Detailed analysis of excited state systematics in a lattice QCD calculation of g_A

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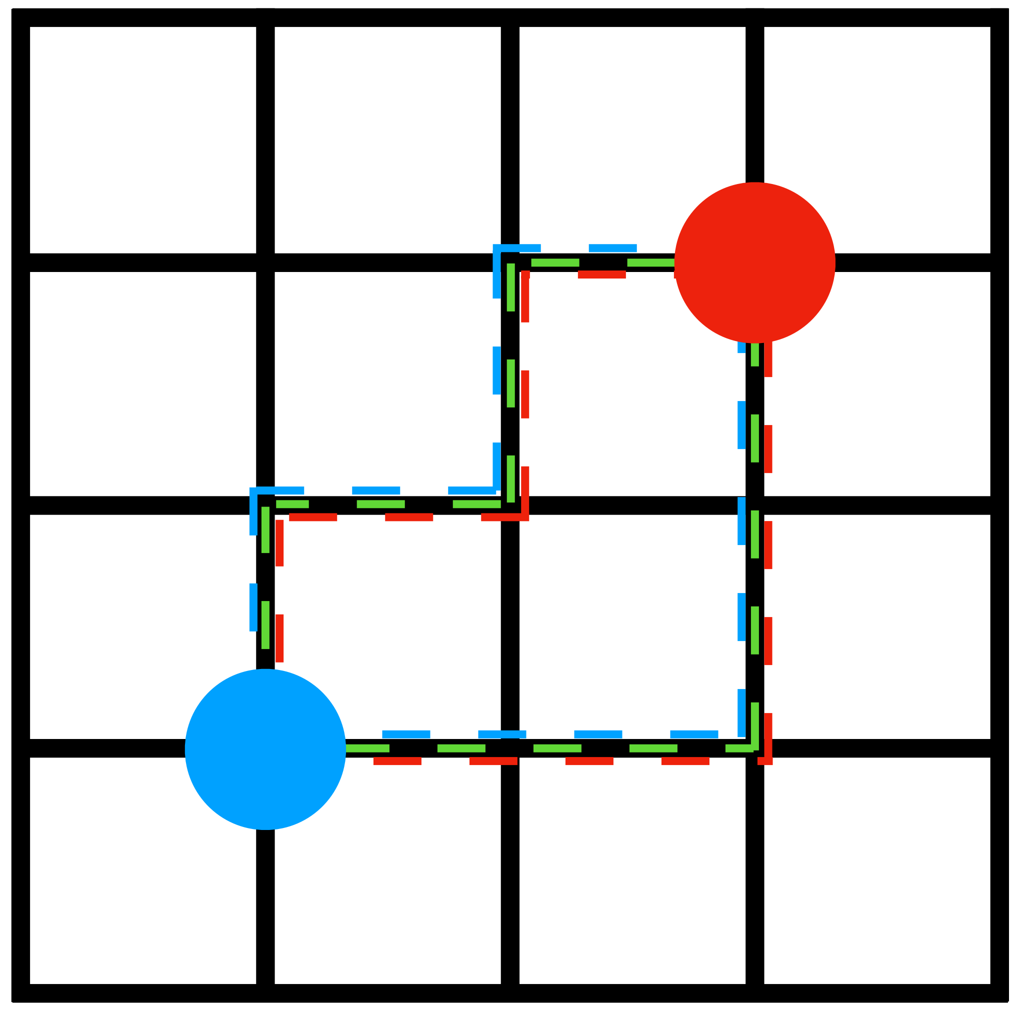
Lattice Quantum Chromodynamics

Discretize QCD in spacetime and put in finite box
    Non-perturbatively defines both UV and IR cutoff

Observables (mass, form factors) calculated from path integral
    Euclidean spacetime can be evaluated with Monte Carlo

Hadronic creation operators are unknown
    Best guesses overlap with a **tower** of hadronic states

This talk is about understanding this tower of states
in the context of the nucleon

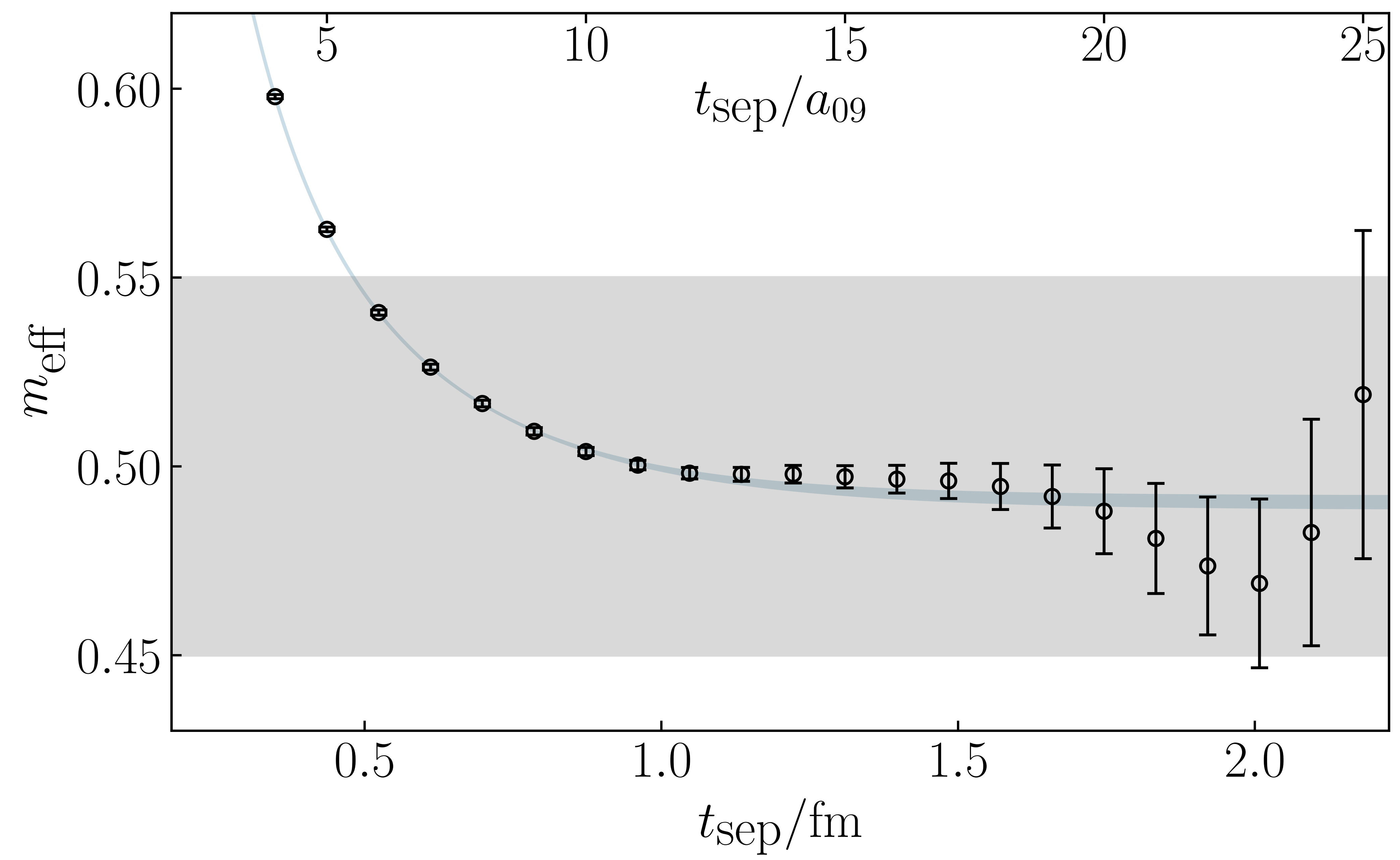


Challenges of Baryon Structure for LQCD

Signal-to-Noise ratio as a function of **time**

Signal $\langle N\overline{N}\rangle = \sum_{\text{states}} |\langle N|i\rangle|^2 \, e^{E_i t} \simeq e^{-M_N t}$

Noise $\sigma(\langle N\overline{N}\rangle) = \sqrt{\langle |N\overline{N}|^2\rangle - |\langle N\overline{N}\rangle|^2}$
$\simeq e^{-3M_\pi t/2}$



