Mass sum rules for the proton

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Based on

Rodini, Metz, Pasquini, JHEP 09 (2020) 067

Metz, Pasquini, Rodini Phys. Rev. D102 (2020) 114042

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Outline

Renormalization procedure for multiple local operators

Energy-Momentum Tensor matrix elements

Different mass decompositions

The Energy-Momentum Tensor

Lagrangian renormalization is understood

$$T^{\mu\nu} = \mathcal{O}_1 + \frac{\mathcal{O}_2}{4} + \mathcal{O}_3$$

$$\mathcal{O}_1 = -F^{\mu\alpha}F^{\nu}{}_{\alpha} \qquad \mathcal{O}_2 = g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$$

$$\mathcal{O}_3 = \frac{i}{4}\,\bar{\psi}\,\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\,\psi \qquad \mathcal{O}_4 = g^{\mu\nu}m\bar{\psi}\psi$$

Renormalize fields, coupling and masses is not enough.

We must also renormalize the operators:

$$\mathcal{O}_{1,R} = Z_T\mathcal{O}_1 + Z_M\mathcal{O}_2 + Z_L\mathcal{O}_3 + Z_S\mathcal{O}_4$$

$$\mathcal{O}_{2,R} = Z_F\mathcal{O}_2 + Z_C\mathcal{O}_4$$

$$\mathcal{O}_{3,R} = Z_\psi\mathcal{O}_3 + Z_K\mathcal{O}_4 + Z_Q\mathcal{O}_1 + Z_B\mathcal{O}_2$$

$$\mathcal{O}_{4,R} = \mathcal{O}_4$$

Rodini et al., JHEP 09 (2020) 067

Metz et al., Phys. Rev. D102 (2020) 114042 | Hatta et al., JHEP 12 (2018) 008 | Tanaka, JHEP 01 (2019) 120

A few simple definitions

We work in $1 + (3 - 2\epsilon)$

We can consider the 00 component of a bare operator without problems

$$E^2 = -\sum_i F^{0i} \qquad\qquad F^2 = 2(B^2 - E^2)$$

$$\mathcal{O}_1 + \frac{1}{4}\mathcal{O}_2 = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \frac{g^{\mu\nu}}{4}F^2$$

$$\mathrm{Tr}_4(\mathcal{O}_1 + \frac{1}{4}\mathcal{O}_2) = -\mathrm{Tr}_4(F^{\mu\alpha}F^{\nu}{}_{\alpha}) + F^2$$

$$\mathrm{Tr}_d(\mathcal{O}_1 + \frac{1}{4}\mathcal{O}_2) = -\mathrm{Tr}_d(F^{\mu\alpha}F^{\nu}{}_{\alpha}) + \frac{d}{4}F^2 = -F^2 + \frac{d}{4}F^2$$

Rodini et al., JHEP 09 (2020) 067

Metz et al., Phys. Rev. D102 (2020) 114042

The total EMT is not affected by the additional renormalization

$$T_R^{\mu\nu} = T^{\mu\nu}$$

$$T^{\mu}{}_{\mu} = (T_R)^{\mu}{}_{\mu} = (T^{\mu}{}_{\mu})_R = (1+\gamma_m)(m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^{\alpha\beta}F_{\alpha\beta})_R$$

However, in general, trace and renormalization do not commute

$$\mathrm{Tr}[O_R^{\mu\nu}] \neq (\mathrm{Tr}[O^{\mu\nu}])_R$$

In particular $\quad T_{q,R}^{\mu\nu} = \mathcal{O}_{3,R} \qquad T_{g,R}^{\mu\nu} = \mathcal{O}_{1,R} + \frac{\mathcal{O}_{2,R}}{4}$

$$(T_{q,R})^{\mu}{}_{\mu} = (1+y)(m\bar{\psi}\psi)_R + x\,(F^{\alpha\beta}F_{\alpha\beta})_R$$

$$(T_{g,R})^{\mu}{}_{\mu} = (\gamma_m - y)(m\bar{\psi}\psi)_R + \left(\frac{\beta}{2g} - x\right)(F^{\alpha\beta}F_{\alpha\beta})_R$$

