QED corrections to the hadronic spectrum from lattice QCD with massive photons

Amy Nicholson
UNC, Chapel Hill

In collaboration with:
Kate Clark (NVIDIA), Michele Della Morte (CP3-SDU), Andrea Shindler (FRIB/MSU), Andre Walker-Loud (LBNL), Justus Tobias Tsang (CP3-SDU), Zack Hall (UNC), Haobo Yan (Peking U), Henry Monge-Camacho (UCR), Ben Hoerz (LBNL), Dean Howarth (LBNL), Pavlos Vranas (LLNL)

GHP (2021)
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Precision nuclear physics from LQCD

- Sub-percent precision LQCD now possible

Neutron-proton mass difference: accurate to 300 KeV (BMW 2015)

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REPORT

Ab initio calculation of the neutron-proton mass difference

Sz. Borsanyi[1], S. Durr[1,2], Z. Fodor[1,2,3,*], C. Hoelbling[1], S. D. Katz[3,4], S. Krieg[1,2], L. Lellouch[5], T. Lippert[1,2], A. Portelli[5,6], K. K. Szabo[1,2], B. C. Toth[1]

[1]Department of Physics, University of Wuppertal, D-42119 Wuppertal, Germany.
[2]Jülich Supercomputing Centre, Forschungszentrum Jülich, D-52428 Jülich, Germany.
[3]Institute for Theoretical Physics, Eötvös University, H-1117 Budapest, Hungary.
[4]Lendület Lattice Gauge Theory Research Group, Magyar Tudományos Akadémia–Eötvös Loránd University, H-1117 Budapest, Hungary.
[5]CNRS, Aix-Marseille Université, Université de Toulon, CPT UMR 7332, F-13288, Marseille, France.
[6]School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK.
*Corresponding author. E-mail: fodor@bodri.elte.hu

Science 27 Mar 2015:
Vol. 347, Issue 6229, pp. 1452-1455

Axial charge of the nucleon: $g_A = 1.271(13)$ (CalLat 2018)

nature
International journal of science

Altmetric: 114 | More detail »

Letter | Published: 30 May 2018

A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics

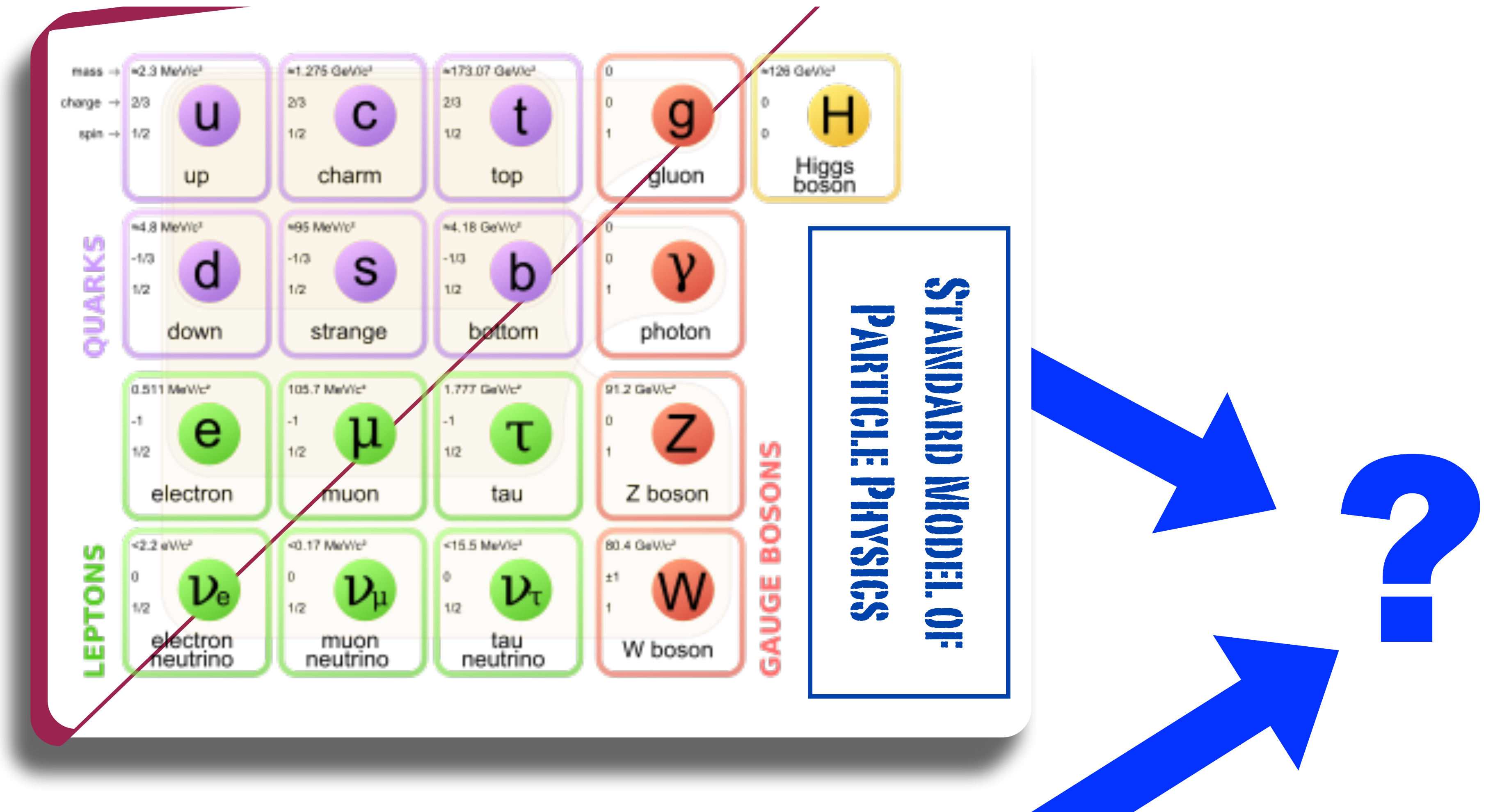
C. C. Chang, A. N. Nicholson, E. Rinaldi, E. Berkowitz, N. Garron, D. A. Brantley, H. Monge-Camacho, C. J. Monahan, C. Bouchard, M. A. Clark, B. Joó, T. Kurth, K. Orginos, P. Vranas & A. Walker-Loud

Nature **558**, 91–94 (2018) | Download Citation

Precision nuclear physics from LQCD

- Sub-percent precision LQCD now possible
- Look for new physics
  - isospin-breaking effects will become important

Look for discrepancies between the SM and experiment: neutron lifetime, proton radius, muon g-2