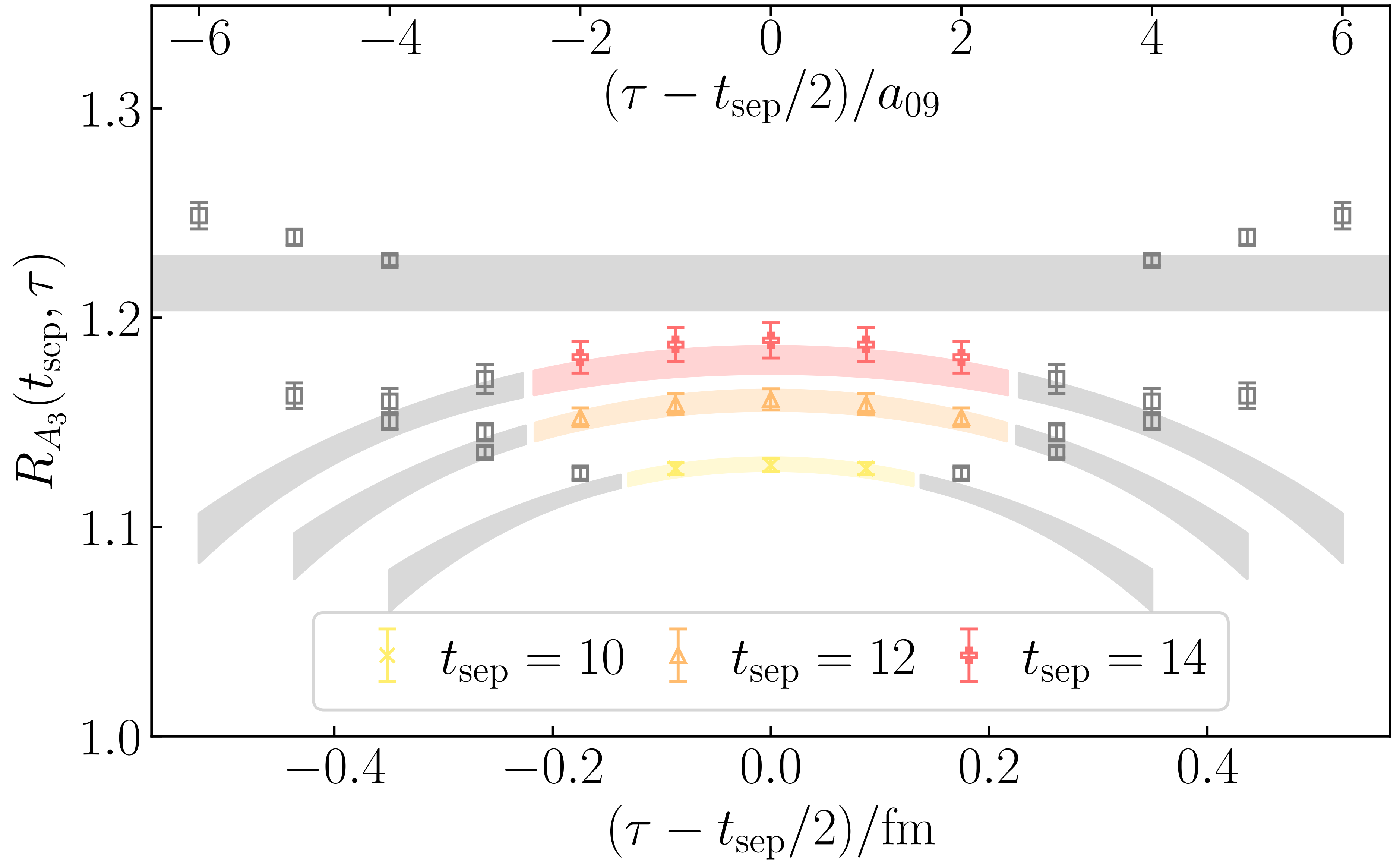
Signal decays with GeV scale
Noise decays with MeV scale

This is further exacerbated for excited states

Possible strategies around this issue

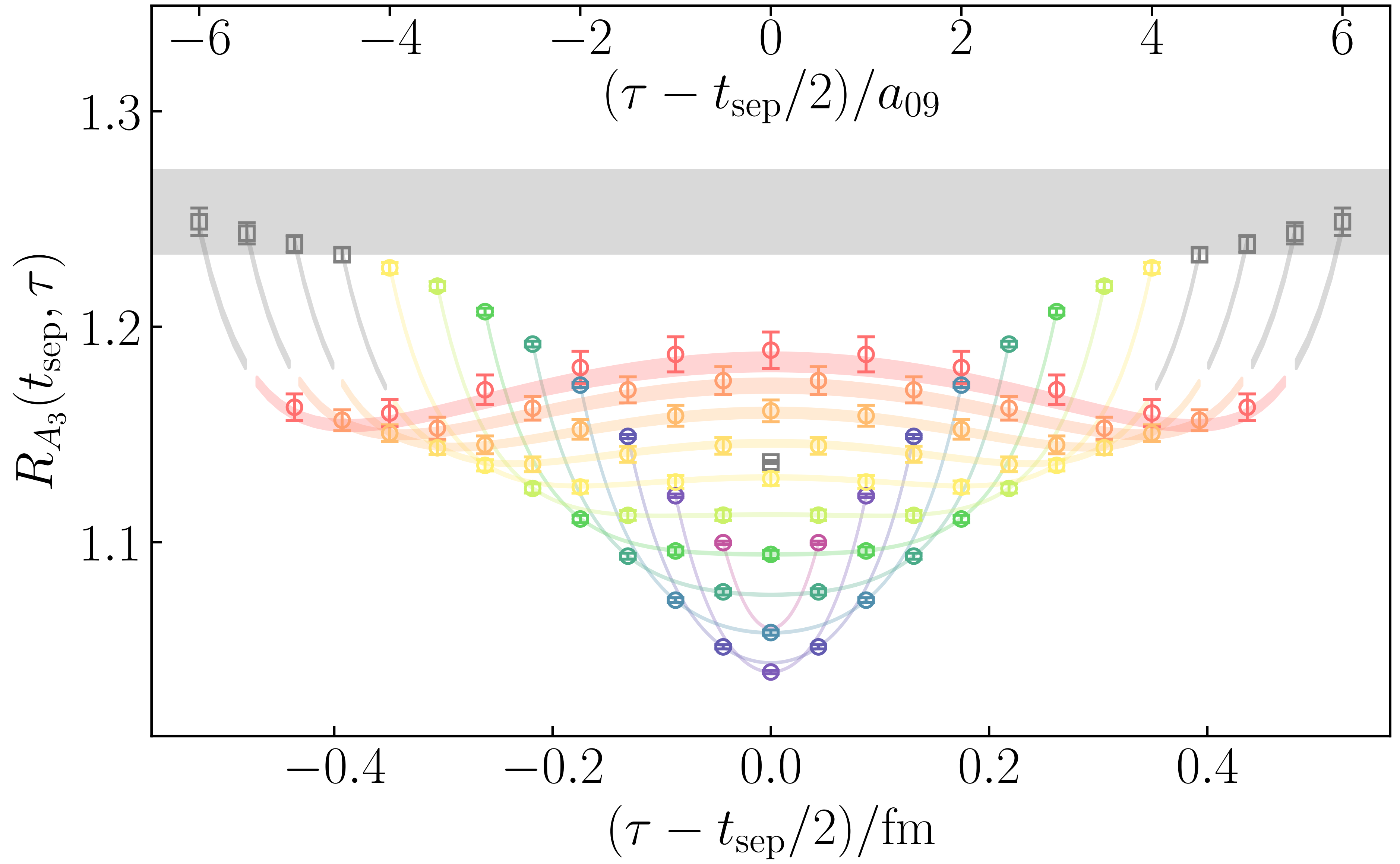
Use large Euclidean time as a filter

How large is enough?
Requires exponential amount of MC samples.
    What happens when there is not quite enough?



Disentangle excited-states at early time

Much more complicated!
How to do this reliably?

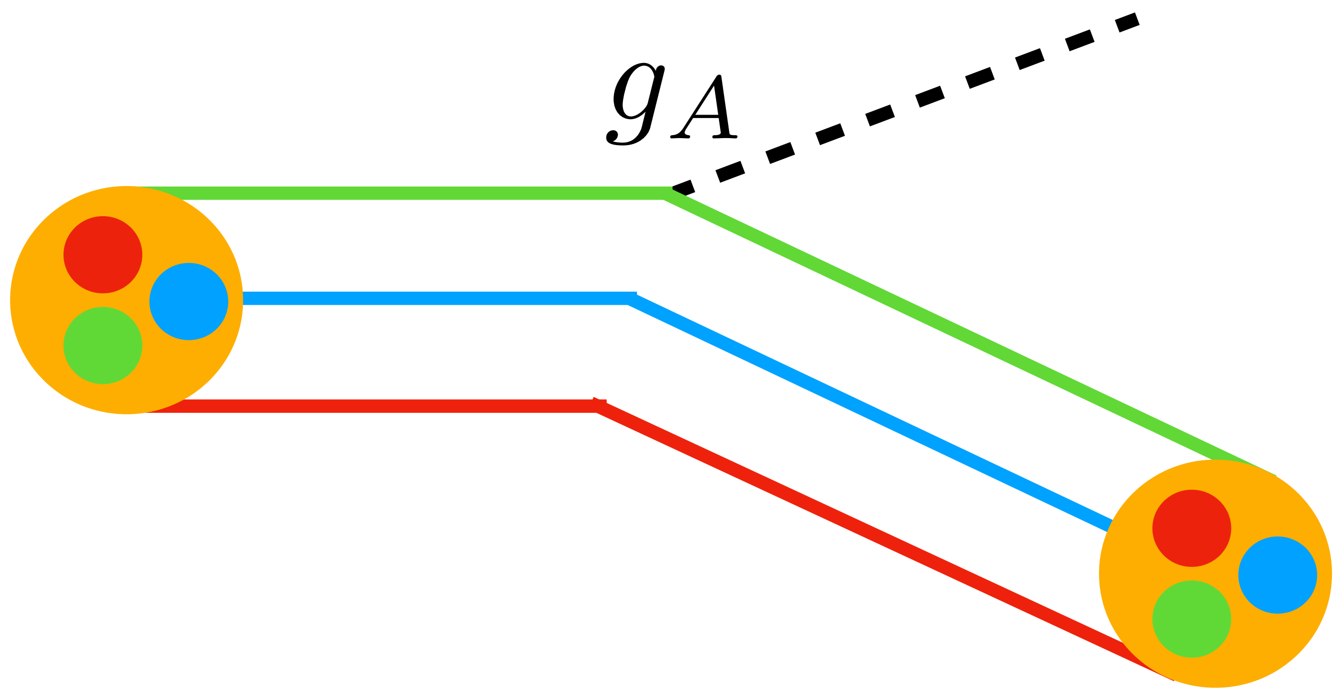


This talk focuses on latter for the axial-vector correlation function

Data: Post-processed correlation functions

Correlation function from Path integral

$$C_A(t,\tau) = \sum_n |z_n|^2 g^A_{nn} e^{-E_n t} + 2\sum_{n<m} z_n z_m^\dagger g^A_{nm} e^{-(E_n+\Delta_{mn}/2)t}$$



