First lattice QCD study of proton twist-3 GPDs

Martha Constantinou

Temple University

In collaboration with:

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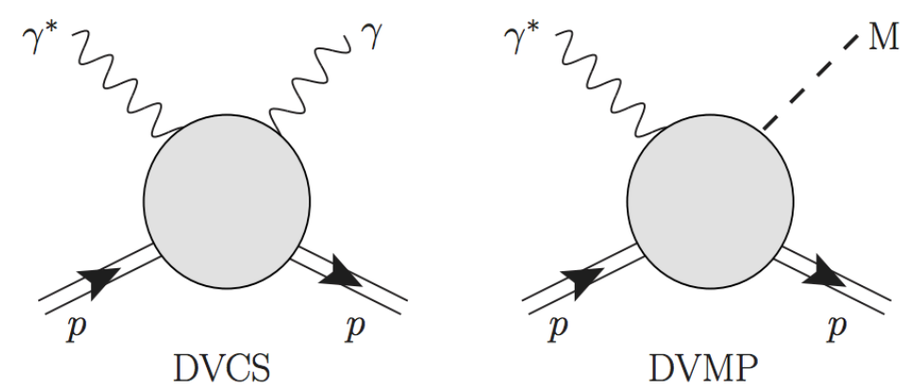
9th Workshop
APS Topical Group on Hadronic Physics (GHP)

April 13, 2021

Why GPDs?

- ★ GPDs provide information on spatial distribution of partons inside the hadron, and its mechanical properties (OAM, pressure, etc.)
  [M. Burkardt, Phys.Rev.D62 071503 (2000), hep-ph/0005108]
  [M. V. Polyakov, Phys. Lett. B555 (2003) 57, hep-ph/0210165]

- ★ Experimentally accessed in DVCS and DVMP
  [X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]
  (Halls A,B,C (JLab), PHENIX, STAR, HERMES, COMPASS, GSI, BELLE, J-PARC)



- ★ Experimentally, GPDs are not well-constrained:
  - independent measurements to disentangle GPDs
  - limited coverage of kinematic region
  - data on certain GPDs
  - indirectly related to GPDs through the Compton FFs
  - GPDs phenomenology more complicated than PDFs (multi-dimensionality)

Why twist-3 GPDs?

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 Twist-3

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Twist-2 Twist-3

Higher-twist distributions:

- ★ Lack density interpretation, but can be sizable
- ★ Sensitive to soft dynamics
- ★ challenging to probe experimentally and isolate from leading-twist

[Defurne et al., PRL 117, 26 (2016); Defurne et al., Nature Commun. 8, 1 (2017)]

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- ★ Needed for proton tomography
- ★ Related to certain spin-orbit correlations [C. Lorce, PLB 735 (2014) 344, arXiv:1401.7784]
- ★ Estimation on power corrections in hard exclusive processes (DVCS)
- ★ $[\widetilde{H} + \widetilde{G}_2](x, \xi, t)$ related to tomography of $F_\perp$ acting on the active q in DIS off a transversely polarized N right after the virtual photon absorbing

[M. Burkardt, PRD 88 (2013) 114502, arXiv:0810.3589]

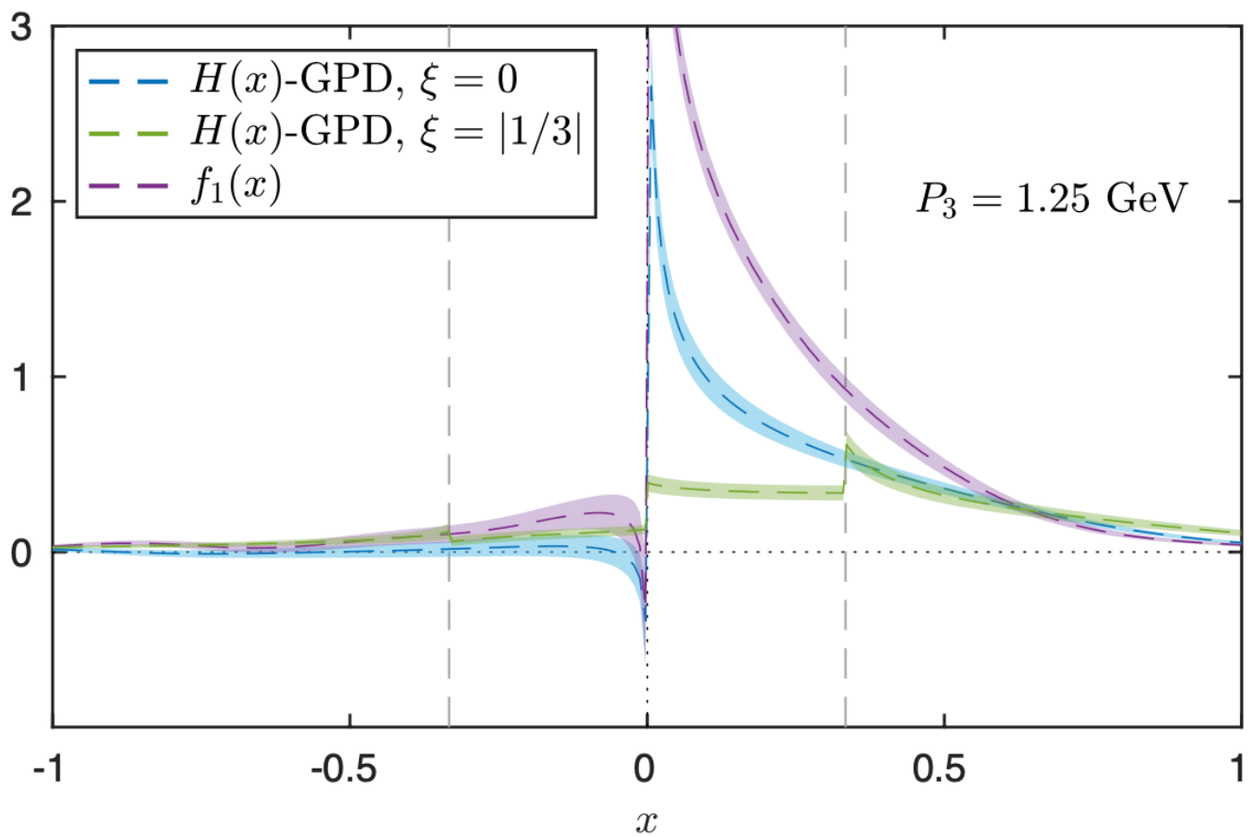
- ★ $G_2(x, \xi, t)$ related to L_q :

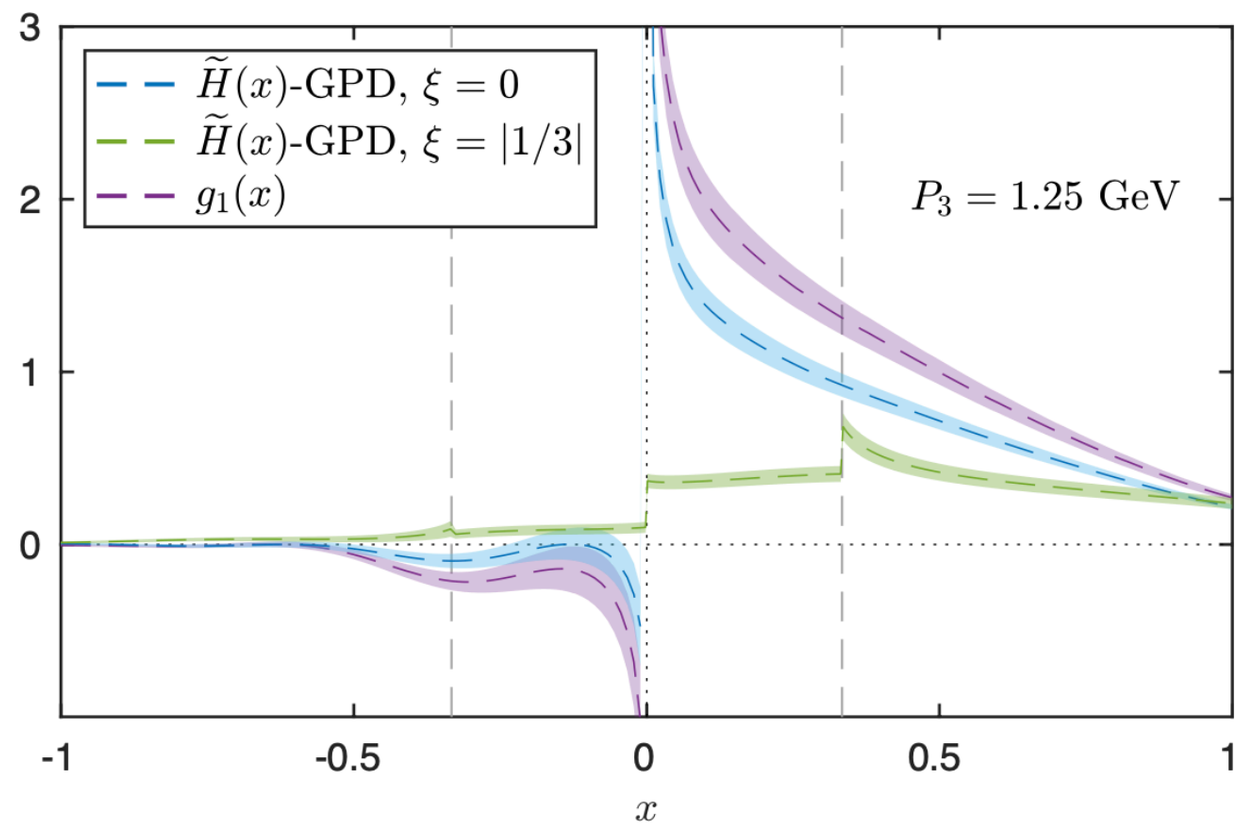
$$L_q = -\int_{-1}^{1} dx\, x\, G_2^q(x, \xi, t = 0)$$

