Precision Lattice Calculation of the *x*-dependence of Pion Valence PDF

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Argonne NATIONAL LABORATORY

In collaboration with Xiang Gao, Andrew Hanlon, Nikhil Karthik, Swagato Mukherjee, Peter Petreczky, Philipp Scior, Sergey Syritsyn, in preparation.

The Electron-Ion Collider

"A machine that will unlock the secrets of the strongest force in Nature"

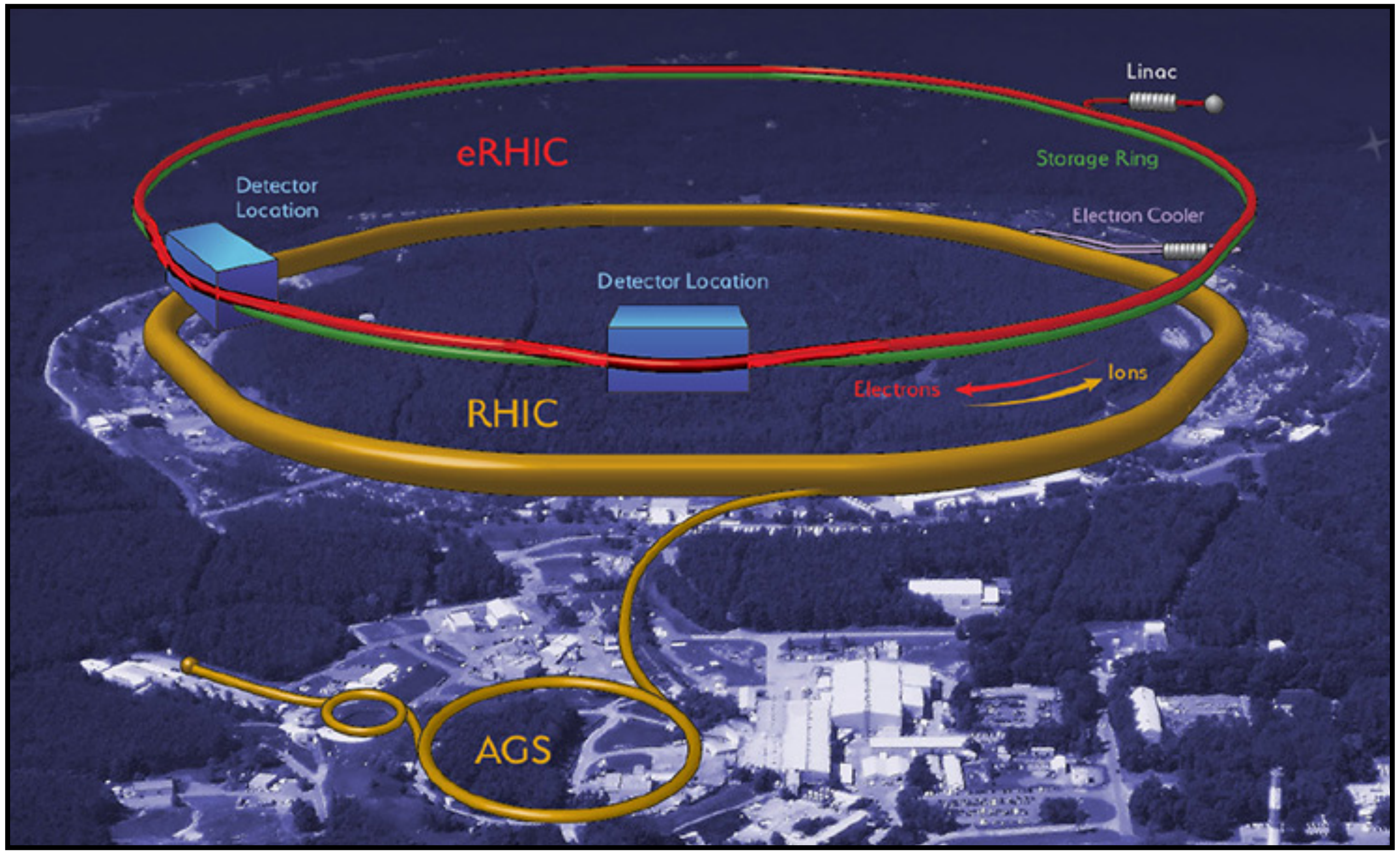


Image credit, BNL.

- Precision 3D imaging of protons and nuclei
- Solving the proton spin puzzle
- Search for saturation
- Quark and gluon confinement
- Quarks and gluons in nuclei

Outline

- Methodology
  - Large-momentum effective theory
  - Hybrid renormalization scheme

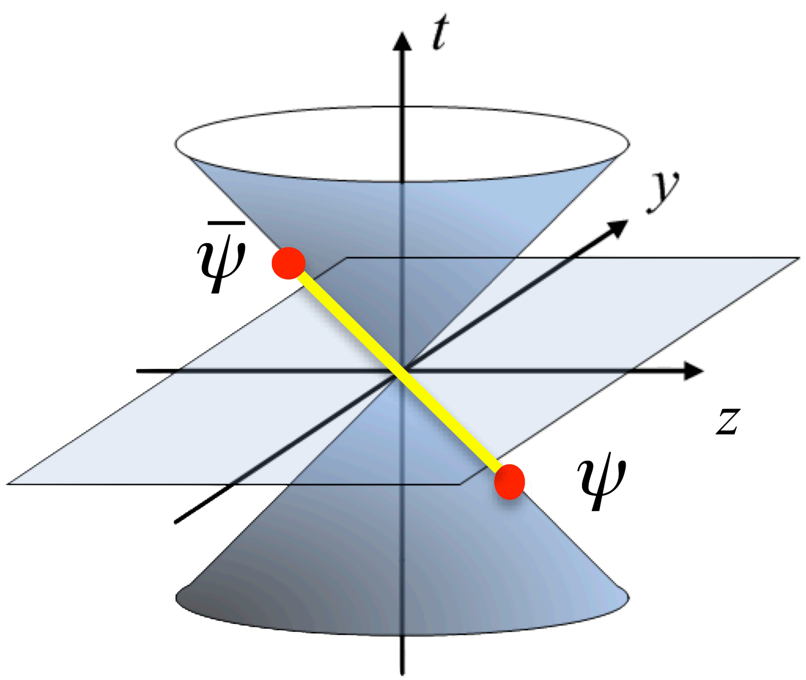
- Lattice calculation
  - Wilson line mass renormalization
  - Fourier transform and perturbative matching
  - Final results