Ratio correlation function

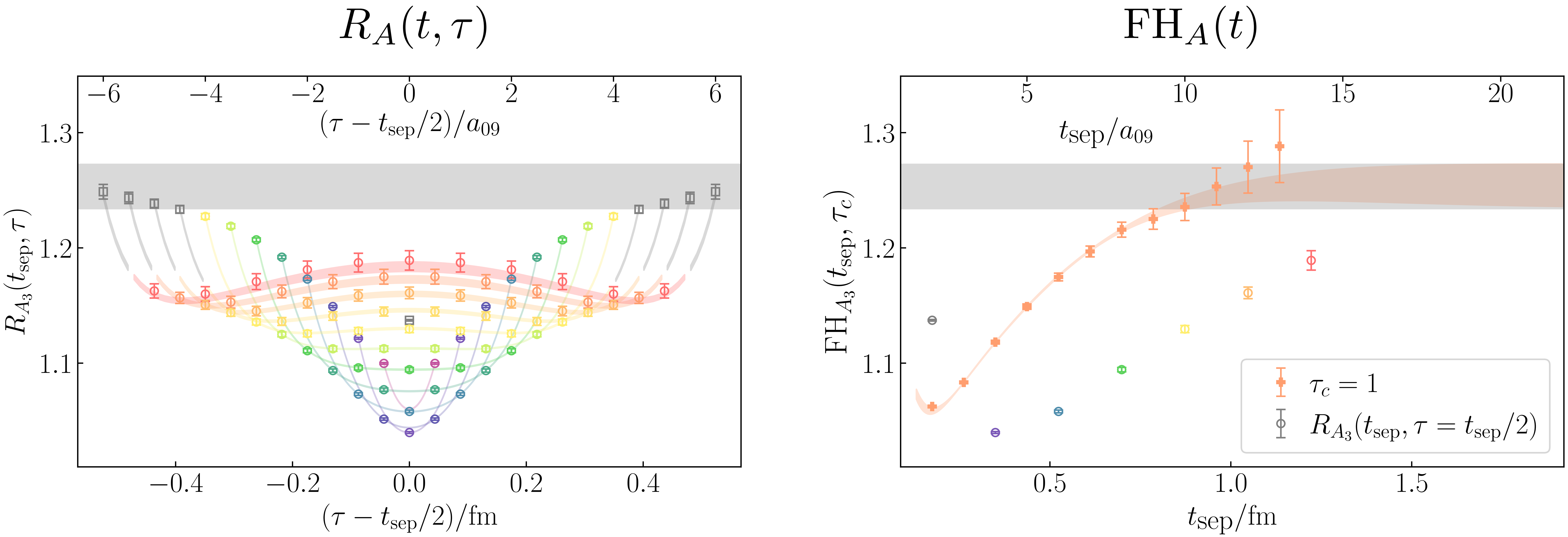
$$R_A(t,\tau) = C_A(t,\tau)/C_2(t) \to \lim_{t\to\infty} R_A(t,\tau \simeq t/2) = g^A_{00}$$

Feynman-Hellmann correlation function

$$\mathrm{FH}_A(t) = \left.\frac{\partial E^{\mathrm{eff}}_\lambda}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{dt}\left[\frac{\sum_\tau C_A(t+dt,\tau)}{C_2(t+dt)} - \frac{\sum_\tau C_A(t,\tau)}{C_2(t)}\right]$$

Let's compare these

Data: Post-processed correlation functions



1) The FH correlator goes to the infinite-time limit much sooner

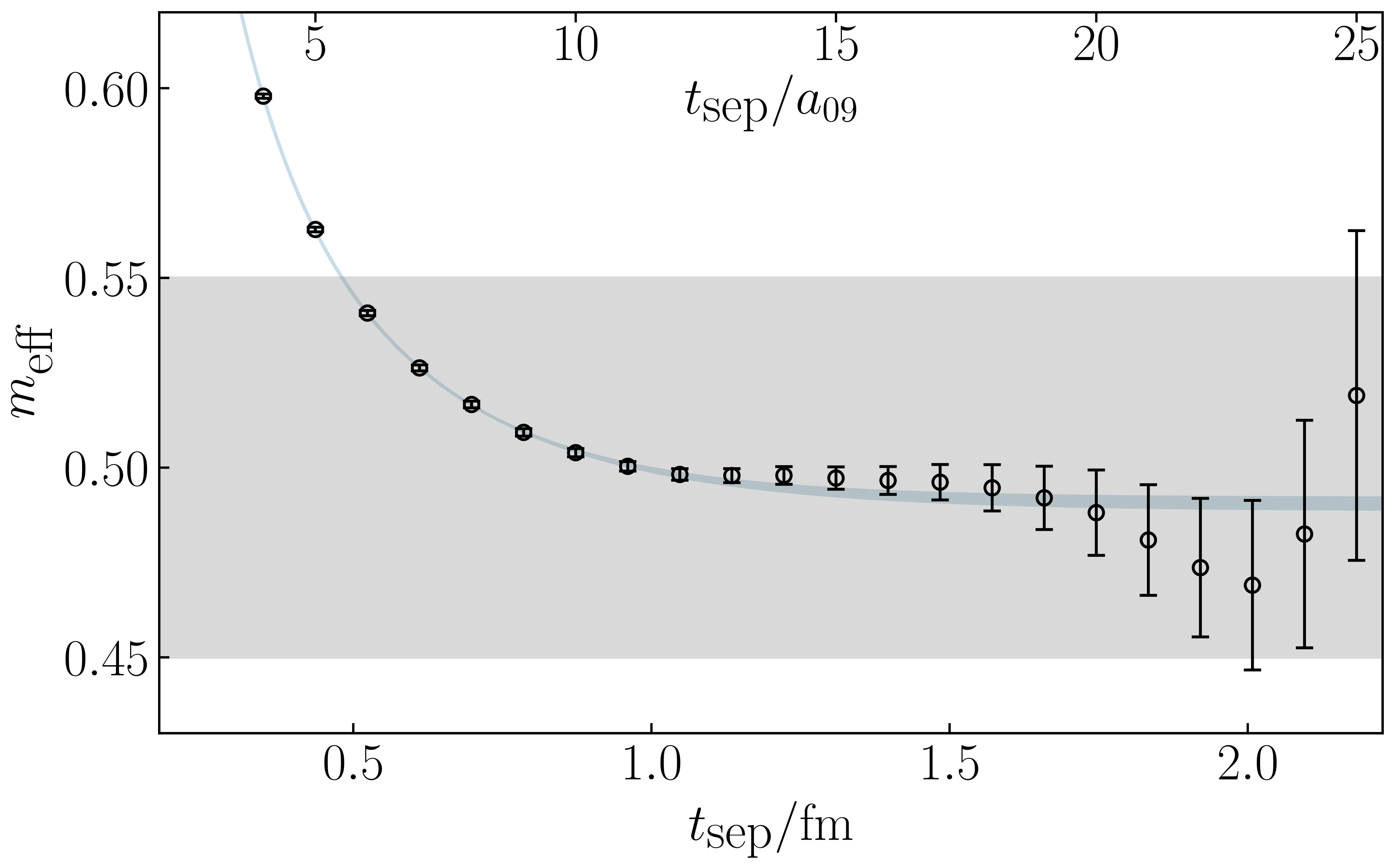
Why should you believe our infinite-time extrapolation? I show this first…
and if you accept it then… How much sooner and why?