Rodini et al., JHEP 09 (2020) 067

Metz et al., Phys. Rev. D102 (2020) 114042 | Hatta et al., JHEP 12 (2018) 008 | Tanaka, JHEP 01 (2019) 120

How to fix the counterterms (1)

$Z_{F,C}$ Are known from R. Tarrach, Nucl. Phys. B 196 (1982) 45

$Z_{L,T,Q,\psi}$ Are given by the evolution equations for the second moment of the flavor-singlet unpolarized parton distributions. Tanaka, JHEP 01 (2019) 120

$$\mathrm{AMF}(\mathrm{R}[q]) = Z_1 \mathrm{AMF}(q) + Z_2 \mathrm{AMF}(g)$$
$$\mathrm{AMF}(\mathrm{R}[g]) = Z_3 \mathrm{AMF}(q) + Z_4 \mathrm{AMF}(g)$$

Average Moment Fractions (AMF) from PDFs

$$\mathrm{R}[\mathrm{AMF}(q)] = Z_\psi \mathrm{AMF}(q) + Z_Q \mathrm{AMF}(g)$$
$$\mathrm{R}[\mathrm{AMF}(g)] = Z_L \mathrm{AMF}(q) + Z_T \mathrm{AMF}(g)$$

Average Moment Fractions (AMF) from EMT (++ components)

$$Z_\psi^{[\epsilon]} = Z_1^{[\epsilon]} \quad Z_Q^{[\epsilon]} = Z_2^{[\epsilon]}$$
$$Z_T^{[\epsilon]} = Z_3^{[\epsilon]} \quad Z_L^{[\epsilon]} = Z_4^{[\epsilon]}$$

The counterterms have the same divergent part

How to fix the counterterms (2)

The other counterterms are not independent!

$$Z_T + Z_Q = 1 \qquad\qquad Z_L + Z_\psi = 1$$

$$Z_M + Z_B + \frac{Z_F}{4} = \frac{1}{4} \qquad\qquad Z_S + Z_K + \frac{Z_C}{4} = 0$$

$$Z_M = \frac{Z_T}{d} - \frac{Z_F}{d}\left(1 - \frac{\beta}{2g} + x\right)$$

$$Z_S = -\frac{Z_L}{d} - \frac{Z_C}{d}\left(1 - \frac{\beta}{2g} + x\right) - \frac{y - \gamma_m}{d}$$

$$Z_B = \frac{Z_Q}{d} + \frac{x}{d} Z_F$$

$$Z_K = -\frac{Z_\psi}{d} + \frac{x}{d} Z_C + \frac{1+y}{d}$$

Rodini et al., JHEP 09 (2020) 067

Metz et al., Phys. Rev. D102 (2020) 114042 | Hatta et al., JHEP 12 (2018) 008 | Tanaka, JHEP 01 (2019) 120

$$\widetilde{\mathcal{O}}_{1,R} = \mathcal{O}_{1,R} + \frac{1}{4}\left(1 - \frac{\beta}{2e} + x\right)\mathcal{O}_{2,R} + \frac{y-\gamma_m}{4}\mathcal{O}_{4,R},$$

$$\widetilde{\mathcal{O}}_{3,R} = \mathcal{O}_{3,R} - \frac{x}{4}\mathcal{O}_{2,R} - \frac{1+y}{4}\mathcal{O}_{4,R}$$

Fixing x and y it corresponds to choose a scheme

Diagonal schemes:

D1: $x = 0 \quad y = \gamma_m$ Rodini et al., JHEP 09 (2020) 067

D2: $x = 0 \quad y = 0$ Metz et al., Phys. Rev. D102 (2020) 114042

MS-like schemes: Tanaka, JHEP 01 (2019) 120 Hatta et al., JHEP 12 (2018) 008

Alternative construction of MSbar scheme Metz et al., Phys. Rev. D102 (2020) 114042

