Delta}_T}{(2\pi)^2} e^{i\Delta_T\cdot b} \int dz^- d^2\mathbf{z}_T\, e^{ikz} \langle P-\Delta, \Lambda' \mid \bar{q}(0)\gamma^+\mathcal{U}(0,z)q(z) \mid P,\Lambda\rangle \,|_{z^+=0}$$



Two types of Fourier conjugates

Distance between two photons

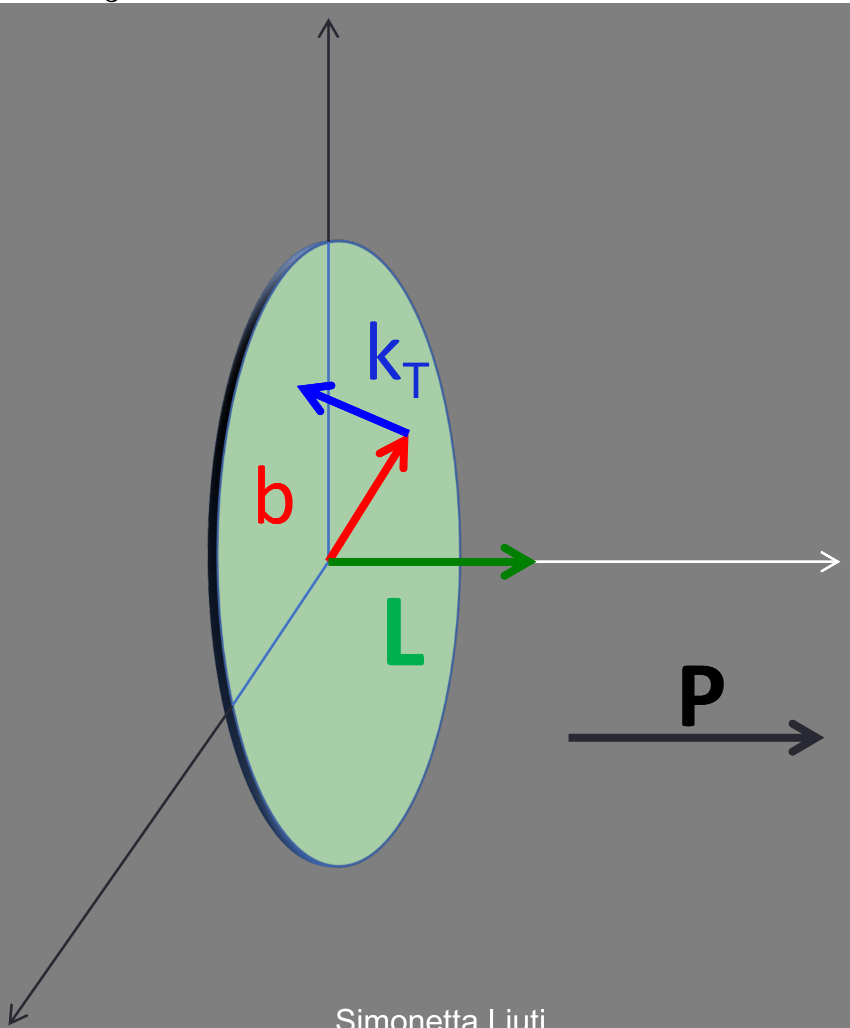
$$\begin{cases} b = \dfrac{b_{in} + b_{out}}{2} \\ \Delta = p - p' \end{cases}$$

$$\begin{cases} z_T = b_{in} - b_{out} \\ k_T \end{cases}$$

Longitudinal OAM definition through Wigner Distributions

$$L_q^{\mathcal{U}} = \int dx \int d^2\mathbf{k}_T \int d^2\mathbf{b}\,(\mathbf{b} \times \mathbf{k}_T)_z\, \mathcal{W}^{\mathcal{U}}(x, \mathbf{k}_T, \mathbf{b})$$

