Gluon Gravitational Form Factors for Hadrons of Different Spins

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Gravitational Form Factors

- Form factors of transition matrix elements of energy momentum tensor, i.e $\langle h(p,s)| T_{\mu\nu} |h(p',s')\rangle = \sum_i^n C_i \mathrm{GFF}_i(t)$
- Functions with four momentum transfer
- Distributions: energy, pressure, shear force (e.g Polyakov, Schweitzer arXiv:1805.06596)

Moments of GPDs

- How can we access GFFs from experiments?
- **Symmetric, traceless** piece of EMT coincides with term in OPE for PDFs
- Off-forward -> GFFs related to Mellin moments of Generalized Parton distributions (i.e Ji arXiv:hep-ph/9603249)
- Can access exclusive processes like deeply virtual compton scattering
- Quark GFFs from experiment: nucleon [Burkert Elouadrhiri Girod], pion [Kumano Song Teryaev]

QCD Symmetric Traceless Energy Momentum Tensor

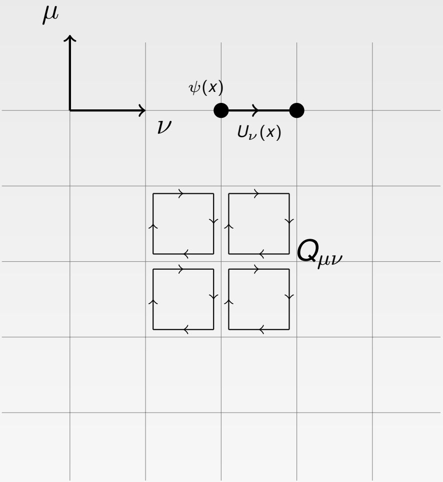
$$T_{\mu\nu} = \sum_q T^q_{\mu\nu} + T^g_{\mu\nu} \quad \partial^\mu T_{\mu\nu} = 0$$

$$T_q^{\mu\nu} = S[i\bar{\psi}_q \overleftrightarrow{D}^\mu \gamma^\nu \psi_q], \; T_g^{\mu\nu} = S[G^{\mu\alpha} {G^\nu}_\alpha]$$

- Trace anomaly gives rise to additional GFFs, but we can't access them in the same way in the off-forward limit on the lattice
- See e.g Lorcé, Mantovani, Pasquini arXiv:1704.08557 for discussion on asymmetric terms
- Our work: use $T^g_{\mu\nu}$ to extract symmetric traceless gluon GFFs for spin 0, 1/2, 1, 3/2 on the lattice
- Extract partial gluon densities from GFFs (energy, pressure, shear force)

Operators

- Gluon strength tensor - Clover term
  $G_{\mu\nu} = \frac{1}{8}(Q_{\mu\nu} - Q_{\nu\mu})$
- $T_g^{\mu\nu} = G^{\mu}{}_{\alpha} G^{\alpha\nu}$
- Choose hypercubic irreps that are safe from power divergent mixing with lower dimensional operators ($\tau_1^{(3)}$, $\tau_3^{(6)}$)



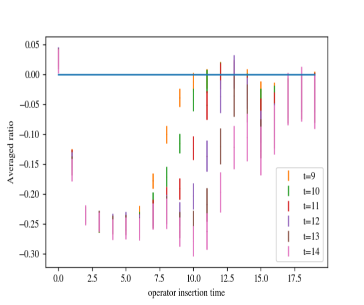
Ensemble and calculation

- Isoclover ensemble, $N_f = 2 + 1$
- Unphysical quark mass $m_\pi \sim 450\text{MeV}$
- 2821 configurations $32^3 \times 96$, 203 sources for each one
- All sink momenta with $|\vec{p}|^2 \leq 5(2\pi/L)^2$ and operator momenta with $|\vec{\Delta}|^2 \leq 18(2\pi/L)^2$
- All independent spin combinations
- $C_{3pt}^{ss'}$, $C_{2pt}^{s's'}$ bootstrap resampling