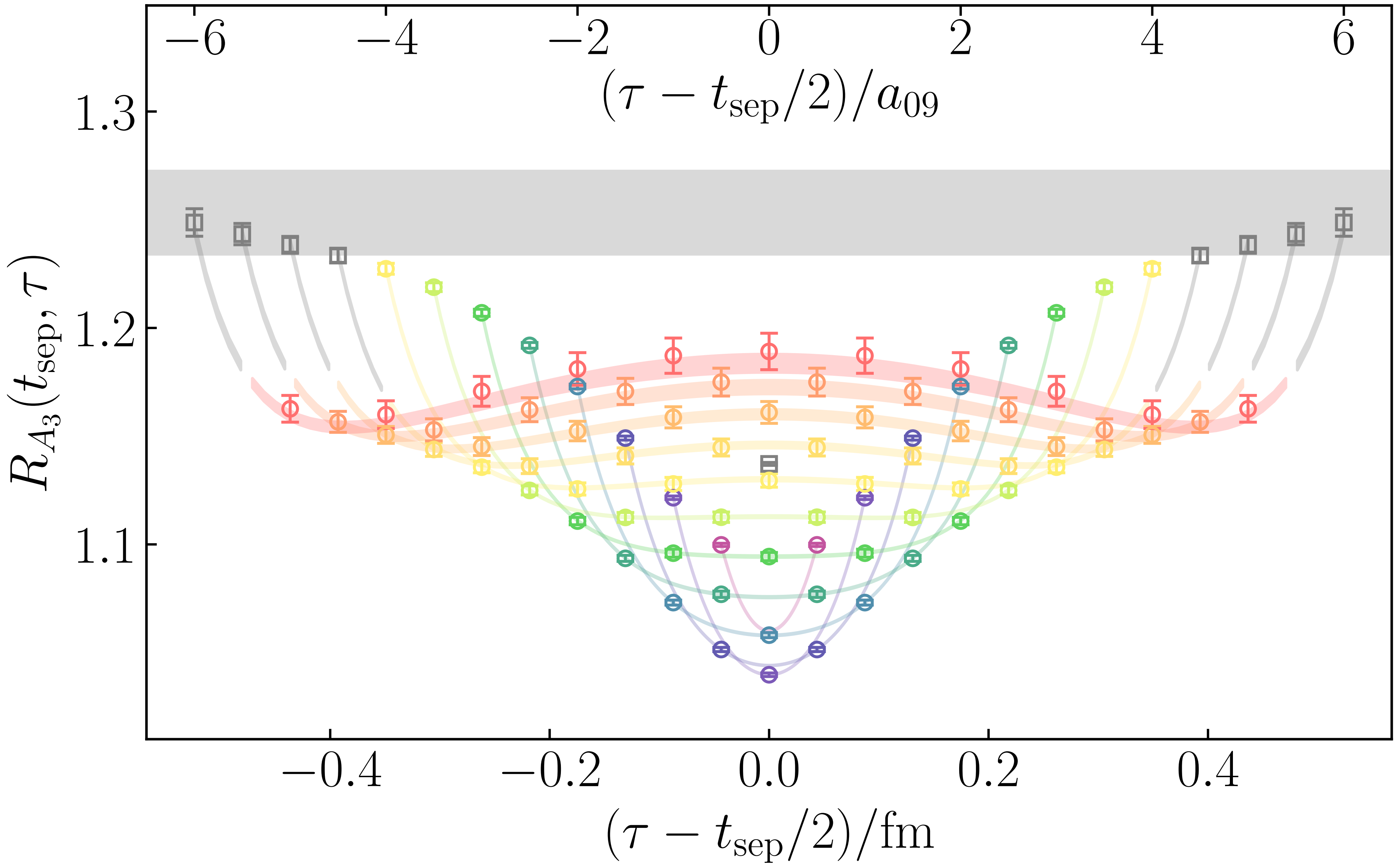
Analysis strategies

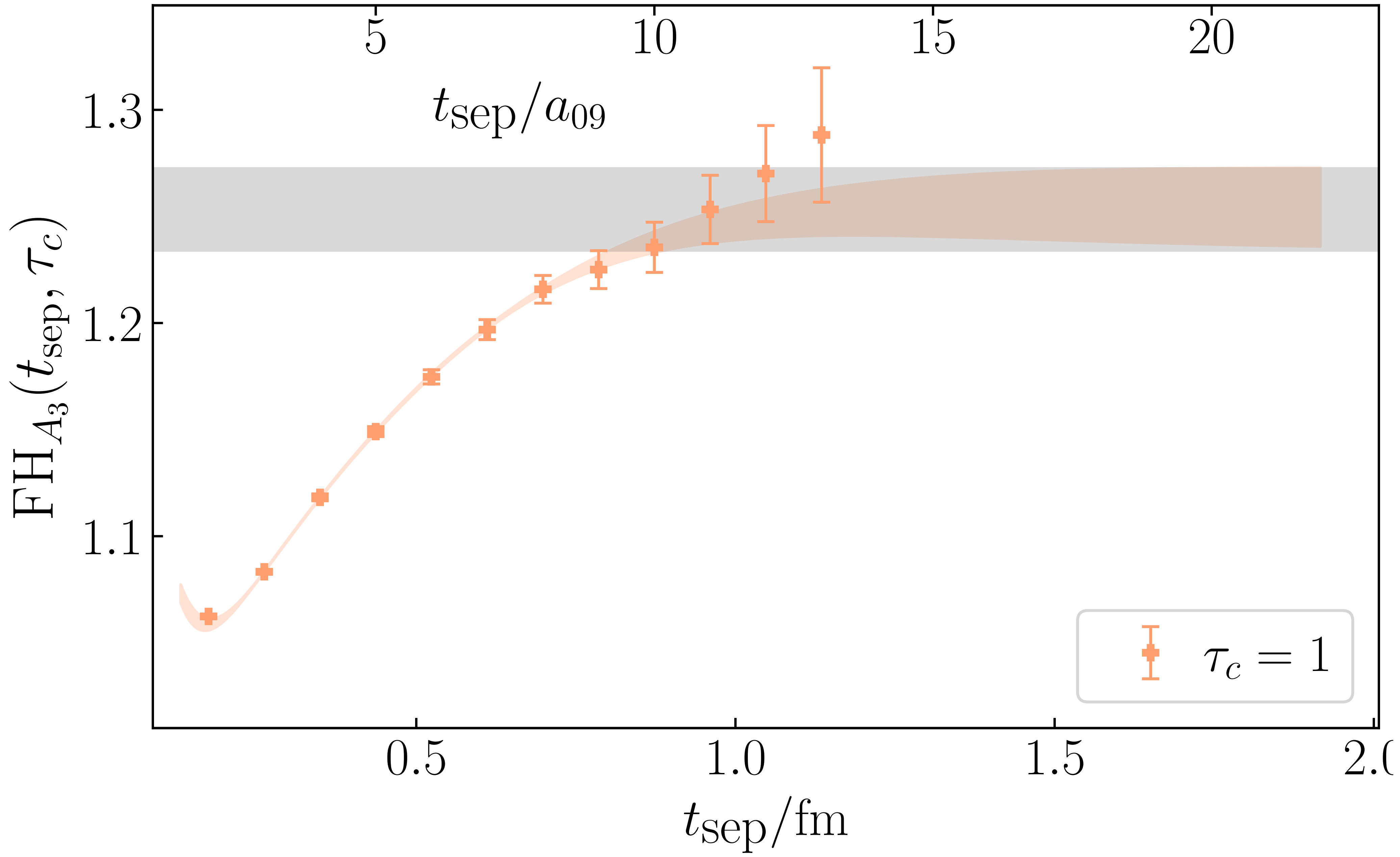
2-point correlator

3-point correlator at t=[2,14]
makes $R_A(t,\tau)$ and $\mathrm{FH}_A(t)$

We show 3 ways to fit this dataset







Sensitivity analysis

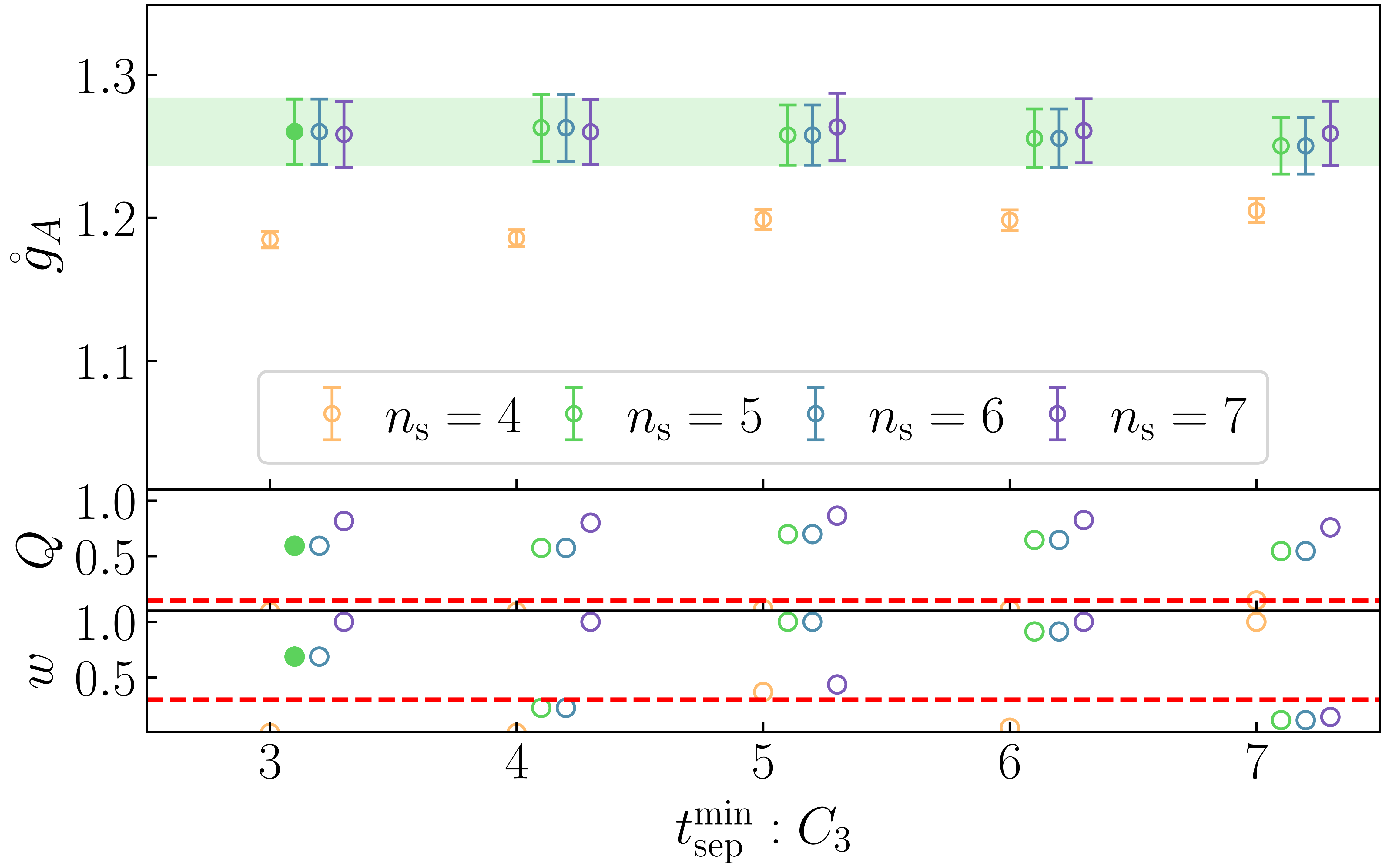
We are fitting a physics model to data

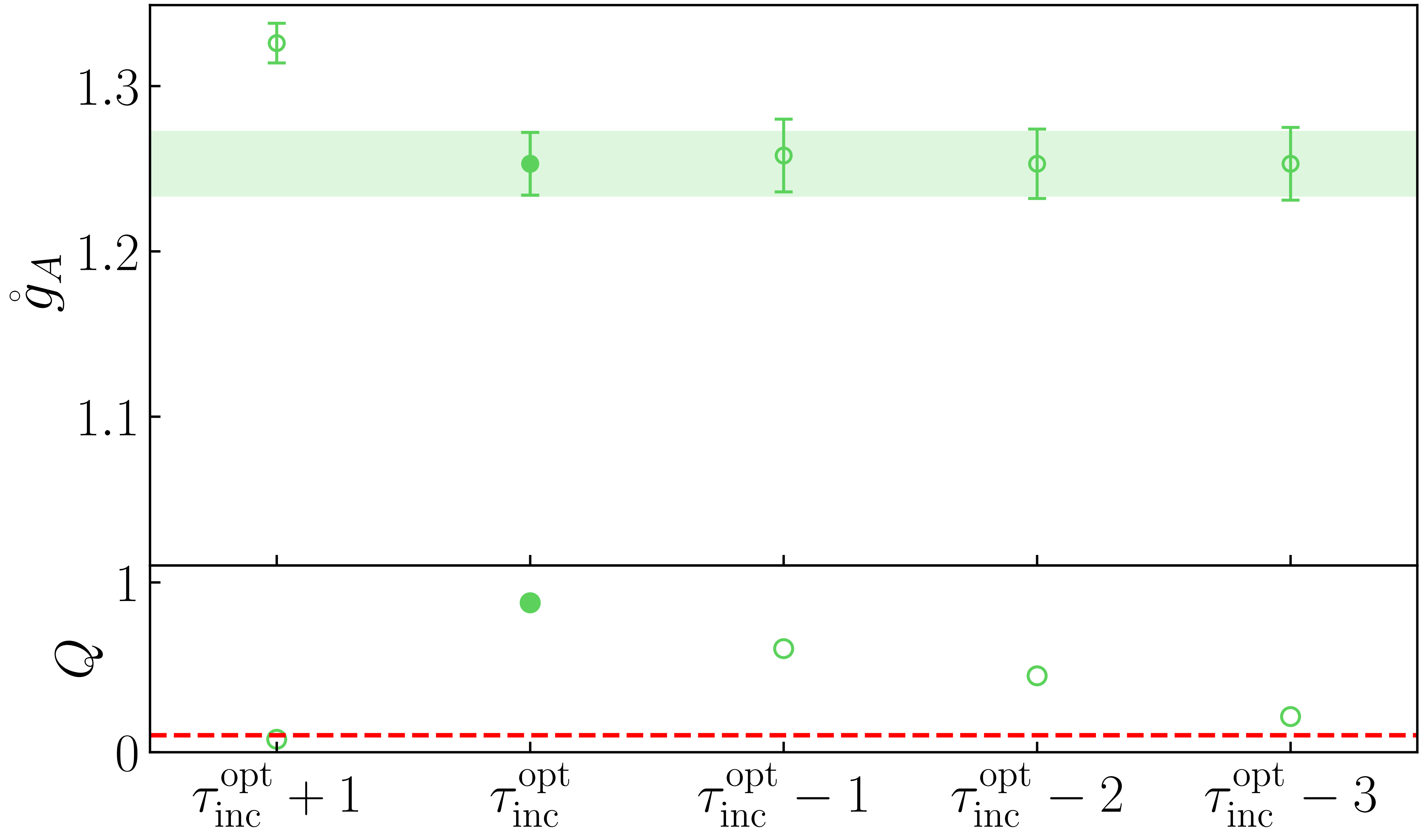
Data specifications for this work
2-point correlator extracts overlap factor (z_n) and energy (E_n)
~20 data points to constrain 2 parameters per state

3-point correlator extracts matrix elements (g_{nm})
13 data points to constrain n=m, 1 parameter per state
~50 data points to constraint n!=m, n(n+1)/2 parameters per state

Things to worry about
Underfitting and overfitting
Sensitivity of posterior distribution to various hyper parameters
    number of states, fit region, spectrum model