Hatta
Burkardt
Lorce, Pasquini,
Ji, Xiong, Yuan
Mukherjee,
Courtoy, Engelhardt, Rajan, SL



$$\langle L_z \rangle = \lim_{\xi \to 0, \Delta_T \to 0} \int dx \int d^2 k_T \left(i \varepsilon_{ij} \frac{\partial}{\partial \Delta_i} k_j \right) W_{(\Lambda=+)}(x, \xi, k_T^2, k_T \cdot \Delta_T, \Delta_T^2)$$

- Lorce, Pasquini, Xiong, [redacted]uan
- Hatta, Yoshida
- Ji, Xiong, Yuan

$$L_q = -\int_0^1 dx \int d^2 k_T \, \frac{k_T^2}{M^2} \, F_{14} = -\int_0^1 dx \, F_{14}^{(1)}$$

$$L_q \cdot S_q = -\int_0^1 dx \int d^2 k_T \, \frac{k_T^2}{M^2} \, G_{11} = -\int_0^1 dx \, G_{11}^{(1)}$$

- Lorentz Invariance Relations establish a connection between k_T^2 moments of TMDs/GTMDs and twist 3 PDFs/GPDs

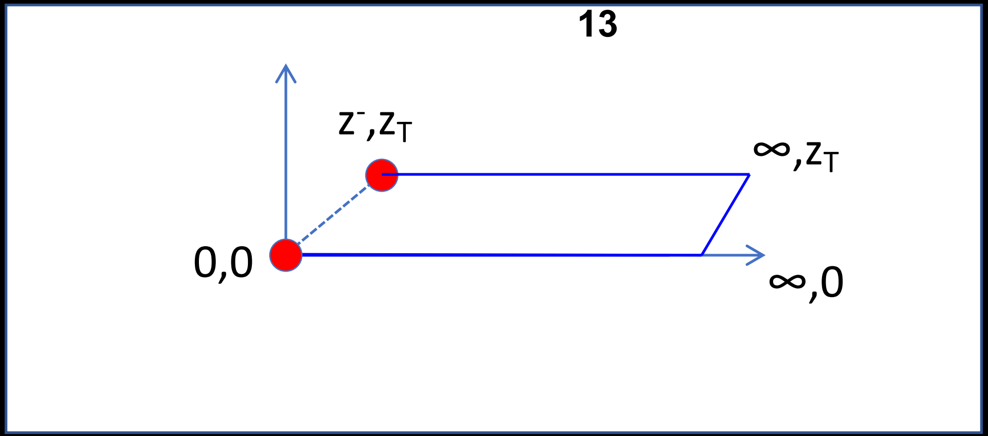
$L_q(x)$

L_q

$$F_{14}^{(1)} = -\int_x^1 dy \left(\tilde{E}_{2T} + H + E\right) \Rightarrow -L_q = \int_0^1 dx F_{14}^{(1)} = \int_0^1 dx x G_2$$

- ✓ Twist 3 GPD notation from Meissner, Metz and Schlegel, JHEP(2009)
- ✓ We confirm and corroborate the global/integrated OAM result deducible from Ji, Xiong, Yuan PRD88 (2013)

Generalized LIR for a staple link



$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14} = \tilde{E}_{2T} + H + E + \mathcal{A}$$

LIR violating term

$$0 = \int \frac{d^4x}{(2\pi)^4} e^{ik\cdot x} \langle PS|\overline{\psi}(0)\left[i\not{D}(0) - m\right] i\sigma^{i+}\gamma_5\psi(x)|PS\rangle$$

How does this OAM fit into a sum rule?

Equations of Motion (EoM) relation

Longitudinal Angular Momentum Sum Rule

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$

- A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)
- A. Rajan, M. Engelhardt, S.L., PRD (2018)

$$0 = \int \frac{d^4x}{(2\pi)^4} e^{ik\cdot x} \langle PS|\overline{\psi}(0)\left[i\not{D}(0) - m\right] i\sigma^{i+}\gamma_5\psi(x)|PS\rangle$$

$$\Downarrow$$

$$-\frac{\Delta^+}{2} W^{\gamma^i\gamma^5}_{\Lambda'\Lambda} + ik^+\epsilon^{ij} W^{\gamma^j}_{\Lambda'\Lambda} + \frac{\Delta^i}{2} W^{\gamma^+\gamma^5}_{\Lambda'\Lambda} - i\epsilon^{ij}k^j W^{\gamma^+}_{\Lambda'\Lambda} + \mathcal{M}^{i,S}_{\Lambda'\Lambda} = 0$$