Method

- Form ratios of 3-pt and 2-pt functions to get rid of exponential time dependence and overlap factors
- $R_{ss'}(p, p', t, \tau) = \frac{C_{3pt}^{ss'}}{C_{2pt}^{s's'}(p',t)} \sqrt{\frac{C_{2pt}^{ss}(p,t-\tau) C_{2pt}^{s's'}(p',t) C_{2pt}^{s's'}(p',\tau)}{C_{2pt}^{s's'}(p',t-\tau) C_{2pt}^{ss}(p,t) C_{2pt}^{ss}(p,\tau)}}$
- Linear combination of GFFs, coefficients determined by operator, momenta and spins
- Average over ratios that are expected to be equal up to overall sign
- Overconstrained systems of linear equations for each 4-momentum squared t
- # linear equations: 4 (pion) - 2000 (delta)

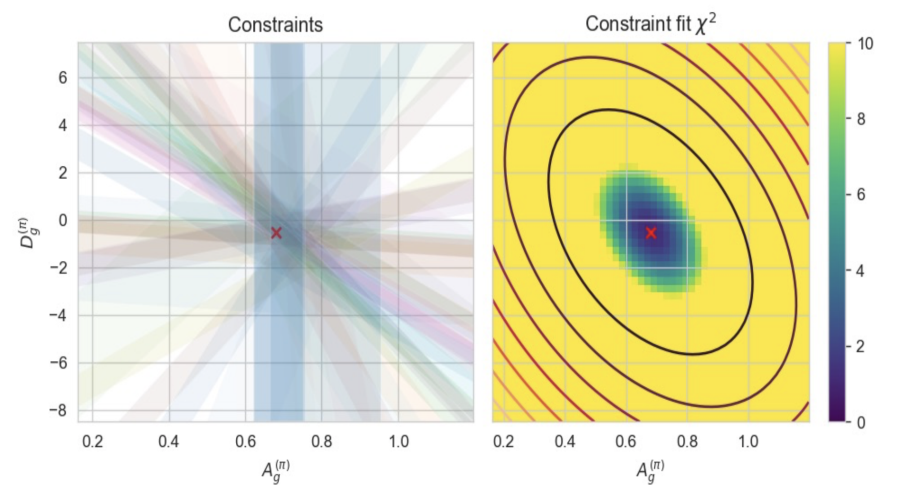
Plateau fitting



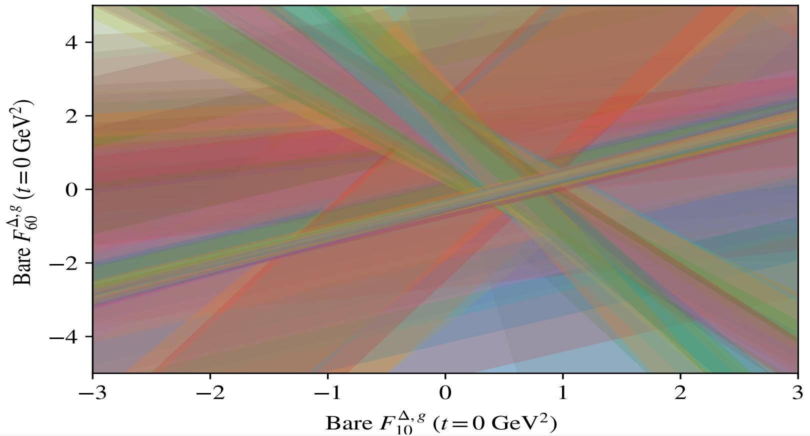
- each $\{t_{start}, t_{end}, \tau_{start}, \tau_{end}\}$ corresponds to a different 'model'
- Penalize noisy data by p-value averaging: each model has weight $w_m \propto Prob(\chi^2_{\mathrm{Ndof}} < \chi^2_m)(\delta x_m)^{-2}$