Sensitivity analysis : fit region and states

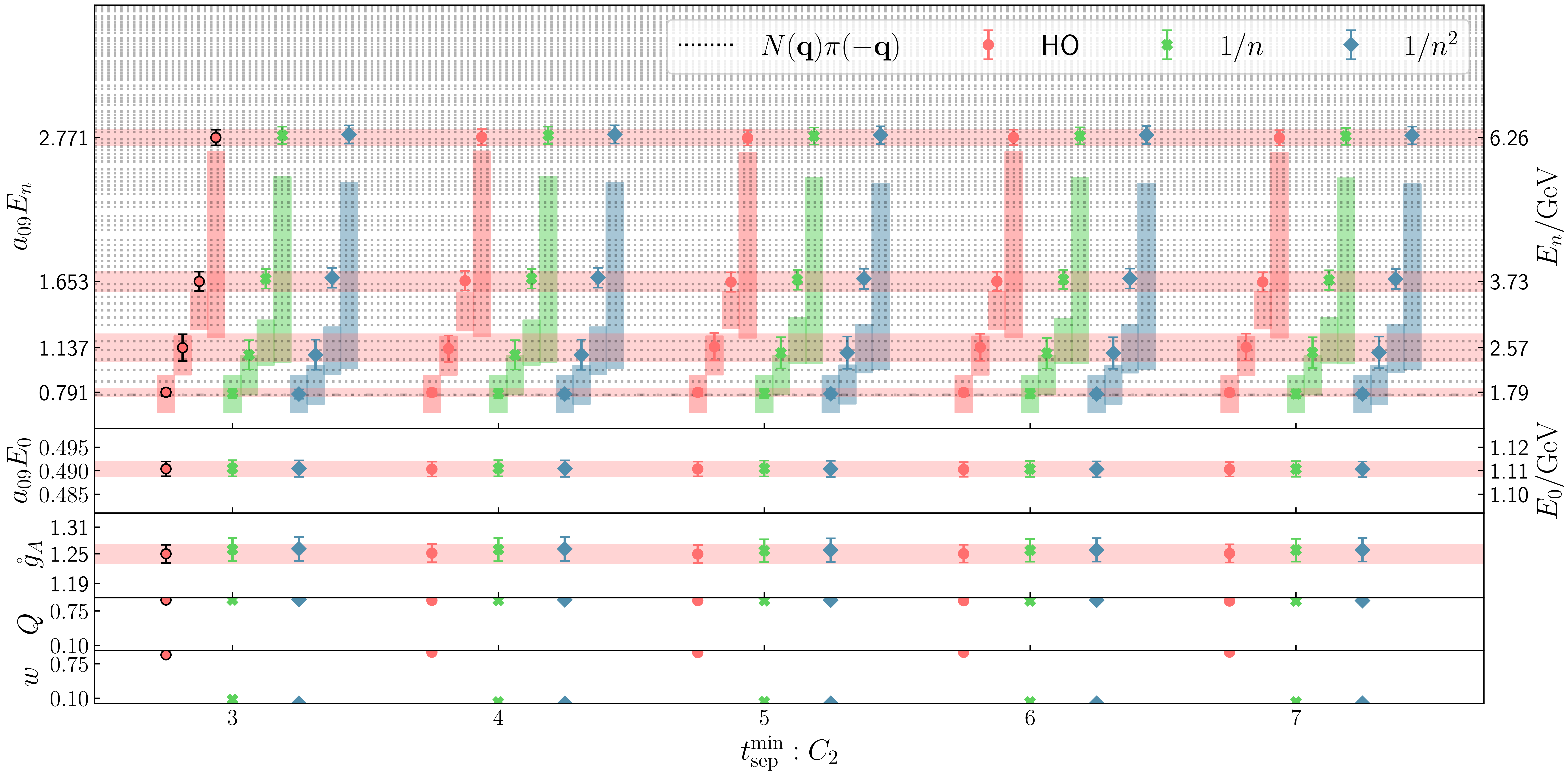




Posterior distribution is insensitive to choice… so choice does not matter

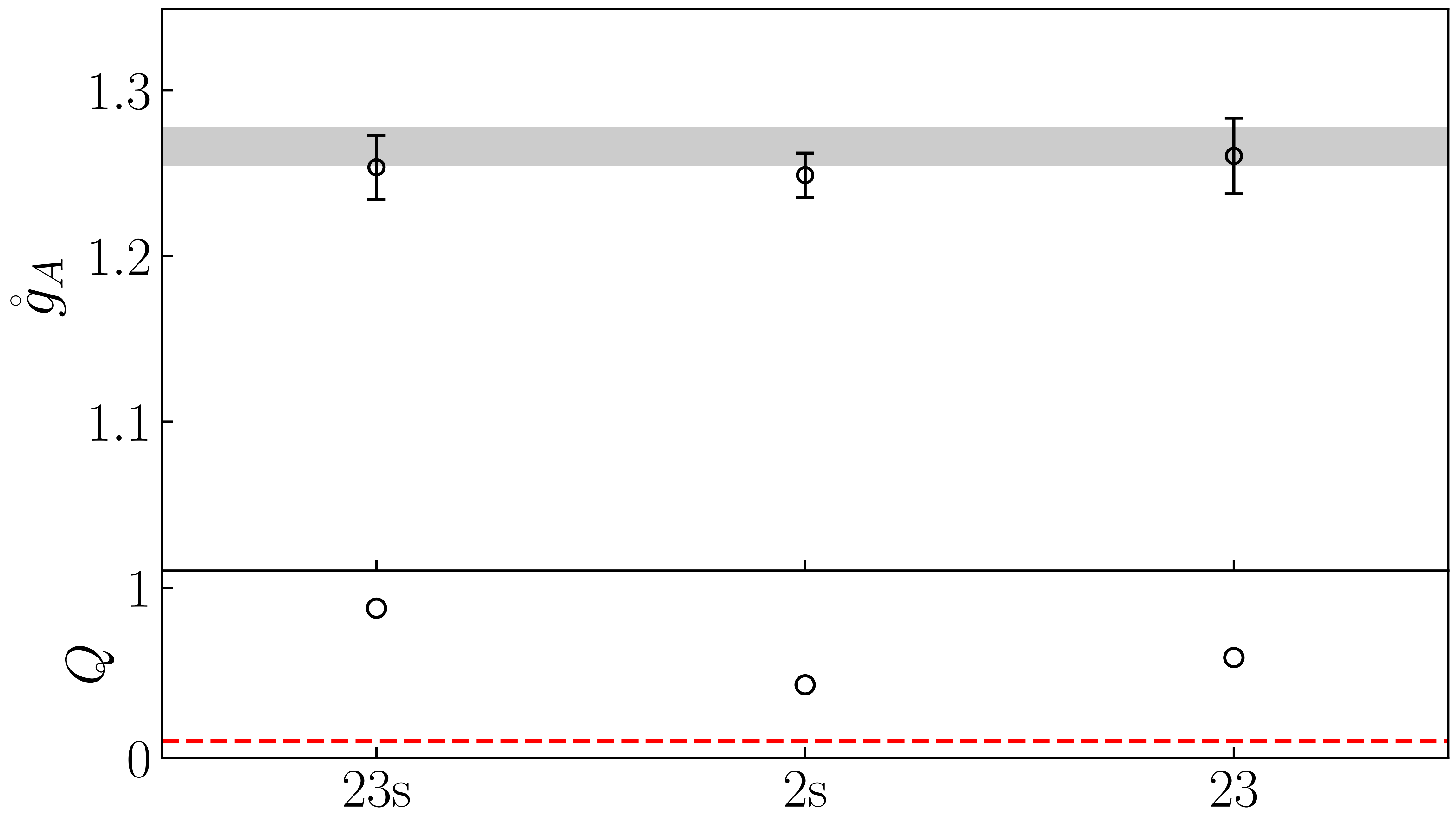
13 source-sink separation times allow for proper study
- enough range to vary *t* and *τ*
- enough data to over-constrain a 7 state fit

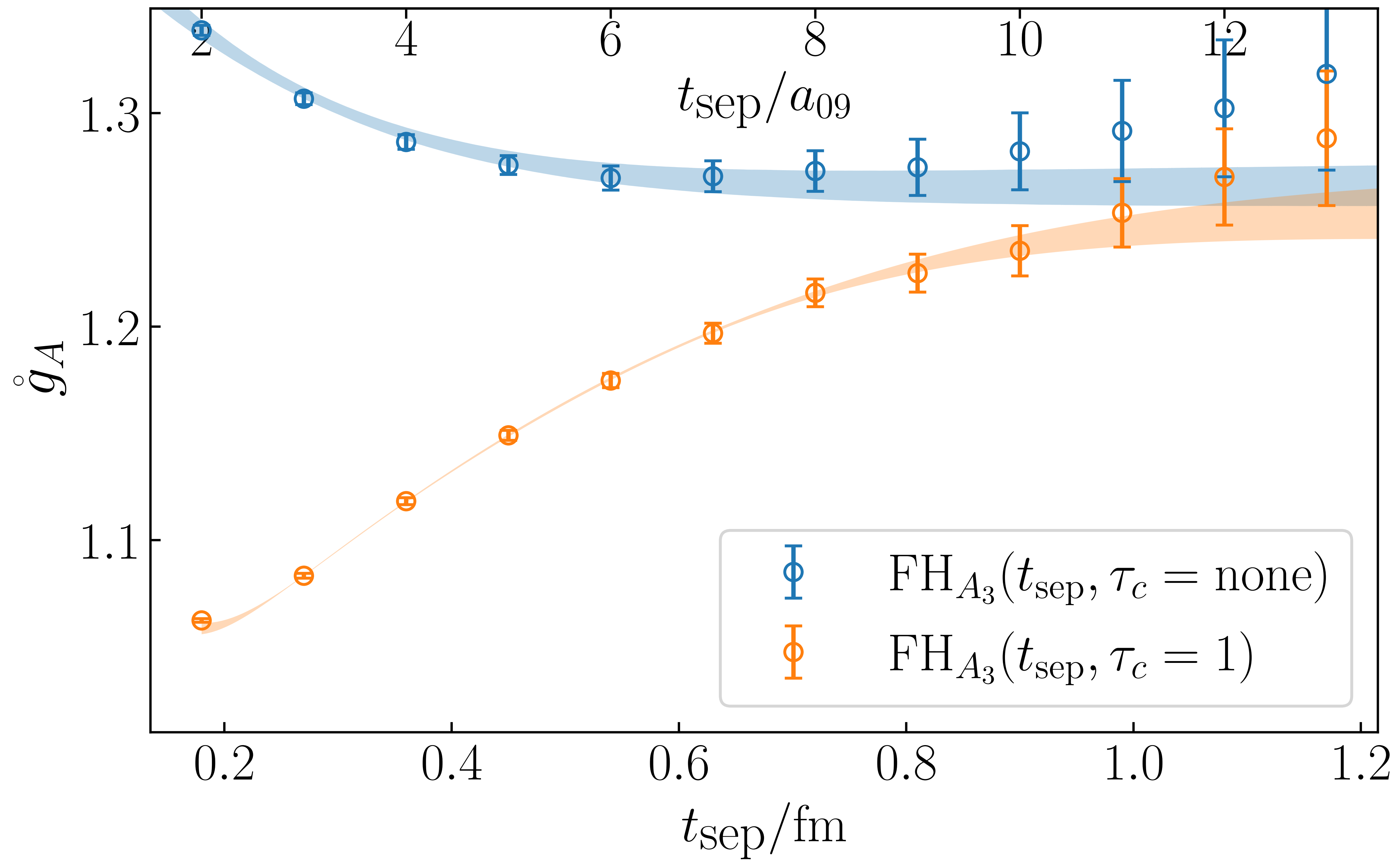
Sensitivity analysis : excited state spectrum



The first 5 states inferred from data (because we fit all the curvature in 3pt fcn)

Best fit consistency



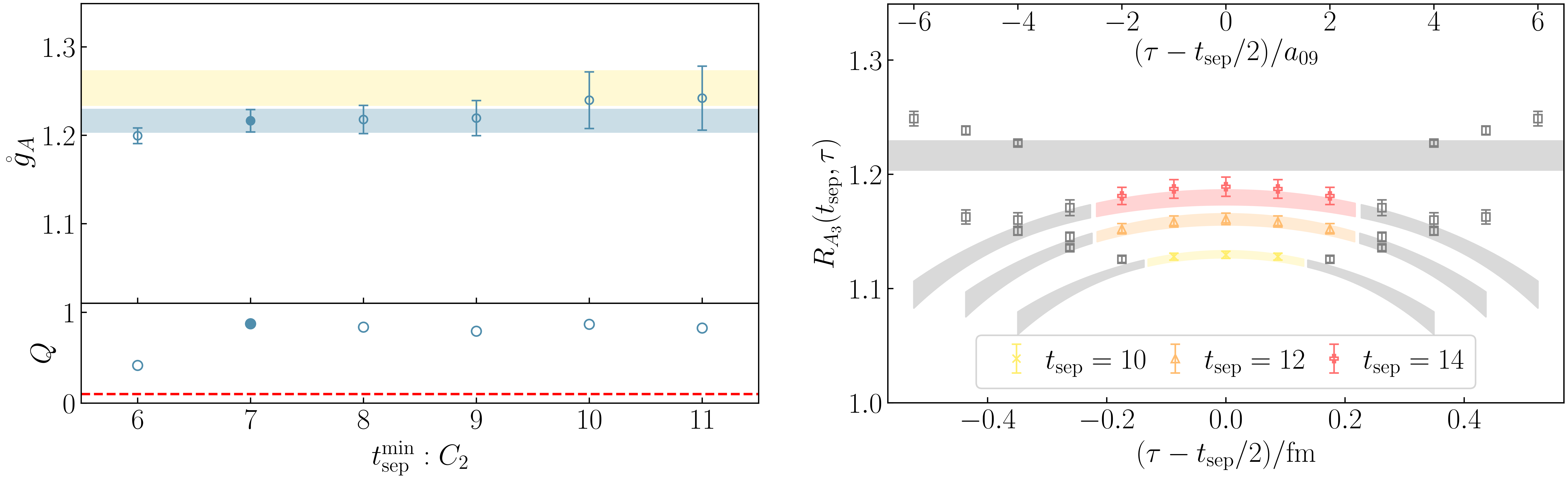


The 3 analysis strategies are consistent within 1 standard deviation

Results consistent with publication [Nature 558, 91-94 (2018)]