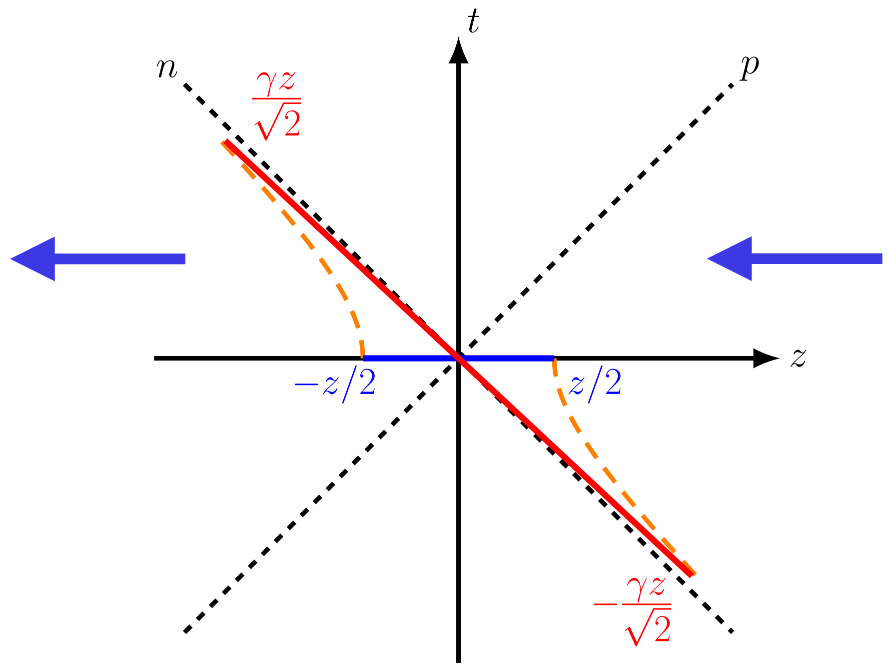
Large-Momentum Effective Theory (LaMET)

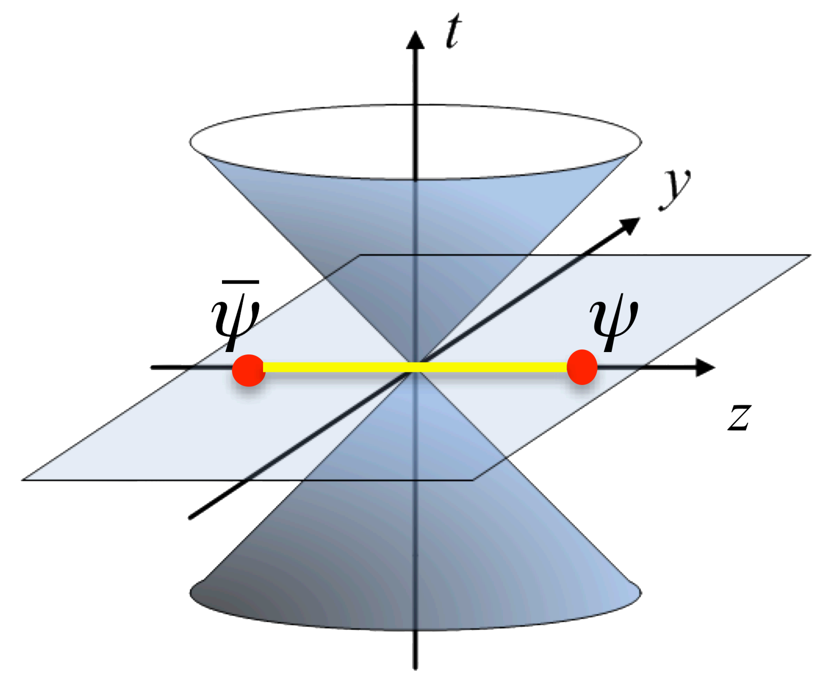
$z + ct = 0, \quad z - ct \neq 0$

Related by Lorentz boost

$t = 0, \quad z \neq 0$







PDF $f(x)$:
Cannot be calculated on the lattice

$$f(x) = \int \frac{dz^-}{2\pi} e^{-ib^-(xP^+)} \langle P | \bar{\psi}(z^-) \times \frac{\gamma^+}{2} W[z^-,0] \psi(0) | P \rangle$$

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, arXiv: 2004.03543.

Quasi-PDF $\tilde{f}(x, P^z)$:
Directly calculable on the lattice

$$\tilde{f}(x, P^z) = \int \frac{dz}{2\pi} e^{iz(xP^z)} \langle P | \bar{\psi}(z) \times \frac{\gamma^z}{2} W[z,0] \psi(0) | P \rangle$$

Large-momentum effective theory (LaMET)

- Large-momentum expansion:

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) \tilde{f}(y, P^z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

  - X. Ji, PRL 110 (2013); SCPMA57 (2014);
  - X. Xiong, X. Ji, J.-H. Zhang and YZ, PRD 90 (2014);
  - Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
  - T. Izubuchi, X. Ji, L. Jin, I. Stewart, and YZ, PRD98 (2018).
  - X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, 2004.03543.
  - X. Ji, YZ, et al., Nucl.Phys.B 964 (2021).

- Matching coefficient:

$$C^{(1)}\left(\xi, \frac{\mu}{yP^z}\right) = -\frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left[\frac{3}{2}\ln\frac{\mu^2}{4x^2P_z^2} + \frac{5}{2}\right]$$

$\xi = \frac{x}{y}$

$$-\frac{\alpha_s C_F}{2\pi}\begin{cases} \left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} + 1\right)_+ & \xi > 1 \\ \left(\frac{1+\xi^2}{1-\xi}\left[-\ln\frac{\mu^2}{y^2P_z^2} + \ln\left(4\xi(1-\xi)\right) - 1\right] + 1\right)_+ & 0 < \xi < 1 \\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi} - 1\right)_+ & \xi < 0 \end{cases}$$

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- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, 2004.03543.
- X. Ji, YZ, et al., Nucl.Phys.B 964 (2021).

State-of-the-art: next-to-next-to-leading order (NNLO) matching for the non-singlet quark quasi-PDF.

- L.-B. Chen, R.-L. Zhu and W. Wang, PRL126 (2021);
- Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, PRL126 (2021).

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$$\xi = \frac{x}{y}$$

$$\ln\frac{\mu^2}{y^2P_z^2} = \ln\frac{\mu^2}{x^2P_z^2} + \ln\frac{x^2}{y^2}$$

Similar to DGLAP evolution:

$$\frac{dC(\xi, \mu/(xP^z))}{d\ln(xP^z)} = \frac{\alpha_s C_F}{\pi}\left[P_{qq}^{(0)}(\xi) - \frac{3}{2}\delta(1-\xi)\right]$$

X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, arXiv: 2004.03543.

