Long-range processes in QCD

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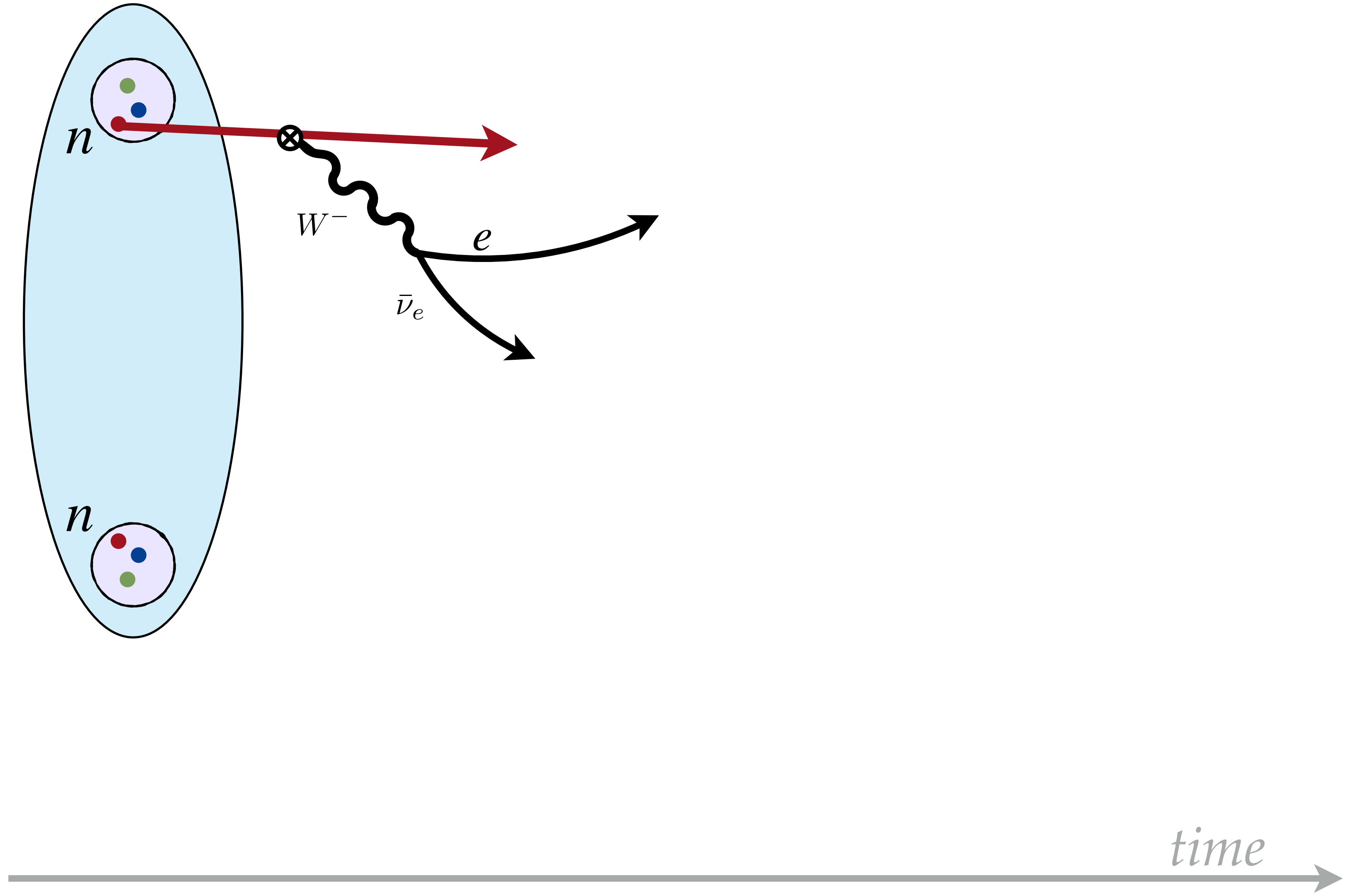
GHP, 2021

Long-range processes

- Double beta decays



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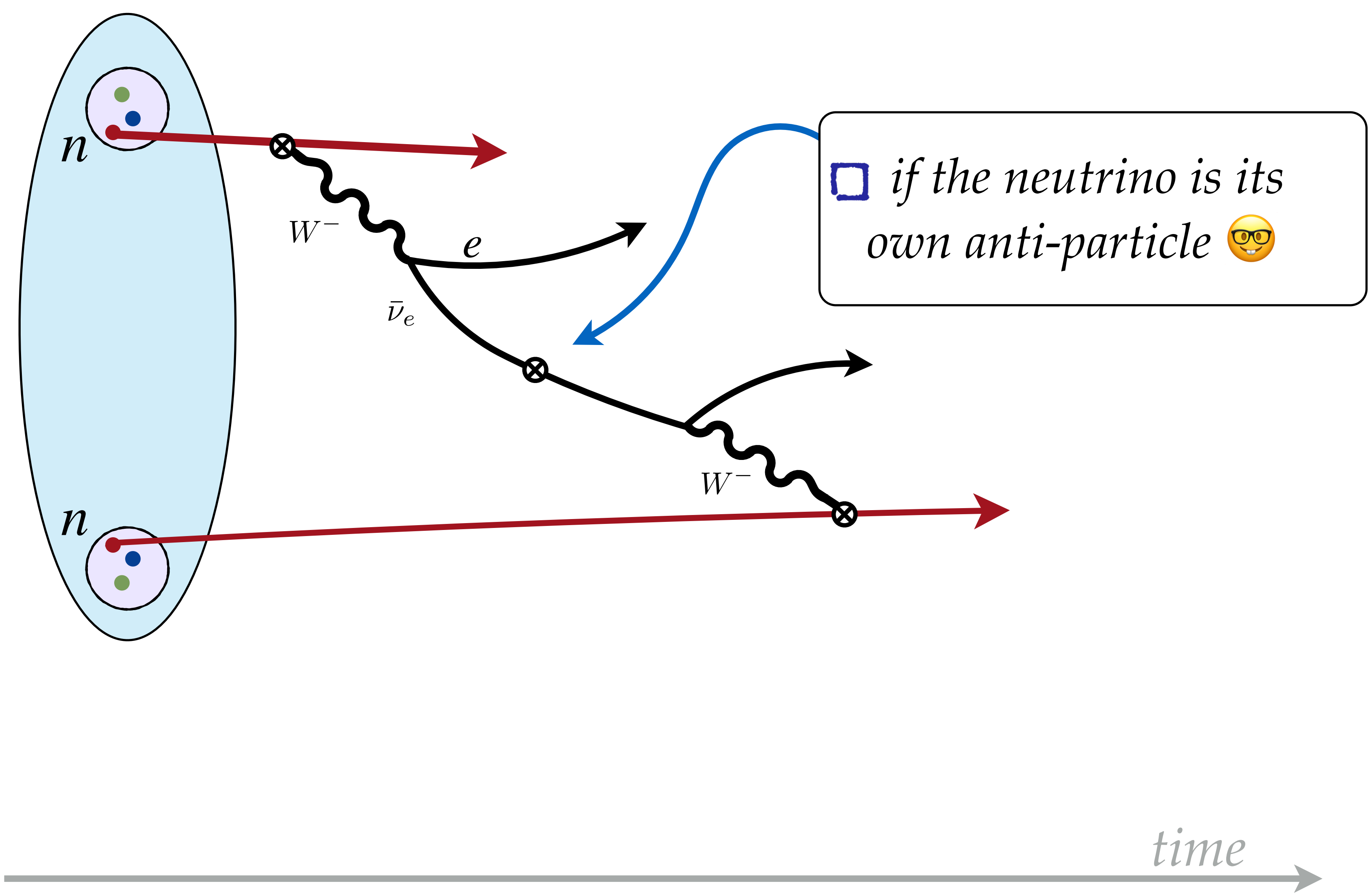