- $\hat{x} = \langle x \rangle_w, (\delta \hat{x}^{stat})^2 = \langle (\delta x^{stat})^2 \rangle_w, (\delta x^{sys})^2 = \langle (x - \hat{x})^2 \rangle_w$
- Take weighted average within each bootstrap and rescale results such that the diagonal of the covariance matrix = $(\delta \hat{x}^{stat})^2 + (\delta x^{sys})^2$

Band fitting



Band fitting



Renormalization

- irreps $\tau_1^{(3)}$, $\tau_3^{(6)}$ renormalize separately
- Shanahan Detmold 1810.04626:

$$Z^{\overline{\text{MS}}}_{\tau_1^{(3)}}(\mu = 2\text{GeV}) = 0.9(2) \quad Z^{\overline{\text{MS}}}_{\tau_2^{(6)}}(\mu = 2\text{GeV}) = 0.78(7)$$

- Large correlated uncertainties -> d’Agostini bias
- Penalty trick -> marginalize over nuisance parameter

Fitting of GFFs

$$F_{\text{multipole}}(t) = \frac{\alpha}{(1 - t/\Lambda^2)^n}$$

$$F_{\text{z-exp}}(t) = \frac{1}{(1 - t/\Lambda^2)^n} \sum_{k=0}^{k_{max}} a_k [z(t)]^k$$

$$z(t) = \frac{\sqrt{4m_\pi^2 - t} - \sqrt{4m_\pi^2 - t_0}}{\sqrt{4m_\pi^2 - t} + \sqrt{4m_\pi^2 - t_0}}, \quad t_0 = 4m_\pi^2(1 - \sqrt{1 + (2\text{GeV})^2/(4m_\pi^2)})$$

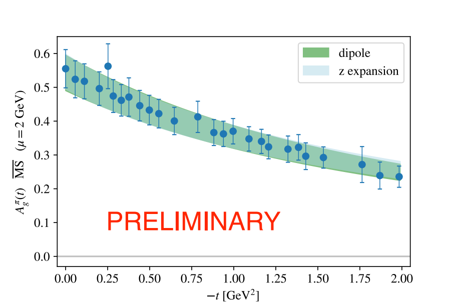
- Hill, Paz arXiv:1008.4619

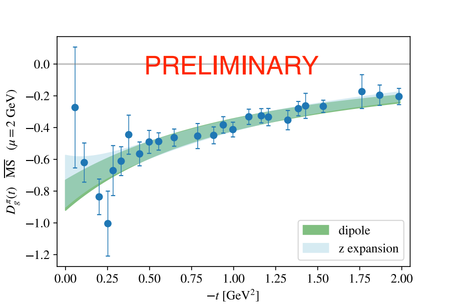
Spin 0 - Pion [Shanahan Detmold 1810.04626]

$$\langle p'| T_i^{\mu\nu} |p\rangle = 2P^\mu P^\nu \mathbf{A}_i(t) + \frac{1}{2}\Delta^\mu \Delta^\nu \mathbf{D}_i(t)$$

- $P = \frac{p+p'}{2}$, $\Delta = p' - p$, $t = -\Delta^2$, $i = \{q, g\}$, $\mathbf{F}(t) \equiv \mathbf{F}_q(t) + \mathbf{F}_g(t)$
- $\mathbf{A}(0) = 1$ momentum faction (Poincare invariance)
- $\mathbf{D}(0) \sim -1$ χPT (Hudson Schweitzer 1712.05316)

Pion Gluon GFFs



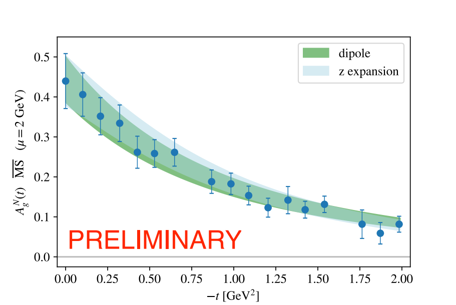


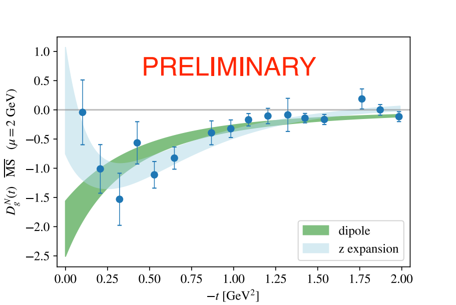
Spin 1/2 - Proton [Shanahan Detmold 1810.04626]