MS-like schemes

Impose vanishing finite contributions to the derived counterterms

$$Z_X = \delta_{X,T} + \delta_{X,\psi} + \delta_{X,F} + \frac{a_X}{\epsilon} + \frac{b_X}{\epsilon^2} + \frac{c_X}{\epsilon^3} + \dots$$

From the definition of $Z_{M,S}$

$$\frac{1}{32}\left[(8 + 4a_T + 2b_T + c_T + \dots) - \left(1 + x - \frac{\beta}{2e}\right)(8 + 4a_F + 2b_F + c_F + \dots)\right] = 0$$

$$\frac{1}{32}\left[-(4a_L + 2b_L + c_L + \dots) - \left(1 + x - \frac{\beta}{2e}\right)(4a_C + 2b_C + c_C + \dots) + 8(\gamma_m - y)\right] = 0$$

Tanaka, JHEP 01 (2019) 120

EMT matrix elements

$$\langle \mathcal{O} \rangle = \langle P | \mathcal{O} | P \rangle$$

$$\langle p', s' | \, T_i^{\mu\nu} \, | p, s \rangle$$

$$= \bar{u}(p', s') \left(A_i(t) \, \frac{P^\mu P^\nu}{M} + J_i(t) \, \frac{i P^{\{\mu} \sigma^{\nu\}\rho} \Delta_\rho}{2M} \right.$$

$$\left. + D_i(t) \, \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} + M \, \bar{C}_i(t) \, g^{\mu\nu} \right) u(p, s)$$

Two form factors for the forward limit

$$\langle \, T_{i,R}^{\mu\nu} \, \rangle = 2 P^\mu P^\nu A_i(0) + 2 M^2 g^{\mu\nu} \bar{C}_i(0)$$

$$i = q, g$$

$$\sum_i A_i(0) = 1 \quad \sum_i \bar{C}_i(0) = 0$$

Ji, Phys. Rev. Lett. 78 (1997) 610

How to obtain the form factors

CT18NNLO $\mu = 2\ \text{GeV}$

Hou et al.,1908.11394

$$A_q(0) \equiv a_q = 0.586 \pm 0.013$$

$$A_g(0) \equiv a_g = 1 - a_q = 0.414 \pm 0.013$$

$$\sigma_u + \sigma_d = \sigma_{\pi N} = \frac{\langle P | \hat{m}\, \bar{u}u \,| P \rangle}{2M} \qquad \sigma_s = \frac{\langle P | m_s\, \bar{s}s \,| P \rangle}{2M} \qquad \sigma_c = \frac{\langle P | m_c\, \bar{c}c \,| P \rangle}{2M}$$

Alarcon et al., Phys.Rev.D 85 (2012) 051503

Alarcon et al., Phys.Lett.B 730 (2014) 342

Hoferichter et al., Phys.Rev.Lett. 115 (2015) 092301

$\boldsymbol{\pi N}$ scattering data analysed with ChPT

$$\sigma_{\pi N}\big|_{\text{ChPT}} = (59 \pm 7)\ \text{MeV} \qquad \sigma_s\big|_{\text{ChPT}} = (16 \pm 80)\ \text{MeV}$$

Lattice QCD

Alexandrou et al., 1909.00485

$$\sigma_{\pi N}\big|_{\text{LQCD}} = (41.6 \pm 3.8)\ \text{MeV} \qquad \sigma_s\big|_{\text{LQCD}} = (39.8 \pm 5.5)\ \text{MeV}$$

$$\sigma_c\big|_{\text{LQCD}} = (107 \pm 22)\ \text{MeV}$$

Hatta sum rule

Hatta et. al., JHEP 12 (2018) 008

$$M = \bar{M}_q + \bar{M}_g = \frac{1}{\langle P|P\rangle}\int d^3x \Big(\langle\, (T_{q,R}(x))^{\mu}{}_{\mu}\,\rangle + \langle\, (T_{g,R}(x))^{\mu}{}_{\mu}\,\rangle \Big)$$