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Threshold logarithms:

$$C(\xi, \mu/(yP^z)) \sim \frac{\alpha_s C_F}{2\pi}\left[\frac{2\ln|1-\xi|}{|1-\xi|} - \frac{2}{1-\xi}\ln\frac{\mu^2}{P_z^2} - \frac{2}{1-\xi}\right]_+$$

Large-x behavior of the extracted PDF is sensitive to the large threshold logarithms.

X. Gao, YZ et al., 2102.01101.

Hybrid renormalization scheme

$$O_B^\Gamma(z,a) = \bar{\psi}_0(z)\Gamma W_0[z,0]\psi_0(0) = e^{\delta m|z|}\, Z_{j_1}(a) Z_{j_2}(a) O_R^\Gamma(z)$$

See X. Ji, YZ, et al., NPB 964 (2021) and references therein.

A minimal subtraction:

- Wilson-line mass subtraction

$$O_R^\Gamma(z,\mu_R) = Z_{\text{hybrid}}(a,\mu_R) e^{-\delta m|z|} O_B^\Gamma(z,a)$$

- Overall renormalization

Ratio-type schemes:

- RIMOM

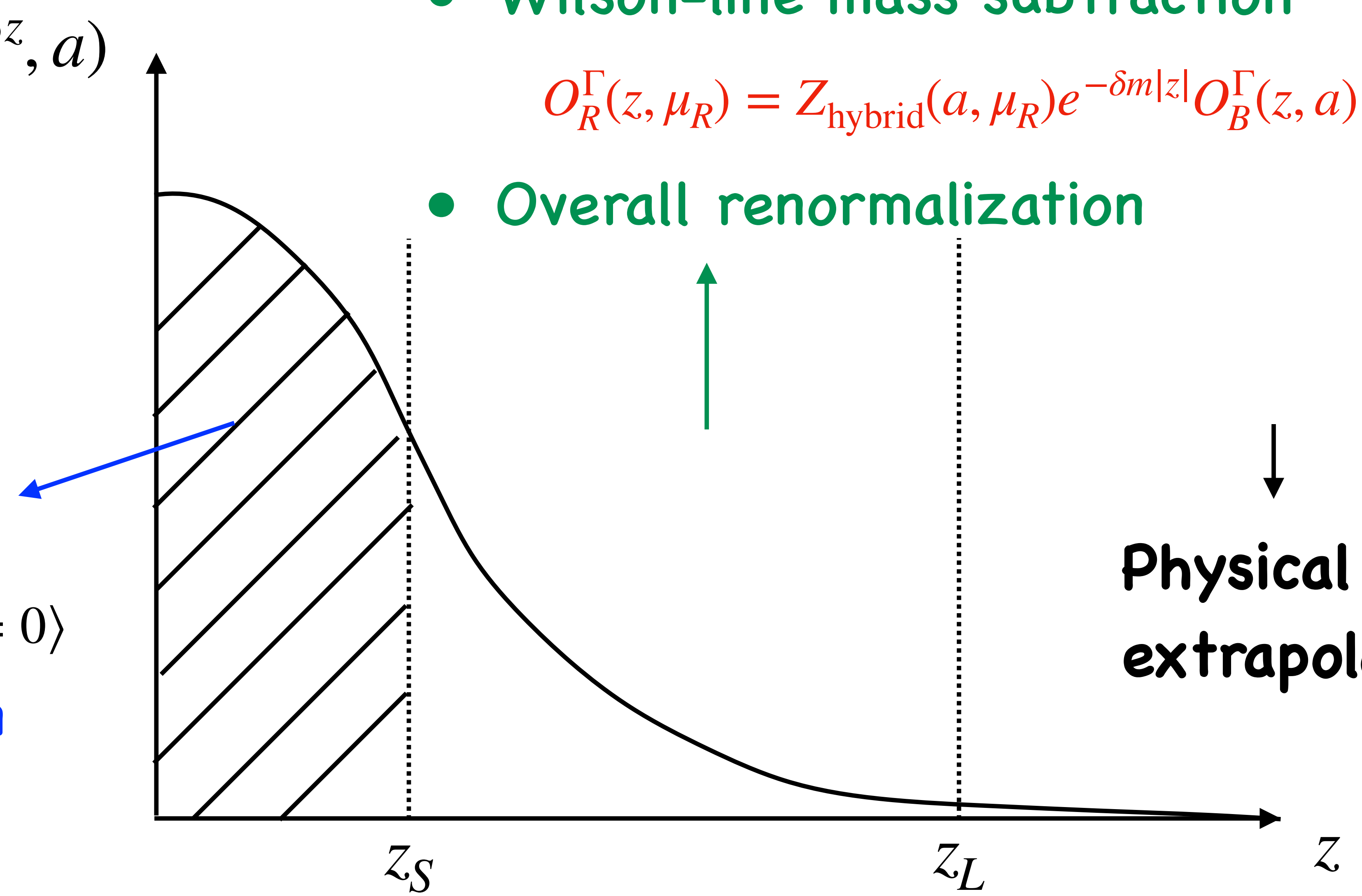
$$Z_X = \langle q | O^\Gamma(z) | q \rangle$$

- Hadron matrix elements

$$Z_X = \langle P_0^z = 0 | O^\Gamma(z) | P_0^z = 0 \rangle$$

- Vacuum expectation value

$$Z_X = \langle \Omega | O^\Gamma(z) | \Omega \rangle$$



Physical extrapolation

$a \ll z_S \ll \Lambda_{\text{QCD}}^{-1}$

$z_L \sim \Lambda_{\text{QCD}}^{-1}$

Outline

- Methodology
  - Large-momentum effective theory
  - Hybrid renormalization scheme
- Lattice calculation
  - Wilson line mass renormalization
  - Fourier transform and perturbative matching
  - Final results

Lattice data

- Wilson-clover fermion on 2+1 flavor HISQ configurations.

n_z	P_z (GeV)		ζ
	$a = 0.06$ fm	$a = 0.04$ fm	
0	0	0	0
1	0.43	0.48	0
2	0.86	0.97	1
3	1.29	1.45	2/3
4	1.72	1.93	3/4
5	2.15	2.42	3/5

$48^3 \times 64$ $64^3 \times 64$

$m_\pi = 300$ MeV

$a = 0.076$ fm

$P_z = 0$ GeV, 0.25 GeV, 0.51 GeV, 0.76 GeV, 1.02 GeV, 1.27 GeV, 1.53 GeV, 1.78 GeV, 2.04 GeV, 2.29 GeV