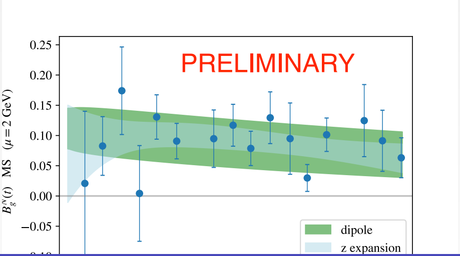
$$\langle p', s'| T_i^{\mu\nu} |p, s\rangle = S[\gamma^\mu P^\nu \mathbf{A}_i(t) + \frac{iP^\mu \sigma^{\nu\rho}\Delta_\rho}{2M}\mathbf{B}_i(t) + \frac{\Delta^\mu \Delta^\nu}{4M}\mathbf{D}_i(t)]$$

- $\mathbf{A}(0) = 1$ (always from Poincare invariance)
- $\mathbf{B}(0) = 0$ (vanishing of anomalous gravitomagnetic moment of spin 1/2)
- **D** unconstrained (0 for free spin 1/2 field theory Hudson Schweitzer 1712.05317)

Proton Gluon GFFs



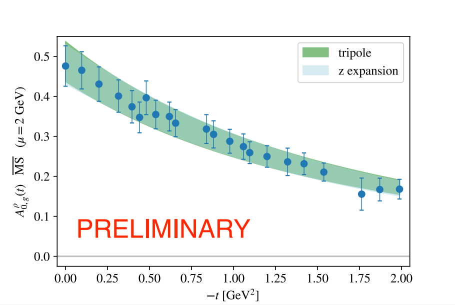


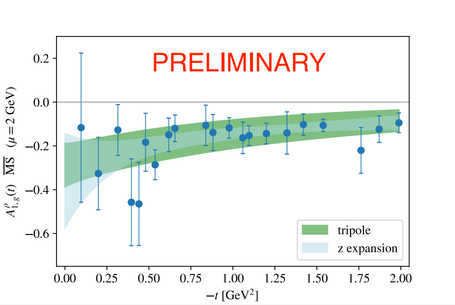


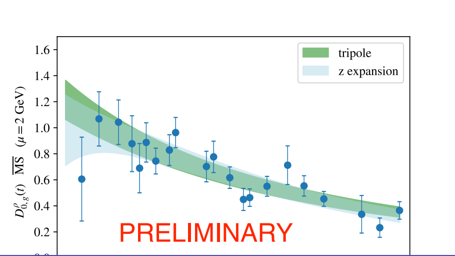
Spin 1 - Rho [Polyakov Sun 1903.02738]