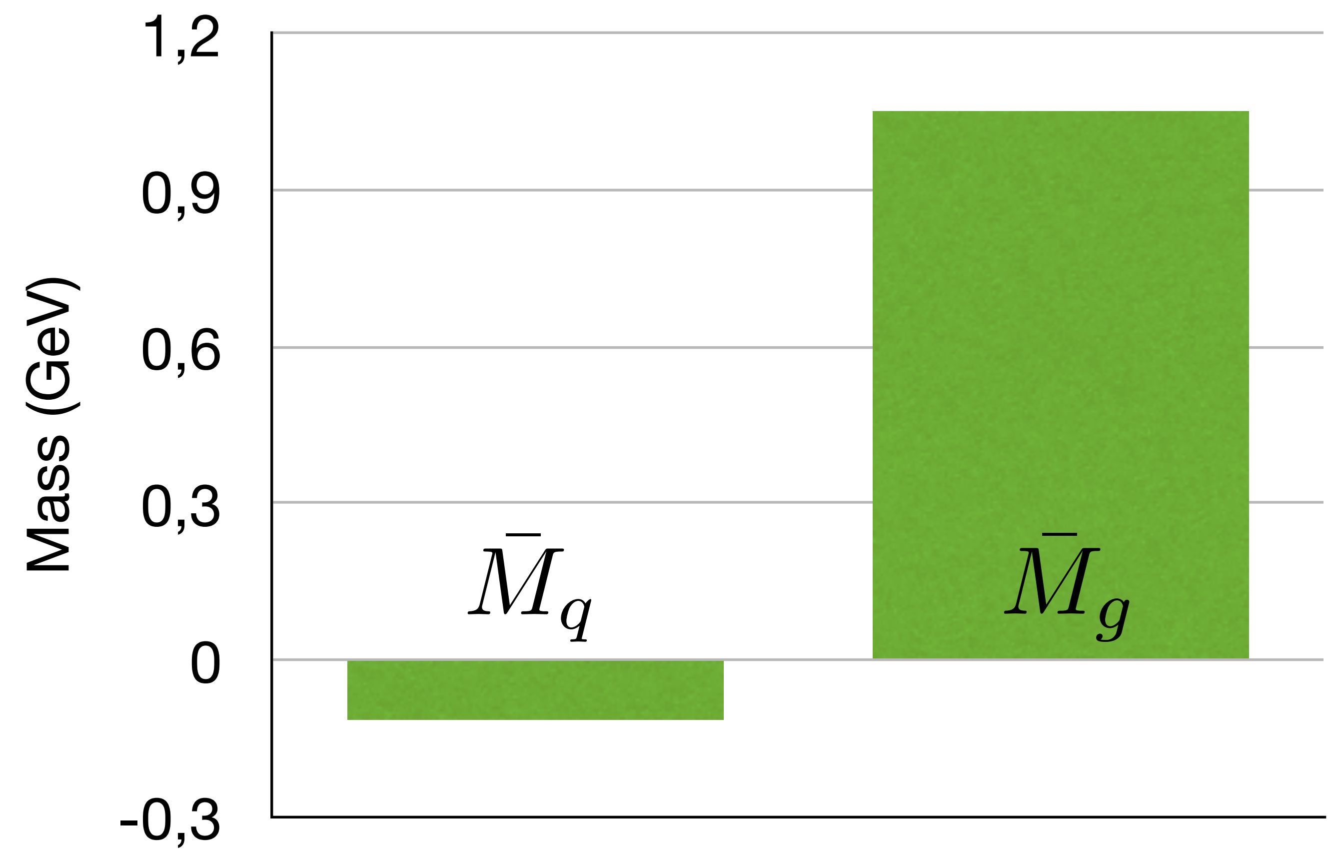
In the rest frame

$$\bar{M}_i = M\,\big(A_i(0) + 4\bar{C}_i(0)\big)$$

In a moving frame

$$\bar{M}_i = \frac{M^2}{E}\,\big(A_i(0) + 4\bar{C}_i(0)\big)$$

MS scheme



Rodini et al., JHEP 09 (2020) 067

Metz et al., Phys. Rev. D102 (2020) 114042

Lorcé two terms sum rule

Lorcé, Eur.Phys.J.C 78 (2018) 2, 120

$$U_i = M\big(A_i(0) + \bar{C}_i(0)\big) \quad H_i = \int d^3x\; T_i^{00}(x) = \int d^3x\; \mathcal{H}_i(x)$$

$$M = U_q + U_g = \frac{1}{\langle P|P\rangle}\Big(\langle\, H_{q,R}\,\rangle + \langle\, H_{g,R}\,\rangle\Big)$$