[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249], [M. Penttinen et al., PLB 491 (2000) 96, arXiv:hep-ph/0006321]

Results on twist-2 GPDs

★ Unpolarized ($H(x,\xi,t), E(x,\xi,t)$) and helicity ($\widetilde{H}(x,\xi,t)$, $\widetilde{E}(x,\xi,t)$) GPDs for zero and nonzero skewness

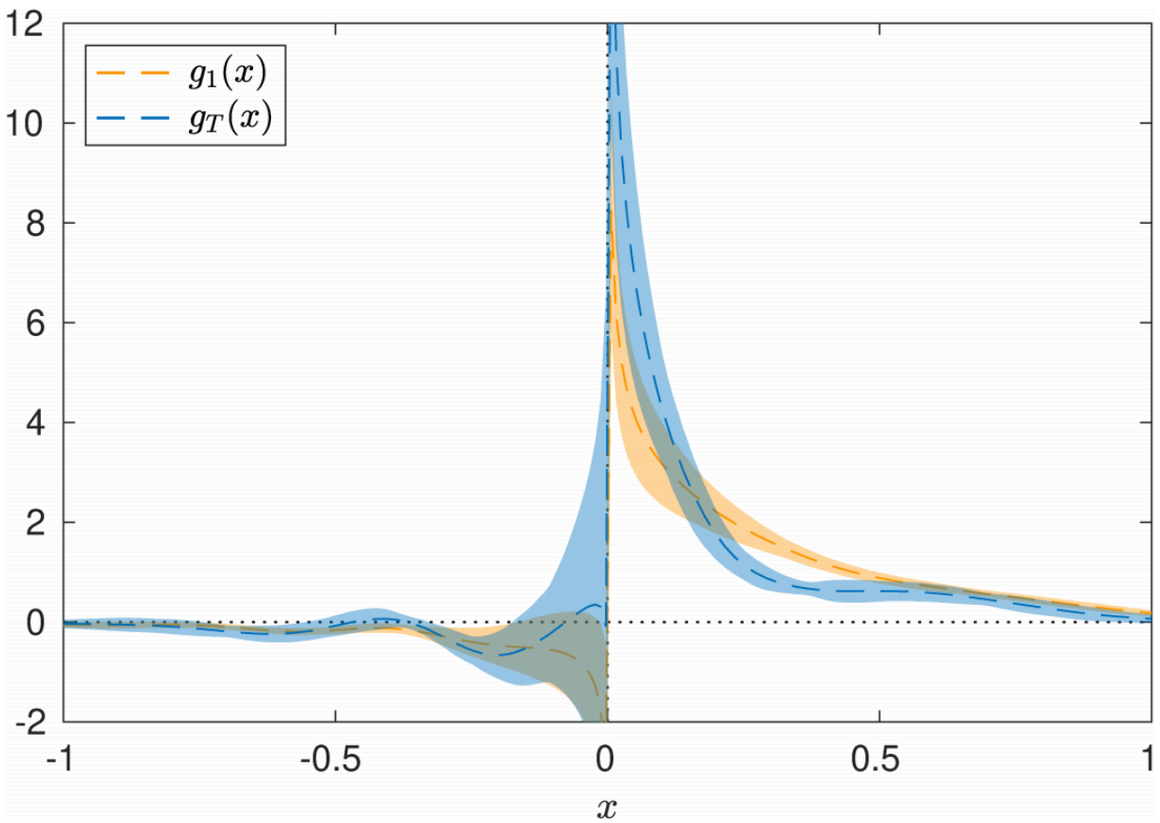




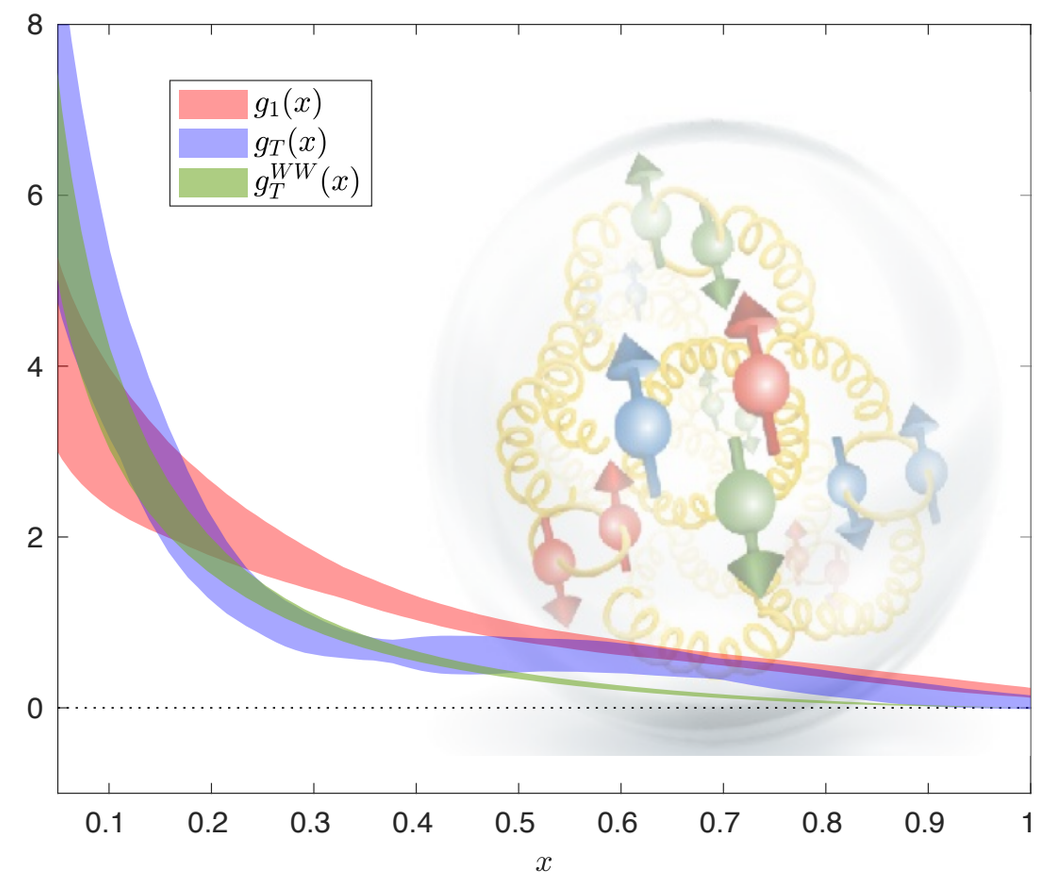
[C. Alexandrou et al. (ETMC), PRL 125 (2020) 26, 262001, arXiv:2008.10573]

Results on twist-3 PDFs

★ Helicity flip ($g_T(x)$) and transversity ($h_L(x)$) twist-3 PDFs



PRD Editors' Suggestion Highlight



[S. Bhattacharya et al., PRD 102 (2020) 11, arXiv:2004.04130]

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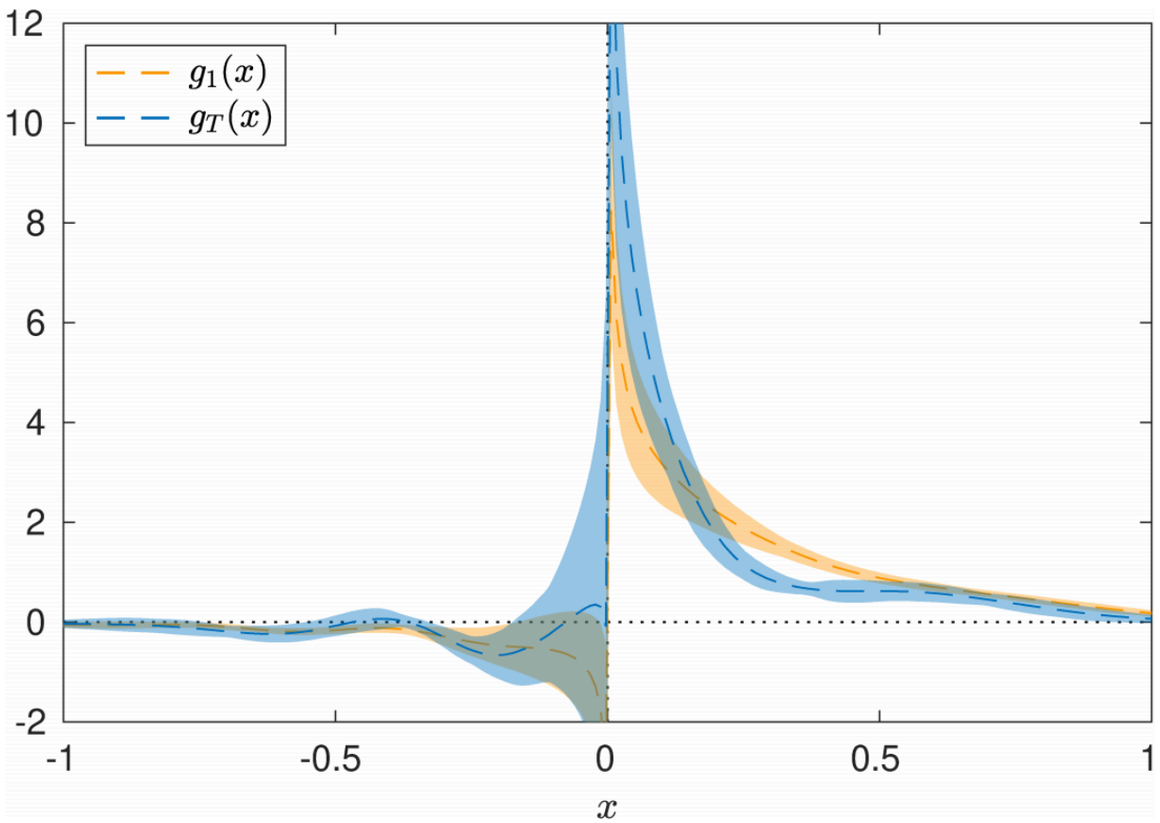