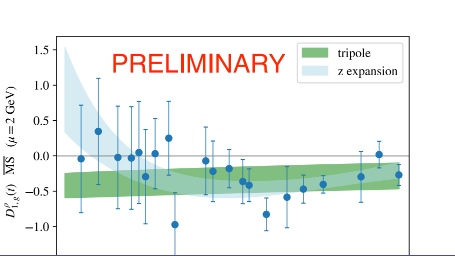
$$
\begin{aligned}
\langle p', \lambda' | \hat{T}_i^{\mu\nu} | p, \lambda \rangle = \epsilon^*_{\alpha'}(\vec{p}', \lambda')\epsilon_\alpha(\vec{p}, \lambda) \Bigg[& 2P^\mu P^\nu \left(-g^{\alpha\alpha'} \mathbf{A}_0^i(t) + \frac{P^\alpha P^{\alpha'}}{M^2} \mathbf{A}_1^i(t) \right) \\
& + \frac{1}{2}(\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2) \left(g^{\alpha\alpha'} \mathbf{D}_0^i(t) + \frac{P^\alpha P^{\alpha'}}{M^2} \mathbf{D}_1^i(t) \right) \\
& + 4[P^{\{\mu} g^{\nu\}\alpha'} P^\alpha + P^{\{\mu} g^{\nu\}\alpha} P^{\alpha'}] \mathbf{J}^i(t) \\
& + [g^{\alpha\{\mu} g^{\nu\}\alpha'} \Delta^2 - 2g^{\alpha'\{\mu} \Delta^{\nu\}} P^\alpha \\
& + 2g^{\alpha\{\mu} \Delta^{\nu\}} P^{\alpha'} - 4g^{\mu\nu} P^\alpha P^{\alpha'}] \mathbf{E}^i(t) \\
& + [2g^{\alpha\{\mu} g^{\nu\}\alpha'} - \frac{1}{2} g^{\alpha\alpha'} g^{\mu\nu}] M^2 \bar{\mathbf{f}}^i(t) \Bigg]
\end{aligned}
$$

Rho Gluon GFFs [Detmold **DP** Shanahan 1703.08220]

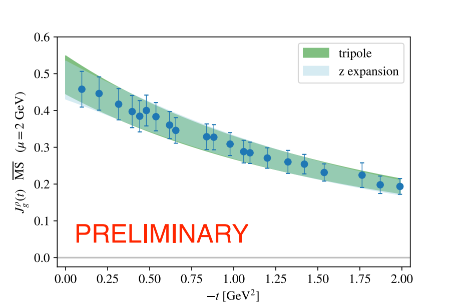


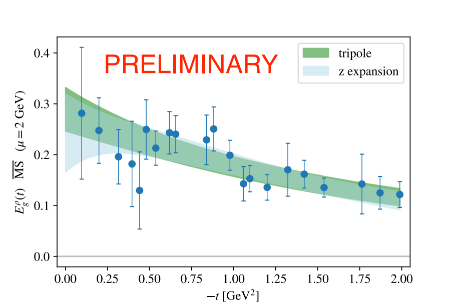


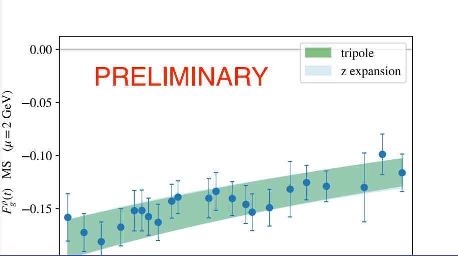




Rho Gluon GFFs



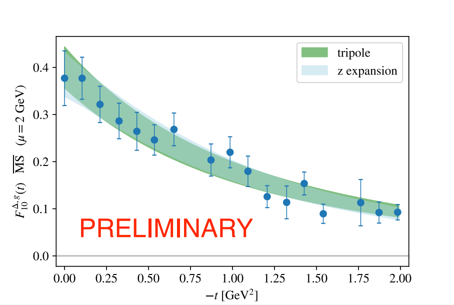


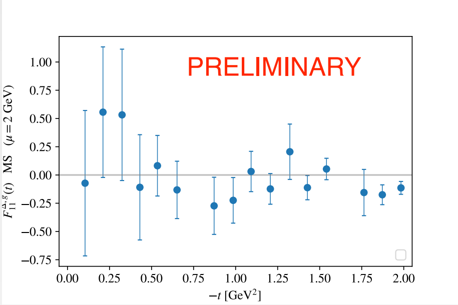


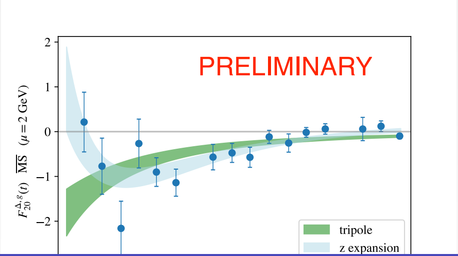
Spin 3/2 - Delta [Kim Sun 2011.00292]