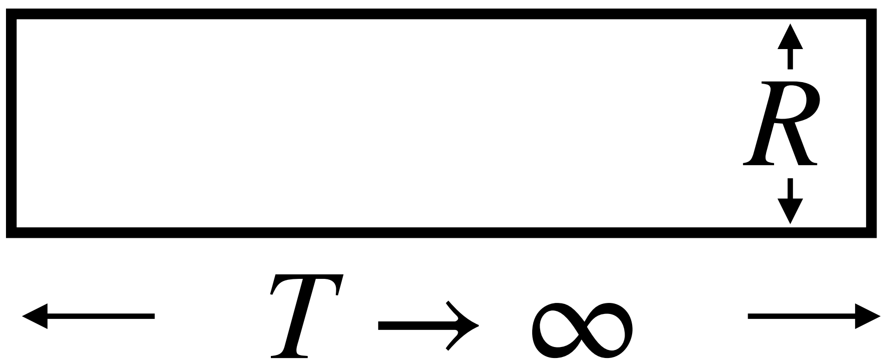
$64^3 \times 64$

$m_\pi = 140$ MeV

- X. Gao, YZ, et al., PRD102 (2020).
- X. Gao, YZ, et al., 2102.01101.

Wilson-line mass renormalization

- Polyakov loop

$\langle \Omega |$  $| \Omega \rangle \propto \exp[-V(R)T]$

- Renormalization condition:

$$V^{\mathrm{lat}}(r,a)\Big|_{r=r_0} + 2\delta m(a) = 0.95/r_0$$

$$\delta m(a) = \frac{1}{a}\sum_n c_n \alpha_s^n(1/a) + \delta m_0^{\mathrm{lat}}$$

$$\delta m_0^{\mathrm{lat}} \sim \Lambda_{\mathrm{QCD}}$$

C. Bauer, G. Bali and A. Pineda, PRL108 (2012).

$$a\delta m(a = 0.04\ \mathrm{fm}) = 0.1508(12)$$

$$a\delta m(a = 0.06\ \mathrm{fm}) = 0.1586(8)$$

$$a\delta m(a = 0.076\ \mathrm{fm}) = 0.1597(16)$$

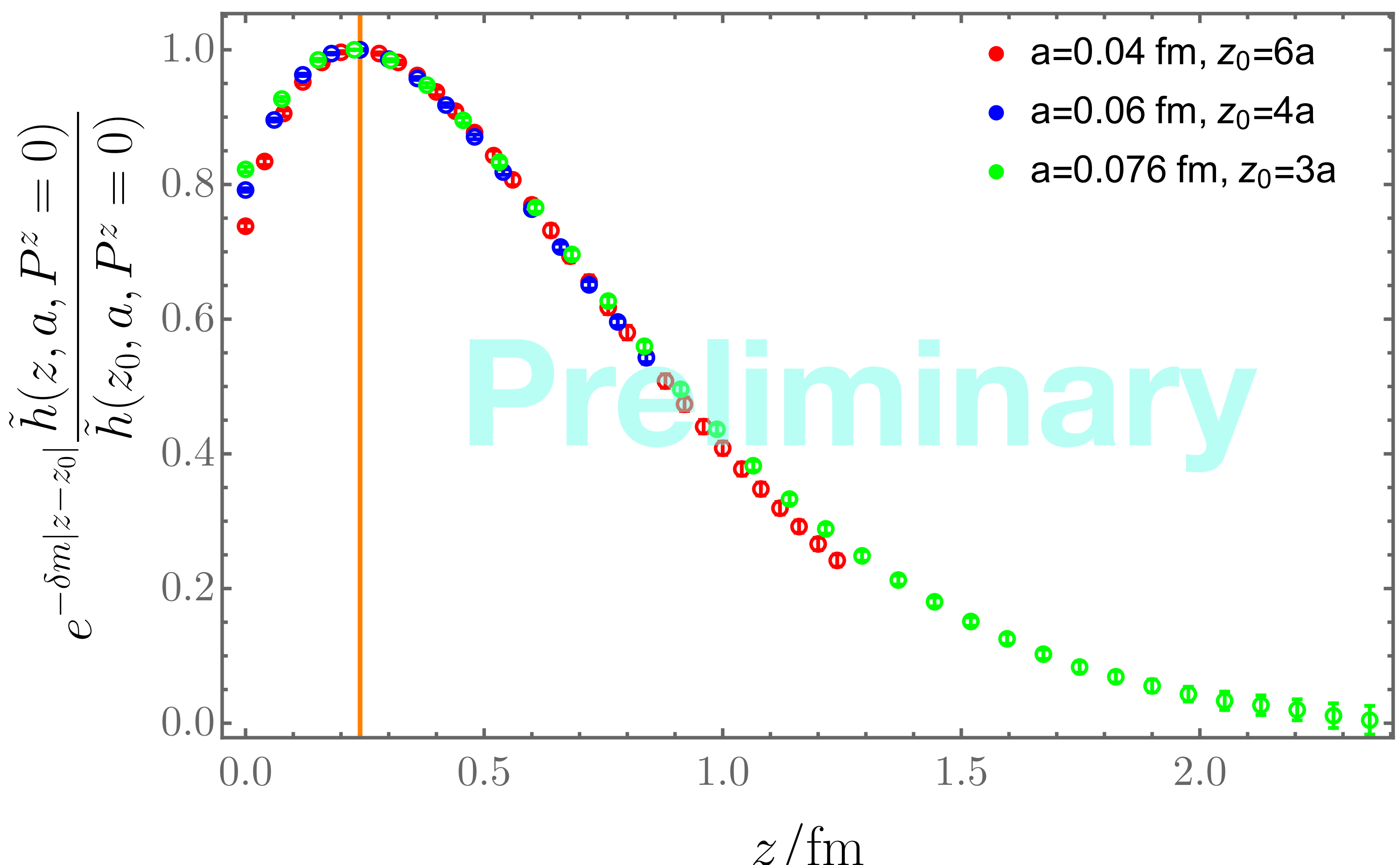
A. Bazavov et al., TUMQCD, PRD98 (2018).

Wilson-line mass renormalization

- Check of continuum limit:

$$O_B^\Gamma(z,a) = e^{\delta m|z|}\, Z_{j_1}(a) Z_{j_2}(a) O_R^\Gamma(z)$$

$$\lim_{a\to 0} e^{-\delta m(z-z_0)} \frac{\tilde{h}(z,a,P^z=0)}{\tilde{h}(z_0,a,P^z=0)} = \frac{\tilde{h}(z,P^z=0,\ \mu)}{\tilde{h}(z_0,P^z=0,\ \mu)} \qquad z, z_0 \gg a$$



Wilson-line mass renormalization

- Matching to the MSbar scheme:

$$e^{\delta m_0^{\overline{\rm MS}}(z-z_0)} \lim_{a\to 0} e^{-\delta m(z-z_0)} \frac{\tilde{h}(z, a, P^z=0)}{\tilde{h}(z_0, a, P^z=0)} = \frac{\tilde{h}(z, P^z=0,\ \mu)}{\tilde{h}(z_0, P^z=0,\ \mu)} \quad z, z_0 \gg a$$

Renormalon in the Wilson-line mass correction.