Precision nuclear physics from LQCD

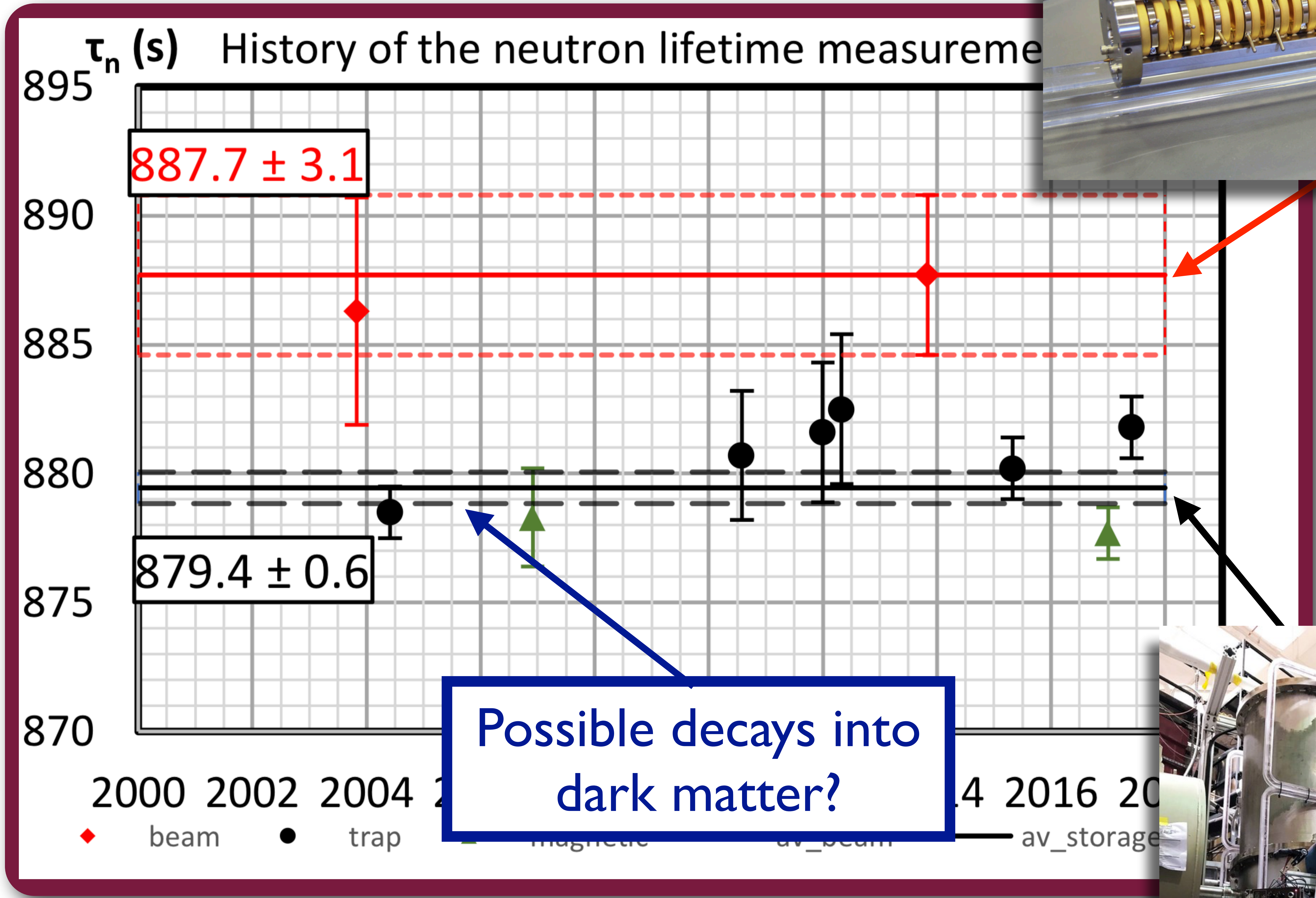
- Sub-percent precision LQCD now possible
- Look for new physics
  - isospin-breaking effects will become important

Neutron lifetime puzzle

Need ~0.2-0.5% theoretical uncertainty on gA to discriminate between measurements

$$g_A = 1.2711(103)^{s}(39)^{\chi}(15)^{a}(19)^{v}(04)^{I}(55)^{M}$$

Can we help understand radiative corrections?



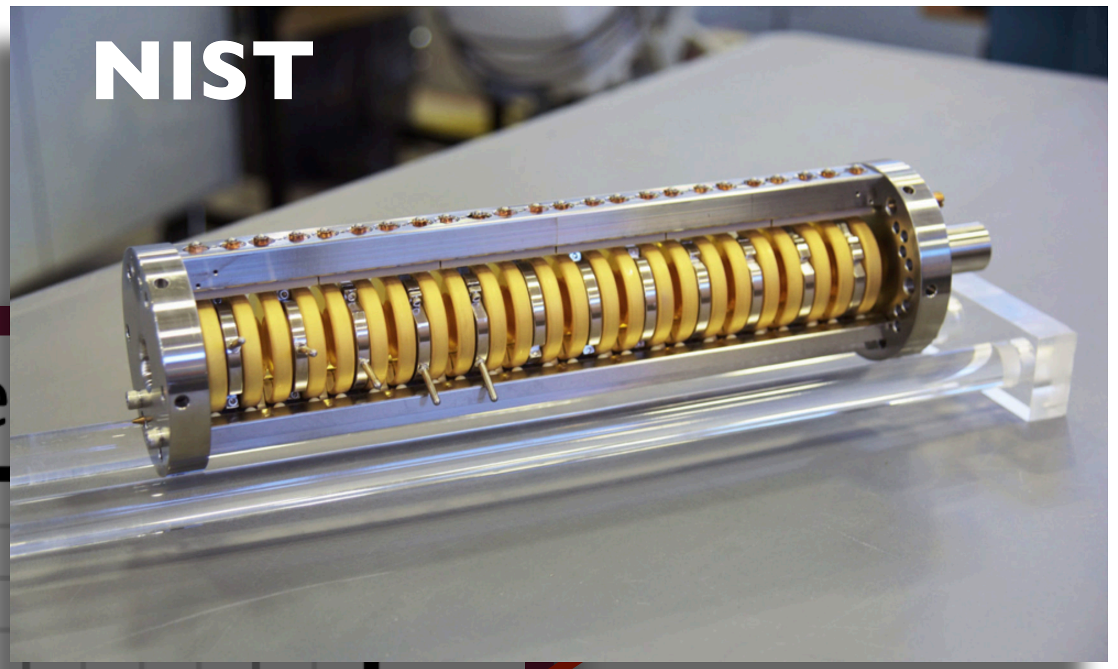
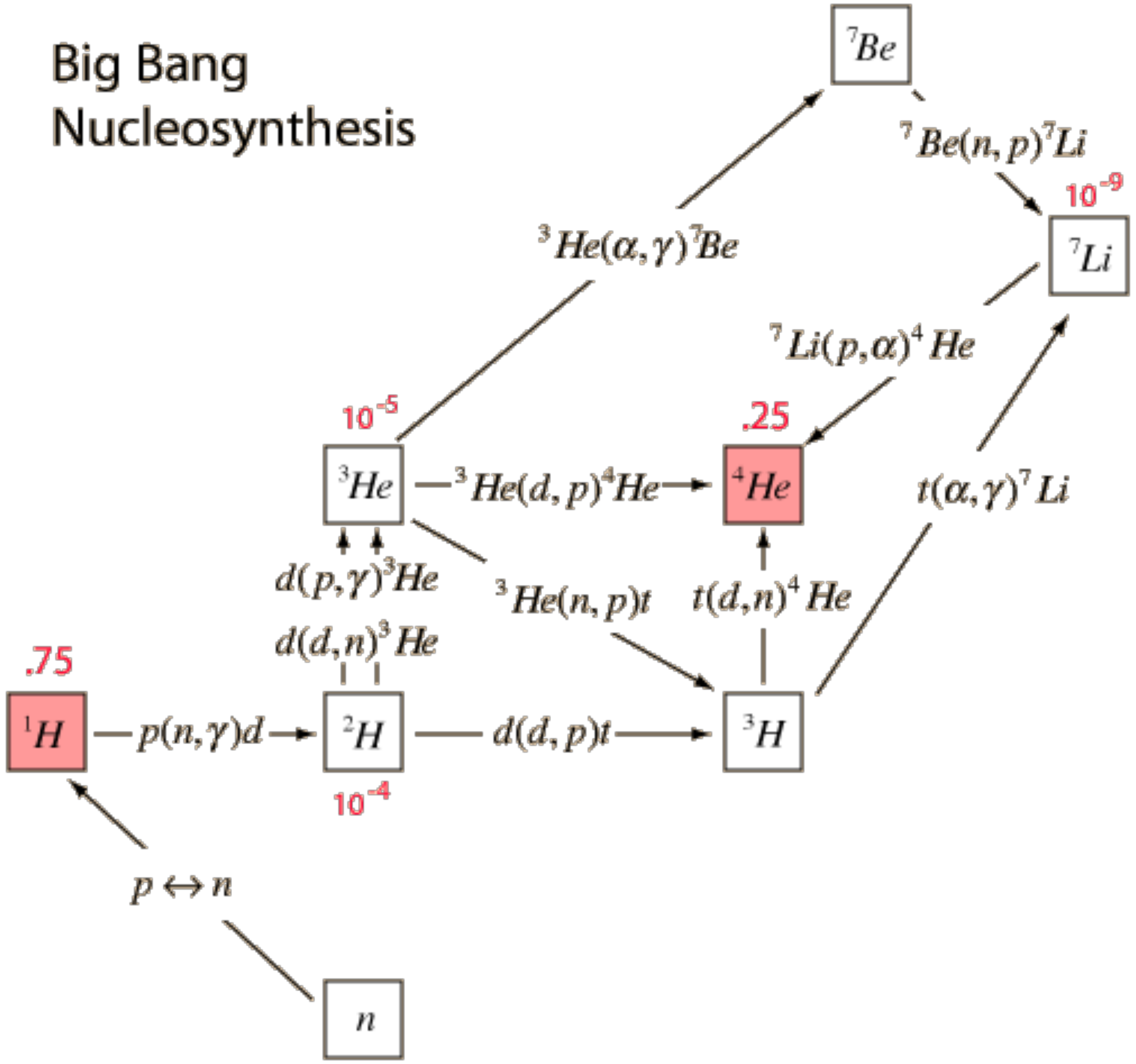