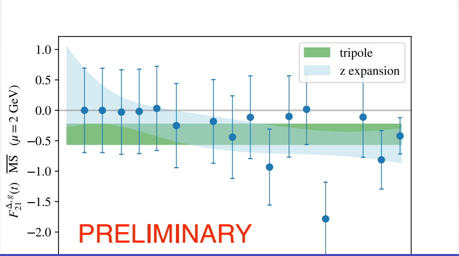
$$\langle p', s'| \, T^{\mu\nu} \, |p, s\rangle = \bar{u}_{\alpha'}(p', s') \Bigg[\frac{P^\mu P^\nu}{M} \left(-g^{\alpha\alpha'} \mathbf{F}^{\Delta,i}_{1,0}(t) + \frac{\Delta^\alpha \Delta^{\alpha'}}{2M^2} \mathbf{F}^{\Delta,i}_{1,1}(t) \right)$$
$$+ \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{4M} \left(-g^{\alpha\alpha'} \mathbf{F}^{i}_{2,0}(t) + \frac{\Delta^\alpha \Delta^{\alpha'}}{2M^2} \mathbf{F}^{\Delta,i}_{2,1}(t) \right)$$
$$+ \frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{M} \left(-g^{\alpha'\alpha} \mathbf{F}^{\Delta,i}_{4,0}(t) + \frac{\Delta^{\alpha'} \Delta^{\alpha}}{2M^2} \mathbf{F}^{\Delta,i}_{4,1}(t) \right)$$
$$+ \frac{2}{M} \left(\Delta^{\{\mu} g^{\nu\}\{\alpha'} \Delta^{\alpha\}} + -g^{\mu\nu} \Delta^\alpha \Delta^{\alpha'} - g^{\alpha'\{\mu} g^{\nu\}\alpha} \Delta^2 \right) \mathbf{F}^{\Delta,i}_{5,0}($$
$$- 2Mg^{\alpha'\{\mu} g^{\nu\}\alpha} \mathbf{F}^{\Delta,i}_{6,0}(t) \Bigg] u_\alpha(p, s)$$