Long-range processes

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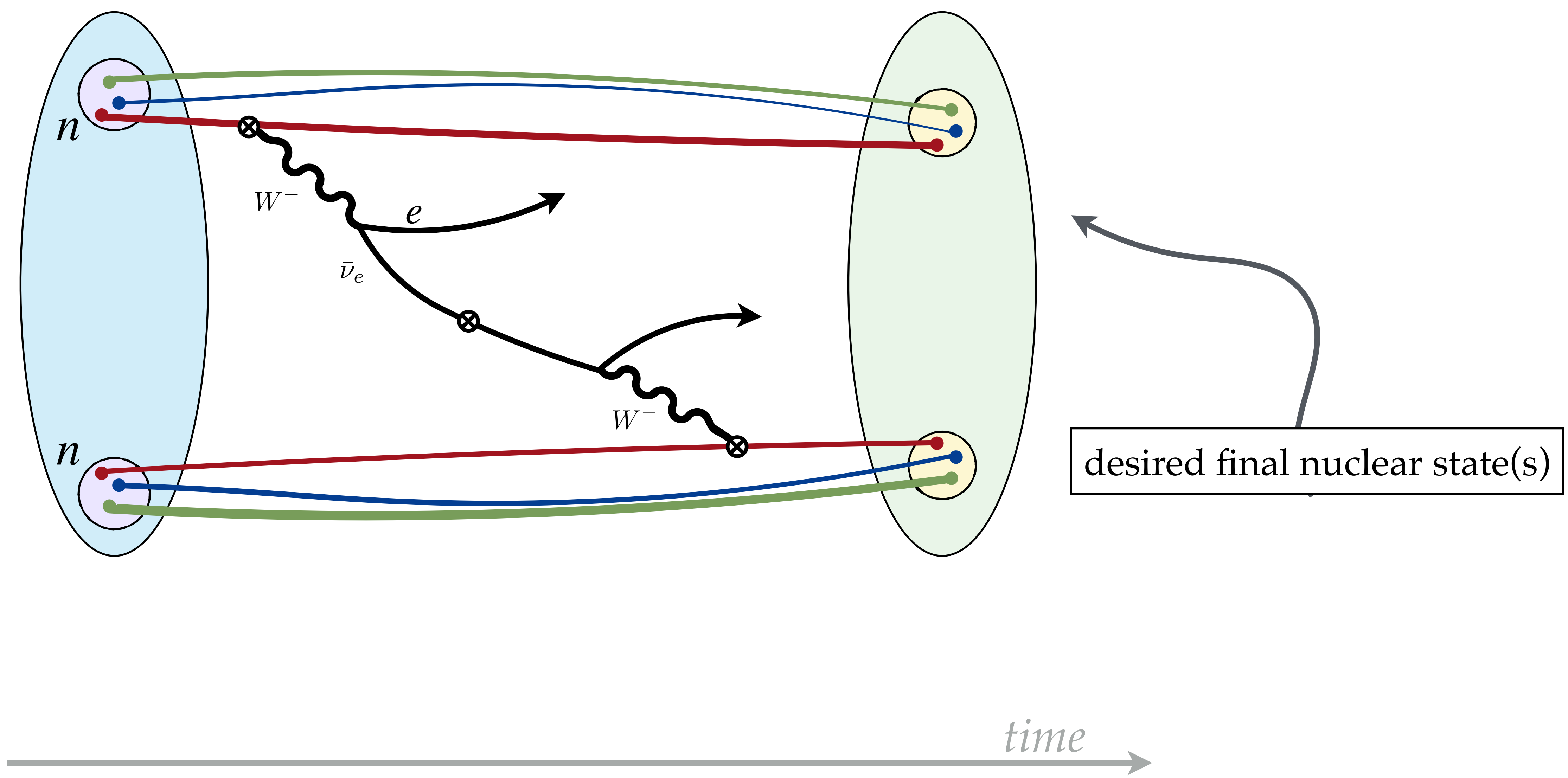
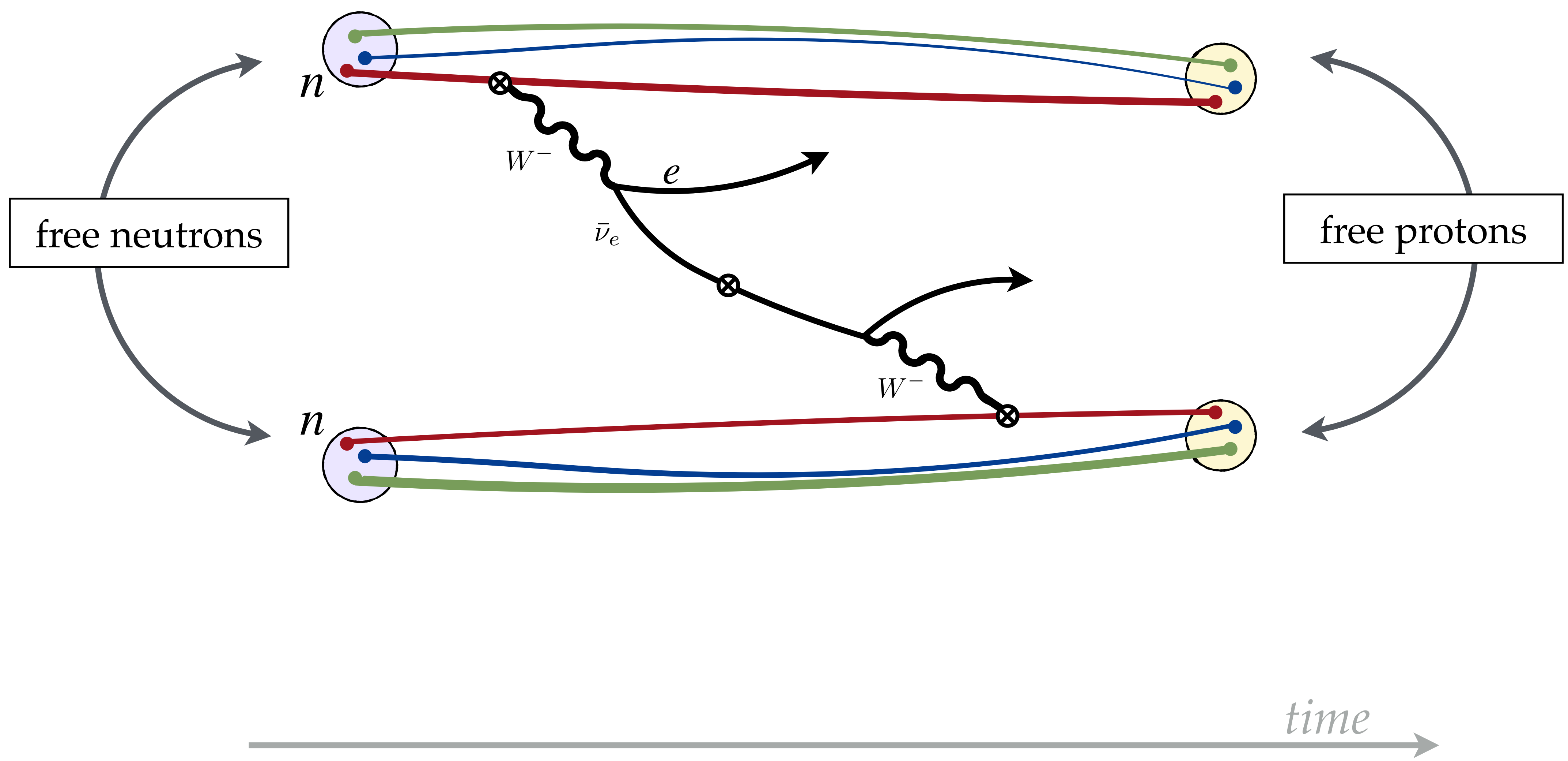


Long-range processes

- Double beta decays

Like what will be able to be determined from the Standard Model:



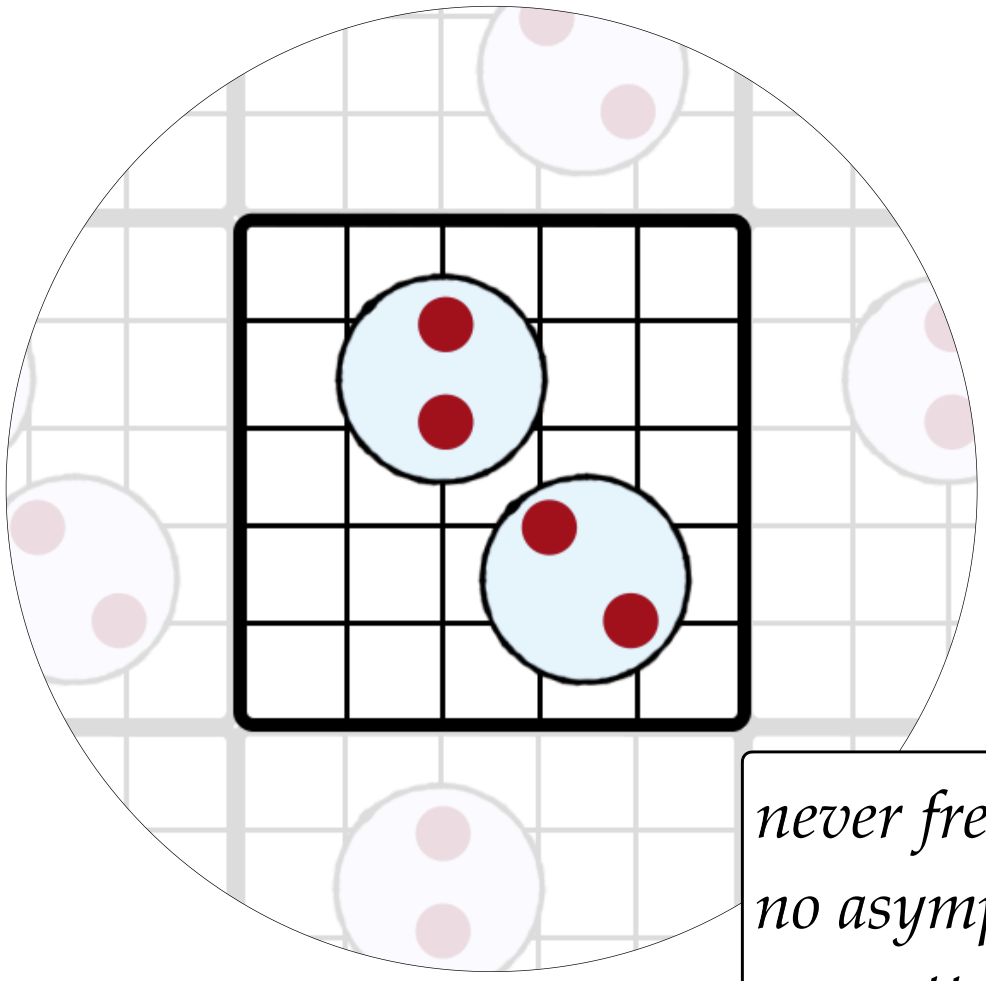
Long-range processes

- Double beta decays,
- $K^0 - \overline{K^0}$ mixing $\iff$ $K_L - K_S$ mass splitting,
- Compton scattering,
- Neutrino-nucleus scattering,
- $\gamma^\star\gamma^\star \to \pi\pi$ / / Glueball structure,
- Radiative corrections in weak decays,
- …

All can be defined as: $\mathcal{T} \sim \int d^4x\, e^{ix\cdot q}\, \langle n_f |\, T\left[\mathcal{J}_{2,M}(t)\, \mathcal{J}_1(0)\right] | n_i \rangle_\infty$

lattice QCD

- Euclidean spacetime: $t_M \to -it_E$
- Monte Carlo sampling
- lattice spacing
- finite volume: $L \sim 1 - 10\,\mathrm{fm}$
- quark masses: $m_q \to m_q^{\text{phys}}$



never free
no asymptotic states
no scattering

lattice QCD

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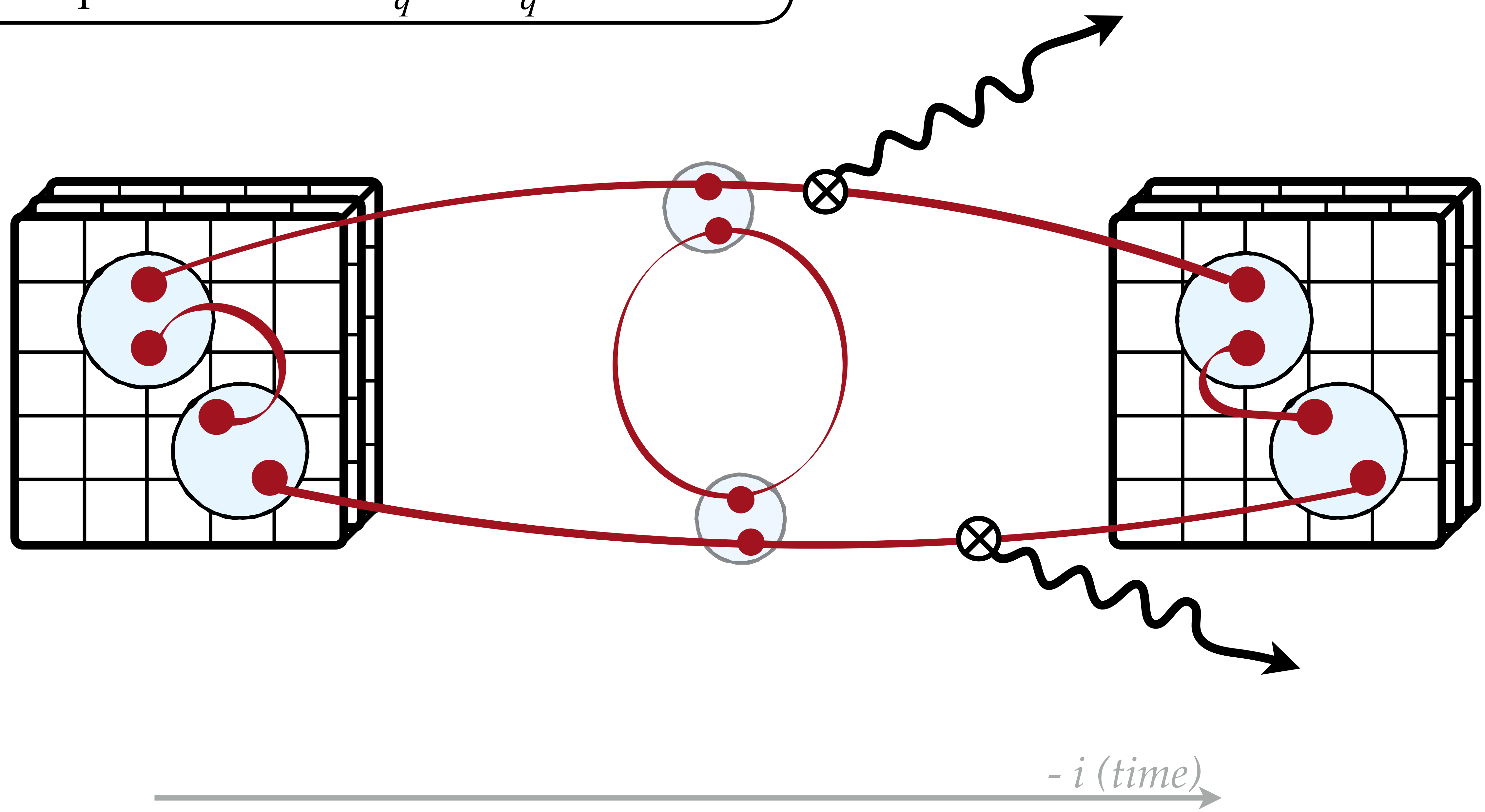
strongly correlated issues:
"time evolution operator $\sim e^{-t\hat{H}_L m}$ depends on both the time-signature and size of the volume"