Longitudinal Angular Momentum Sum Rule

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$

A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)
A. Rajan, M. Engelhardt, S.L., PRD (2018)

$$-\frac{\Delta^+}{2} W^{\gamma^i\gamma^5}_{\Lambda'\Lambda} + ik^+\epsilon^{ij} W^{\gamma^j}_{\Lambda'\Lambda} + \frac{\Delta^i}{2} W^{\gamma^+\gamma^5}_{\Lambda'\Lambda} - i\epsilon^{ij}k^j W^{\gamma^+}_{\Lambda'\Lambda} + \mathcal{M}^{i,S}_{\Lambda'\Lambda} = 0$$

Twist – 3 GPD

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

New! From DVCS

DVCS (Ji,'97)

Polarized ep
$g_1(x)$

Color force/gauge link

Integrated in x

L = J - S + 0

Transverse Angular Momentum Sum Rule

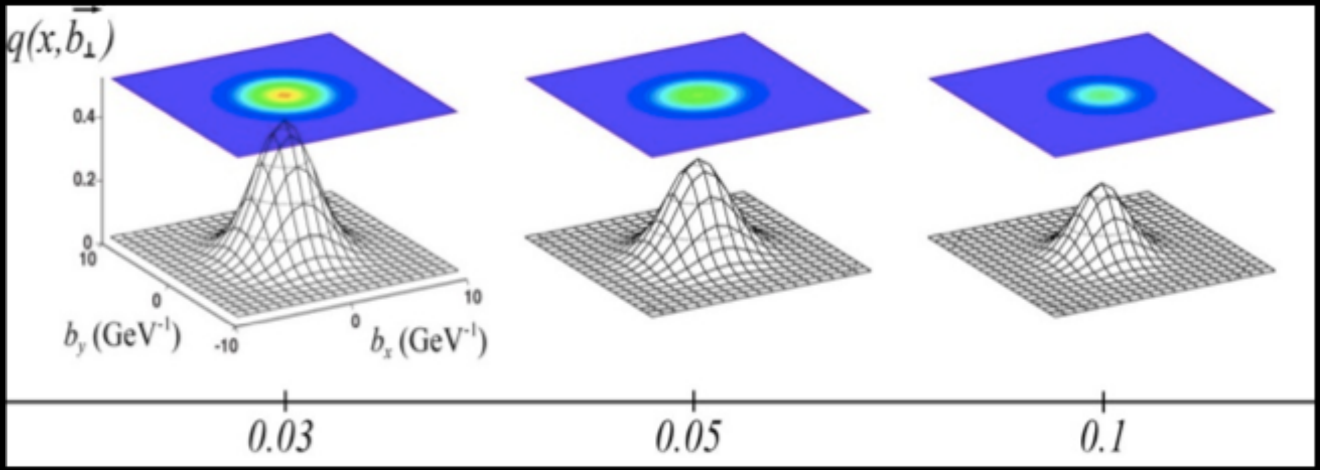
$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$

A. Rajan, M. Engelhardt, SL, *to be submitted*

$$\underbrace{\frac{1}{2}\int dx\, x(H+E) - \frac{1}{2}\int dx\, \mathcal{M}_T}_{J_T} = \underbrace{\int dx\, x\big(\widetilde{E}_{2T} + H + E + \frac{H_{2T}}{\xi}\big)}_{L_T} + \underbrace{\frac{1}{2}\int dx\, g_T}_{S_T} - \frac{1}{2}\int dx\, x\mathcal{A}_T$$

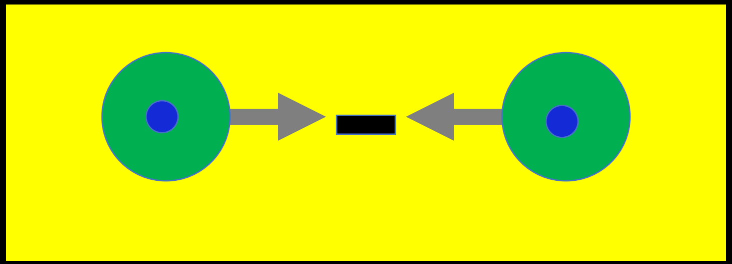
See also recent paper by Guo, Ji, Shiells 2101.05243