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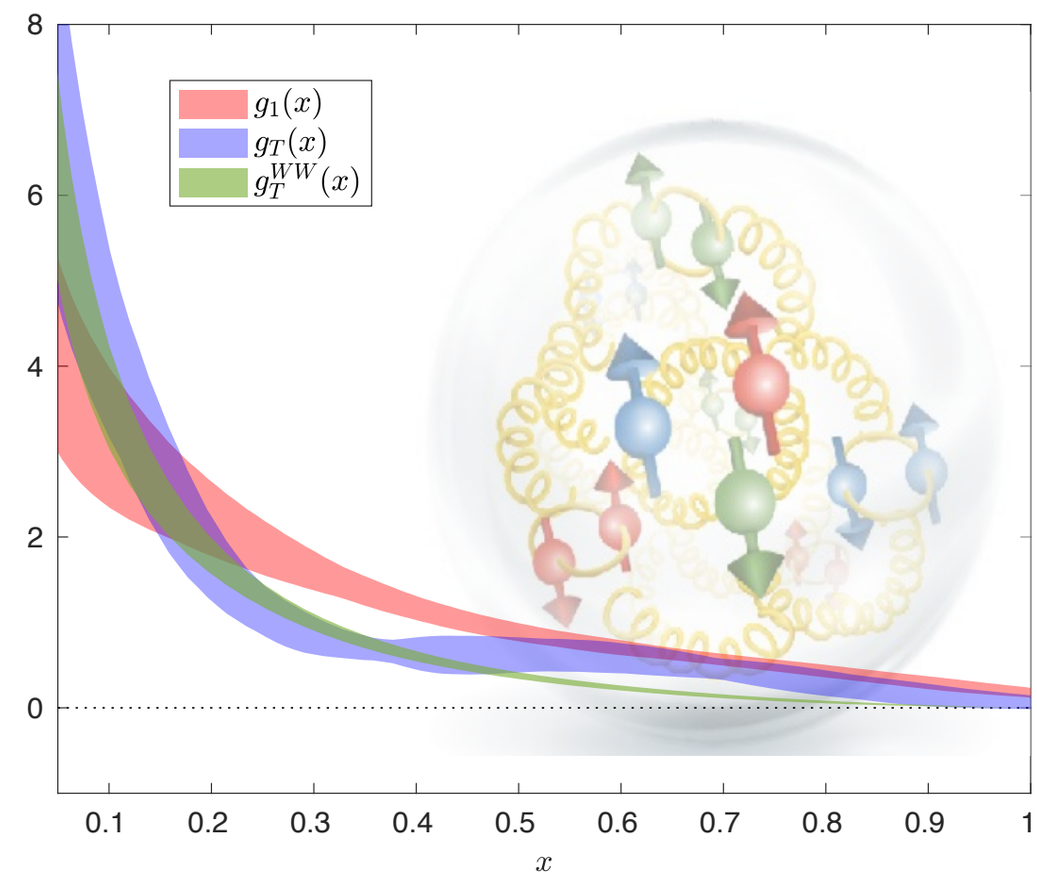

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See talks by:

S. Bhattacharya: Tue @ 4:50 pm (on matching)

A. Scapellato: Tue @ 5:10 pm (on lattice results)

Twist-3 GPDs from lattice QCD

(quasi-distributions method)

Useful Reviews:

[K. Cichy, M. Constantinou, Advances in HEP, Volume 2019, Article ID 3036904, arXiv:1811.07248]

[X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, and Y. Zhao (2020), 2004.03543]

[M. Constantinou (invited review) Eur. Phys. J. A 57 (2021) 2, 77, arXiv:2010.02445]

Access of GPDs on a Euclidean Lattice

★ Relies on Large Momentum Effective Theory (LaMET) to reconstruct GPDs

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002; X. Ji, Sci. China Phys. Mech. Astron. 57, 1407 (2014)]

★ Matrix elements of spatial operators with fast moving hadrons

$$\tilde{q}_{\Gamma}^{\mathrm{GPD}}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_{\mu}$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

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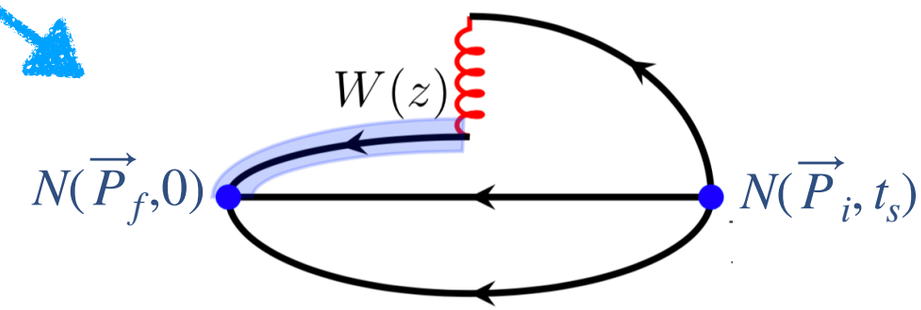

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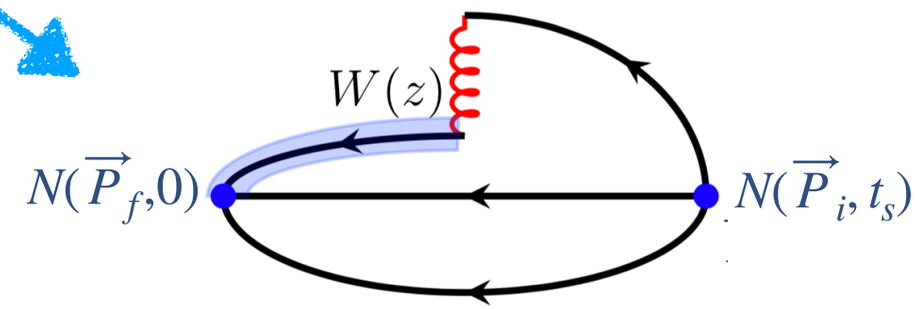

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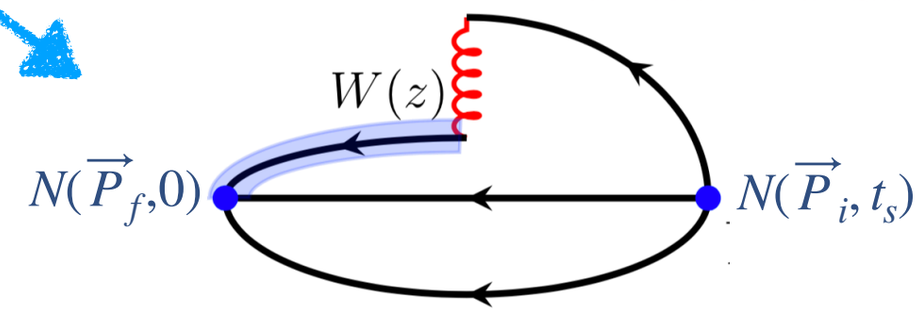

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} also in PDFs

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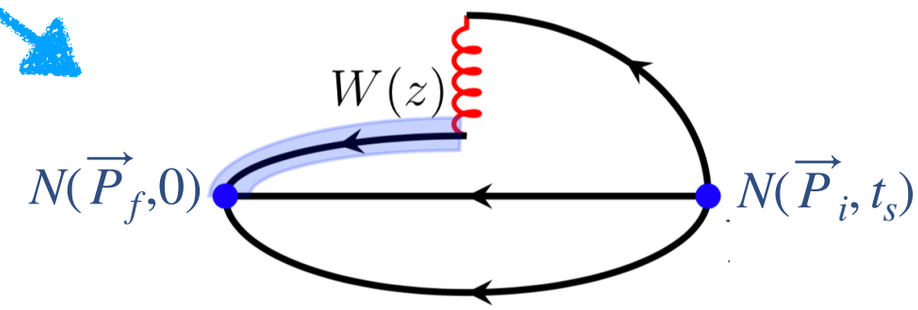