Figure: A. P. Serebrov, E .A. Kolomensky, A. K. Fomin, I. A. Krasnoschekova, A. V. Vassiljev, D. M. Prudnikov, I. V. Shoka, A. V. Chechkin, M. E. Chaikovskiy, V. E. Varlamov, S. N. Ivanov, A. N. Pirozhkov, P. Geltenbort, O. Zimmer, T. Jenke, M. Van der Grinten, M. Tucker, arXiv:1712.05663

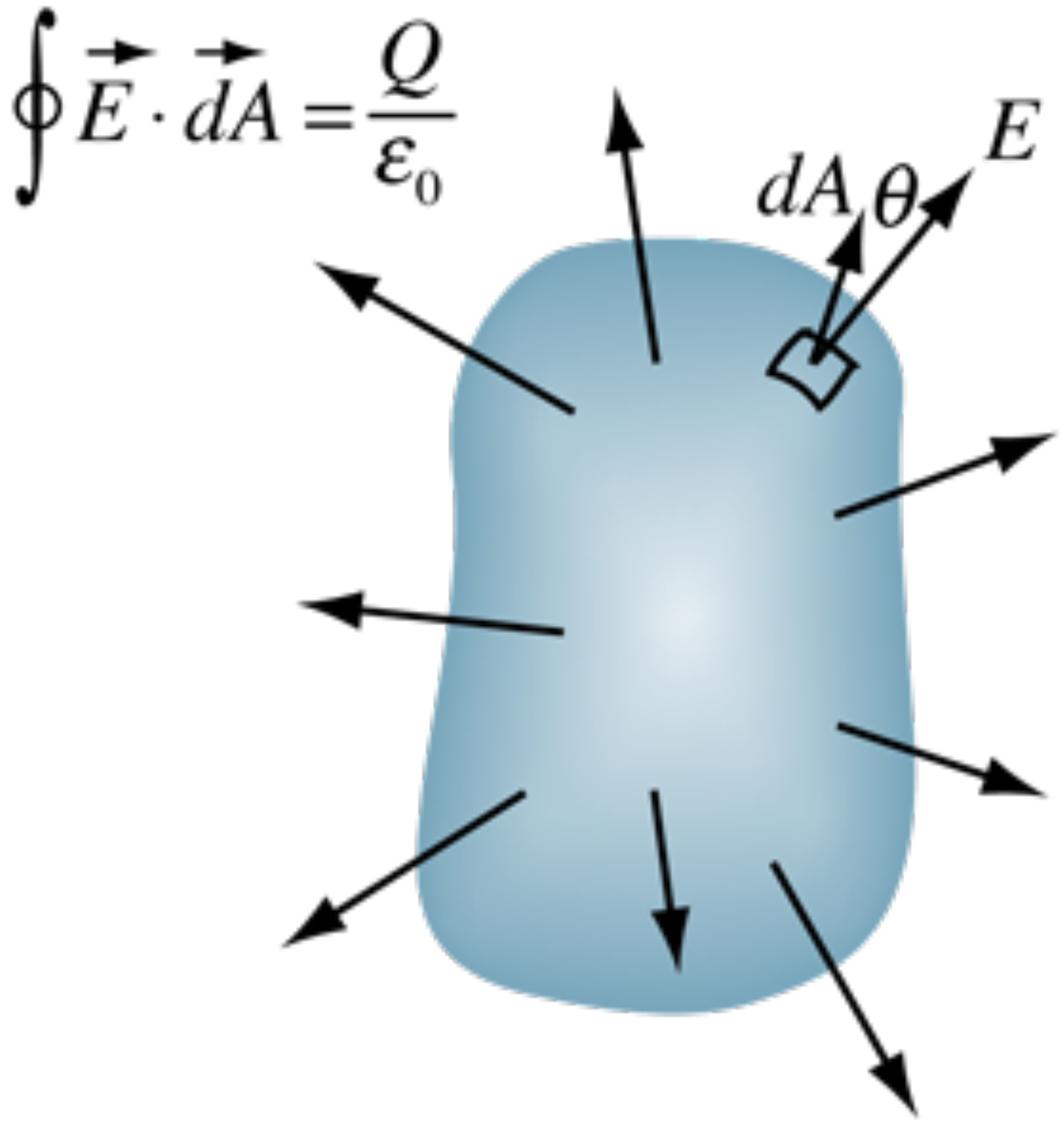
Precision nuclear physics from LQCD

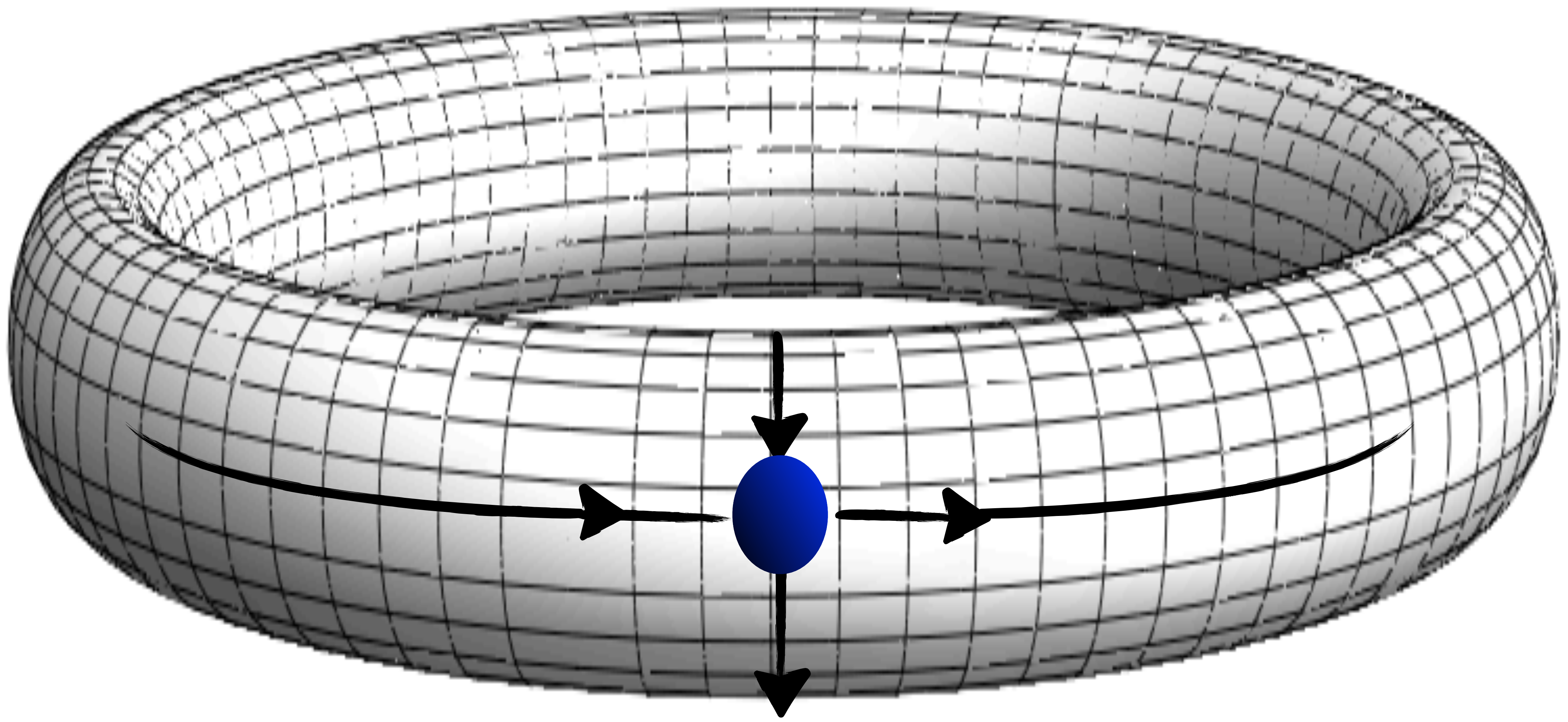
- Sub-percent precision LQCD now possible
- Look for new physics