C. Bauer, G. Bali and A. Pineda, PRL108 (2012);
C. Alexandrou et al. (ETMC), 2011.00964.

$$\tilde{h}^{\overline{\rm MS}}(z, P^z=0,\ \mu) = \left[C_0(\alpha_s(\mu), z^2\mu^2) + \mathcal{O}(z^2\Lambda^2_{\rm QCD})\right] \qquad z \ll \Lambda^{-1}_{\rm QCD}$$

Perturbative:

- Known to NNLO
  - L.-B. Chen, R.-L. Zhu and W. Wang, PRL126 (2021);
  - Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, PRL126 (2021).
- 3-loop anomalous dimension available.
  - V. Braun and K. G. Chetyrkin, JHEP 07 (2020).

Non-perturbative:

- Leading infrared renormalon contribution is quadratic

$$\propto z^2\Lambda^2_{\rm QCD}$$

- V. Braun, A. Vladimirov and J.-H. Zhang, PRD99 (2019).

Wilson-line mass renormalization

- Matching to the MSbar scheme:

$$e^{-\delta m(z-z_0)} \frac{\tilde{h}(z, a, P^z = 0)}{\tilde{h}(z_0, a, P^z = 0)} = e^{-\delta m_0^{\overline{\rm MS}}(z-z_0)} \frac{C_0(\alpha_s(\mu), z^2\mu^2) + \Lambda z^2}{C_0(\alpha_s(\mu), z_0^2\mu^2) + \Lambda z_0^2} \qquad z_0 \gg a$$

  - Use both fixed-order and renormalization-group improved (RGI) NNLO OPE formulae;

X. Gao, YZ et al., 2102.01101.

  - Two parameter fit to a wide range of z.

Wilson-line mass renormalization

- Matching to the MSbar scheme:
  - Fixed-order OPE leads to excellent fits, while RGI OPE does not;
  - The a-dependences of the parameters are negligible.

	$z_{\max} = 0.48$ fm	$z_{\max} = 0.60$ fm	$z_{\max} = 0.72$ fm
$a = 0.04$ fm, $\mu = 2.0$ GeV	$\chi^2_{dof} = 0.33$, $\delta m_0 \to 0.166$, $\Lambda \to -0.0475$	$\chi^2_{dof} = 0.26$, $\delta m_0 \to 0.164$, $\Lambda \to -0.0485$	$\chi^2_{dof} = 0.45$, $\delta m_0 \to 0.167$, $\Lambda \to -0.0472$
$a = 0.06$ fm, $\mu = 2.0$ GeV	$\chi^2_{dof} = 0.0015$, $\delta m_0 \to 0.169$, $\Lambda \to -0.0485$	$\chi^2_{dof} = 0.024$, $\delta m_0 \to 0.173$, $\Lambda \to -0.0468$	$\chi^2_{dof} = 0.24$, $\delta m_0 \to 0.179$, $\Lambda \to -0.044$
	$z_{\max} = 0.532$ fm	$z_{\max} = 0.608$ fm	$z_{\max} = 0.684$ fm
$a = 0.076$ fm, $\mu = 2.0$ GeV	$\chi^2_{dof} = 0.011$, $\delta m_0 \to 0.171$, $\Lambda \to -0.0449$	$\chi^2_{dof} = 0.11$, $\delta m_0 \to 0.174$, $\Lambda \to -0.0436$	$\chi^2_{dof} = 0.38$, $\delta m_0 \to 0.178$, $\Lambda \to -0.0421$

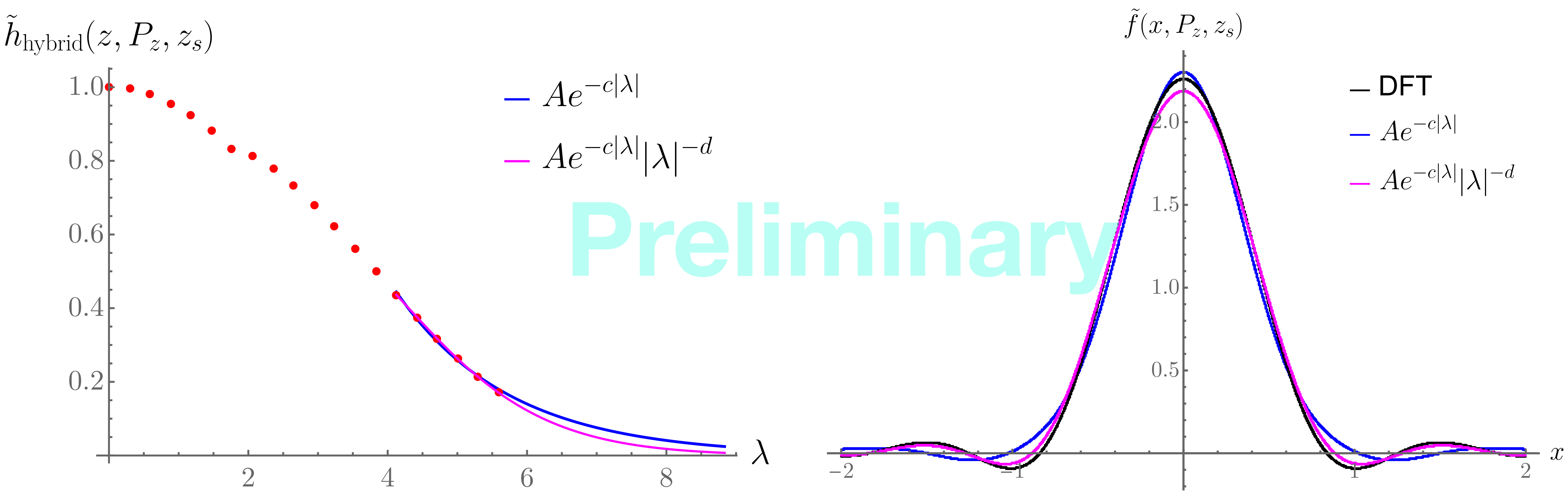
Preliminary

Table 4: δm_0 in unit of GeV, Λ in unit of GeV^2, and λ in unit of GeV^4.

$z_0 = 4a$

Fourier transform

- Extrapolation with models featuring an exponential decay:



$a = 0.04$ fm, $P_z = 1.45$ GeV, $z_S = 6a$, $z_L = 19a$.