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} also in PDFs

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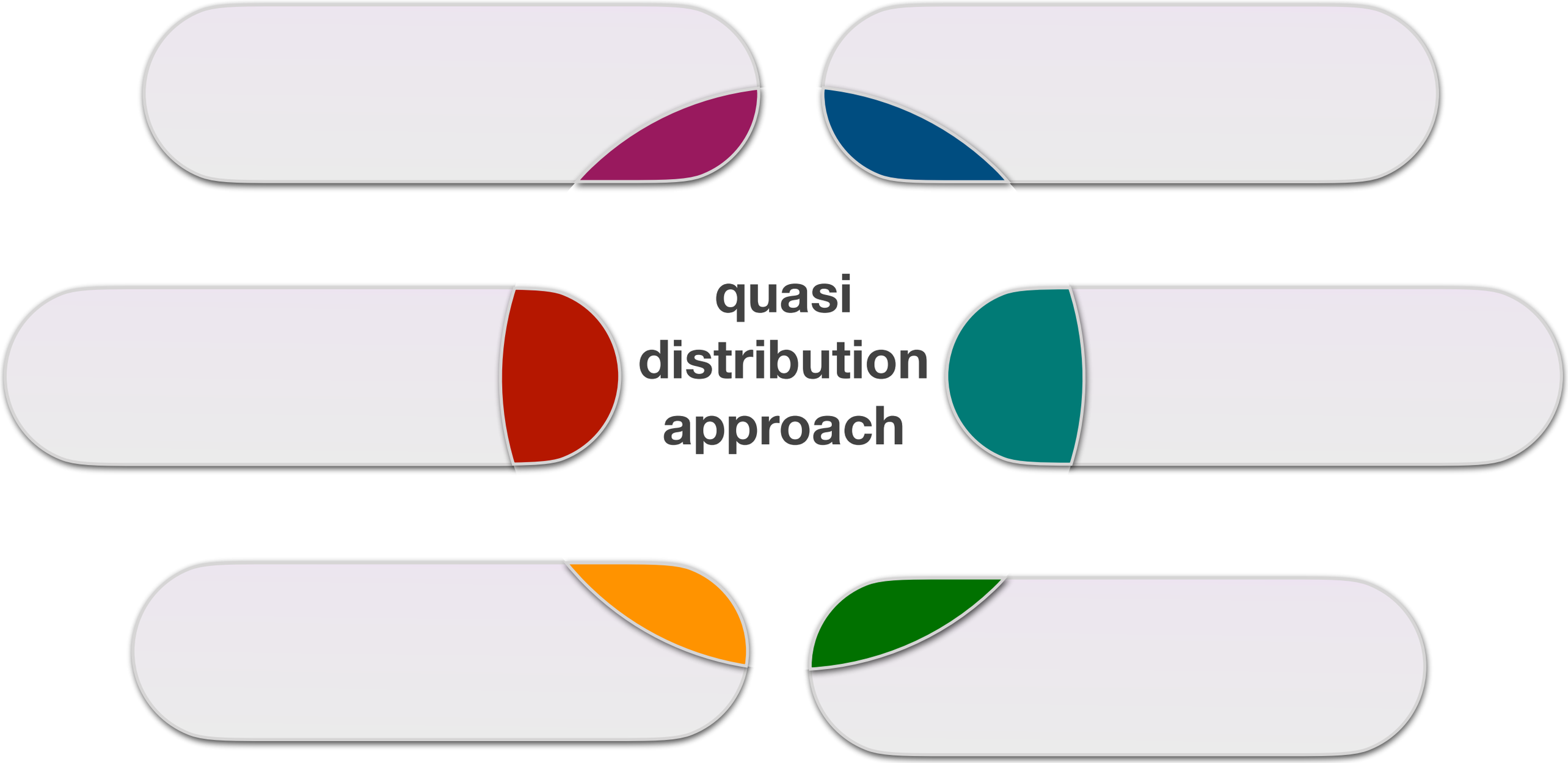
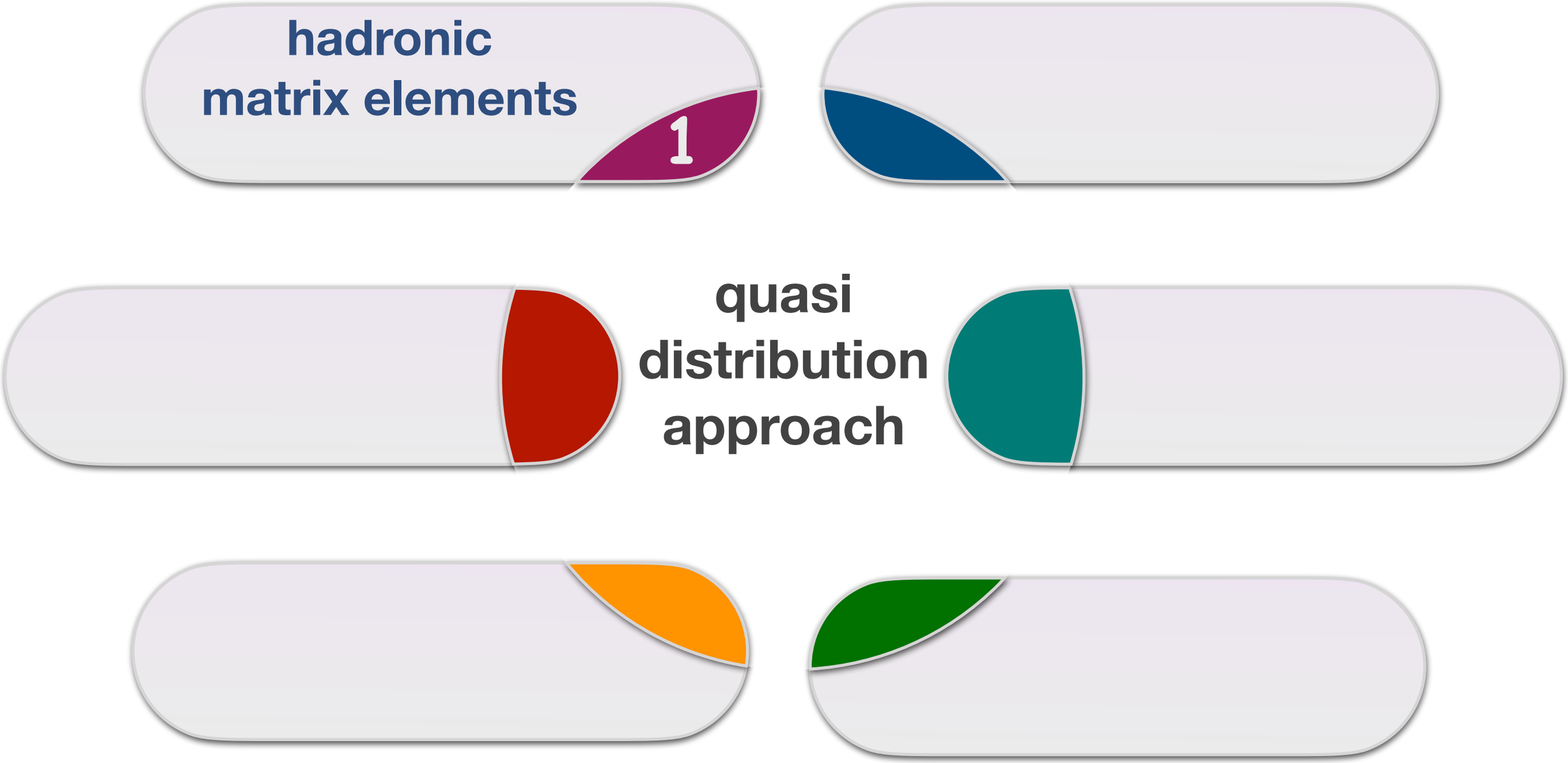
} for GPDs

★ Challenges of calculation:

- Increased statistical uncertainties due to momentum transfer
- Need for multiple matrix elements to disentangle GPDs
- Frame dependence
- Matching for nonzero skewness


quasi
distribution
approach



$$C^{2pt} = \langle N | N \rangle \qquad C^{3pt} = \langle N | \overline{\psi}(z)\Gamma\mathscr{A}(z,0)\psi(0) | N \rangle$$

hadronic matrix elements

1

Identification of ground state

2

quasi distribution approach

$$R_\Gamma(\mathscr{P}_\kappa, \mathbf{P_f}, \mathbf{P_i}; t, \tau) = \frac{C^{3pt}_\Gamma(\mathscr{P}_\kappa, \mathbf{P_f}, \mathbf{P_i}; t, \tau)}{C^{2pt}(\mathscr{P}_0, \mathbf{P_f}; t)} \times \sqrt{\frac{C^{2pt}(\mathscr{P}_0, \mathbf{P_i}; t-\tau) C^{2pt}(\mathscr{P}_0, \mathbf{P_f}; \tau) C^{2pt}(\mathscr{P}_0, \mathbf{P_f}; t)}{C^{2pt}(\mathscr{P}_0, \mathbf{P_f}; t-\tau) C^{2pt}(\mathscr{P}_0, \mathbf{P_i}; \tau) C^{2pt}(\mathscr{P}_0, \mathbf{P_i}; t)}} \xrightarrow[\tau \gg a]{t-\tau \gg a} h_{\mathscr{O},\mathscr{P}}(z, t, \xi, P_3)$$

hadronic matrix elements 1

Identification of ground state 2

quasi distribution approach

Renormalization 3

$$h^R_{\mathcal{O},\mathcal{P}}(z,t,\xi,P_3,\mu) = Z_{\mathcal{O}}(z,\mu)\, h_{\mathcal{O},\mathcal{P}}(z,t,\xi,P_3)$$

[M. Constantinou , H. Panagopoulos, Phys. Rev. D96, 054506 (2017), arXiv:1705.11193]

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4 “form factors” disentanglement

$$\widetilde{F}^{\mu} = P^{\mu}\frac{\tilde{h}^{+}}{P^{+}}\widetilde{H} + P^{\mu}\frac{\tilde{e}^{+}}{P^{+}}\widetilde{E}$$

$$+ \Delta_{\perp}^{\mu}\frac{\tilde{b}}{2m}\left(\widetilde{E}+\widetilde{G}_1\right) + \tilde{h}_{\perp}^{\mu}\left(\widetilde{H}+\widetilde{G}_2\right) + \Delta_{\perp}^{\mu}\frac{\tilde{h}^{+}}{P^{+}}\widetilde{G}_3 + \tilde{\Delta}_{\perp}^{\mu}\frac{h^{+}}{P^{+}}\widetilde{G}_4$$

hadronic matrix elements 1

Identification of ground state 2

quasi distribution approach

5 x-dependence reconstruction

Renormalization 3

4 “form factors” disentanglement

$$G_i(x, t, \xi, \mu, P_3) = \int \frac{dz}{4\pi} e^{-ixP_3 z} F_{G_i}(z, P_3, t, \xi, \mu)$$