Measuring Angular Momentum



graph from M. Defurne

- Beam Target Spin Correlation UL: unpolarized quark density in a longitudinally polarized proton



Observables

GPD	Twist	$P_q P_p$	TMD	$P_{Beam} P_p$ (DVCS)	$P_{Beam} P_p$ ($\mathcal{I}$)
$\mathbf{H} + \frac{\xi^2}{1-\xi} E$	2	UU	f_1	$UU,\ LL,\ UT^{\sin(\phi-\phi_s)},\ LT^{\cos(\phi-\phi_s)}$	$UU^{\cos\phi},\ LU^{\sin\phi}$
$\widetilde{\mathbf{H}} + \frac{\xi^2}{1-\xi} \widetilde{E}$	2	LL	g_1	$UU,\ LL,\ UT^{\sin(\phi-\phi_s)},\ LT^{\cos(\phi-\phi_s)}$	$UU^{\cos\phi},\ UL^{\cos\phi},\ LU^{\sin\phi},\ LL^{\sin\phi},\ UT^{\frac{\cos\phi}{\sin\phi}},\ LT^{\cos\phi}$
$\mathbf{E}$	2	UT	$f_{1T}^{\perp(*)}$	$UT^{\sin(\phi-\phi_s)},\ LT^{\cos(\phi-\phi_s)}$	$UU^{\cos\phi},\ LU^{\sin\phi},\ UT,\ LT,\ UT^{\cos\phi},\ UT^{\sin\phi}$
$\widetilde{\mathbf{E}}$	2	LT	g_{1T}	$UT^{\sin(\phi-\phi_s)},\ LT^{\cos(\phi-\phi_s)}$	$UL^{\sin\phi},\ LL^{\cos\phi},\ UT^{\cos\phi},\ UT^{\sin\phi}$
$\mathbf{H}+\mathbf{E}$	2	-	-	-	$UU^{\cos\phi},\ LU^{\sin\phi},\ UL^{\sin\phi},\ LL^{\cos\phi},\ UT^{\cos\phi},\ UT^{\sin\phi}$
$2\widetilde{\mathbf{H}}_{\mathbf{2T}} + \mathbf{E}_{\mathbf{2T}} - \xi \widetilde{E}_{2T}$	3	UU	$f^{\perp}$	$UU^{\cos\phi},\ LU^{\sin\phi}$	UU, LU
$2\widetilde{\mathbf{H}}'_{\mathbf{2T}} + \mathbf{E}'_{\mathbf{2T}} - \xi \widetilde{E}'_{2T}$	3	LL	$g_L^{\perp}$	$UU^{\cos\phi},\ LU^{\sin\phi}$	UU, LU
$\mathbf{H}_{\mathbf{2T}} + \frac{\mathbf{t_o - t}}{\mathbf{4M^2}} \widetilde{\mathbf{H}}_{\mathbf{2T}}$	3	UT	$f_T^{(*)}, f_T^{\perp(*)}$	$UU^{\cos\phi},\ UL^{\cos\phi},\ LU^{\sin\phi},\ LL^{\cos\phi}$	UU, LU
$\mathbf{H}'_{\mathbf{2T}} + \frac{\mathbf{t_o - t}}{\mathbf{4M^2}} \widetilde{\mathbf{H}}'_{\mathbf{2T}}$	3	LT	$g'_T, g_T^{\perp}$	$UU^{\cos\phi},UL^{\cos\phi},\ LU^{\sin\phi},\ LL^{\cos\phi}$	UU, LU
$\mathbf{E}_{\mathbf{2T}} - \xi E_{2T}$	3	UL	$f_L^{\perp(*)}$	$UU^{\cos\phi},\ UL^{\cos\phi},\ LU^{\sin\phi},\ LL^{\cos\phi}$	UU, LU, UT
$\widetilde{\mathbf{E}}'_{\mathbf{2T}} - \xi E'_{2T}$	3	LU	$g^{\perp(*)}$	$UU^{\cos\phi},\ UL^{\cos\phi},\ LU^{\sin\phi},\ LL^{\cos\phi}$	UU, LU, UT
$\widetilde{\mathbf{H}}_{\mathbf{2T}}$	3	UT_x	$f_T^{\perp(*)}$	$UU^{\cos\phi},\ UL^{\cos\phi},\ LU^{\sin\phi},\ LL^{\cos\phi}$	UU, LU, UT
$\widetilde{\mathbf{H}}'_{\mathbf{2T}}$	3	LT_x	$g_T^{\perp}$	$UU^{\cos\phi},\ UL^{\cos\phi},\ LU^{\sin\phi},\ LL^{\cos\phi}$	UU, LU, UT