We are still in the process of finishing the hybrid scheme analysis, so to show some preliminary results, we opt to use the ratio scheme for now.

Temporary choice: ratio scheme

- Taking advantage of the fact that the NNLO OPE with leading IR renormalon contribution can fit to a wide range of z:

$$\lim_{a\to 0}\frac{\tilde{h}(z,a,P^z)}{\tilde{h}(z,a,P^z=0)}=\frac{\tilde{h}(z,P^z,\ \mu)}{\tilde{h}(z,P^z=0,\ \mu)}=\frac{\tilde{h}(z,P^z,\ \mu)}{C_0(\alpha_s(\mu),z^2\mu^2)+\Lambda z^2}$$

$$z\leq 0.72\ \mathrm{fm}$$

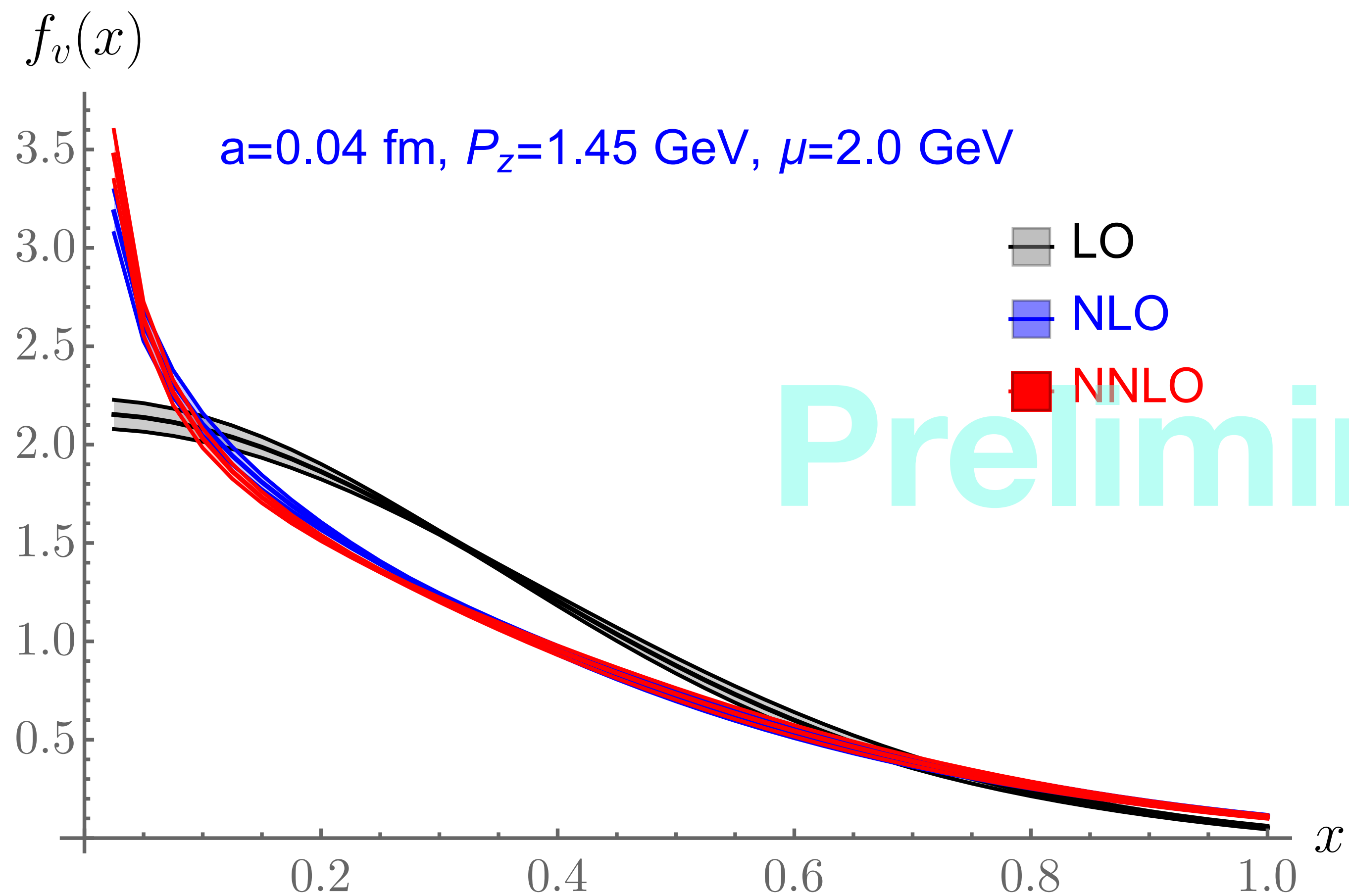
$$\lim_{a\to 0}\frac{\tilde{h}(z,a,P^z)}{\tilde{h}(z,a,P^z=0)}\frac{C_0(\alpha_s(\mu),z^2\mu^2)+\Lambda z^2}{C_0(\alpha_s(\mu),z^2\mu^2)}=\frac{\tilde{h}(z,P^z,\ \mu)}{C_0(\alpha_s(\mu),z^2\mu^2)}$$