In this work: Backus-Gilbert

1 hadronic matrix elements

6 Matching to light-cone GPDs

2 Identification of ground state

quasi distribution approach

5 x-dependence reconstruction

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$$G_i(x,t,\xi,\mu_0,(\mu_0)_3,P_3) = \int_{-1}^{1} \frac{dy}{|y|} C_{G_i}\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{yP_3},\frac{(\mu_0)_3}{yP_3},r\right) F_{G_i}(y,t,\xi,\mu) + \mathcal{O}\left(\frac{m^2}{P_3^2},\frac{t}{P_3^2},\frac{\Lambda_{\rm QCD}^2}{x^2P_3^2}\right)$$

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In this work: ξ=0 (matching of gT)

S. Bhattacharya: Tue @ 4:50 pm

Parameters of calculation

(u-d flavor combination)

- ★ Nf=2+1+1 twisted mass fermions & clover term

- ★ Ensemble parameters:

Pion mass:	260 MeV
Lattice spacing:	0.093 fm
Volume:	$32^3 \times 64$
Spatial extent:	3 fm

- ★ Kinematical setup:

P_3 [GeV]	$\vec{Q} \times \frac{L}{2\pi}$	$-t$ [GeV^2]	ξ	N_{meas}
0.83	(2,0,0)	0.69	0	4288
1.25	(2,0,0)	0.69	0	4288
1.25	(2,2,0)	1.39	0	4288
1.67	(2,0,0)	0.69	0	4288

- ★ Excited states: T_{sink} ~1 fm

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We utilize

$\pm P_3$

$\pm \vec{Q}$

$\gamma^j \gamma^5, \quad j = 1,2$

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Helicity flip twist-3 GPDs

- ★ **Transverse matrix element of axial operator (proton boost: $\vec{P} = (0,0,P_3)$)**

Helicity flip twist-3 GPDs

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[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

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★ **Sum Rules (generalization of Burkhardt-Cottingham)**

$$\int_{-1}^{1} dx\,\widetilde{H}(x,\xi,t) = G_A(t)\,, \qquad \int_{-1}^{1} dx\,\widetilde{E}(x,\xi,t) = G_P(t)\,, \qquad \int_{-1}^{1} dx\,\widetilde{G}_i(x,\xi,t) = 0$$

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$\mu = 1, 2$

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[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]

★ **Sum Rules (generalization of Efremov-Leader-Teryaev)**

[A. Efremov, O. Teryaev , E. Leader, PRD 55 (1997) 4307, hep-ph/9607217]

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_1(x,\xi,t) = \frac{1}{2}\left[F_2(t) + \left(\xi\frac{\partial}{\partial\xi} - 1\right)\int_{-1}^{1} dx\, x\, \widetilde{E}(x,\xi,t)\right], \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_2(x,\xi,t) = \frac{1}{2}\left[\xi^2 G_E(t) - \frac{t}{4m^2}F_2(t) - \widetilde{A}_{20}(t)\right],$$

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_3(x,\xi,t) = \frac{\xi}{4}G_E(t)\,, \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4}G_E(t)$$

F_2 : Pauli FF
G_E : electric FF
$\widetilde{A}_{20}$, $\widetilde{B}_{20}$: axial GFFs

Sum rules simplify for zero skewness

Decomposition of matrix elements

- **Decomposition in Minkowski space** [F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

$$h^\mu = \bar{u}(p')\,\gamma^\mu\,u(p)\,, \qquad e^\mu = \bar{u}(p')\,\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}\,u(p)\,, \qquad b = \bar{u}(p')\,u(p)\,,$$

$$\tilde{h}^\mu = \bar{u}(p')\,\gamma^\mu\gamma_5\,u(p)\,, \qquad \tilde{e}^\mu = \frac{\Delta^\mu}{2m}\,\tilde{b}\,, \qquad \tilde{b} = \bar{u}(p')\,\gamma_5\,u(p)$$

$$\widetilde{F}^\mu = P^\mu \frac{\tilde{h}^+}{P^+}\widetilde{H} + P^\mu \frac{\tilde{e}^+}{P^+}\widetilde{E}$$

$$+ \Delta_\perp^\mu \frac{\tilde{b}}{2m}(\widetilde{E} + \widetilde{G}_1) + \tilde{h}_\perp^\mu (\widetilde{H} + \widetilde{G}_2) + \Delta_\perp^\mu \frac{\tilde{h}^+}{P^+}\widetilde{G}_3 + \tilde{\Delta}_\perp^\mu \frac{h^+}{P^+}\widetilde{G}_4$$

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- **Kinematic factors defined by calculation setup**
- **All twist-3 helicity GPDs:** **4 x computational cost compared to PDFs !**

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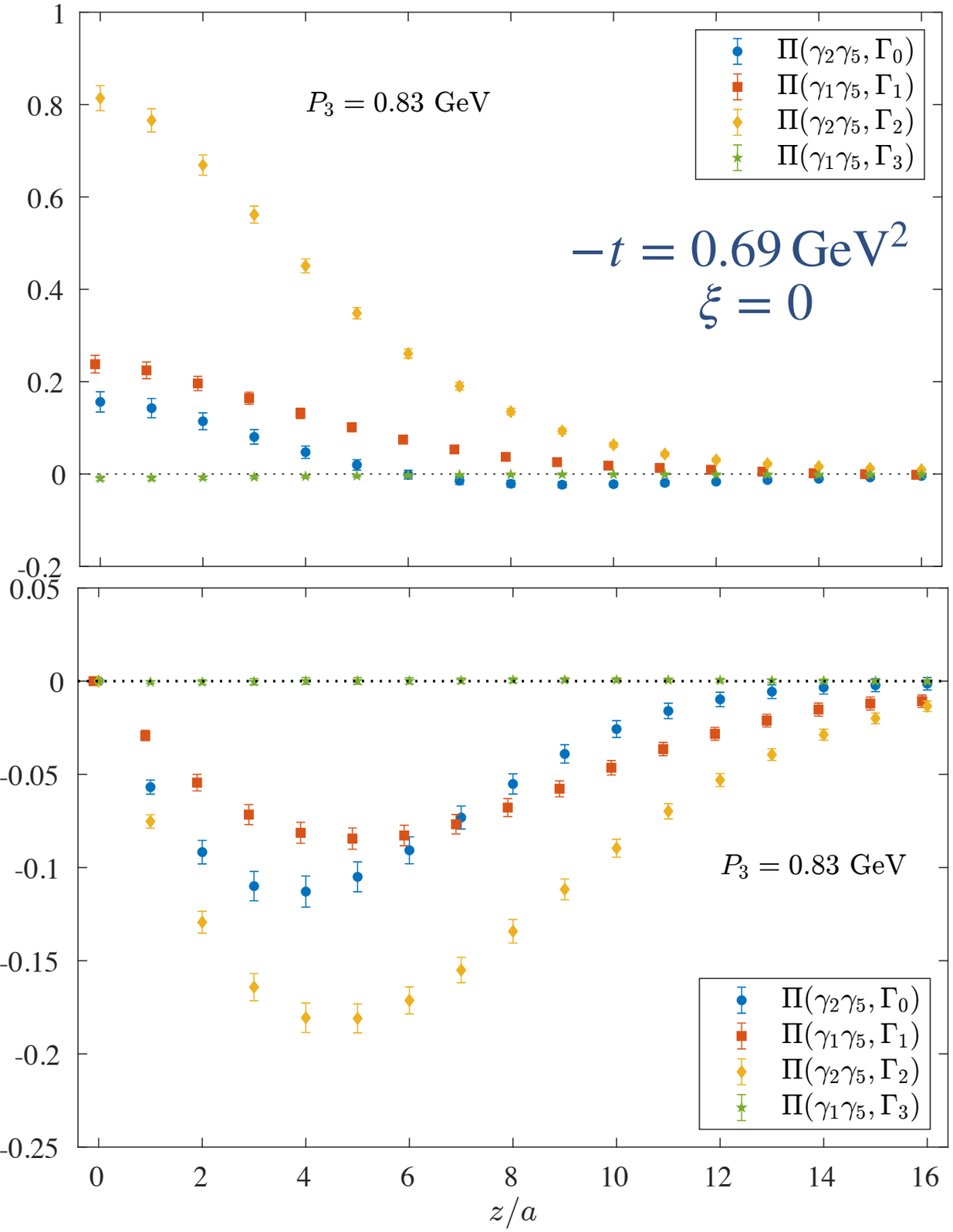
- **Kinematic factors defined by calculation setup**
- **All twist-3 helicity GPDs:** **4 x computational cost compared to PDFs !**
- **For** $\vec{Q} = (Q_x,0,0)$ **the following matrix elements contribute**
  - $\Pi(\gamma^2\gamma^5, \Gamma_0)$: $\widetilde{H} + \widetilde{G}_2\,, \quad \widetilde{G}_4$
  - $\Pi(\gamma^2\gamma^5, \Gamma_2)$: $\widetilde{H} + \widetilde{G}_2\,, \quad \widetilde{G}_4$
  - $\Pi(\gamma^1\gamma^5, \Gamma_1)$: $\widetilde{H} + \widetilde{G}_2\,, \quad \widetilde{E} + \widetilde{G}_1$
  - $\Pi(\gamma^1\gamma^5, \Gamma_3)$: $\widetilde{G}_3$

Parity projectors

$$\Gamma_0 = \frac{1}{4}(1+\gamma^0)$$

$$\Gamma_i = \frac{1}{4}(1+\gamma^0)\gamma^5\gamma^i$$

Bare matrix elements (ME)