2) Control systematics by showing no sensitivity to all hyperparameters

Comparison with late time analysis



Repeat sensitivity analysis on 3-point fit at t=[10, 12, 14]

The best fit late time result is 2 standard deviations in tension

3) Late time data is under sampled (S/N issue) so uncertainty is unreliable

Excited-state contributions

2-point function

$$C_2(t) = C_2^{\mathrm{gs}}(t) + C_2^{\mathrm{es}}(t)$$

$$C_2^{\mathrm{gs}}(t) = |z_0|^2 e^{-E_0 t}$$
$$C_2^{\mathrm{es}} = \sum_{n\geq 1} |z_n|^2 e^{-E_n t}$$

3-point function

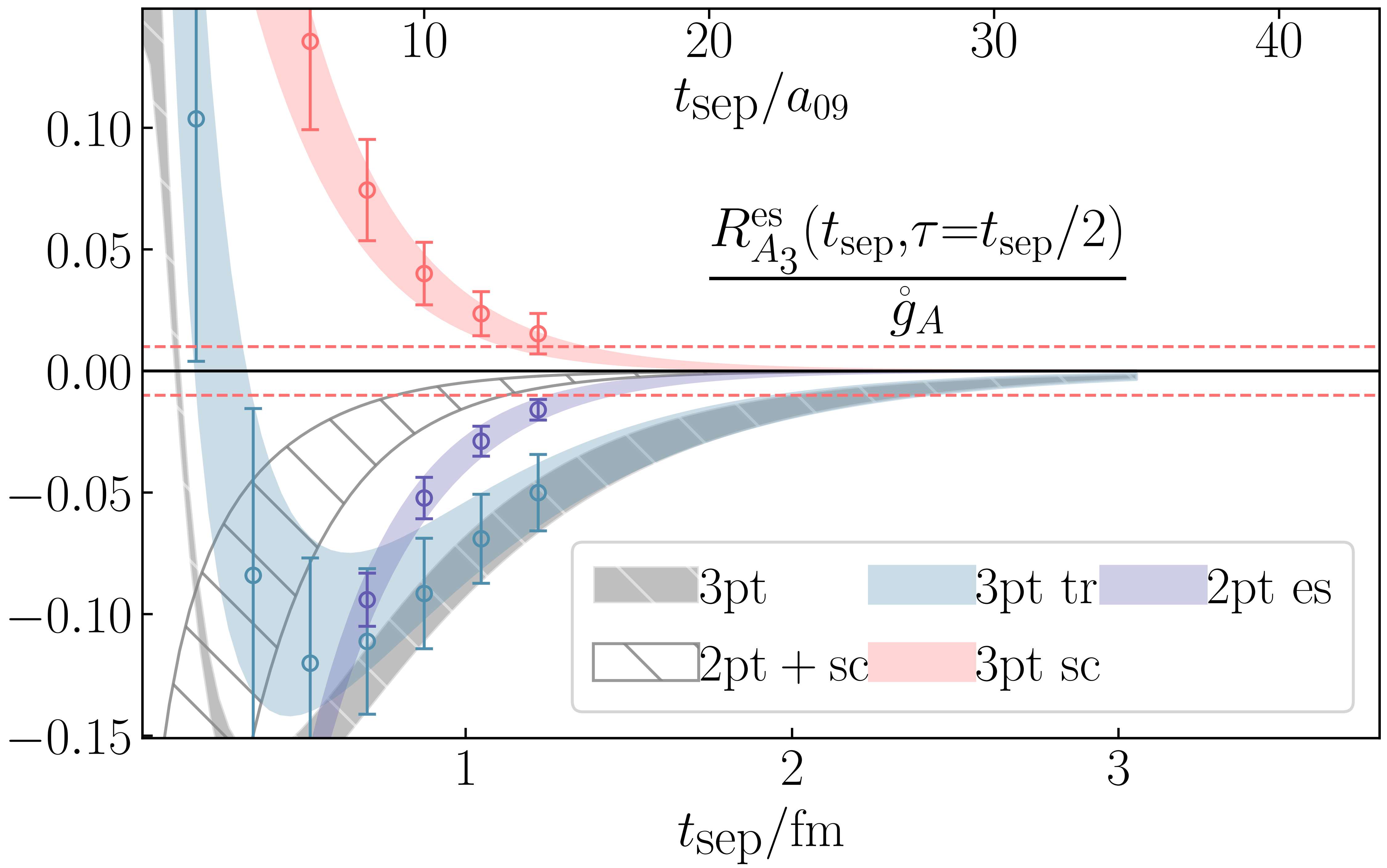
$$C_A(t,\tau) = C_A^{\mathrm{gs}}(t) + C_A^{\mathrm{sc}}(t) + C_A^{\mathrm{tr}}(t,\tau)$$

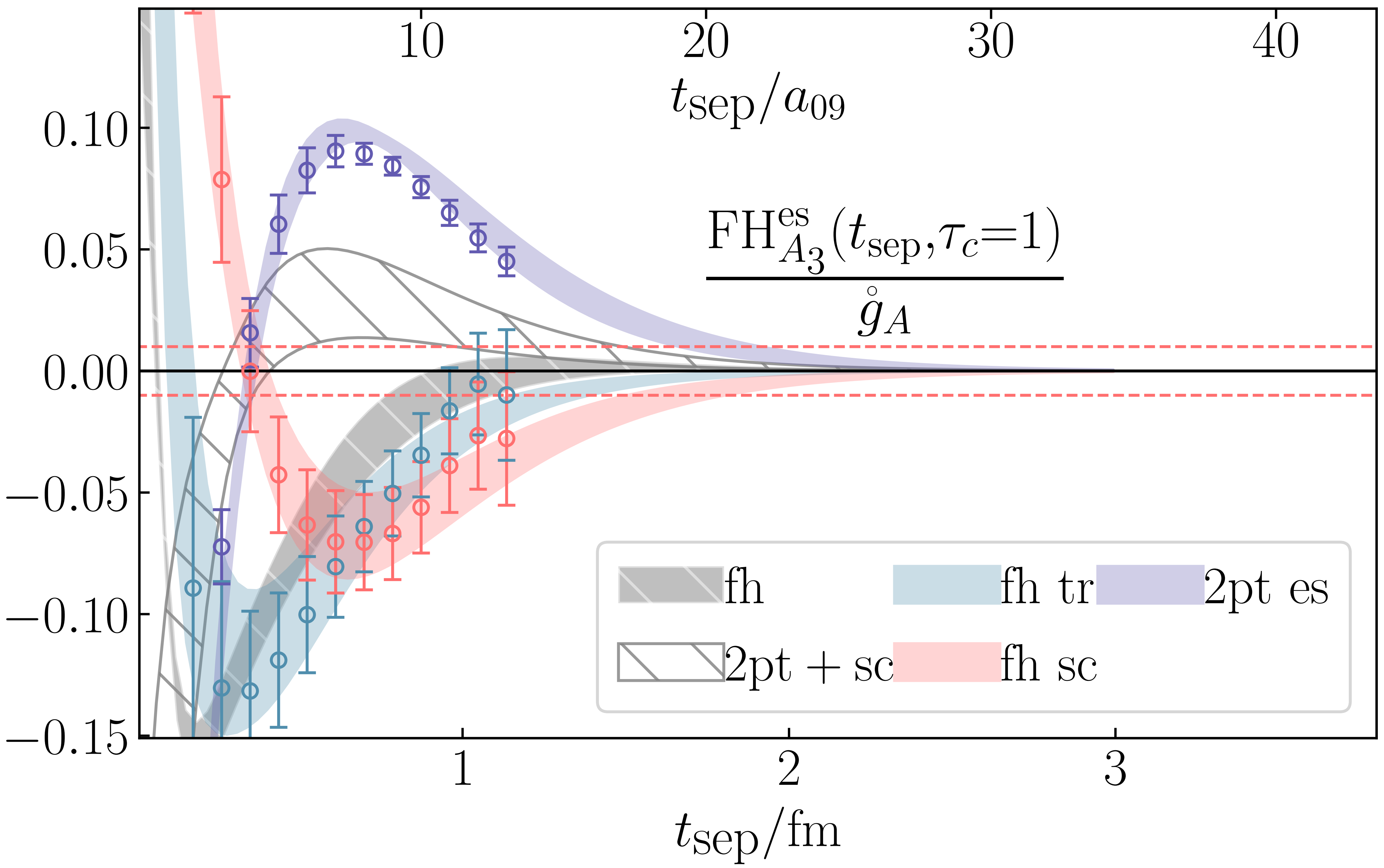
$$\begin{aligned} C_A^{\mathrm{gs}}(t) =& |z_0|^2 g_{00}^A e^{-E_0 t} \\ C_A^{\mathrm{sc}}(t) =& \sum_{n\geq 1} |z_n|^2 g_{nn}^A e^{-E_n t} \\ C_A^{\mathrm{tr}}(t,\tau) =& 2\sum_{n<m} z_n z_m^\dagger g_{nm}^A e^{-(E_n+\Delta_{mn}/2)t} \\ & \times \cosh\left[\Delta_{mn}(\tau - t/2)\right] \end{aligned}$$

Break fit model down to ground state and excited state contributions

Have a data driven fit of first 5 states, what are the contributions?