Four-point functions

Physical long-range processes: $\mathscr{T} \sim \int_{-\infty}^{\infty} dt\, e^{iq_0 t} \langle n_f |\, \mathscr{J}_{2,M}(t)\, \mathscr{J}_1(0)\, | n_i \rangle_\infty$

Naïve guess: $\mathscr{T} \sim \int_{-\infty}^{\infty} d\tau\, e^{q_0 \tau} \langle n_f |\, \mathscr{J}_{2,E}(\tau)\, \mathscr{J}_1(0)\, | n_i \rangle_L$

Can't be that easy, can it? 🤨
Why do I feel like I'm being swindled? 😒

Four-point functions

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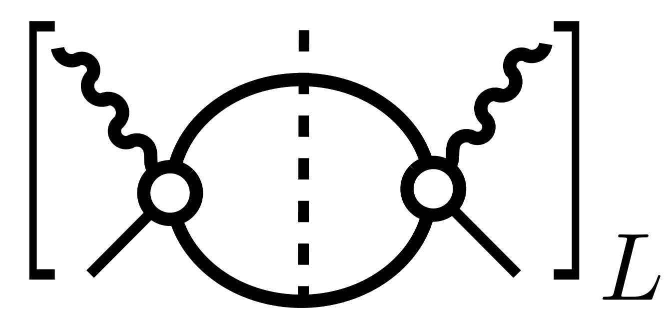
Hidden dragons [🐉]:

- ☐ The integral doesn't always converge!
- ☐ The divergences are volume dependent!

$$\int_{-\infty}^{\infty} d\tau\, e^{q_0 \tau} \langle n_f | \mathscr{J}_{2,E}(\tau)\, \mathscr{J}_1(0) \,|\, n_i \rangle_L \sim \int_{-\infty}^{\infty} d\tau\, e^{(q_0 + E_f - E_n)\tau} \langle n_f | \mathscr{J}_{2,E}(0) \,|\, n \rangle_L \xrightarrow{q_0 + E_f = E_n} \infty$$



associated with intermediate states going on-shell

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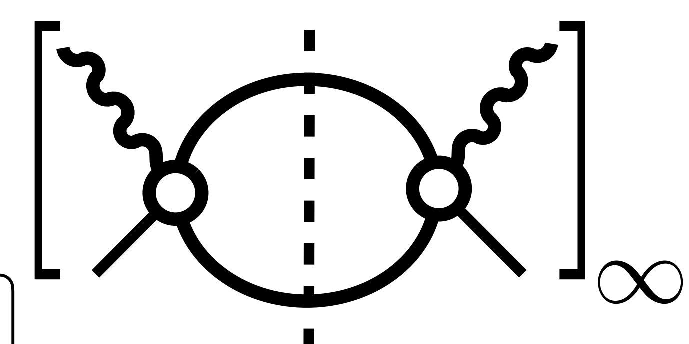
- $\mathscr{T}$ has brach-cuts associated intermediate states



$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

square root singularity

Resolution

- ☑ The divergences depend on $\langle n_f | \mathcal{J}_{2,E}(0) | n \rangle_L$ and $E_n(L)$,
- ☑ Determine $E_n(L)$ and $\langle n_f | \mathcal{J}_{2,E}(0) | n \rangle_L$ from two-point and three-point correlators
- ☑ Define
  $\Delta G(\tau, L) = \langle n_f | \mathcal{J}_{2,E}(\tau)\, \mathcal{J}_1(0) | n_i \rangle_L -$ divergent pieces.

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Matthias Schindler
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RB, Davoudi, Hansen, Schindler, Baroni (2019)

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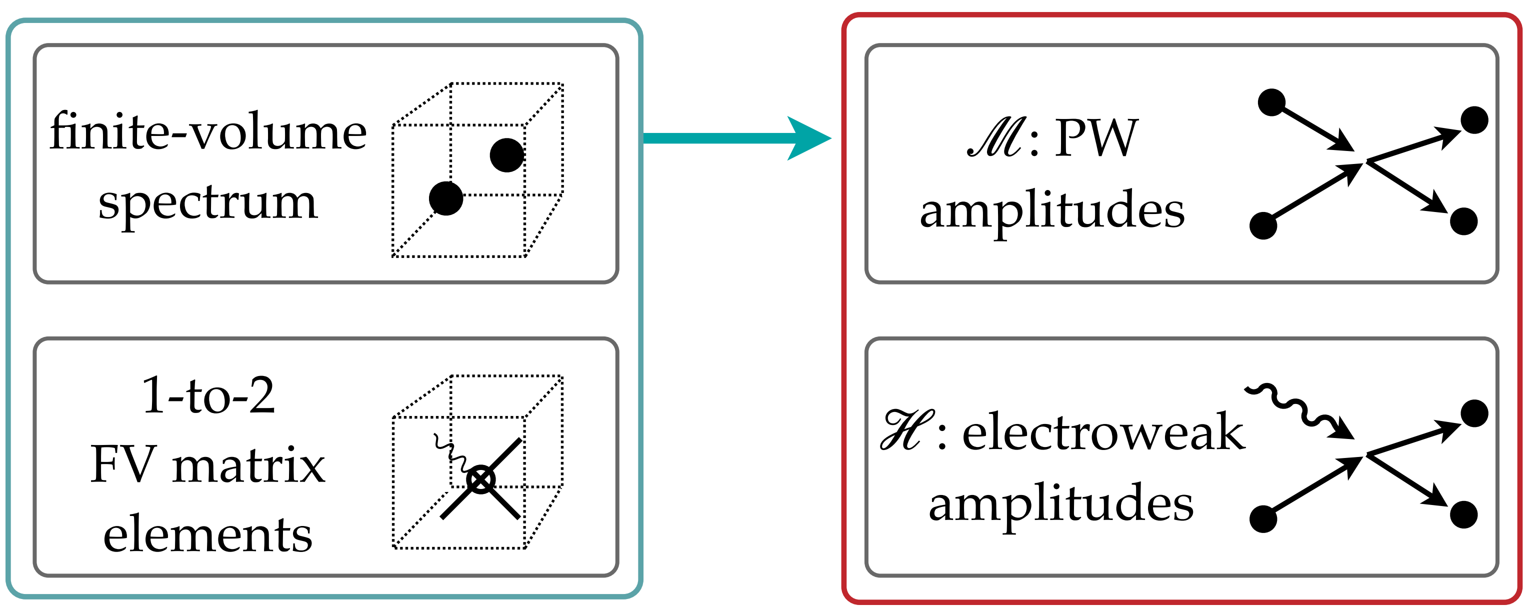
- Use existing finite-volume formalism to determine hadronic ($\mathcal{M}$) and transitions amplitudes ($\mathcal{H}$).



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- ☑ Use existing finite-volume formalism to determine hadronic ($\mathscr{M}$) and transitions amplitudes ($\mathscr{H}$).
- ☑ "Correct" the low-energy behavior
- ☑ Final equation

$$\mathscr{T} = \int_{-\infty}^{\infty} d\tau\, e^{q_0 \tau}\, \Delta G(\tau, L) + \Delta\mathscr{T}[\mathscr{M}, \mathscr{H}, L]$$

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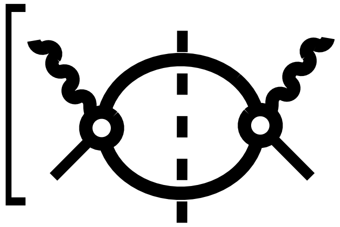
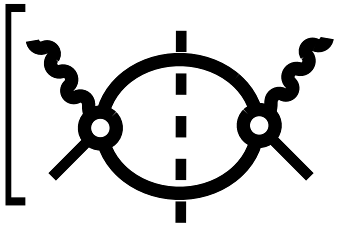
Some remarks

$$\mathscr{T} = \int_{-\infty}^{\infty} d\tau\, e^{q_0 \tau} \Delta G(\tau, L) + \Delta\mathscr{T}[\mathscr{M}, \mathscr{H}, L]$$

- ☑ "Exact" [model/EFT independent, …]
- ☑ Hold for any single-particle external states
- ☑ Holds up to $s = (3m)^2$
- ☑ Intuitive picture:

$\Delta\mathscr{T}[\mathscr{M}, \mathscr{H}, L] = \Big[$  $\Big]_\infty - \Big[$  $\Big]_L$ $-$F.V. pole

in Minkowski

- ☑ Just need $\mathscr{M}$ and $\mathscr{H}$ [talks by David Wilson, Jo Dudek, Luka Leskovec,…]

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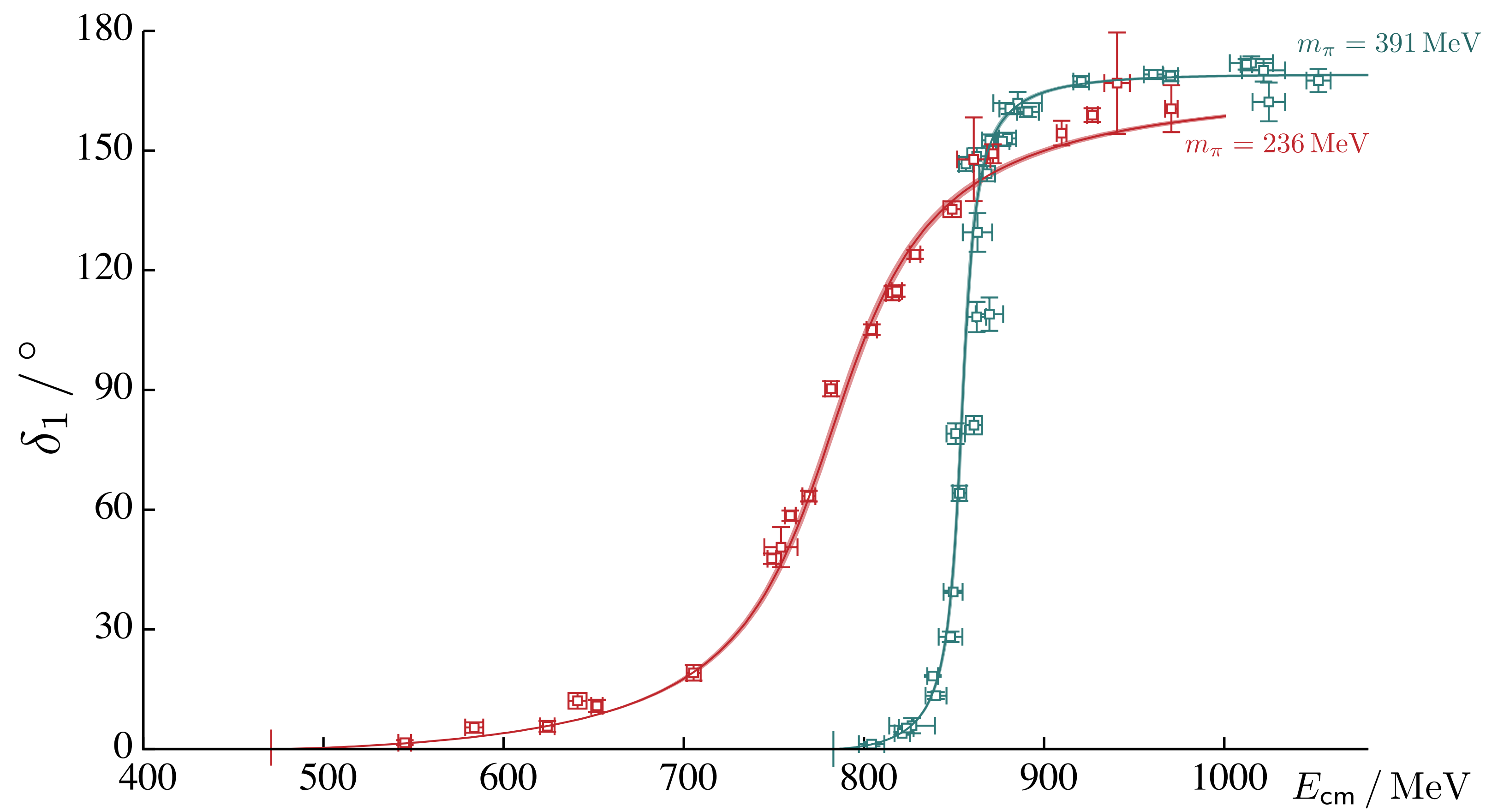
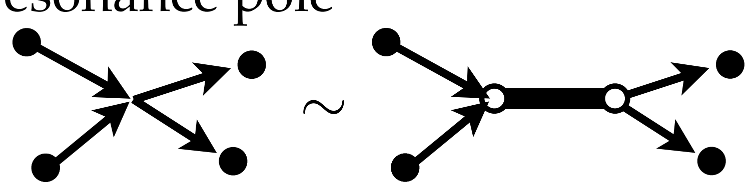
RB, Davoudi, Hansen, Schindler, Baroni (2019)