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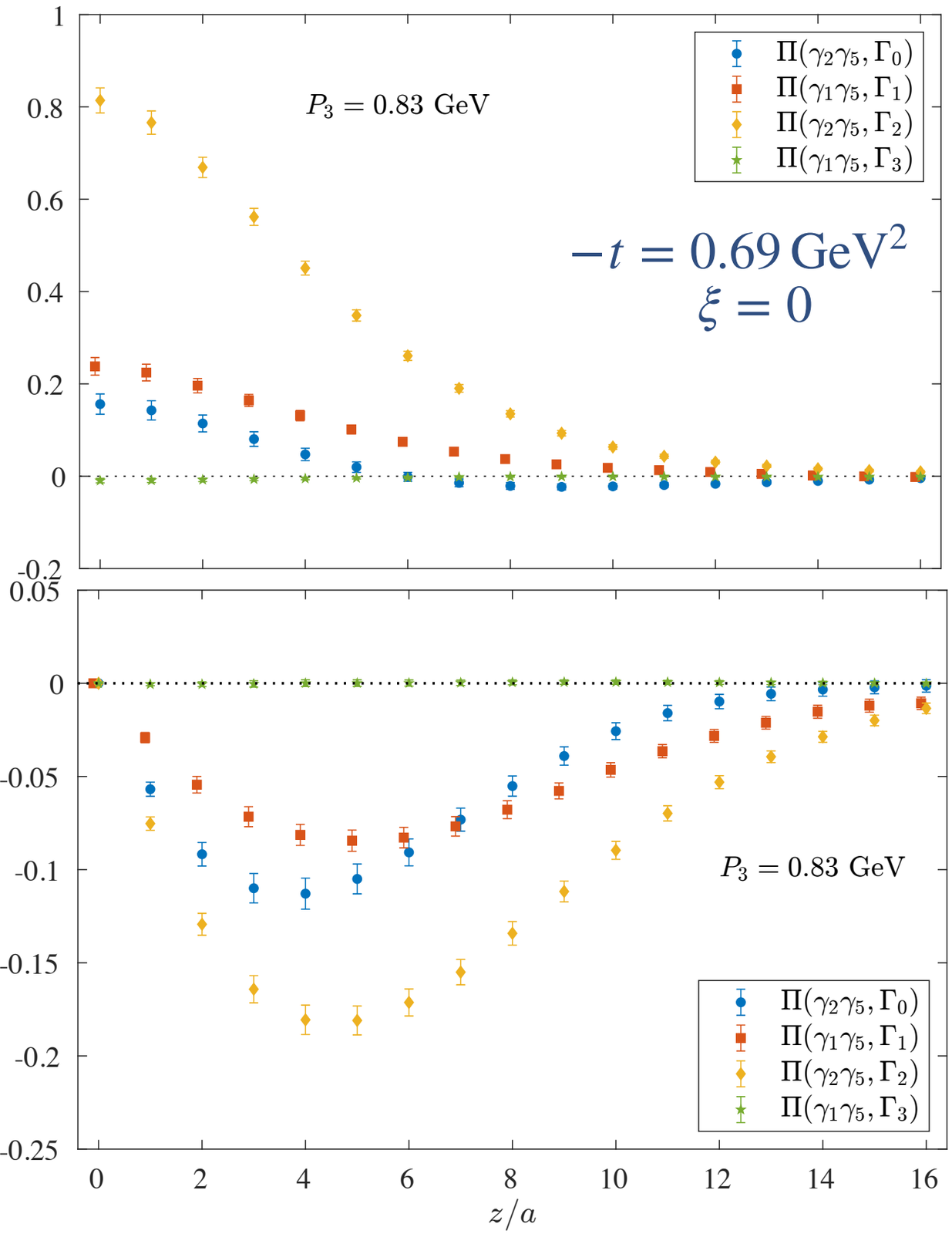
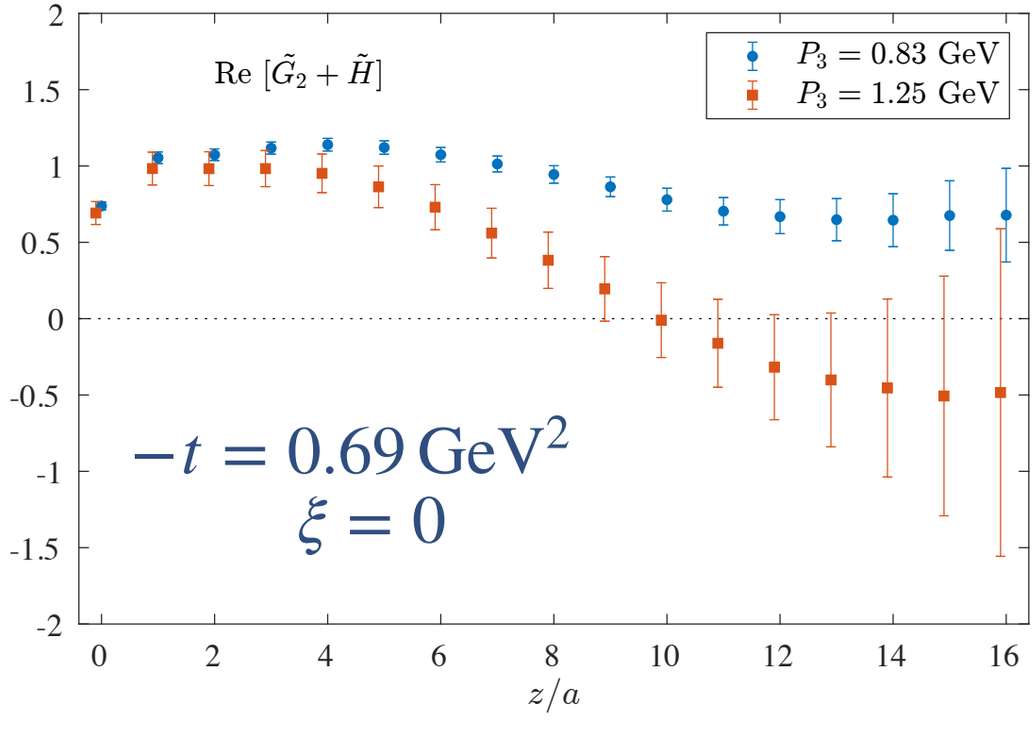
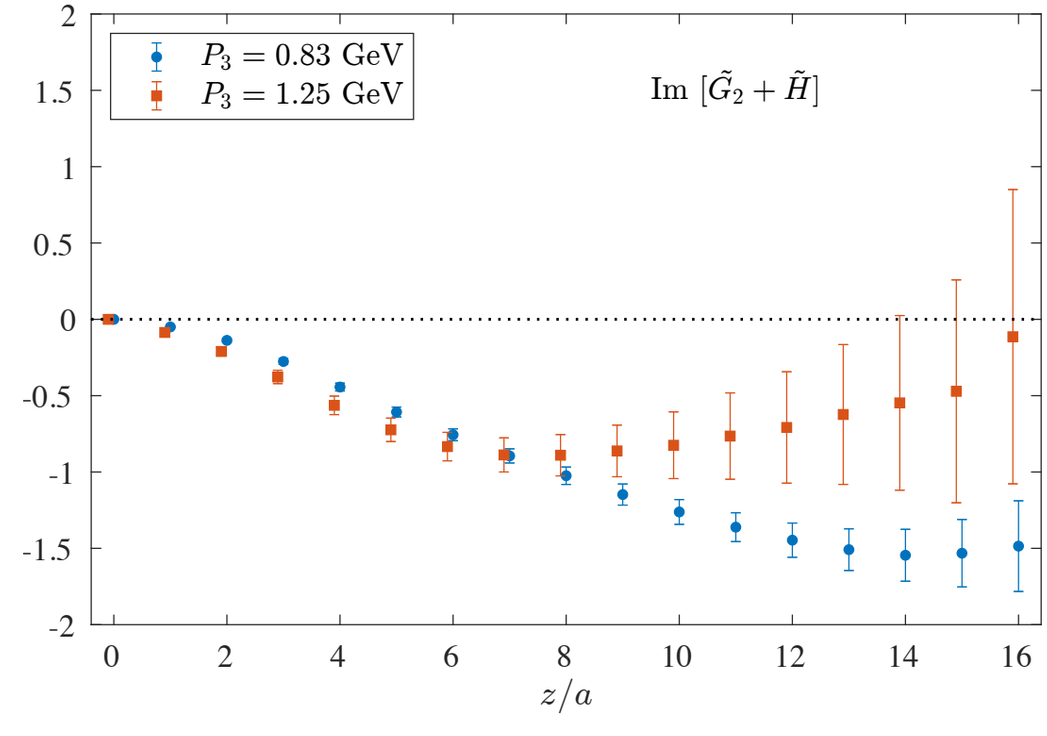
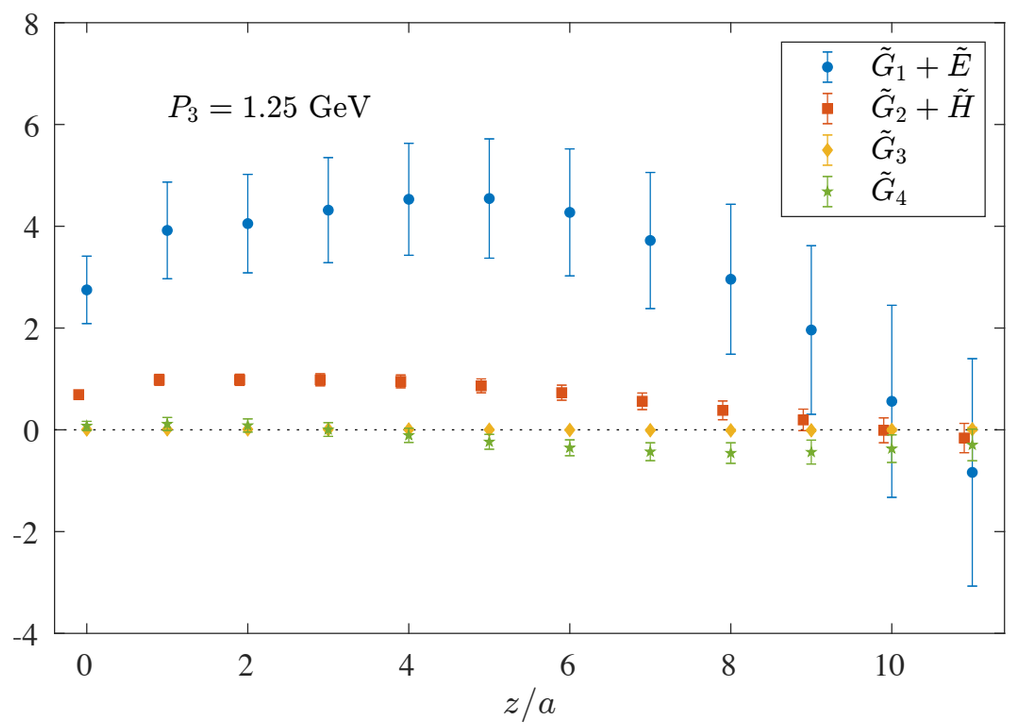