  - isospin-breaking effects will become important
- Build quantitative connection between QCD & nuclear physics
  - requires interplay between LQCD & many-body approaches
  - some processes exquisitely sensitive to isospin-breaking (BBN)



QED on the lattice

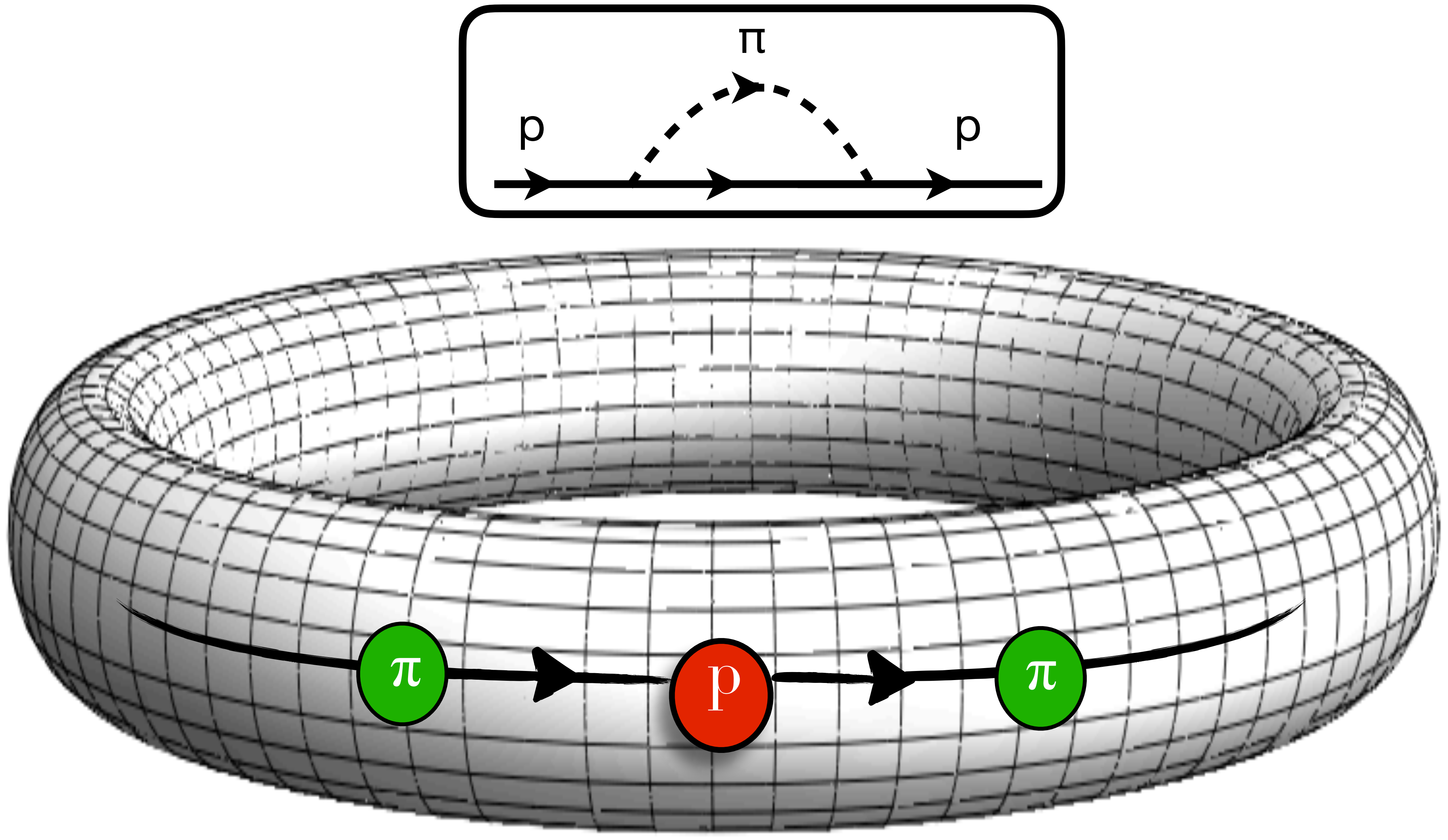
- Lattice calculations necessarily performed in a finite (generally, periodic) volume
- Gauss's law prohibits charged objects within a periodic volume