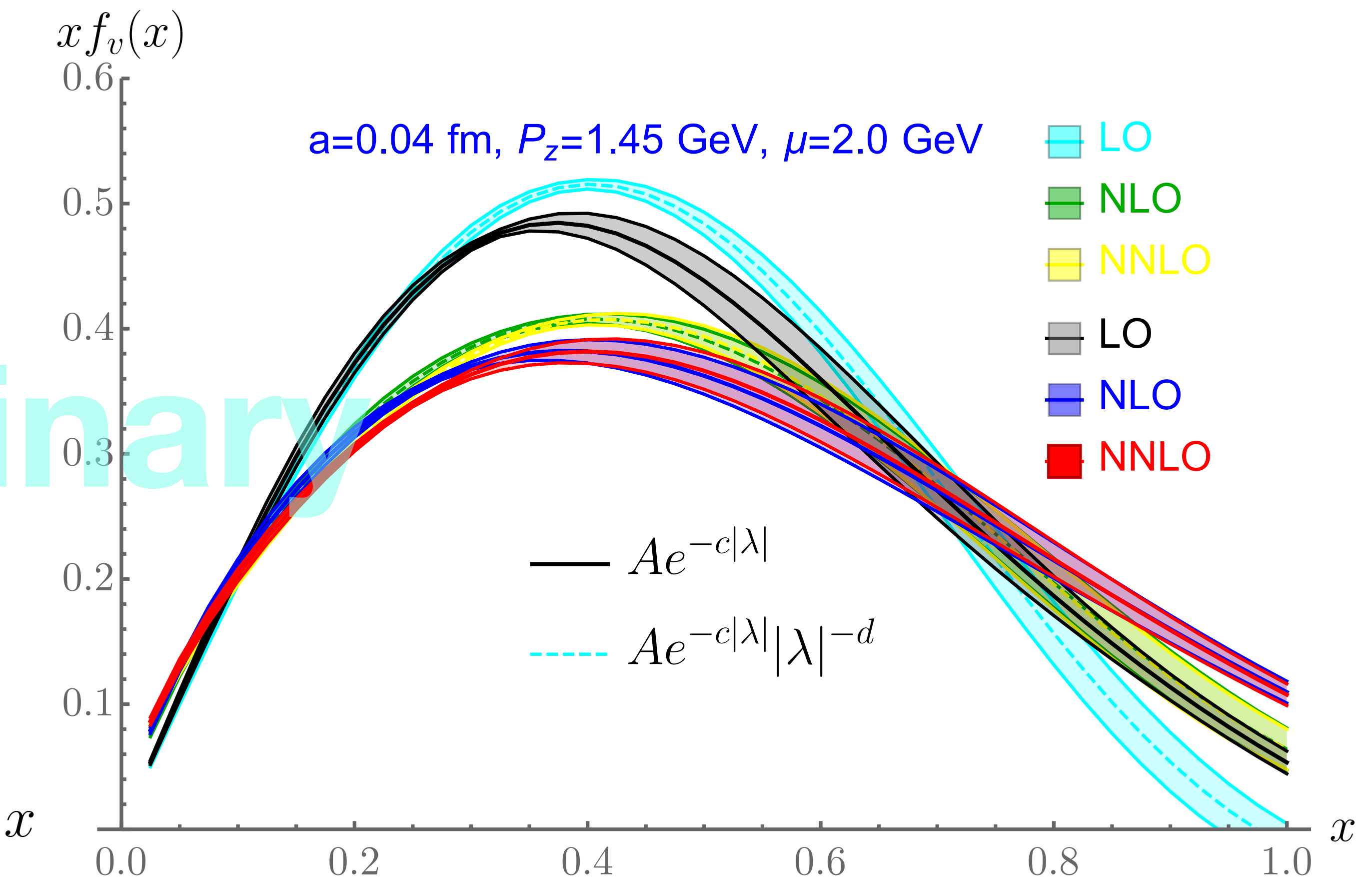
Ratio scheme. But strictly speaking, the factorization formula is in doubt at large z.

- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and YZ, PRD98 (2018);
- A. Radyushkin, Phys.Lett.B 781 (2018).

Perturbative matching at NNLO

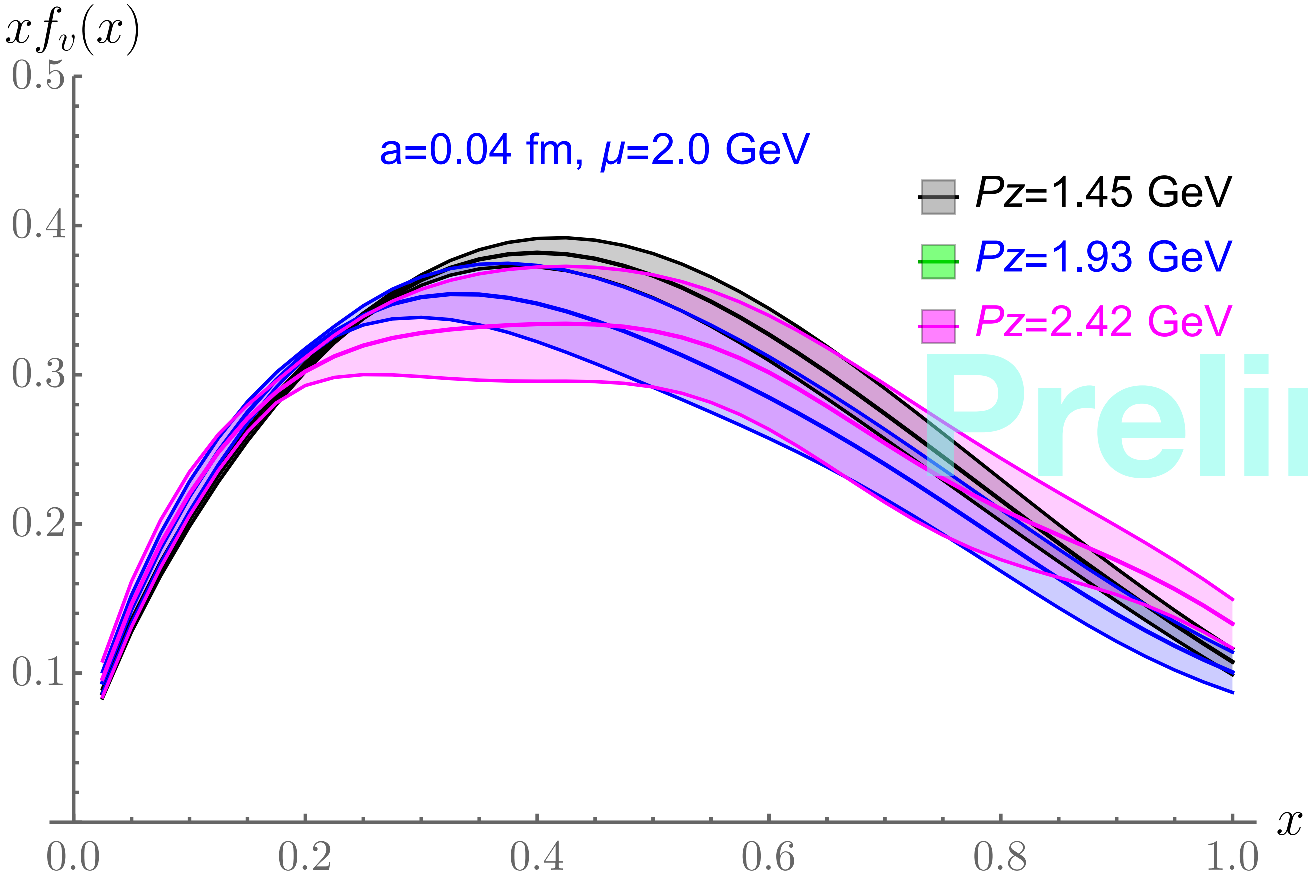
- Perturbative correction shows good convergence.