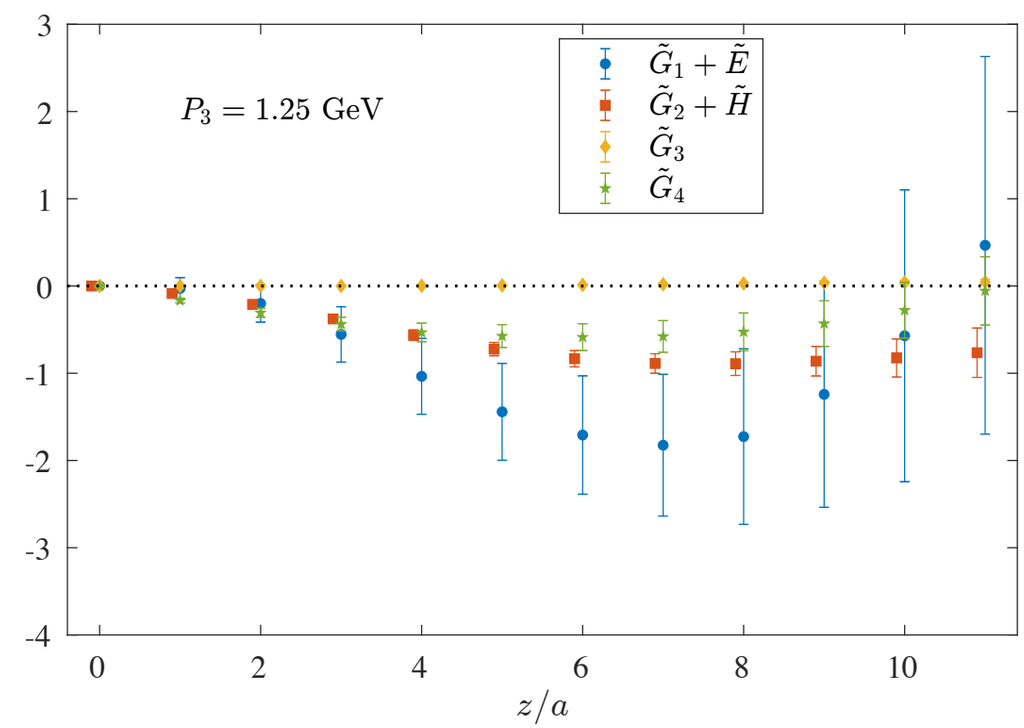
- ★ $\Pi(\gamma^2\gamma^5, \Gamma_0)$ & $\Pi(\gamma^2\gamma^5, \Gamma_2)$: disentangle $\widetilde{H} + \widetilde{G}_2$, $\widetilde{G}_4$
- ★ $\Pi(\gamma^1\gamma^5, \Gamma_1)$ and $\widetilde{H} + \widetilde{G}_2$: disentangle $\widetilde{E} + \widetilde{G}_1$
- ★ $\Pi(\gamma^1\gamma^5, \Gamma_3)$ gives $\widetilde{G}_3$
- ★ Similar picture for $P_3 = 1.25$ GeV
- ★ Real part of ME: dominant
- ★ $\widetilde{G}_3$ is kinematically suppressed

Matrix elements decomposition









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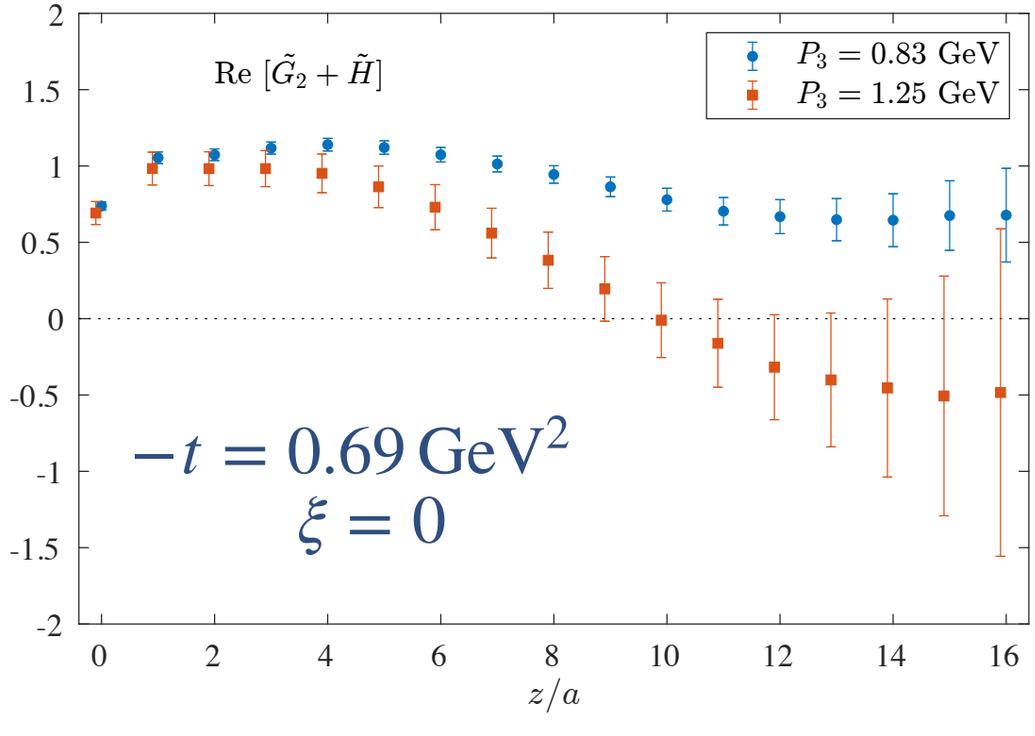
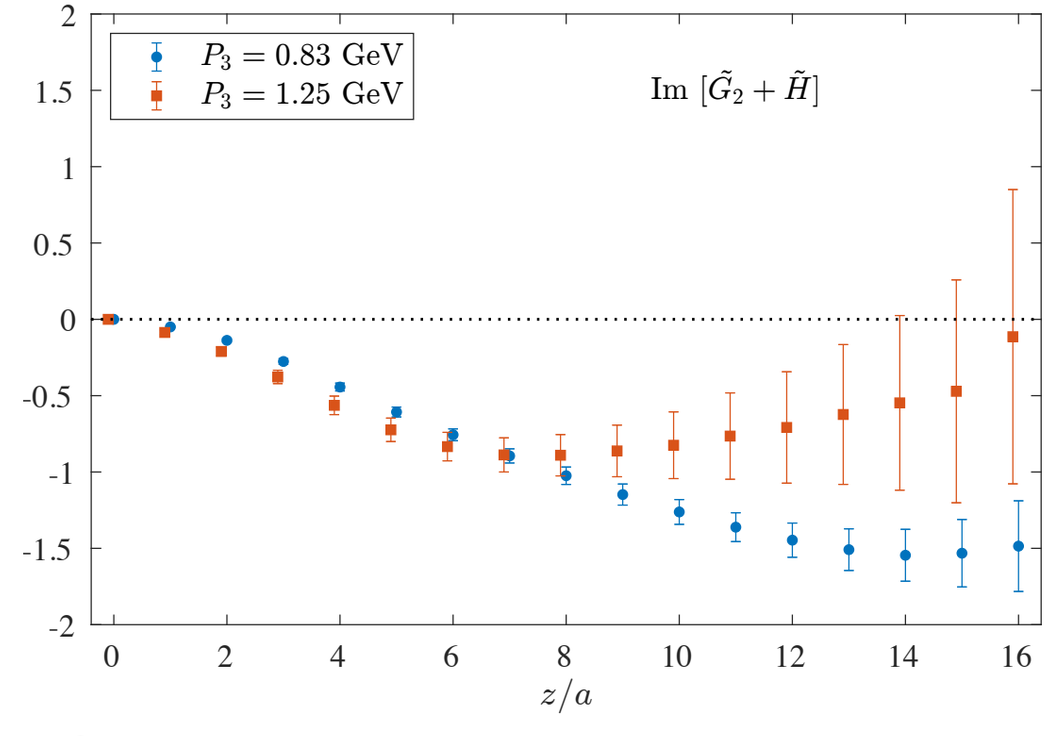
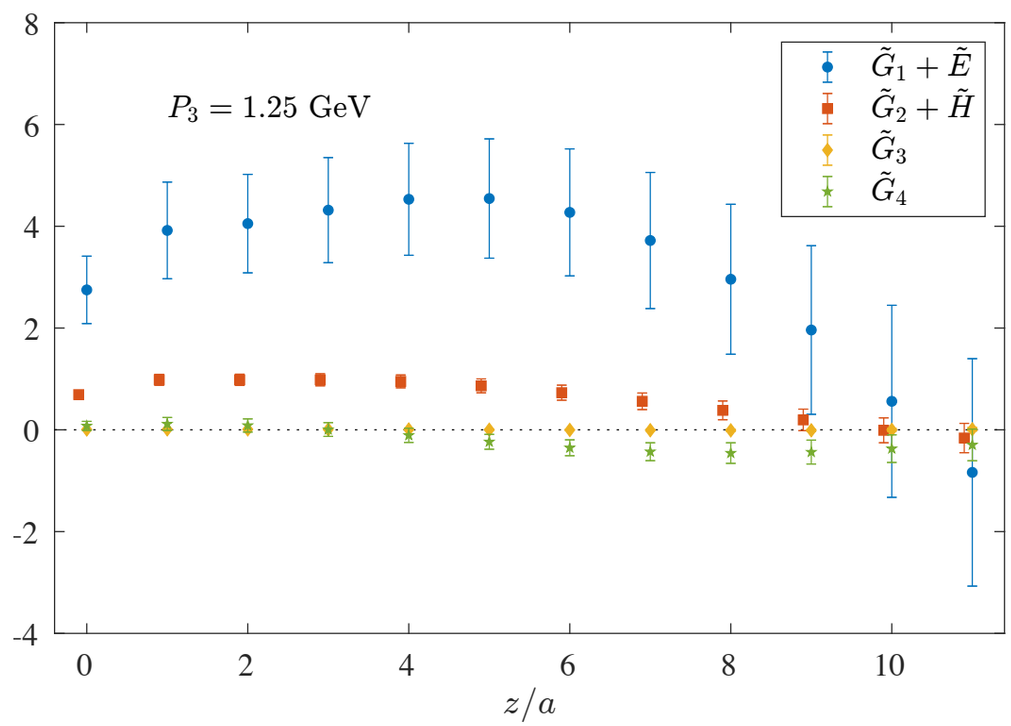
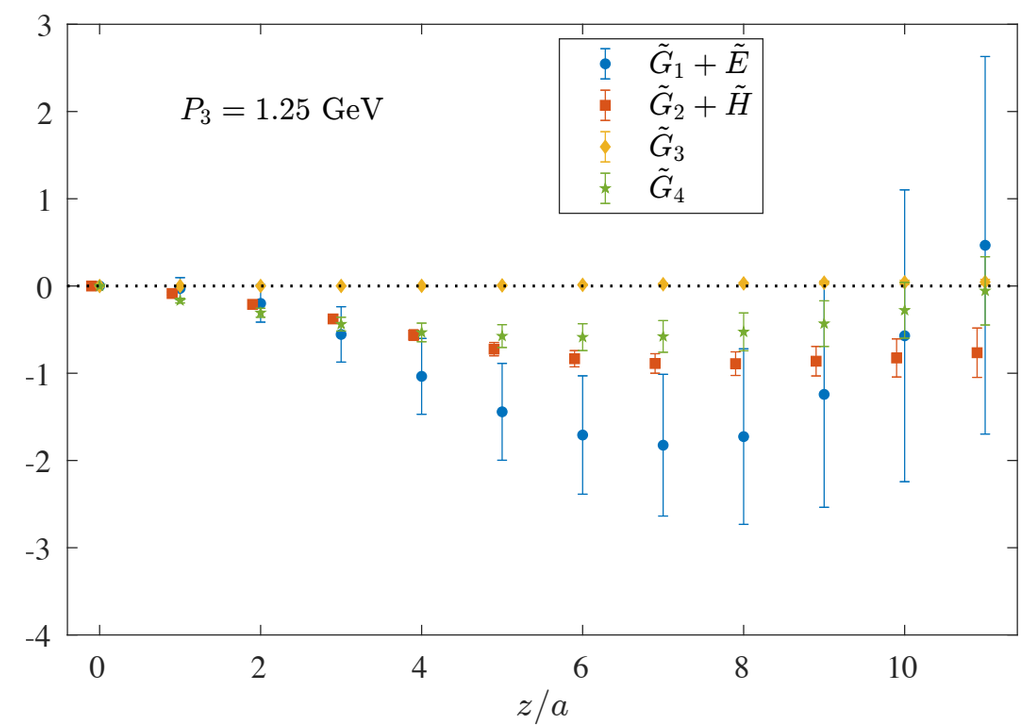