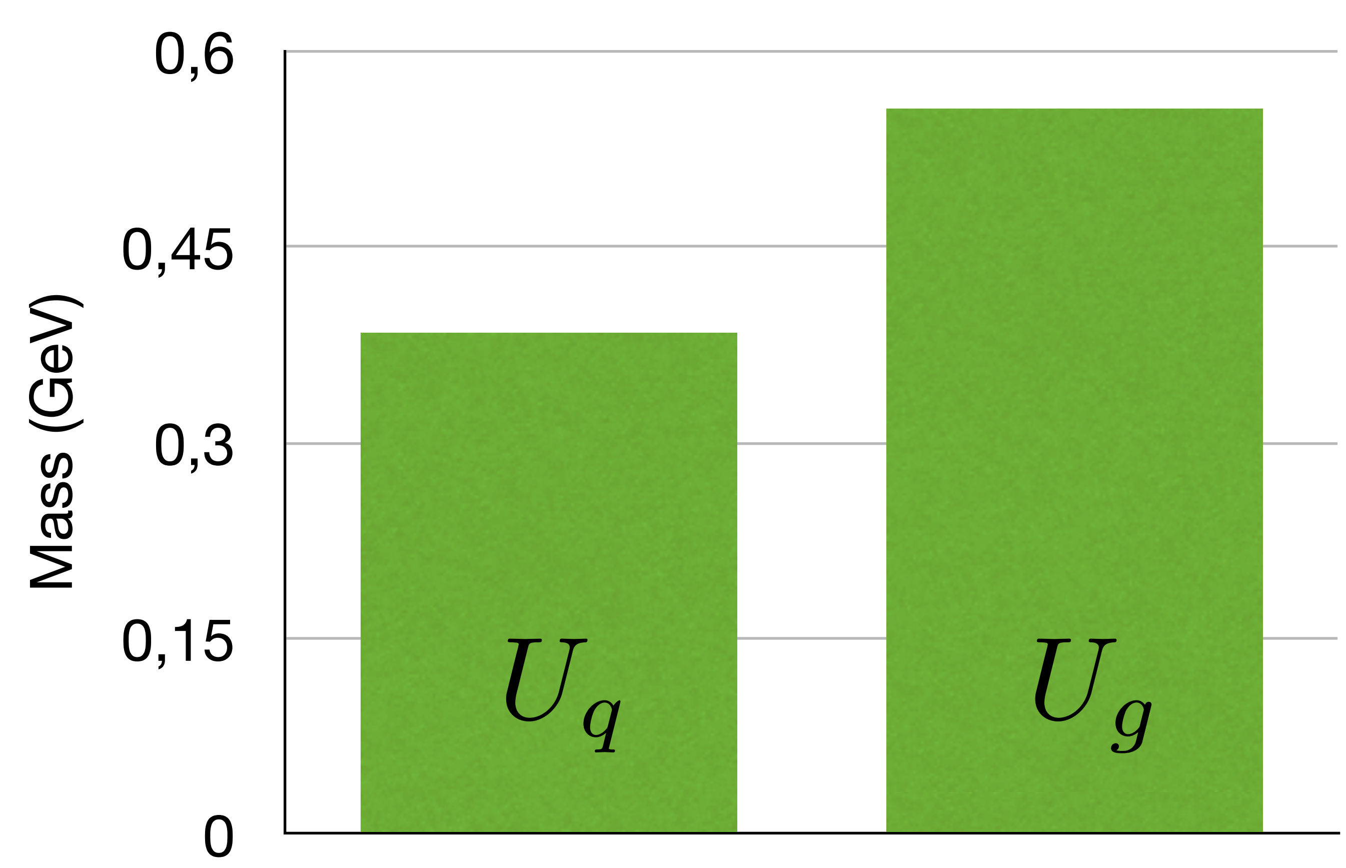
$$T^{00}_{q,R} = (m\bar{\psi}\psi)_R + (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R$$