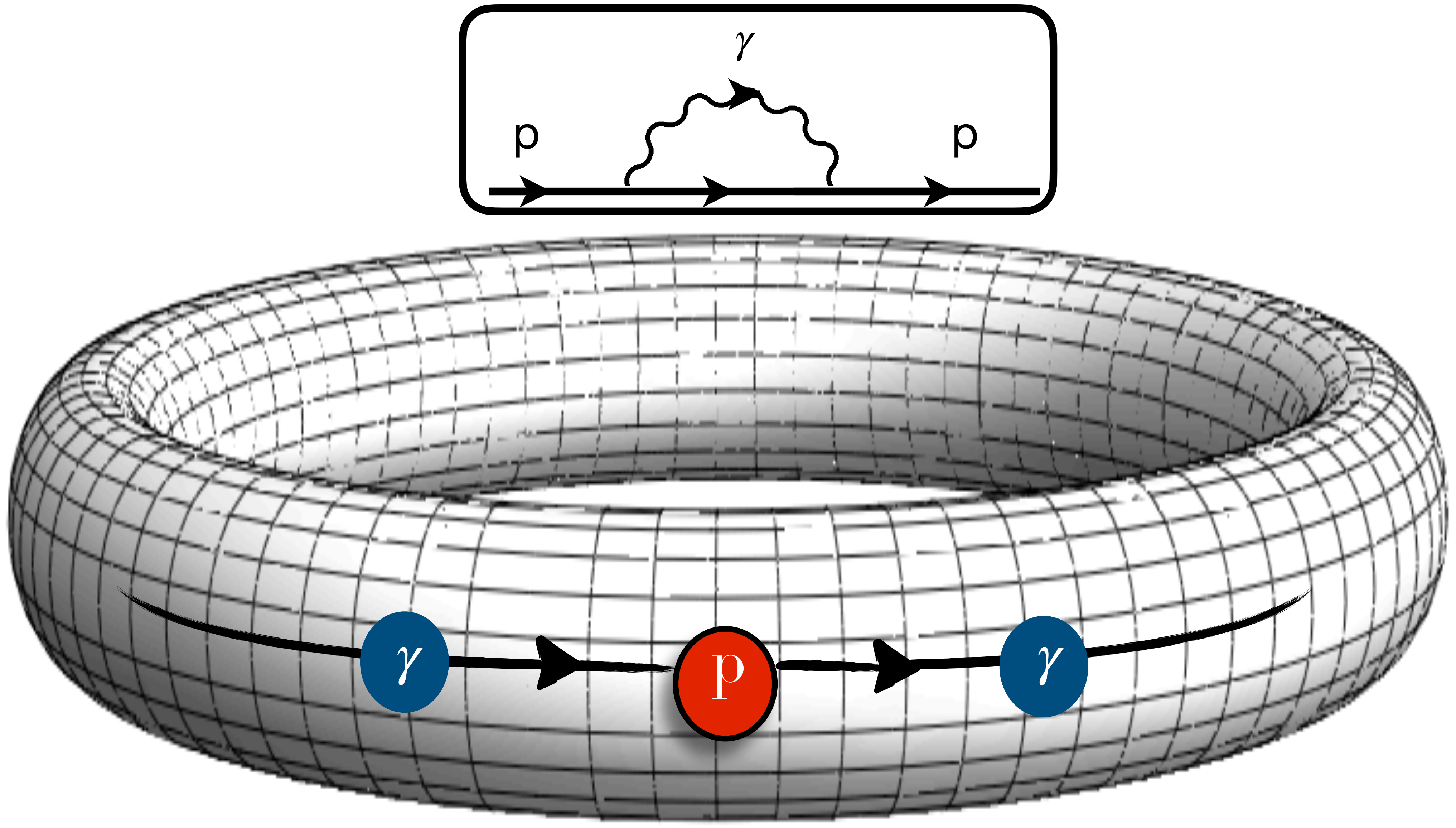


QED on the lattice

- Lattice calculations necessarily performed in a finite (generally, periodic) volume
- Gauss's law prohibits charged objects within a periodic volume
- Massless photon induces unphysical power-law IR effects



$$\delta_{\mathrm{FV}} \sim e^{-m_\pi L}$$



$$\delta_{\mathrm{FV}} \sim L^{-n}$$

QED on the lattice

- Some solutions:
  - QED$_{TL}$: $\tilde{A}_\mu(0) = 0$ — No transfer matrix
  - QED$_{SF}$: $eA_\mu(0)/L^3 \in (-\pi/L, \pi/L)$ — No transfer matrix (probably)
  - QED$_L$: $\tilde{A}_\mu(p_0, \mathbf{0}) = 0$ — Non-local
  - QED$_C$: C-parity BC $A_\mu(x+L_k\hat{k}) = -A_\mu(x)$, $\psi(x+L_k\hat{k}) = C^{-1}\bar{\psi}^T(x)$ — Flavor-mixing

Gauss's Law: invariance under large gauge transformations:

$$\langle \psi(x)\bar{\psi}(y)\rangle = 0\,, \quad \text{for } x \neq y$$

A. Duncan, E. Eichten and H. Thacker, Phys. Rev. Lett. 76 (1996) 3894–3897, [hep-lat/9602005].

M. Gockeler, R. Horsley, E. Laermann, P. E. L. Rakow, G. Schierholz, R. Sommer et al., *Nucl. Phys.* B334 (1990) 527–558.

M. Hayakawa and S. Uno, *Prog. Theor. Phys.* 120 (2008) 413–441, [0804.2044].

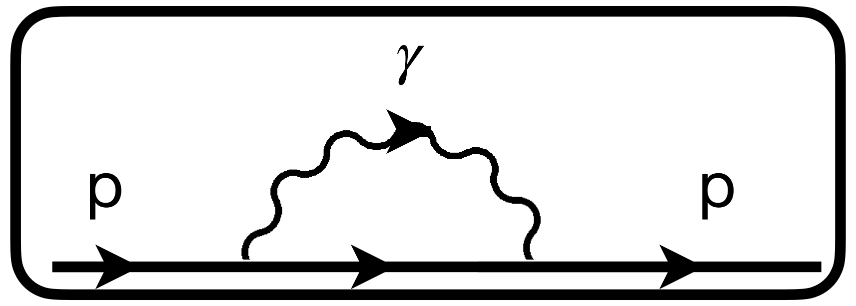
L. Polley and U. Wiese, *Nucl.Phys.* B356 (1991) 629–654.

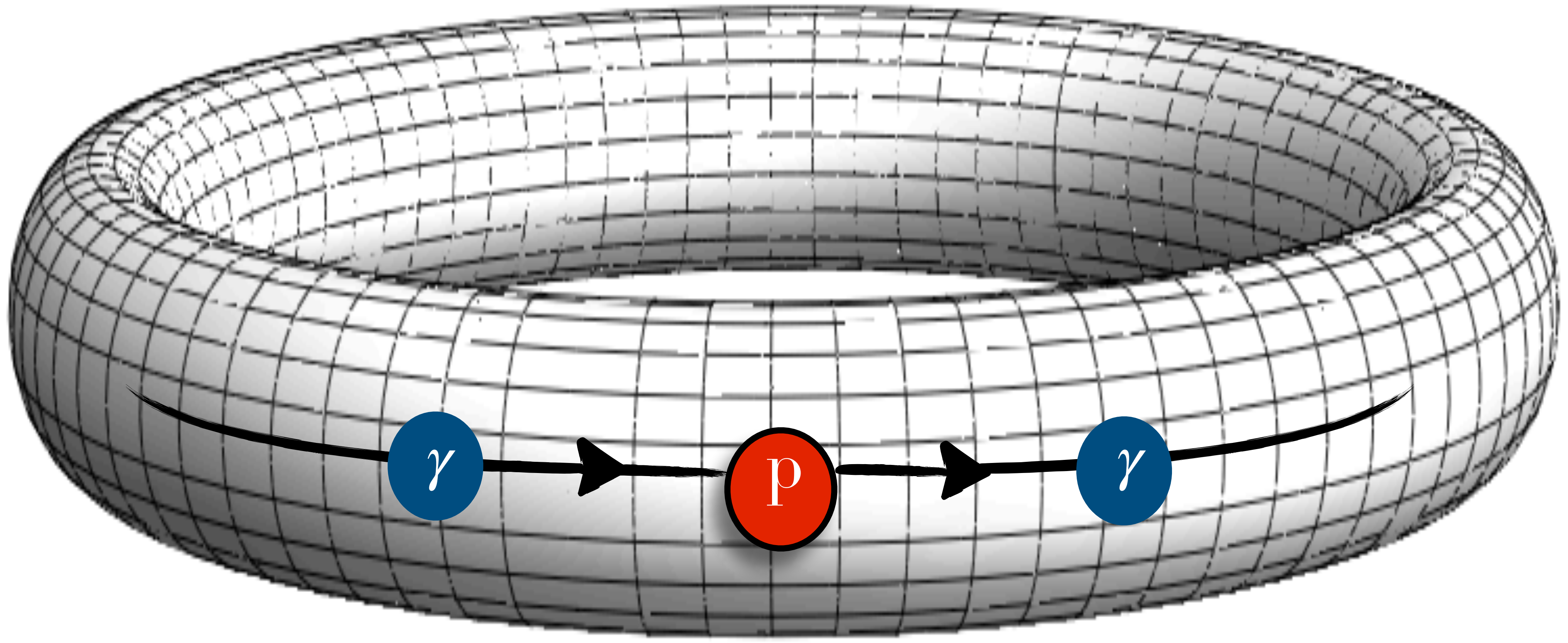
B. Lucini, A. Patella, A. Ramos and N. Tantalo, *JHEP* 02 (2016) 076, [1509.01636].