Transverse OAM

OAM

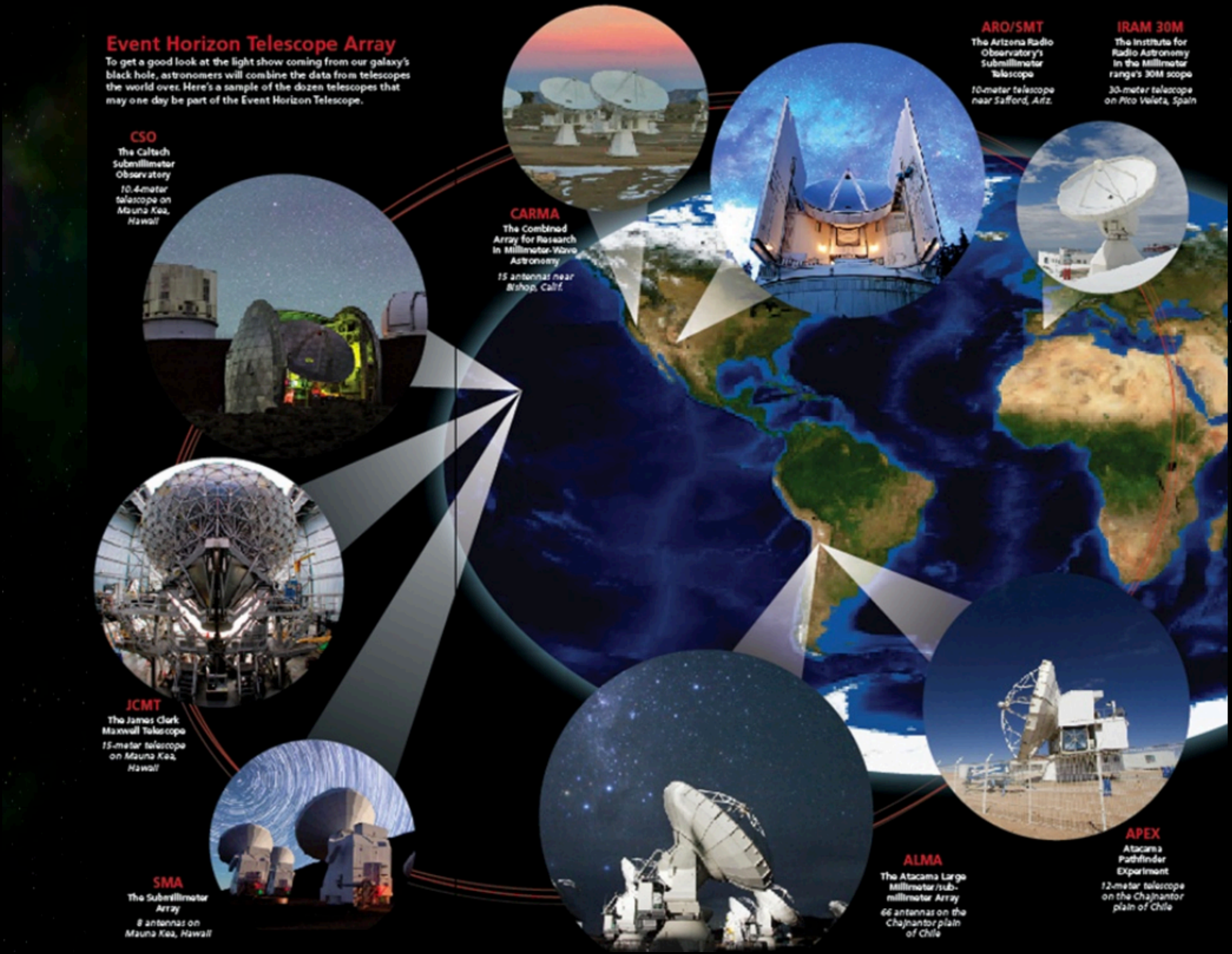
Spin Orbit

Newly accessible configurations!