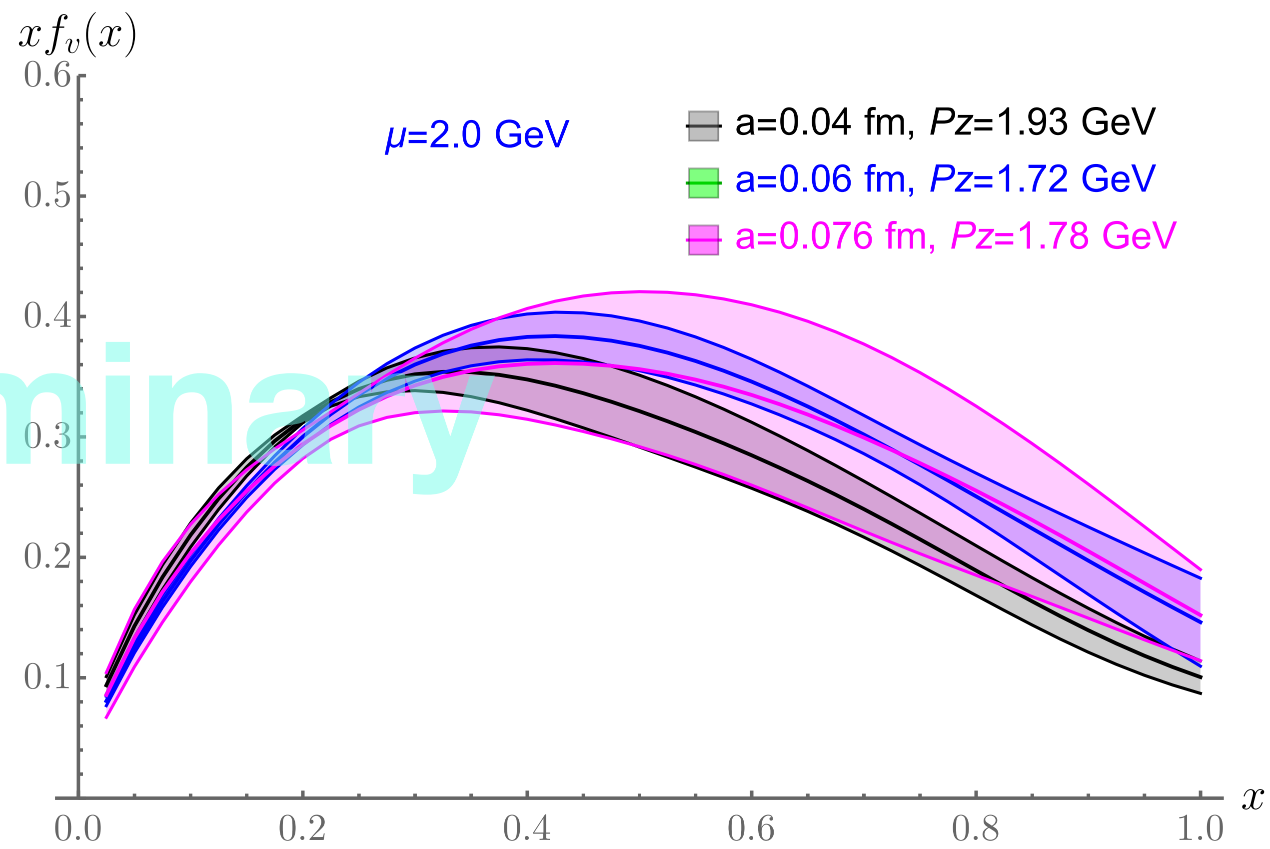




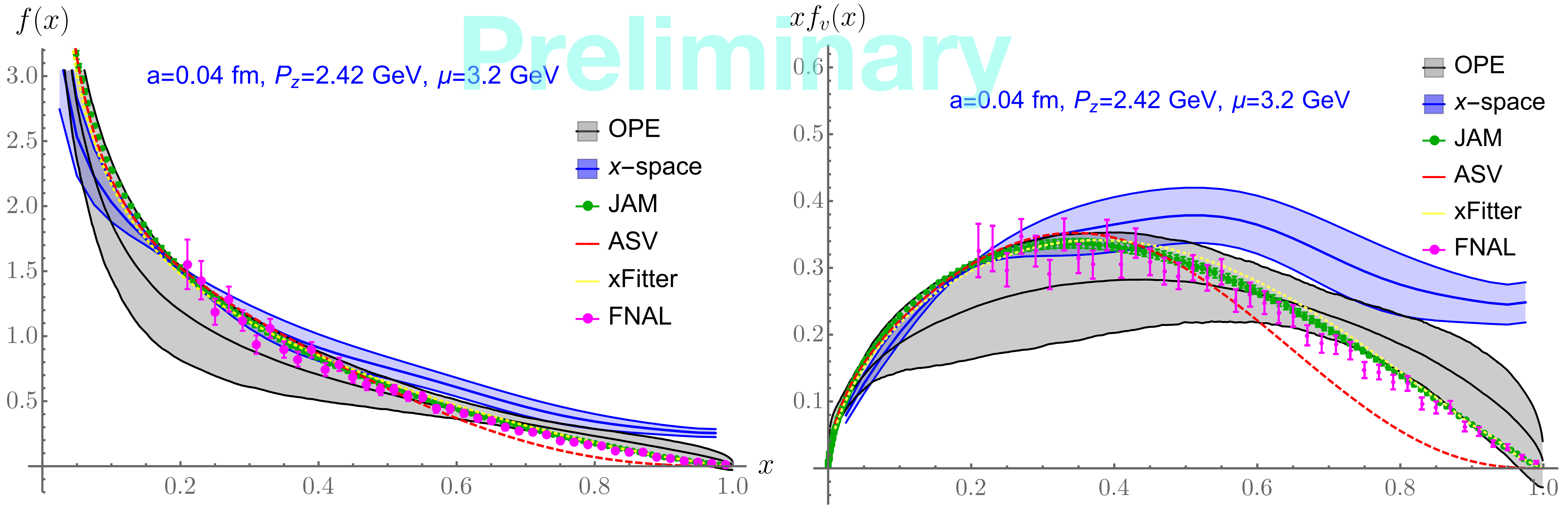
Error band only includes statistical uncertainty.

Dependence on P_z and a





Comparison with previous analysis and phenomenology



Better agreement with experimental fits for 0.1<x<0.5 compared to our previous analysis using OPE in coordinate space.

Conclusion

- We have implemented the hybrid scheme renormalization and NNLO matching correction to obtain the the pion valence PDF;
- The Wilson-line mass renormalization can be well determined from lattice and matched to the MSbar scheme by using the NNLO OPE formula with renormalon-inspired model;
- Compared to the hybrid-scheme matching, the ratio-scheme matching shows excellent perturbative convergence;
- Our final results with ratio scheme show better agreement with phenomenology for $0.1<x<0.5$ compared to previous calculations with same lattice data.