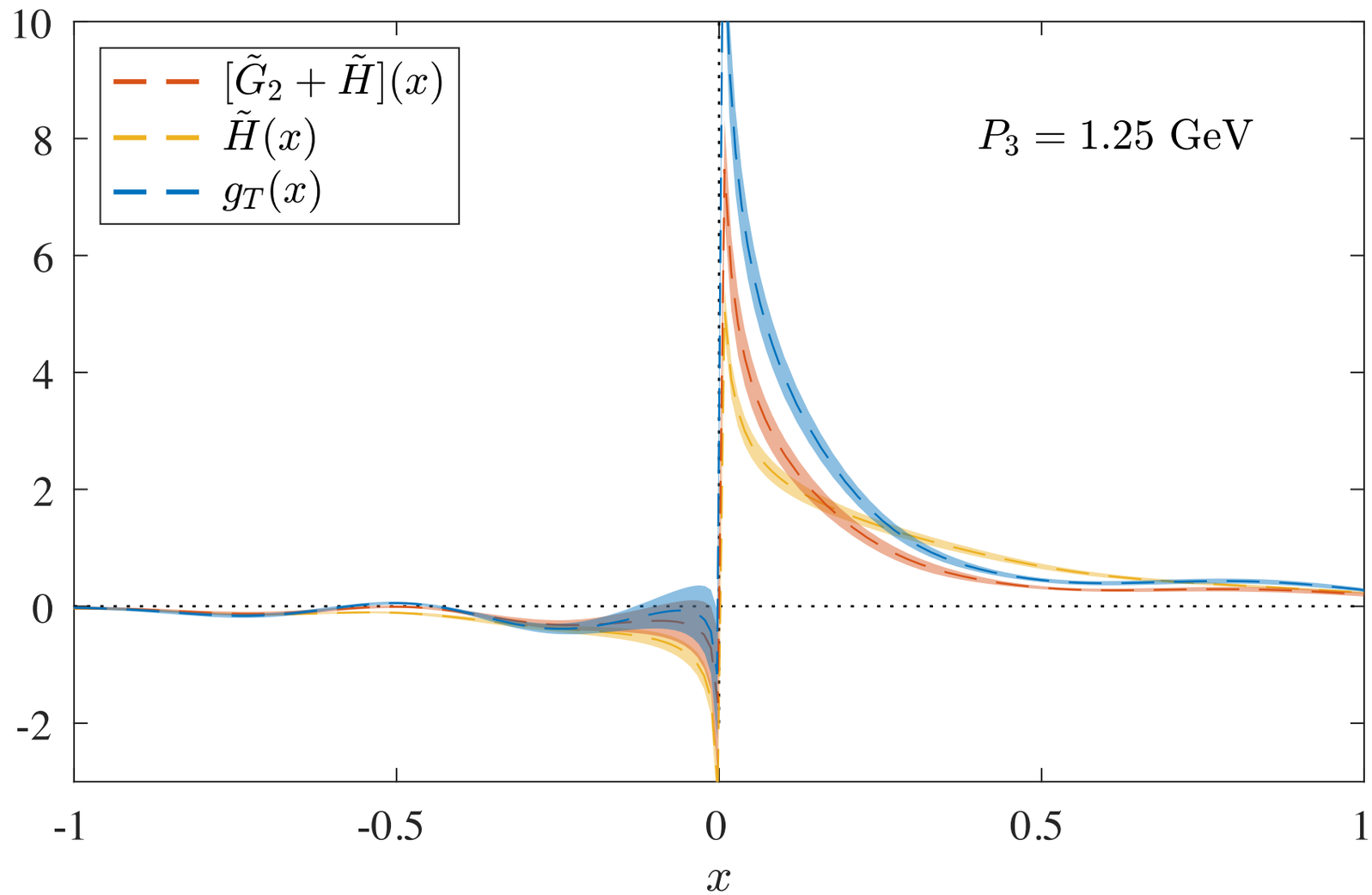


★ $\widetilde{E} + \widetilde{G}_1$: **highest contribution**

★ $\widetilde{G}_3,\ \widetilde{G}_4$: **suppressed**

x-dependence of GPDs

$-t = 0.69\,\mathrm{GeV}^2, \qquad \xi = 0$



★ $g_T(x)$: dominant distribution

★ $\widetilde{H} + \widetilde{G}_2$ similar in magnitude to $\widetilde{H}$

★ $\widetilde{G}_2$ is expected to be small

Concluding Remarks

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- ★ GPDs multi-dimensionality poses computational challenges

- ★ At twist-3 there are 2-parton correlations, as well as 3-parton correlations, such as quark-gluon-quark (qgq)
  - 0th moments of twist-3 GPDs is zero [D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105]
  - 1st moments of twist-3 GPDs have zero qgq contribution
  - alternative matching proposed in Braun, Ji, Vladimirov, arXiv: 2103.12105

- ★ Extraction of twist-3 GPDs is promising with several interesting investigations (WW-approximation, sum rules)

- ★ Nonzero skewness of particular interest: twist-3 GPDs (in models) exhibit discontinuities at $x = \pm\,\xi$

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Thank you

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TMD Collaboration TMD Topical Collab.

DOE Early Career Award (NP)
Grant No. DE-SC0020405