QED$_M$ on the lattice

M. G. Endres, A. Shindler, B. C. Tiburzi and A. Walker-Loud, *Phys. Rev. Lett.* 117 (2016) 072002.

- QED$_M$: Photon mass term $S_{\mathrm{m}}(A,\psi,\bar{\psi}) \stackrel{\text{def}}{=} S(A,\psi,\bar{\psi}) + \frac{m_\gamma^2}{2}\int \mathrm{d}^4x\, A_\mu^2$
  - FV effects are exponential
  - must perform careful extrapolations in two IR scales, L, m_γ
  - zero-mode contribution to masses can be subtracted by hand: $\frac{e^2}{m_\gamma^2 V} t$
- Preliminary checks performed in Endres. et al (2016)
- Our program
  - determine regions of validity for performing extrapolations
  - calculate physical point quantities for a variety of relevant observables
- Lattice details: N_f = 2+1+1 DWF/HISQ
  - for this talk: a=0.12 fm, m_π ~310 MeV, two volumes: L = 2.9, 3.8 fm, 6 values of m_γ





$\delta_{\mathrm{FV}} \sim e^{-m_\gamma L}$

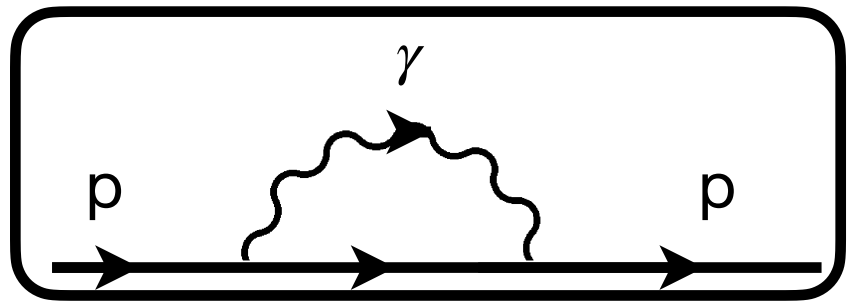
Isospin breaking scheme

Bussone, Della Morte, Janowski, Walker-Loud *PoS* LATTICE2018 (2018) 293

- Isospin breaking in nature from two source: up-down quark mass difference and charges
- Separating the two can be beneficial
  - strong isospin breaking contributions to M_n-M_p related to CP violation
  - disentangling radiative corrections to the neutron lifetime/beta decay
- QED effects renormalize quark masses: can't unambiguously separate contributions
- Would like to fix m_u+m_d, m_u-m_d independently of α
  - fix values of physical quantities that are largely insensitive to α:
    - $m_{\pi 0}$: m_u+m_d, $m_{\Sigma+}-m_{\Sigma-}$: m_u-m_d
    - setting $m_{\Sigma+} = m_{\Sigma-}$ defines an isospin symmetric line for studying QED only effects
    - setting $m_{\pi+} = m_{\pi 0}$ defines an isospin symmetric line for studying strong isospin breaking only effects

Extrapolations to the physical point

- Must take FV/m_γ limits carefully (do not commute!)
  - ChiPT expectations:



$L \to \infty$

$$\frac{\delta_L M^{LO}}{M} = 2\pi\alpha Q^2 \frac{m_\gamma}{M}\left[\mathcal{I}_1(m_\gamma L) - \frac{1}{(m_\gamma L)^3}\right],$$

$$\frac{\delta_L M^{NLO}}{M} = \pi\alpha Q^2 \frac{m_\gamma^2}{M^2}\left[2\mathcal{I}_{1/2}(m_\gamma L) + \mathcal{I}_{3/2}(m_\gamma L)\right]$$

$$\mathcal{I}_n(z) = \frac{1}{2^{(n+1/2)}\pi^{3/2}\Gamma(n)}\sum_{\nu\neq 0}\frac{K_{3/2-n}(z\,|\nu|)}{(z\,|\nu|)^{3/2-n}}.$$

$m_\gamma \to 0$

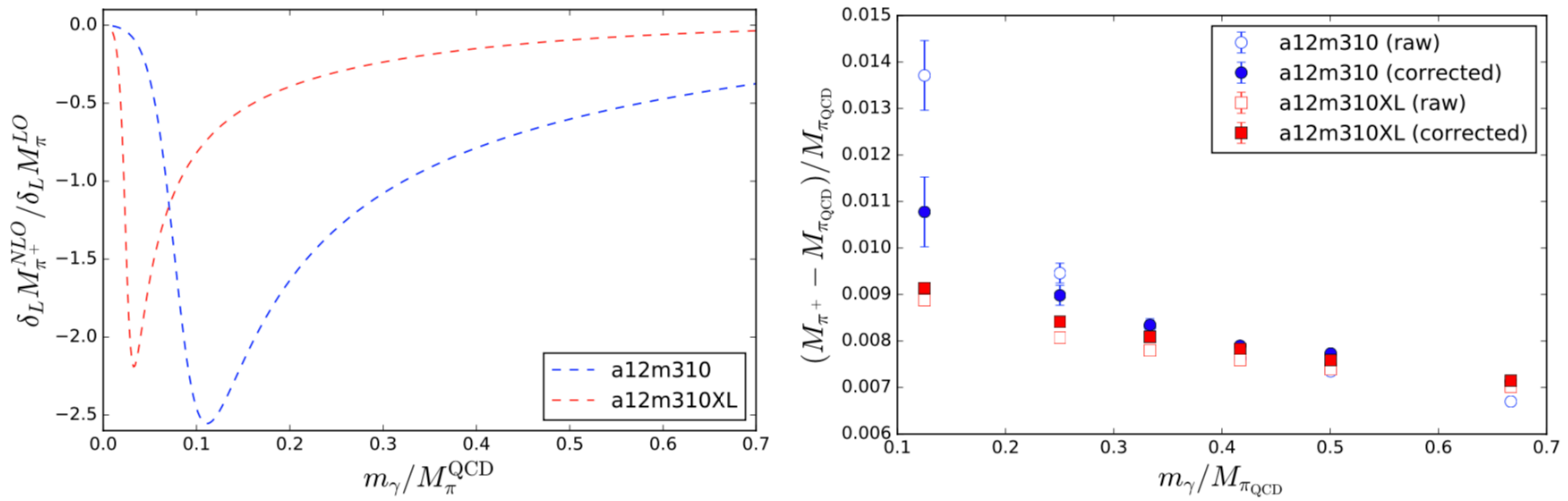
$$M(m_\gamma) - M(0) = \Delta_\gamma M^{LO} + \Delta_\gamma^{NLO} + \mathcal{O}\left(\frac{m_\gamma^3}{M^2}\right)$$