$\pi\pi$ scattering - (I=1 channel)



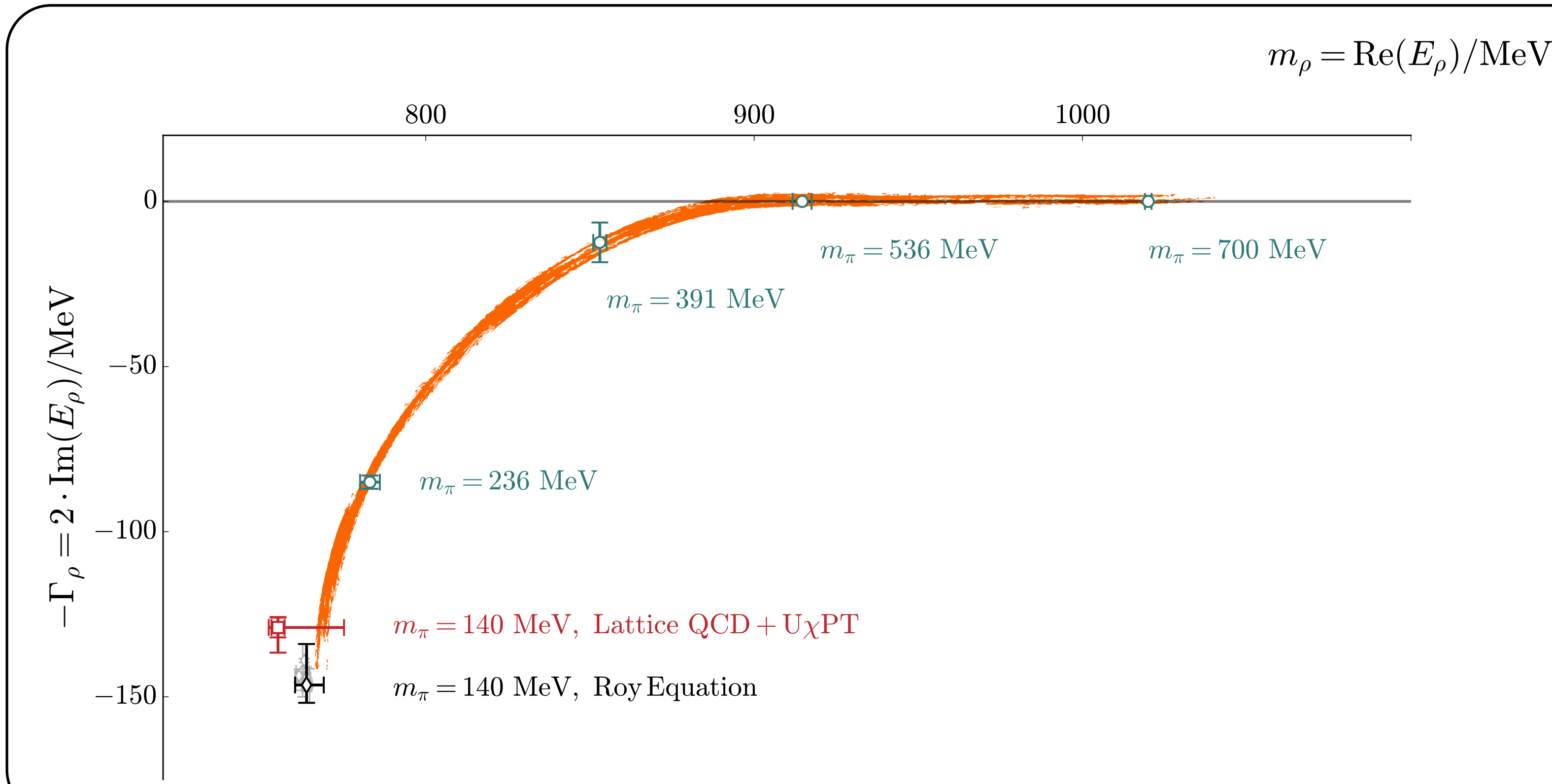
$$\mathcal{M} \sim \frac{1}{p \cot \delta - ip}$$

resonance pole



$$\sim \frac{1}{s - s_0}$$

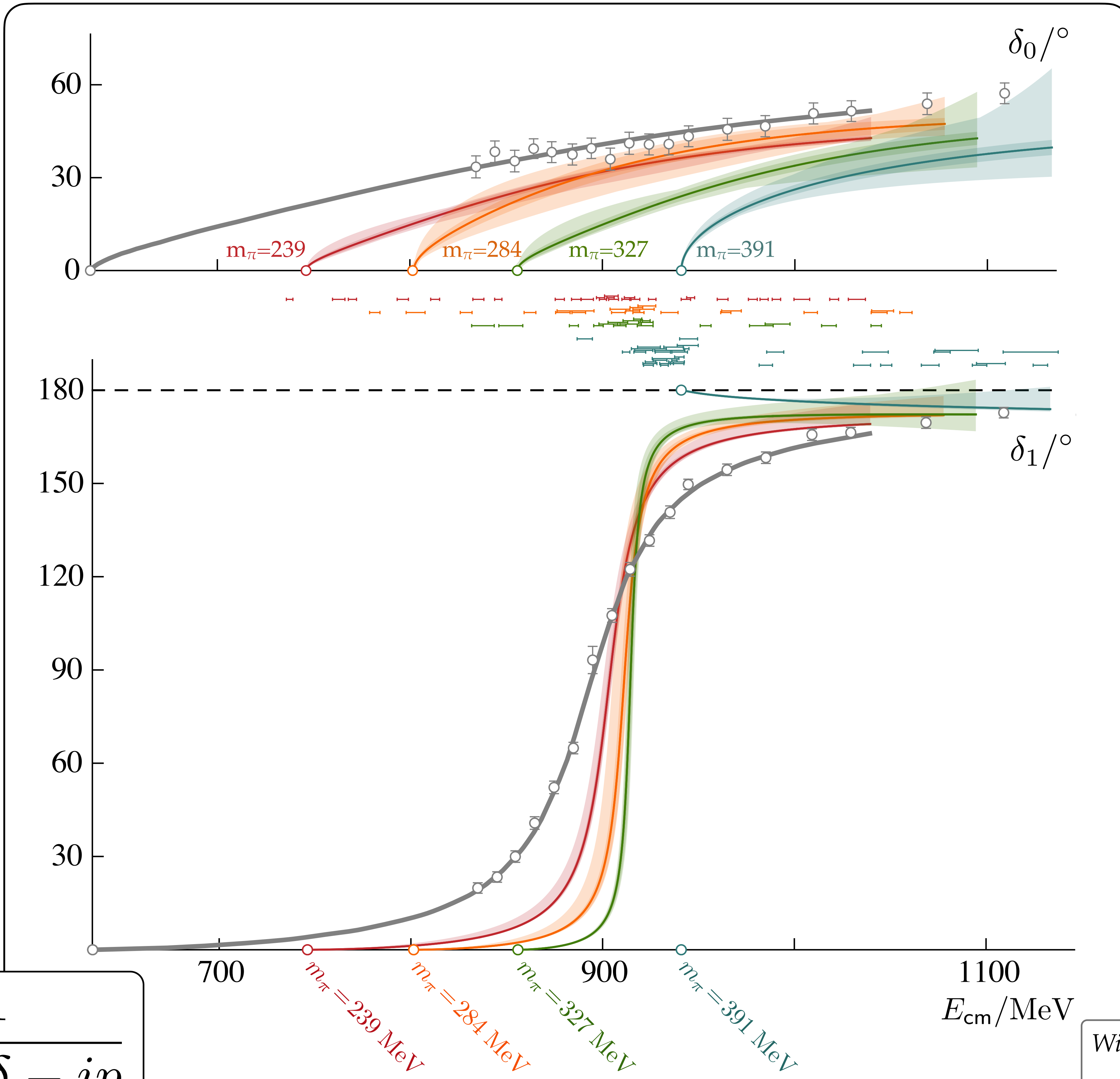
$$s_0 = (m_R - i\Gamma_R/2)^2$$



Dudek, Edwards, & Thomas (2012)

Wilson, RB, Dudek, Edwards, & Thomas (2015)

πK scattering - (I=1/2 channel)



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Wilson, RB, Dudek, Edwards, & Thomas (2019)

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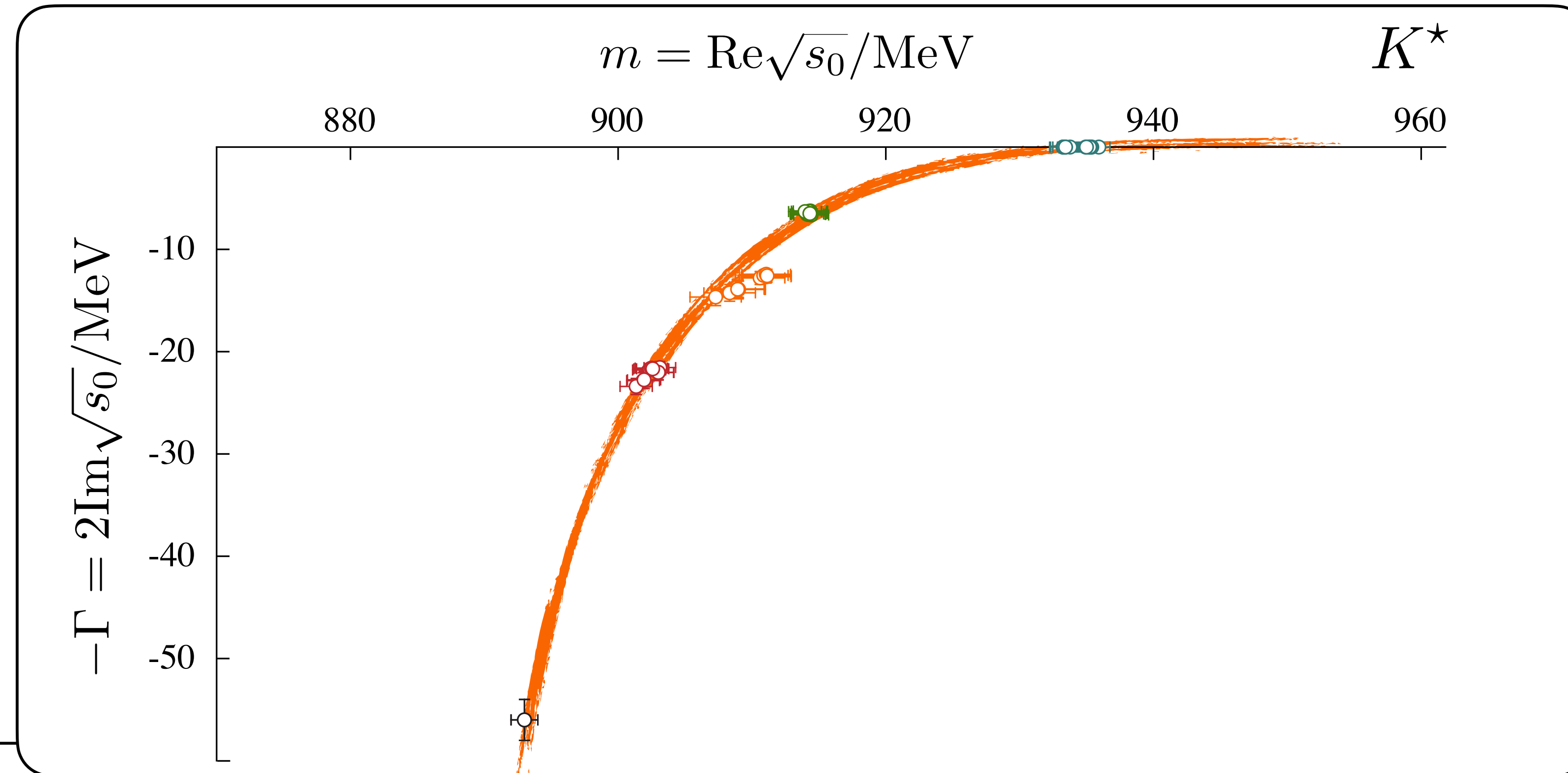