Kriesten et al., Phys.Rev.D 101 (2020)
Kriesten and SL , 2004.08890

A multi-step, multi-prong process that compares to imaging a black hole

Images courtesy of Kent Yagi, UVA



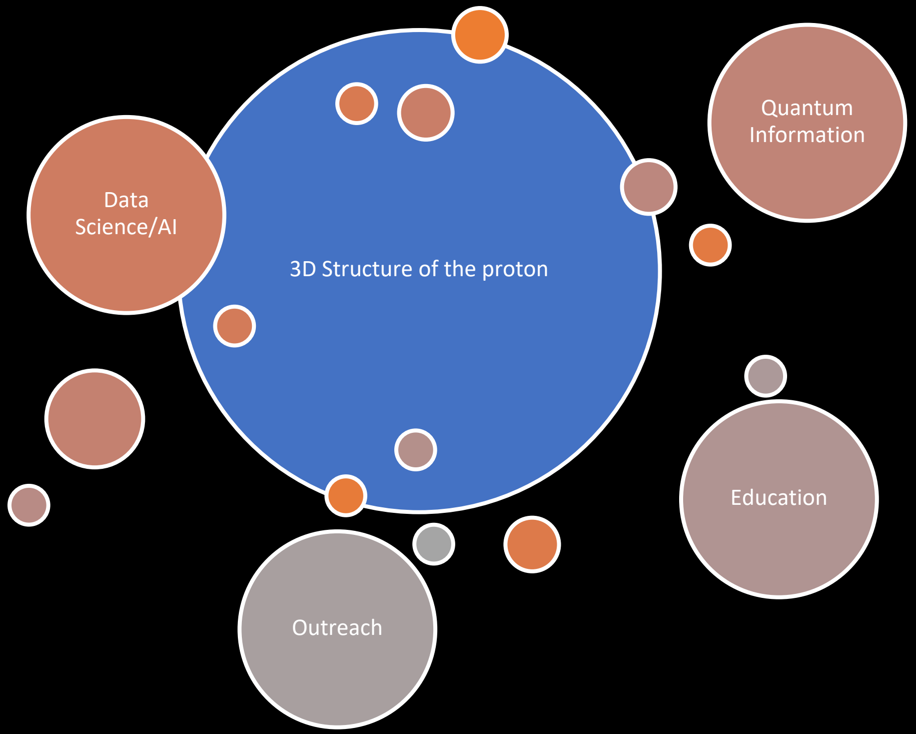
Event Horizon Telescope

Center for Femtography (CNF)

CENTER for NUCLEAR FEMTOGRAPHY

SURA

- December 2018 -- Founded at UVA meeting
- Summer 2019 -- Pilot projects started funded by SURA
- Summer 2020 -- Xiangdong Ji is appointed director



- CNF has funded multiple projects on Femtography through GPDs.
- Covering different areas
  - Experimental data
  - ML & AI
  - Inverse problems
  - Lattice calculations
  - …
- Initial funding $0.5M, per year
- Now it is time to organize different efforts into a larger collaboration, involving people outside VA.

Courtesy of X. Ji

CONCLUSIONS

- We presented an avenue to identify observables sensitive to both longitudinal and transverse OAM

- Jefferson Lab @12 GeV and EIC will make history as the we uncover the mechanical properties the of the proton and observe its spatial images

- To observe, evaluate and interpret GPDs and Wigner distributions at the subatomic level requires stepping up data analyses from the standard methods and developing new numerical/analytic/quantum computing methods

Center for Nuclear Femtography!