Delta Gluon GFFs

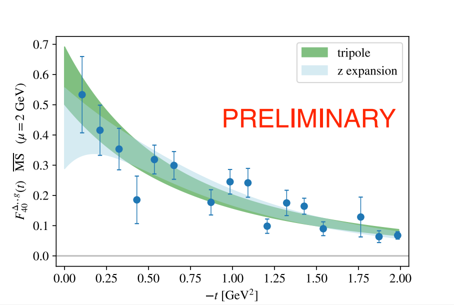


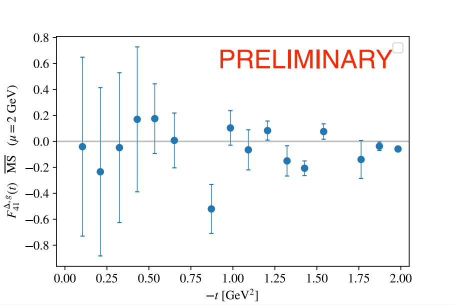


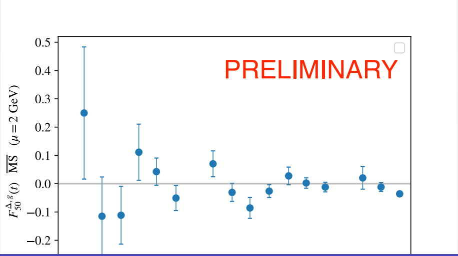


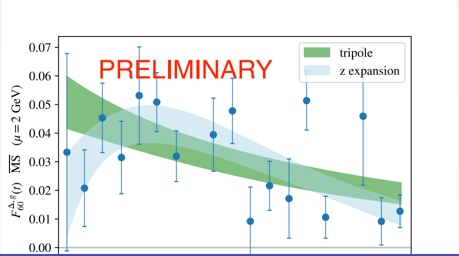


Delta Gluon GFFs





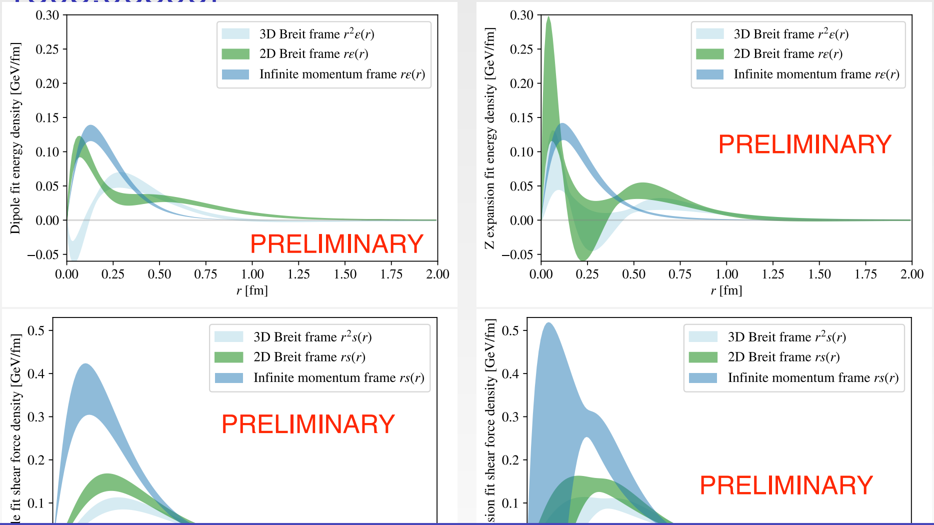




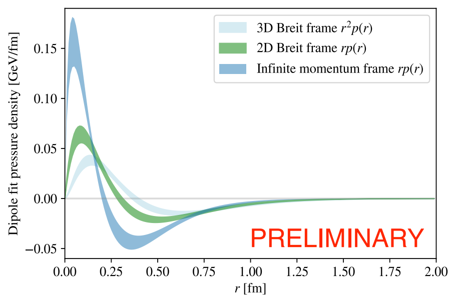
Frame dependence

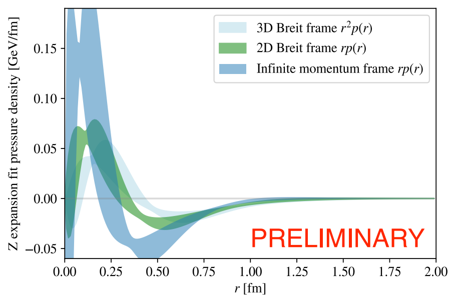
- Spacial densities depend on Fourier transforms of GFFs -> frame dependence (Breit frame, infinite momentum frame)
- Recent literature [Panteleeva Polyakov 2102.10902, Freese Miller 2102.01683, Jaffe 2010.15887, Lorcé 2007.05318, Lorcé Moutarde Trawínski 1810.09837 etc.]
- We study 3D/2D Breit frame and 2D IMF following [Lorcé Moutarde Trawínski 1810.09837] for boost factor conventions
- $[K]^{\text{BF3,BF2,IMF}}$ - Fourier transform in appropriate frame

Pion gluon traceless densities - [Polyakov Schweitzer 1805.06596]