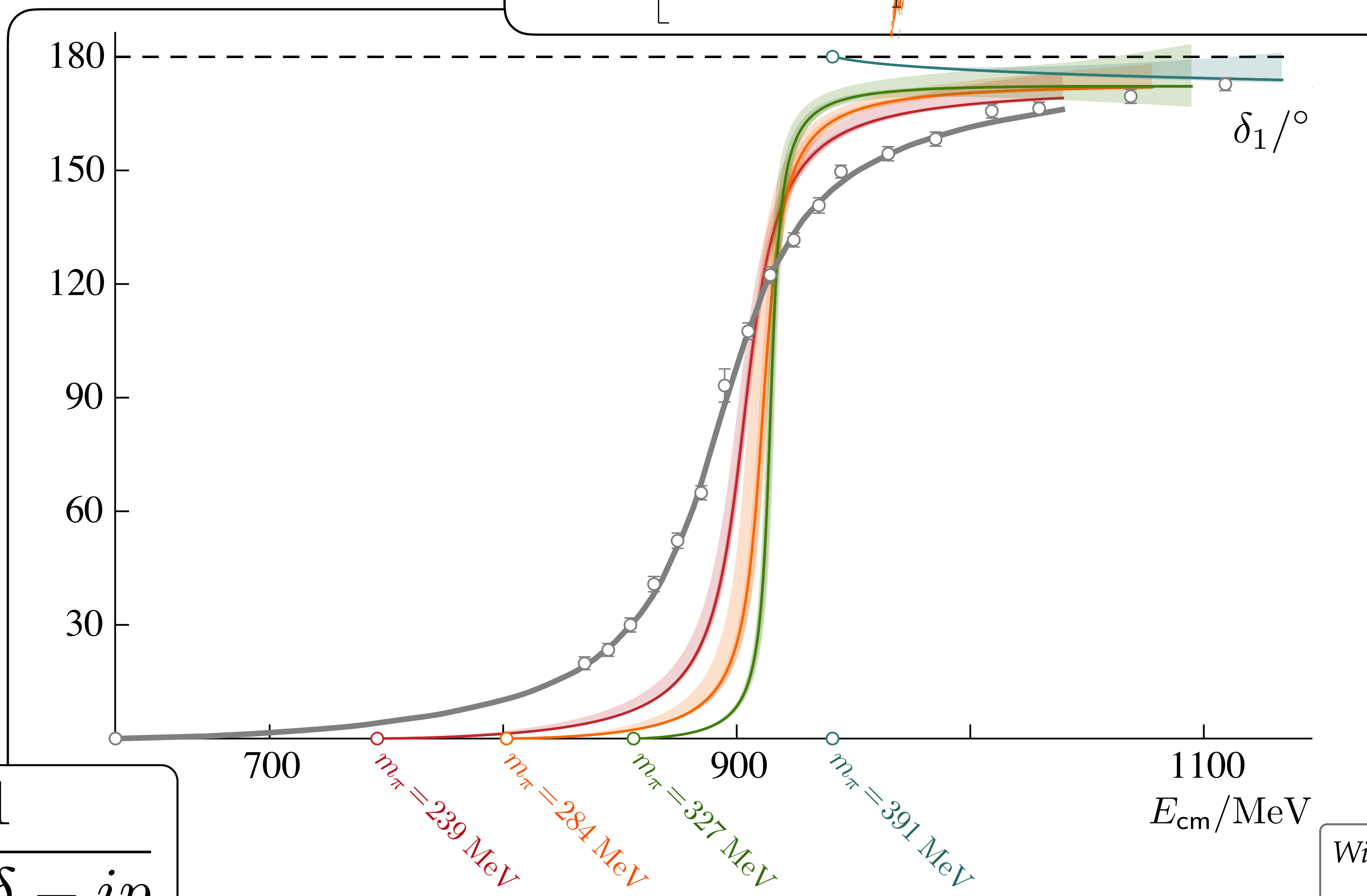

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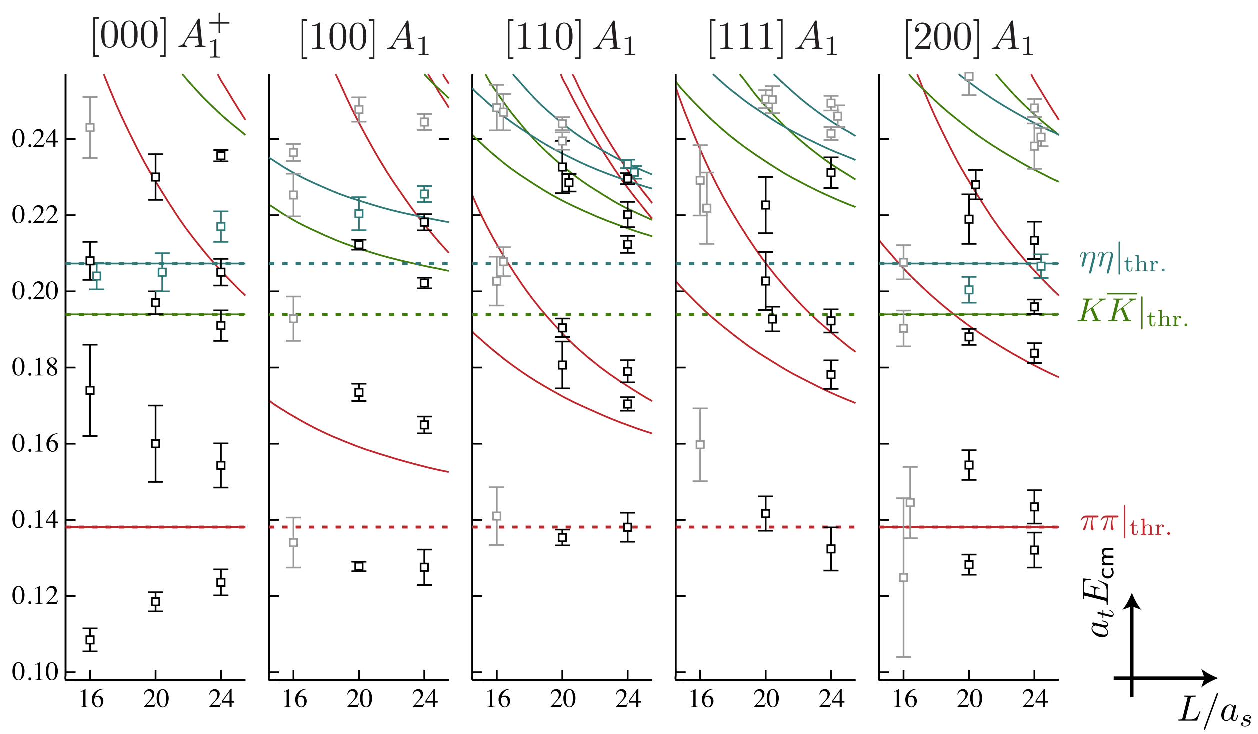
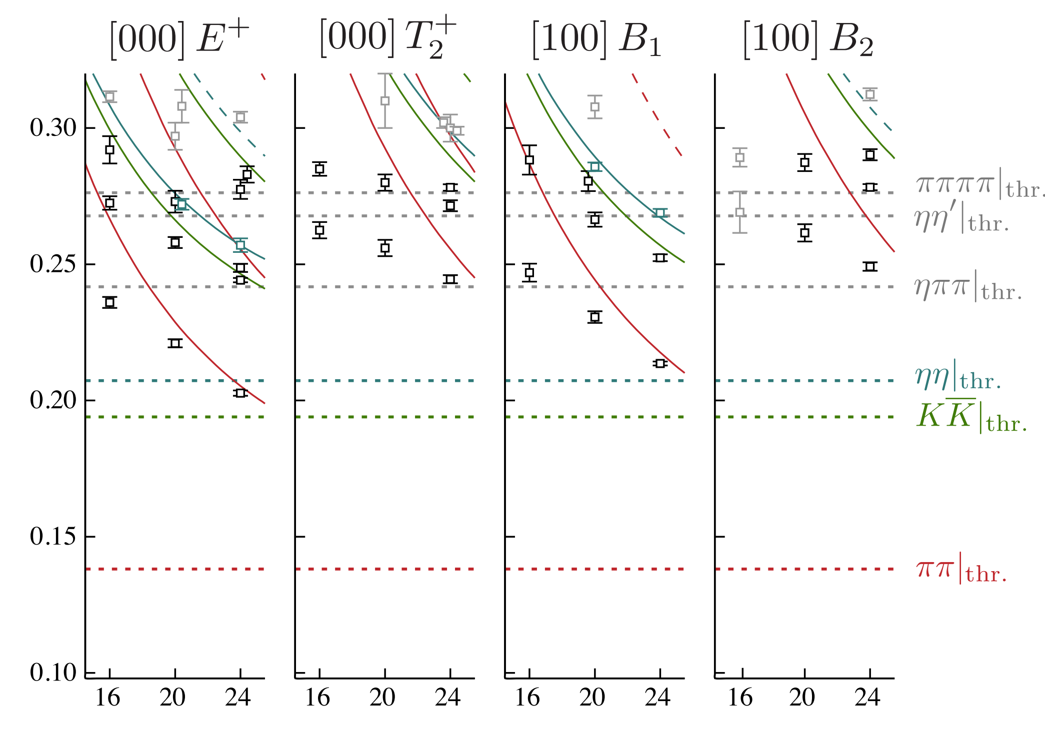
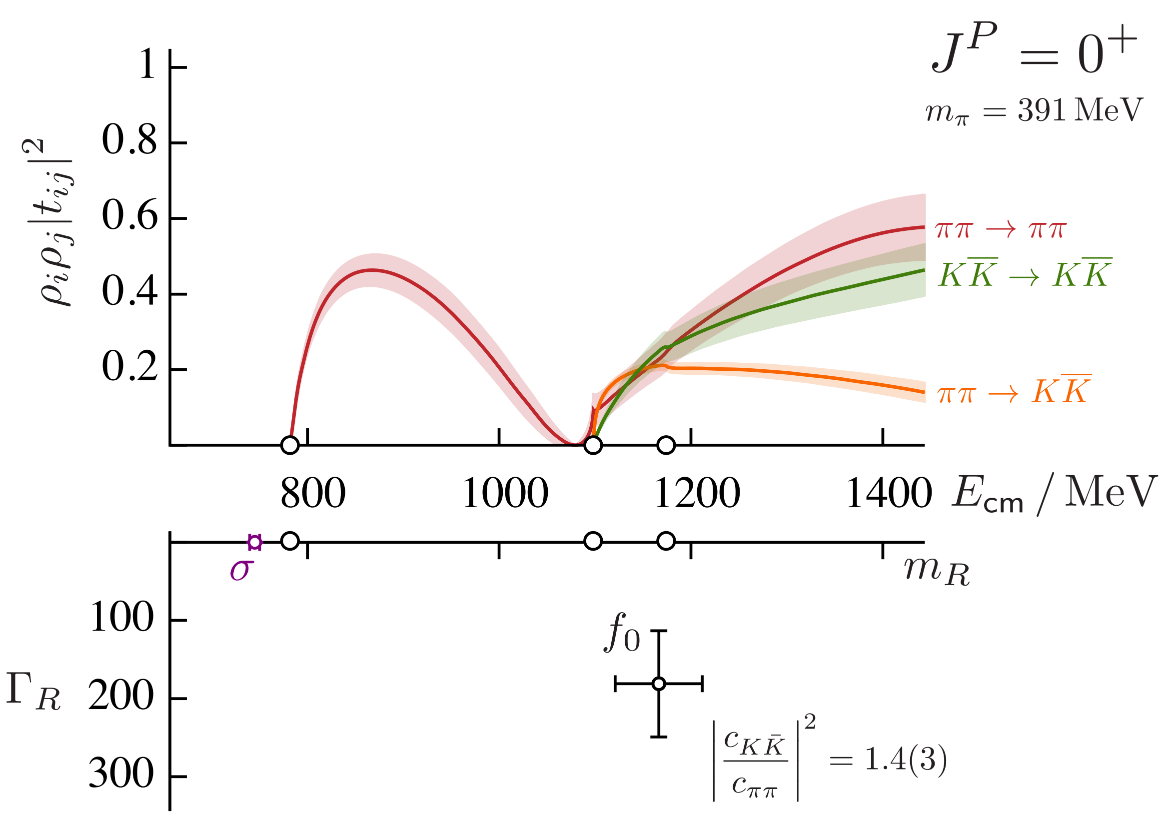
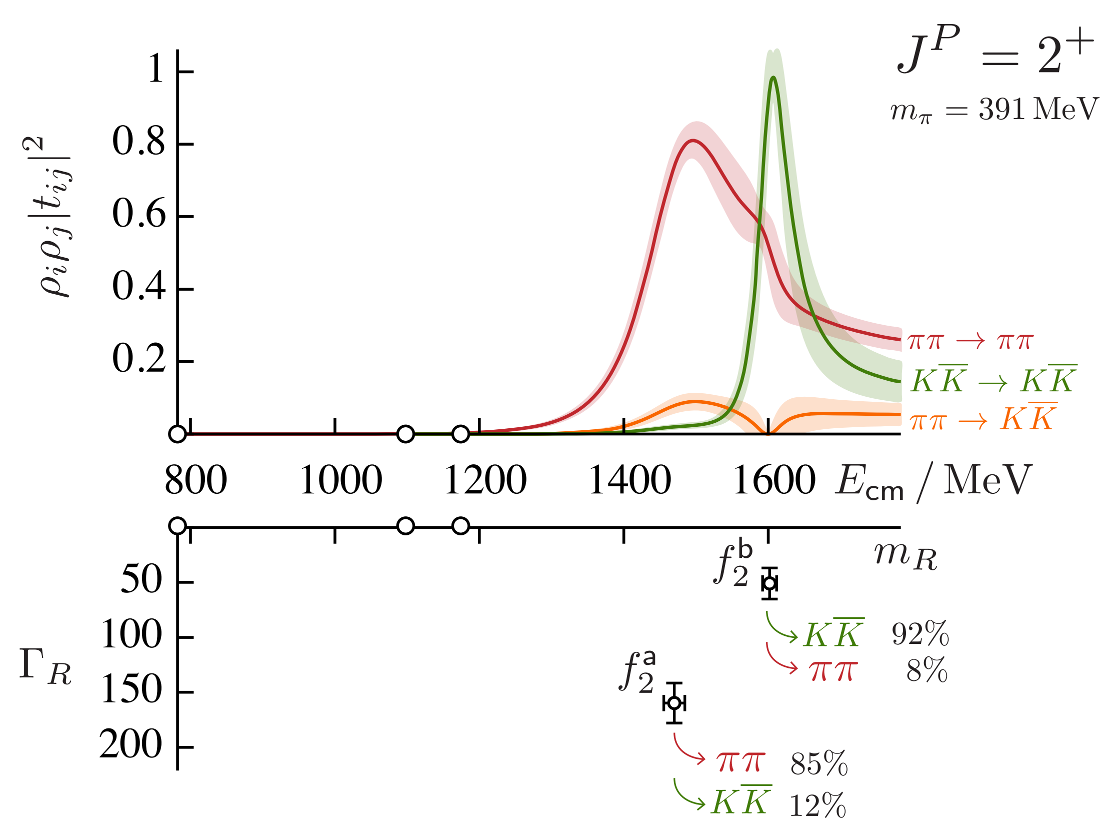
Wilson, RB, Dudek, Edwards, & Thomas (2019)