$$\frac{\Delta_\gamma M^{LO}}{M} = -\frac{\alpha}{2}Q^2\frac{m_\gamma}{M}$$

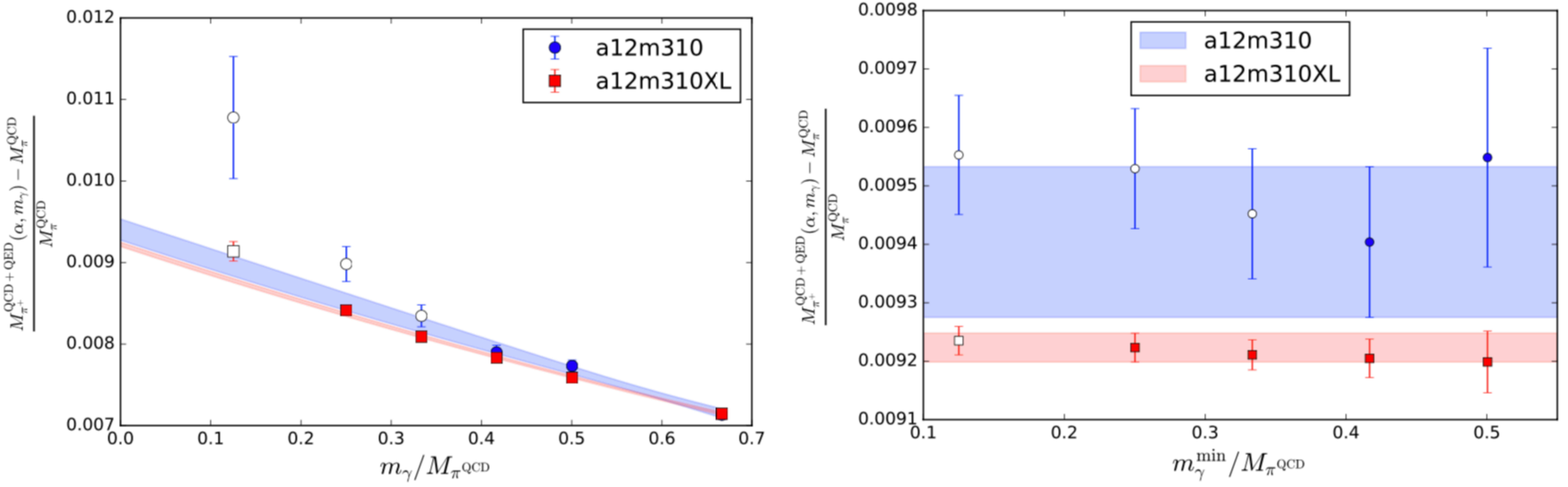
$$\frac{\Delta_\gamma M^{NLO}}{M} = \left(C\alpha - \frac{\alpha}{4\pi}Q^2\right)\frac{m_\gamma^2}{M^2}.$$

  - Must remove zero mode (leads to linear t-dependence in effective mass with a known, analytic coefficient)