$$T^{00}_{g,R} = \frac{1}{2}(E^2 + B^2)_R$$

$$\left\langle (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R + \frac{1}{2}(E^2+B^2)_R \right\rangle$$

$$= \left\langle \gamma_m (m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^{\alpha\beta}F_{\alpha\beta})_R \right\rangle$$

MS scheme and Scenario A



Rodini et al., JHEP 09 (2020) 067

Metz et al., Phys. Rev. D102 (2020) 114042

Ji sum rule

$$T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu} \qquad \hat{T}^{\mu\nu} = \frac{g^{\mu\nu}}{4} T^{\alpha}_{\alpha}$$

$$\mathcal{H}'_q = \bar{T}^{00}_{q,R} = (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R + (m\bar{\psi}\psi)_R - \frac{1+y}{4}(m\bar{\psi}\psi)_R - \frac{x}{4}(F^{\alpha\beta}F_{\alpha\beta})_R$$

$$\mathcal{H}'_m = \hat{T}^{00}_{q,R} = \frac{1+y}{4}(m\bar{\psi}\psi)_R + \frac{x}{4}(F^{\alpha\beta}F_{\alpha\beta})_R$$

$$\mathcal{H}'_g = \bar{T}^{00}_{g,R} = \frac{1}{2}(E^2+B^2)_R + \frac{y-\gamma_m}{4}(m\bar{\psi}\psi)_R - \frac{1}{4}\left(\frac{\beta}{2g} - x\right)(F^{\alpha\beta}F_{\alpha\beta})_R$$

$$\mathcal{H}'_a = \hat{T}^{00}_{g,R} = \frac{\gamma_m - y}{4}(m\bar{\psi}\psi)_R + \frac{1}{4}\left(\frac{\beta}{2g} - x\right)(F^{\alpha\beta}F_{\alpha\beta})_R$$

Ji, Phys.Rev. D52 (1995) 271

Ji, Phys.Rev.Lett. 74 (1995) 1071 | Ji, Liu, 2101.04483 | Ji, 2102.07830

Metz et al., Phys. Rev. D102 (2020) 114042 | Rodini et al., JHEP 09 (2020) 067 | Ji et al, Phys.Rev.D 103 (2021) 074002

Reshuffle of the terms to obtain simple-looking operators

$$M = M_q + M_m + M_g + M_a = \frac{\langle H_q \rangle}{\langle P|P \rangle} + \frac{\langle H_m \rangle}{\langle P|P \rangle} + \frac{\langle H_g \rangle}{\langle P|P \rangle} + \frac{\langle H_a \rangle}{\langle P|P \rangle}$$

$$\mathcal{H}_q = (\psi^\dagger \, i\boldsymbol{D} \cdot \boldsymbol{\alpha} \, \psi)_R$$