Scalar $\pi\pi$-KK



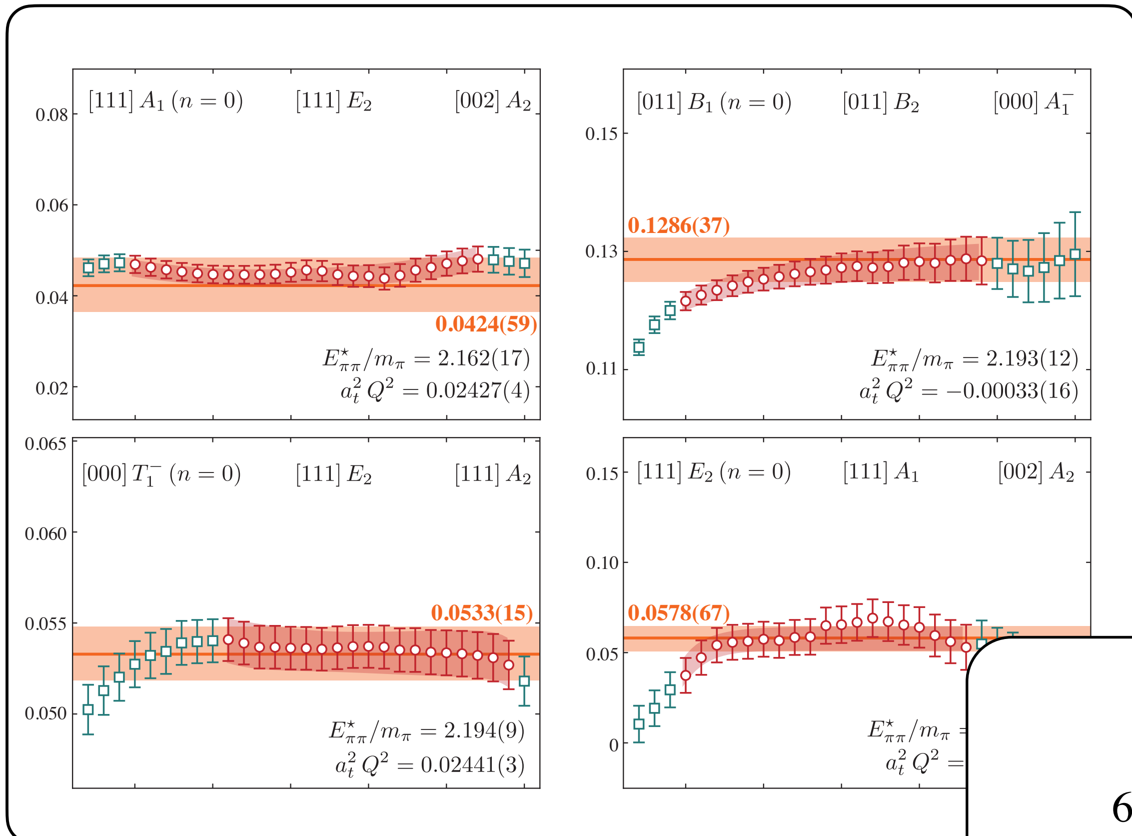


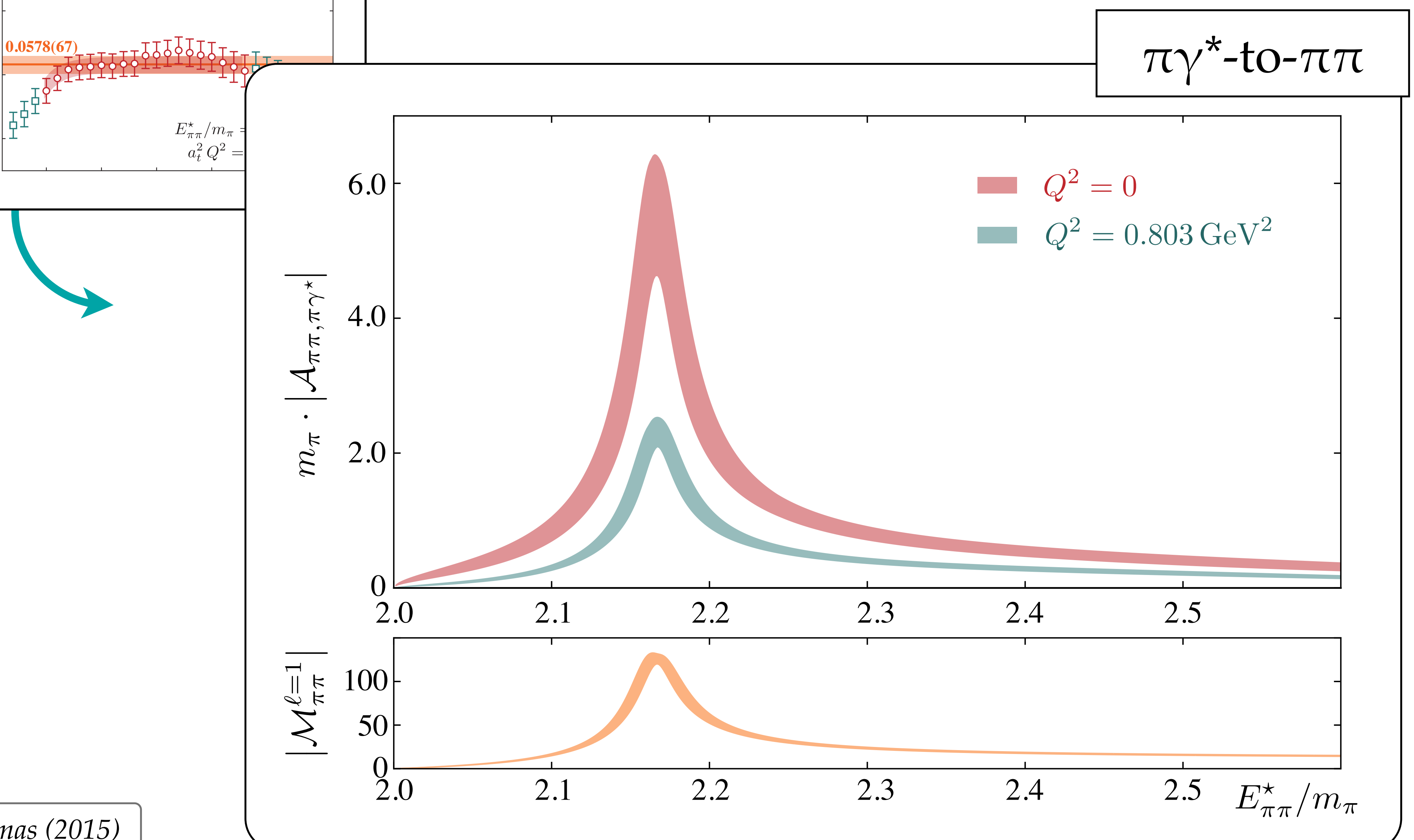




RB, Dudek, Edwards, & Wilson (2017)