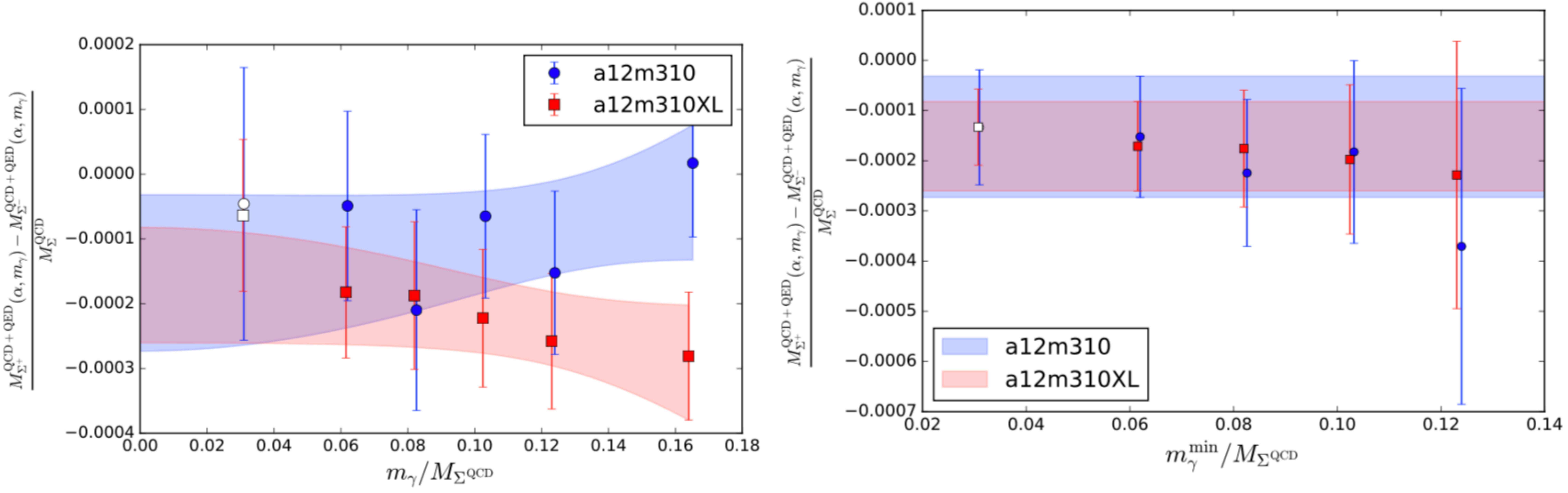
Finite volume corrections



m_γ extrapolations



Σ^+- Σ^- mass splitting



Results

Preliminary

Omega mass used for scale setting in many collaborations

$$M_{\Omega^-} - M_{\Omega_{\rm QCD}} \sim 1.783(53)\ \text{MeV}$$

Important for BBN

$$M_{p^+} - M_{n^0} \sim 0.947(62)\ \text{MeV}$$

obs	ens	ΔQ	ΔaM	C	χ^2/dof	$m_\gamma^{\rm min}$	$m_\gamma^{\rm max}$
$\pi^+ - \pi_{\rm QCD}$	a12m310	1	+0.001774(24)	+0.00098(37)	1.13	0.0786	0.1258
$\pi^+ - \pi_{\rm QCD}$	a12m310XL	1	+0.0017398(46)	+0.001377(71)	0.87	0.0472	0.1258
$\pi^+ - \pi_{\rm QCD}$	comb	1	+0.0017412(45)	+0.001368(69)	1.07		
$\pi^0_{\rm conn} - \pi_{\rm QCD}$	a12m310	0	+0.0002618(72)	−0.00007(13)	0.21	0.0472	0.1258
$\pi^0_{\rm conn} - \pi_{\rm QCD}$	a12m310XL	0	+0.0002632(28)	+0.000070(50)	0.06	0.0472	0.1258
$\pi^0_{\rm conn} - \pi_{\rm QCD}$	comb	0	+0.0002628(26)	+0.000049(45)	0.76		
$\overline{u}u^0_{\rm conn} - \pi_{\rm QCD}$	a12m310	0	+0.000441(12)	−0.00011(21)	0.07	0.0472	0.1258
$\overline{u}u^0_{\rm conn} - \pi_{\rm QCD}$	a12m310XL	0	+0.0004373(44)	+0.000115(78)	0.03	0.0472	0.1258
$\overline{u}u^0_{\rm conn} - \pi_{\rm QCD}$	comb	0	+0.0004376(42)	+0.000086(72)	0.30		
$\overline{d}d^0_{\rm conn} - \pi_{\rm QCD}$	a12m310	0	+0.0001084(30)	−0.000028(53)	0.31	0.0472	0.1258
$\overline{d}d^0_{\rm conn} - \pi_{\rm QCD}$	a12m310XL	0	+0.0001103(12)	+0.000027(21)	0.12	0.0472	0.1258
$\overline{d}d^0_{\rm conn} - \pi_{\rm QCD}$	comb	0	+0.0001099(11)	+0.000019(19)	1.35		
$\pi^+ - \pi^0_{\rm conn}$	a12m310	1	+0.001523(19)	+0.00088(27)	1.17	0.0786	0.1258
$\pi^+ - \pi^0_{\rm conn}$	a12m310XL	1	+0.0014761(37)	+0.001309(56)	1.12	0.0472	0.1258
$\pi^+ - \pi^0_{\rm conn}$	comb	1	+0.0014780(36)	+0.001307(55)	2.66		
$K^+ - K_{\rm QCD}$	a12m310	1	+0.001579(18)	+0.00108(49)	0.82	0.0472	0.1258
$K^+ - K_{\rm QCD}$	a12m310XL	1	+0.0015042(51)	+0.00269(16)	0.05	0.0472	0.1258
$K^+ - K_{\rm QCD}$	comb	1	+0.0015095(50)	+0.00260(15)	2.93		
$K^0 - K_{\rm QCD}$	a12m310	0	+0.0002297(23)	−0.000182(77)	0.22	0.0472	0.1258
$K^0 - K_{\rm QCD}$	a12m310XL	0	+0.0002257(14)	+0.000050(44)	0.60	0.0472	0.1258
$K^0 - K_{\rm QCD}$	comb	0	+0.0002265(12)	−0.000007(38)	1.63		
$K^+ - K^0$	a12m310	1	+0.001350(16)	+0.00125(41)	0.91	0.0472	0.1258
$K^+ - K^0$	a12m310XL	1	+0.0012782(44)	+0.00265(13)	0.05	0.0472	0.1258
$K^+ - K^0$	comb	1	+0.0012829(42)	+0.00259(12)	3.78		
$\Sigma^+ - \Sigma_{\rm QCD}$	a12m310	1	+0.00092(13)	+0.0100(92)	0.89	0.0472	0.1258
$\Sigma^+ - \Sigma_{\rm QCD}$	a12m310XL	1	+0.000833(96)	+0.0030(76)	0.22	0.0472	0.1258
$\Sigma^+ - \Sigma_{\rm QCD}$	comb	1	+0.000858(77)	+0.0067(59)	0.86		
$\Sigma^0 - \Sigma_{\rm QCD}$	a12m310	0	+0.000318(94)	+0.0048(67)	0.87	0.0472	0.1258
$\Sigma^0 - \Sigma_{\rm QCD}$	a12m310XL	0	+0.000338(65)	−0.0037(51)	0.42	0.0472	0.1258
$\Sigma^0 - \Sigma_{\rm QCD}$	comb	0	+0.000330(54)	−0.0001(41)	0.77		
$\Sigma^- - \Sigma_{\rm QCD}$	a12m310	-1	+0.001031(51)	+0.0062(37)	1.33	0.0472	0.1258
$\Sigma^- - \Sigma_{\rm QCD}$	a12m310XL	-1	+0.000963(35)	+0.0075(29)	0.77	0.0472	0.1258
$\Sigma^- - \Sigma_{\rm QCD}$	comb	-1	+0.000981(28)	+0.0076(23)	1.14		
$\Sigma^+ - \Sigma^0$	a12m310	1	+0.000616(76)	+0.0048(56)	0.44	0.0472	0.1258
$\Sigma^+ - \Sigma^0$	a12m310XL	1	+0.000496(48)	+0.0066(39)	0.24	0.0472	0.1258
$\Sigma^+ - \Sigma^0$	comb	1	+0.000526(40)	+0.0067(32)	0.87		
$\Sigma^- - \Sigma^0$	a12m310	1	+0.000740(75)	−0.0008(52)	0.18	0.0472	0.1258
$\Sigma^- - \Sigma^0$	a12m310XL	1	+0.000629(47)	+0.0109(34)	0.19	0.0472	0.1258
$\Sigma^- - \Sigma^0$	comb	1	+0.000657(41)	+0.0072(28)	0.62		
$\Sigma^+ - \Sigma^-$	a12m310	0	−0.000116(92)	+0.0046(63)	0.48	0.0472	0.1258
$\Sigma^+ - \Sigma^-$	a12m310XL	0	−0.000131(68)	−0.0045(51)	0.03	0.0472	0.1258
$\Sigma^+ - \Sigma^-$	comb	0	−0.000129(56)	−0.0004(40)	0.68		
$\Omega^- - \Omega_{\rm QCD}$	a12m310	-1	+0.001199(49)	−0.0059(50)	0.43	0.0472	0.1258
$\Omega^- - \Omega_{\rm QCD}$	a12m310XL	-1	+0.000968(44)	+0.0154(47)	0.62	0.0472	0.1258
$\Omega^- - \Omega_{\rm QCD}$	comb	-1	+0.001070(32)	+0.0057(34)	1.95		
$p^+ - n_{\rm QCD}$	a12m310	1	+0.00097(14)	[illegible]	2.08	0.0472	0.1258
$p^+ - n_{\rm QCD}$	a12m310XL	1	+0.000786(96)	+0.0088(67)	1.03	0.0472	0.1258
$p^+ - n_{\rm QCD}$	comb	1	+0.000825(78)	+0.0115(52)	1.86		
$n^0 - n_{\rm QCD}$	a12m310	0	+0.000356(99)	+0.0038(57)	1.86	0.0472	0.1258
$n^0 - n_{\rm QCD}$	a12m310XL	0	+0.000230(63)	+0.0011(44)	0.64	0.0472	0.1258
$n^0 - n_{\rm QCD}$	comb	0	+0.000254(52)	+0.0022(33)	1.41		
$p^+ - n^0$	a12m310	1	+0.000634(77)	+0.0119(48)	1.53	0.0472	0.1258
$p^+ - n^0$	a12m310XL	1	+0.000556(42)	+0.0079(30)	1.41	0.0472	0.1258
$p^+ - n^0$	comb	1	+0.000568(37)	+0.0095(25)	1.87		

Summary

- QED formulations on the lattice must deal with IR issues related to Gauss's law and the lack of a mass gap
- Adding a massive photon alleviates these
  - exponential scaling with the volume
  - must extrapolate in two IR scales
- We have investigated this formulation in the calculation of various quantities, with results at the per-mille precision level
- Can provide a check against results produced using other QED formulations
- Scattering/resonances: our formulation has a clear separation between exponential (unphysical) and power-law (physical) effects
- Future: add strong isospin breaking and sea quark effects (reweighting)

Interlude: are we in the correct regime?

$$m_{\text{eff}}(t,\mathbf{p}) \stackrel{\text{def}}{=} -\frac{\mathrm{d}}{\mathrm{d}t}\ln C(t,\mathbf{p}) = \frac{e^2}{m_\gamma^2 V}t - \frac{\mathrm{d}}{\mathrm{d}t}\ln\langle\psi(t,\mathbf{0})\bar{\psi}(0^+)\delta_{Q,0}\rangle_{\text{TL}} + O(m_\gamma^2).$$

A. Patella (2017)

0-mode (subtracted by hand)

unphysical, p-independent

physical, p-dependent