$$\mathcal{H}_m = (m\bar{\psi}\psi)_R$$

$$\mathcal{H}_g = \frac{1}{2}(E^2 + B^2)_R$$

$$\mathcal{H}_a = 0$$

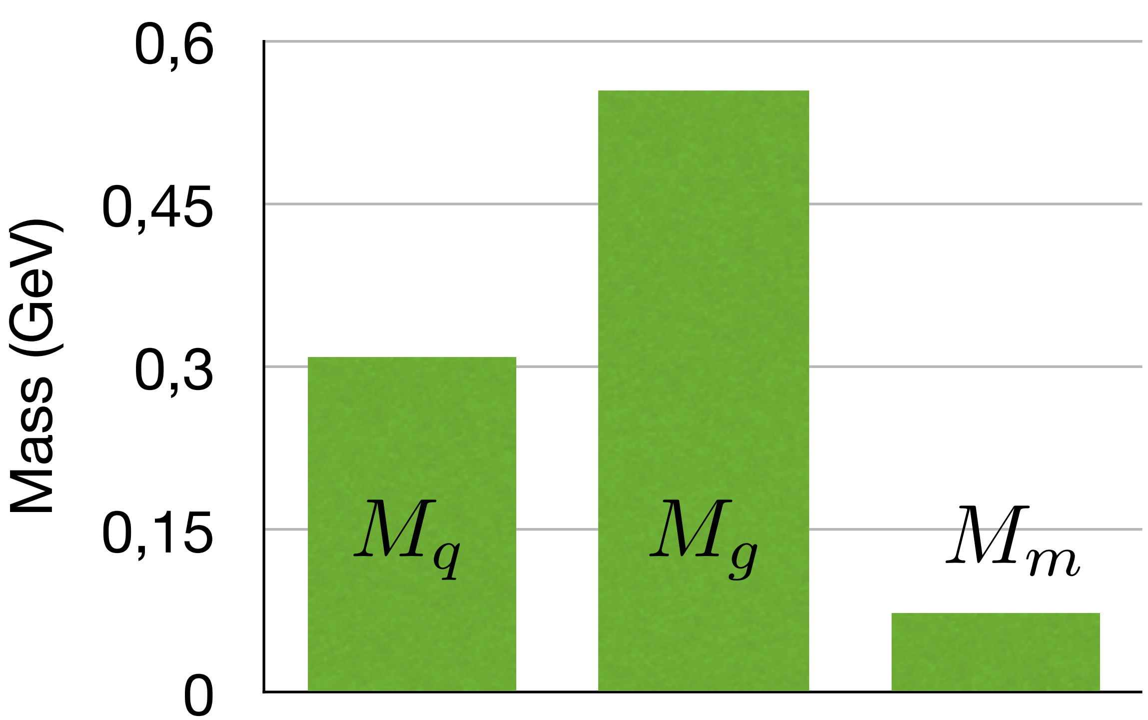
$$(\mathcal{H}_q)_{\mathrm{Ji}} = \left(\psi^\dagger i\boldsymbol{D} \cdot \boldsymbol{\alpha}\psi\right)_R$$

$$(\mathcal{H}_m)_{\mathrm{Ji}} = \left(1 + \frac{\gamma_m}{4}\right)\left(m\bar{\psi}\psi\right)_R$$

$$(\mathcal{H}_g)_{\mathrm{Ji}} = \frac{1}{2}\left(E^2 + B^2\right)_R$$

$$(\mathcal{H}_a)_{\mathrm{Ji}} = \frac{1}{4}\frac{\beta}{2g}\left(F^2\right)_R$$

MS scheme



Mismatch due to difference in computation of the traceless part $\bar{T}^{00}$

Or to some difference in meaning of (some) operators

Ji, 2102.07830 | Ji, Liu, 2101.04483 | Ji et al, Phys.Rev.D 103 (2021) 074002

How strong the scheme dependence is?

Metz et al., Phys. Rev. D102 (2020) 114042

		MS	$\overline{\mathrm{MS}}_1$	$\overline{\mathrm{MS}}_2$	D1	D2
	M_q	0.309 ± 0.044	0.194 ± 0.033	0.178 ± 0.032	0.362 ± 0.045	0.357 ± 0.051
Scenario A	M_m	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080
	M_g	0.555 ± 0.036	0.669 ± 0.047	0.686 ± 0.048	0.502 ± 0.035	0.507 ± 0.029
	M_q	0.234 ± 0.006	0.135 ± 0.003	0.120 ± 0.003	0.286 ± 0.006	0.272 ± 0.008
Scenario B	M_m	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023
	M_g	0.517 ± 0.017	0.617 ± 0.020	0.631 ± 0.020	0.465 ± 0.017	0.479 ± 0.015

Conclusions

Renormalisation of multiple local operators

Different decompositions

Summary

Form factors as fundamental building blocks
for all the decompositions

The specific terms in the mass sum rules are scheme-dependent

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