Transition amplitudes





RB, Dudek, Edwards, Shultz, & Thomas (2015)

Scattering amplitudes using Quantum Computers

For implications of the BBDHS formalism in assessing finite-volume errors for future scattering calculations using quantum computers…

Open Access

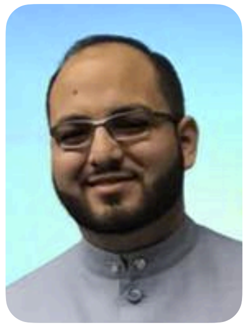
Role of boundary conditions in quantum computations of scattering observables

Raúl A. Briceño, Juan V. Guerrero, Maxwell T. Hansen, and Alexandru M. Sturzu
Phys. Rev. D **103**, 014506 – Published 6 January 2021

Article | References | No Citing Articles | PDF | HTML | Export Citation

ABSTRACT

Quantum computing may offer the opportunity to simulate strongly interacting field theories, such as quantum chromodynamics, with physical time evolution. This would give access to Minkowski-signature correlators, in contrast to the Euclidean calculations routinely performed at present. However, as with present-day calculations, quantum computation strategies still require the restriction to a finite system size, including a finite, usually periodic, spatial volume. In this work, we investigate the consequences of this in the extraction of hadronic and Compton-like scattering amplitudes. Using the framework presented in Briceño *et al.* [Phys. Rev. D **101**, 014509 (2020)], we estimate the volume effects for various $1+1$D Minkowski-signature quantities and show that these can be a significant source of systematic uncertainty, even for volumes that are very large by the standards of present-day Euclidean calculations. We then present an improvement strategy, based in the fact that the finite



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Long-range processes in QCD

$$\mathcal{T} = \int_{-\infty}^{\infty} d\tau\, e^{q_0 \tau} \Delta G(\tau, L) \;+\; \Delta\mathcal{T}[\mathcal{M}, \mathcal{H}, L]$$

RB, Davoudi, Hansen, Schindler, Baroni (2019)

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Extension of previous work by Christ, Feng, Martinelli, & Sachrajda.
Holds for:

- ☑ *single-hadron in/out states,*
- ☑ *arbitrary currents,*
- ☑ *coupled-channels,*
- ☑ *arbitrary angular momentum, and*
- ☑ *systems with non-zero spin.*

For on-going extensions to other systems:

- ☑ *Davoudi & Kadam (2020),*
- ☑ *Feng, Jin, Wang & Zhang (2020)*
- ☑ …

Spectra

- Infinite-volume Hamiltonian

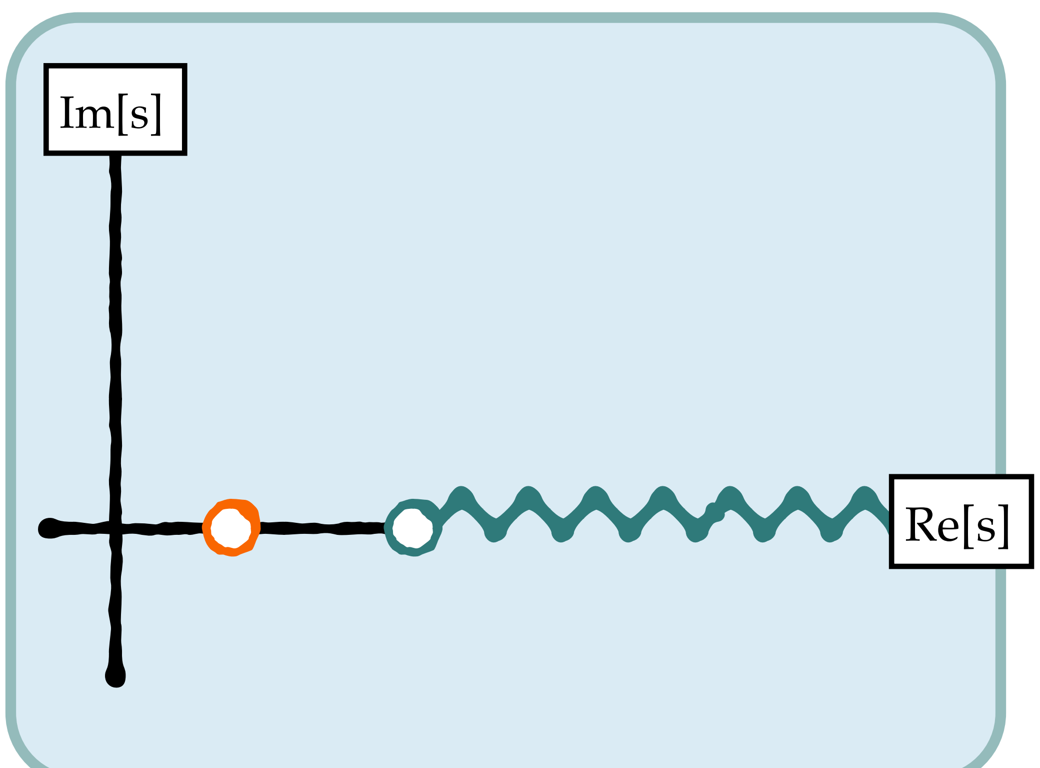
$$\hat{H}_{\infty} = \int_{\infty} d^3x \mathscr{H}$$

- Asymptotic states satisfy:

$$\hat{H}_{\infty,0} |p_1, p_2, \cdots p_n\rangle_0 = |p_1, p_2, \cdots p_n\rangle_0 \sum_{i=0}^{n} \sqrt{p_i^2 + m^2}$$

This is what the interaction picture buys us. At asymptotically large separations in space/time, we have nearly free states — which are the only ones we can generally define. 🤓

- No gap between states [continuum of states]



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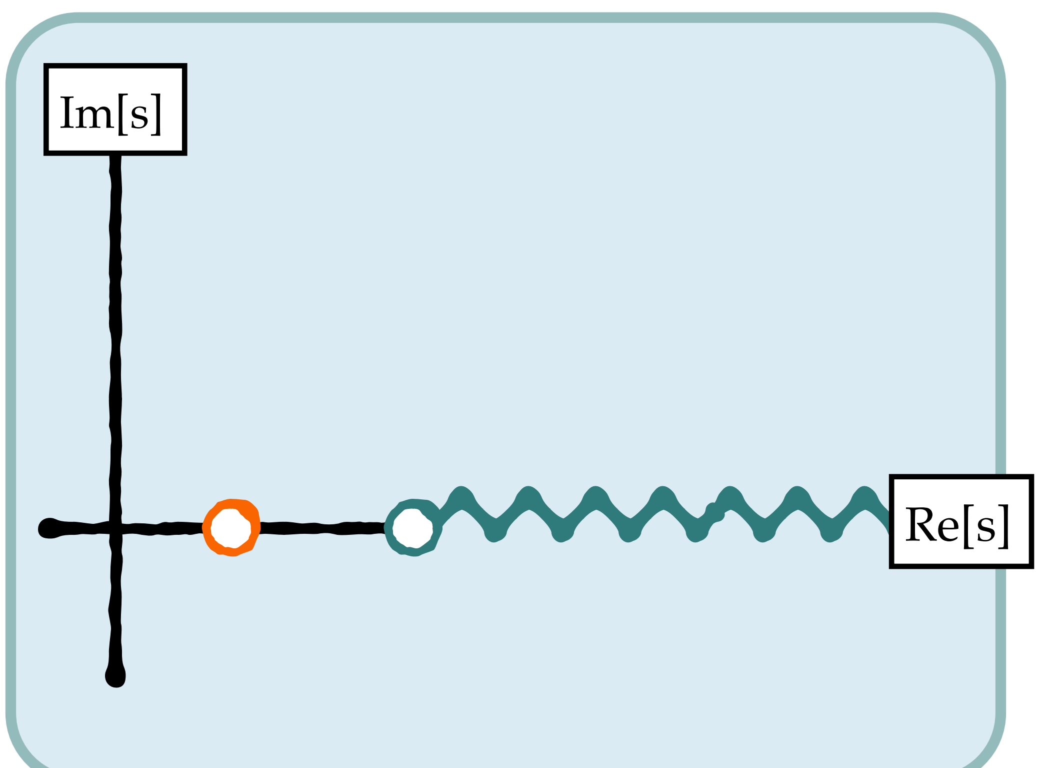


- Finite-volume Hamiltonian

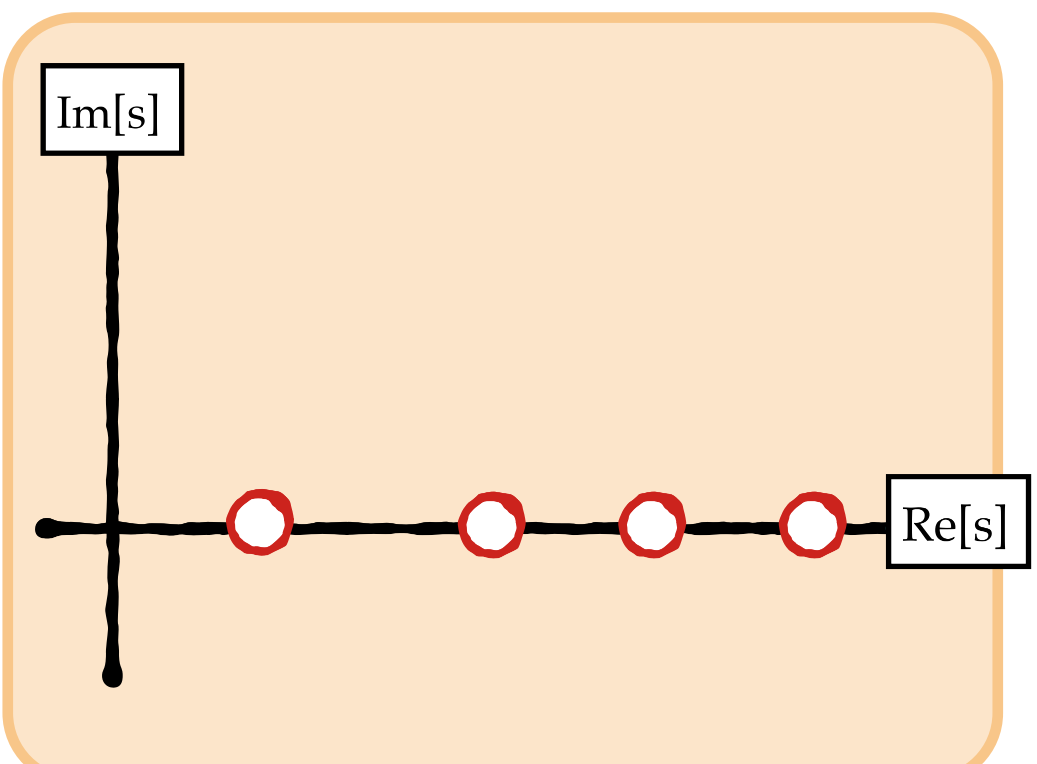
$$\hat{H}_L = \int_{V=L^3} d^3x \mathscr{H}$$