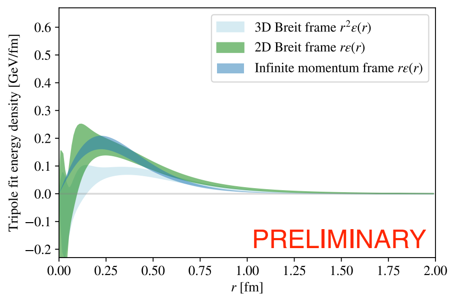


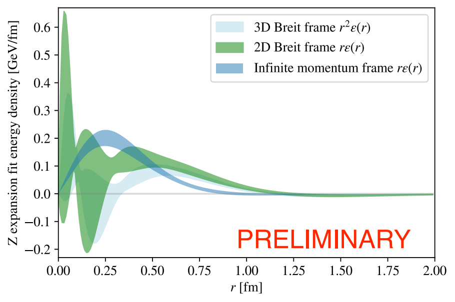
Pion gluon traceless densities

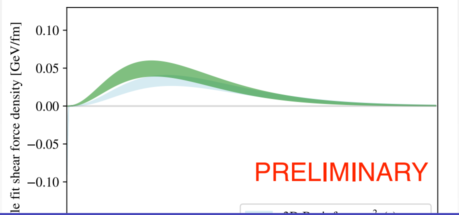


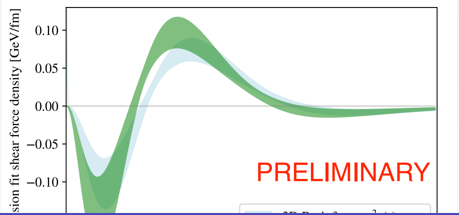


Nucleon gluon traceless densities- [Lorcé Moutarde Trawínski 1810.09837]

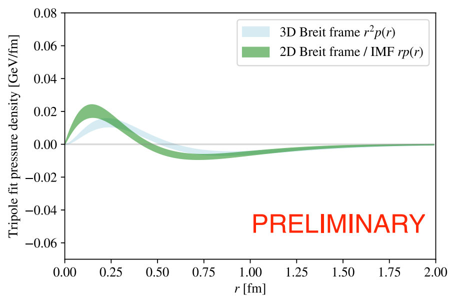


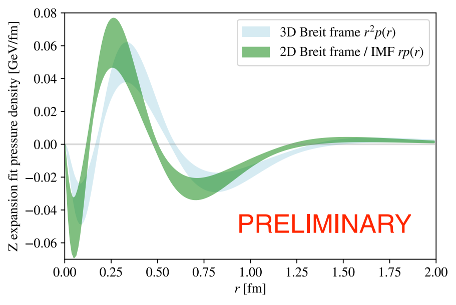




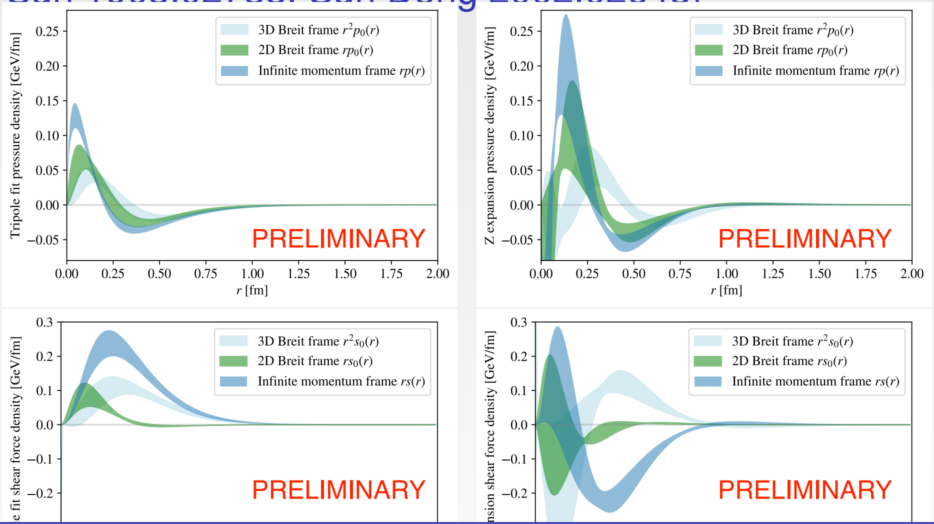


Nucleon gluon traceless densities





Rho gluon traceless monopole densities - [Polyakov Sun 1903.02738, Sun Dong 2002.02648]



Next Steps

- Pion and proton quark and gluon GFFs at $m_\pi = 170 MeV$
- Mixing of quark and gluon pieces under renormalization
- Analyze combined gluon + quark contributions to verify sum rules and obtain total pressure