Excited-state contributions





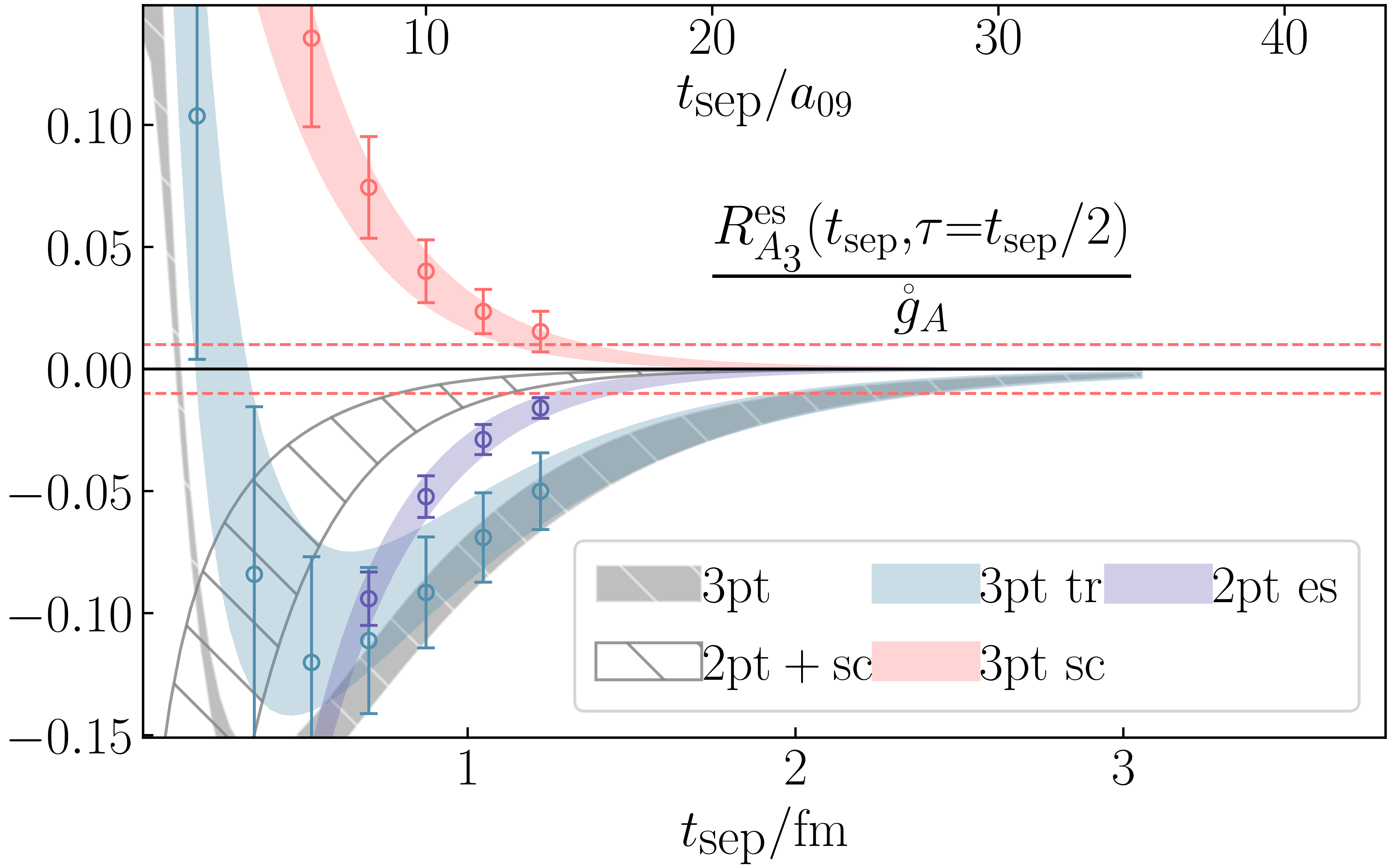
Significant cancellation between 2pt and 3pt sc excited states (red + blue)

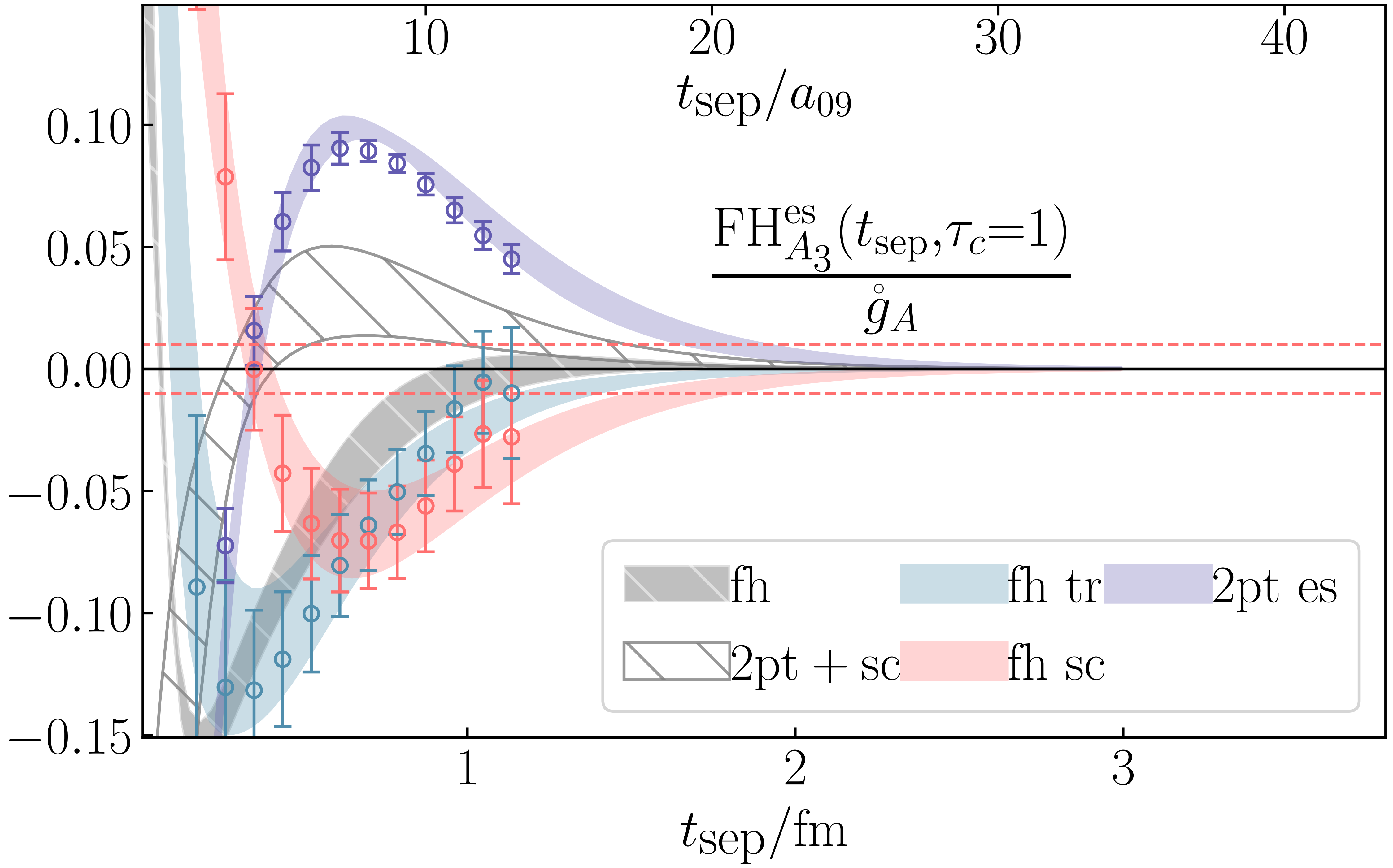
$$C_2^{\mathrm{es}}(t) = \sum_{n \geq 1} |z_n|^2 e^{-E_n t} \qquad C_A^{\mathrm{sc}}(t) = \sum_{n \geq 1} g^A_{nn} |z_n|^2 e^{-E_n t}$$

Result of (partially) conserved currents

4) Excited-state dominated by transition matrix elements (light blue)

Excited-state contributions





4) Excited-state dominated by transition matrix elements (light blue)

Fit and remove the transition excited states (possible with 3pt)
Suppress them and fit (possible with FH)

Or just combine both and do a more involved fit

Conclusions

Nucleon observables suffer from S/N issue

Analysis shows transition as dominant excited-state contribution

1) Calculation can be done at ~2.5fm for 1% systematic
   Requires unreasonable amount of statistics
   or is subject to uncontrollable systematic error

2) Fit and remove the transition contributions
   Standard way of generating correlates makes this possible
   Need enough source-sink separation time for sensible d.o.f. counting

3) Suppress contributions numerically with FH correlation
   Simpler to analyze and is consistent at the 1% level
   The cancellation is not exact
   Unknown how well this works for non-zero momentum transfer

Collaborators

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