- No asymptotic states:

$$\hat{H}_L | n \rangle_L = | n \rangle_L E_n$$

- Intrinsic gap between states:

$$E_{n+1}(L) - E_n(L) \sim \frac{1}{L^\#}$$

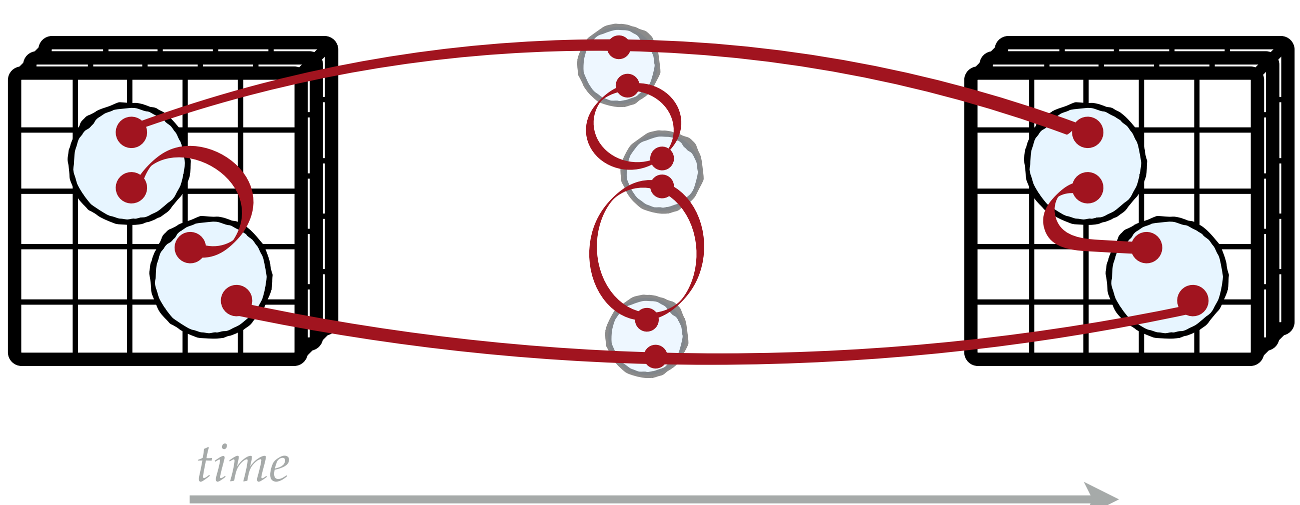


Correlation functions

Easier to think about finite-volume states, where the spectrum is discrete.
Once can always take the $L \to \infty$ limit afterwards.

Minkowski

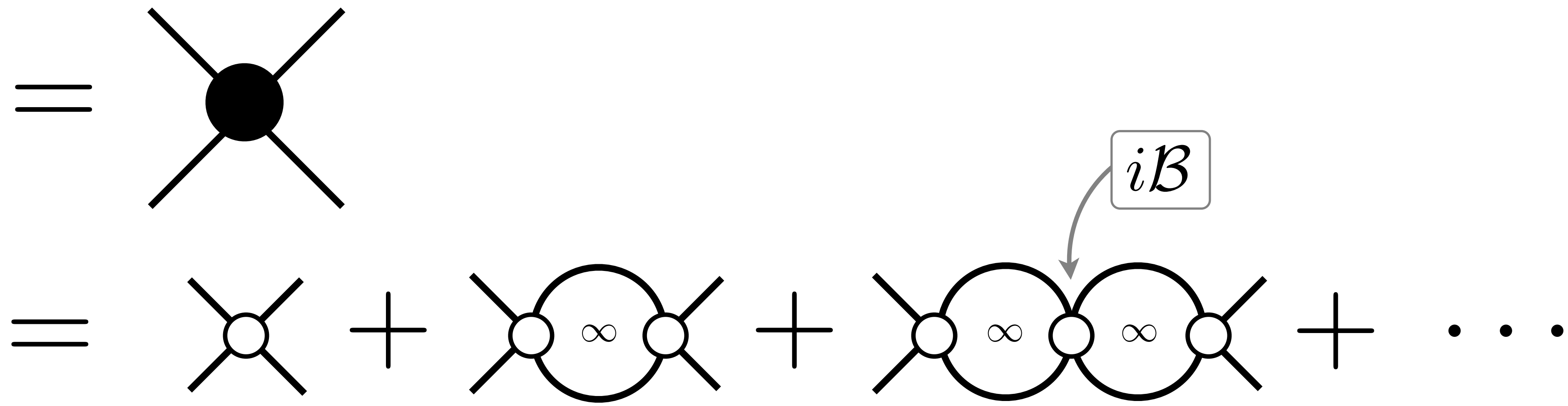
Euclidean



Correlation functions *BBDHS formalism in 1+1D*

Starting with the two-body scattering amplitude:

$i\mathcal{M} =$



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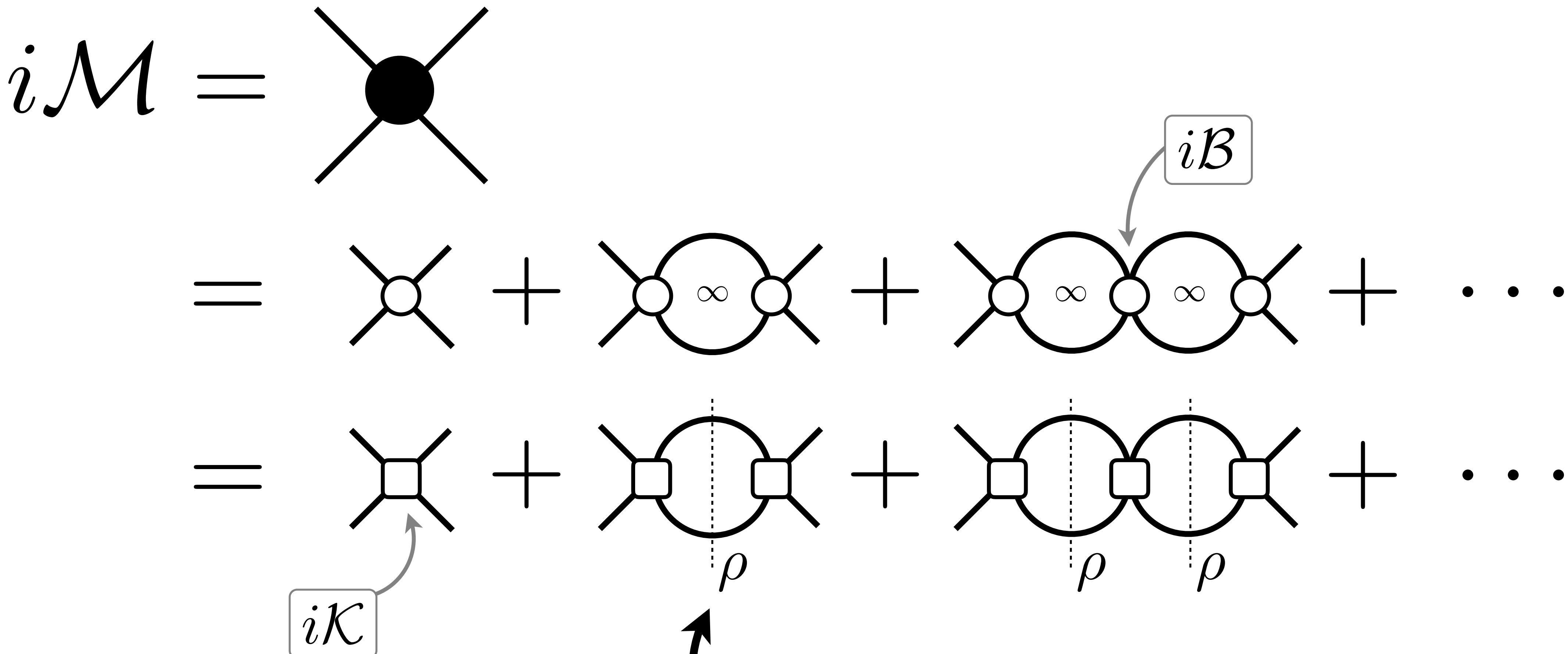

$$\rho = \frac{1}{8E^\star q^\star} \sim \frac{1}{\sqrt{s - s_{\text{th}}}}$$

source of kinematic singularity and imaginary contribution

Correlation functions *BBDHS formalism in 1+1D*

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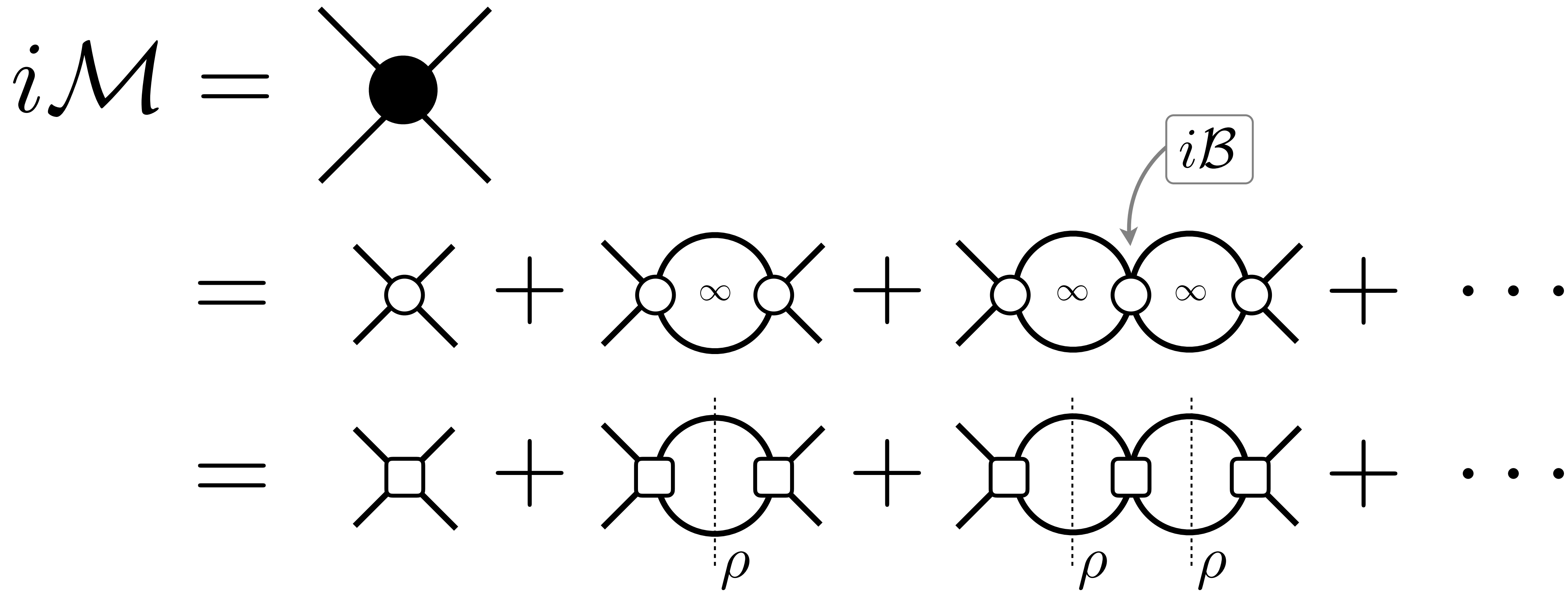

$$= \frac{i}{\mathcal{K}^{-1} - i\rho}$$