Lorentz invariance relation anomalies and intrinsic parton transverse momentum

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Outline

- **What are** Lorentz invariance relations (LIRs)?
- **Derivation** of LIRs
- **Violation** of LIRs
- LIRs in a **renormalizable field theory**
- Summary and conclusion

Lorentz invariance relations (LIRs)

Lorentz invariance relations (LIRs) **connect** the twist-2 and twist-3 parton distribution functions (PDFs) and weighted moments of transverse momentum dependent (TMD) correlation functions

Some examples for LIRs

The distributions on the l.h.s are **twist-3**

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x),$$

$$h_L(x) = h_1(x) - \frac{d}{dx} h_{1L}^{\perp(1)}(x),$$

$$f_T(x) = -\frac{d}{dx} f_{1T}^{\perp(1)}(x),$$

$$h(x) = -\frac{d}{dx} h_1^{\perp(1)}(x).$$

The distributions on the r.h.s are **twist-2**

P.J. Mulders and R.D. Tangerman, Nucl. Phys. B 461, 197-237 (1996).
D. Boer and P.J. Mulders, Phys. Rev. D 57, 5780 (1998).

The superscript (1) indicates the k_T^2 **moment** of the TMD

$$g_{1T}^{(1)}(x) = \int d^2\mathbf{k}_T \frac{\mathbf{k}_T^2}{2M^2} g_{1T}(x, \mathbf{k}_T^2), \quad \text{etc.}$$

Derivation of LIRs

The quark correlator written in a **Lorentz invariant form**

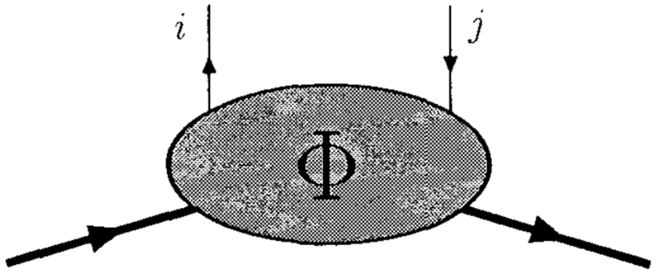
$$\Phi(k,P,S) = a_1 \frac{\mathbf{1}}{M^3} + a_2 \frac{\not{P}}{M^4} + a_3 \frac{\not{k}}{M^4} + a_4 \frac{\gamma_5 \not{S}}{M^4} + a_5 \frac{\gamma_5 [\not{P}, \not{S}]}{M^5} + a_6 \frac{\gamma_5 [\not{k}, \not{S}]}{M^5} + a_7 \frac{(k\cdot S)\gamma_5 \not{P}}{M^6} + a_8 \frac{(k\cdot S)\gamma_5 \not{k}}{M^6} + a_9 \frac{(k\cdot S)\gamma_5 [\not{P}, \not{k}]}{M^7}.$$

k: Parton momentum

P, S: Hadron momentum, spin

M: Hadron mass

$a_j \equiv a_j(k^2, k.P)$



$$\mathrm{Tr}(\Gamma\Phi) = \int d^4\xi e^{ik.\xi} \langle PS|\bar{\psi}(0)\Gamma\psi(\xi)|PS\rangle$$

Projections

$$\Phi^{[\Gamma]}(x, \mathbf{k}_T) = \frac{1}{2}\int dk^- \mathrm{Tr}[\Gamma\Phi],$$

$$\Phi^{[\Gamma]}(x) = \frac{1}{2}\int d^2\mathbf{k}_T \int dk^- \mathrm{Tr}[\Gamma\Phi].$$

Derivation of LIRs

Example

$$\underbrace{g_T(x)}_{\substack{twist-3\\ pdf}} = \underbrace{g_1(x)}_{\substack{twist-2\\ pdf}} + \frac{d}{dx}\underbrace{g_{1T}^{(1)}(x)}_{\substack{twist-2\\ tmd}}$$

$$\int d^2\mathbf{k}_T\, \frac{k_T^2}{2M^2} g_{1T}$$

$$\int d^2\mathbf{k}_T g_{1L}$$

$$\int d^2 k_T \Phi^{[\gamma_5 \gamma^i]}(x, k_T) = \frac{S_T^i}{P^+} g_T(x)$$

$$\Phi^{[\gamma^+\gamma_5]}(x, k_T) = g_{1L}(x, k_T^2)\lambda + g_{1T}(x, k_T^2)\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M}$$

$$\underbrace{\int d^2\mathbf{k}_T dk^- \left(a_4 \frac{2}{M^3} - a_8 \frac{k_T^2}{M^5}\right)}_{g_T(x)} = \underbrace{\int d^2\mathbf{k}_T dk^- \left[a_4 \frac{2}{M^3} + (a_7 - x a_8)\frac{2k^- P^+ - xM^2}{M^5}\right]}_{g_1(x)} + \frac{d}{dx}\underbrace{\int d^2\mathbf{k}_T dk^- \frac{k_T^2}{M^5}(a_7 - x a_8)}_{g_{1T}^{(1)}}$$

Provided that the integrals are **convergent**:

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x)$$

Violation of LIRs

- Inclusion of Wilson lines

K. Goeke, A. Metz, P. Pobylitsa, and M. Polyakov, Phys. Lett. B 567, 27 (2003), hep-ph/0302028.
A. Accardi, A. Bacchetta, W. Melnitchouk, and M. Schlegel, JHEP 11, 093 (2009), 0907.2942.

	No Wilson lines	With Wilson lines
The correlator	$\Phi(k,P,S) = a_1 \frac{1}{M^3} + a_2 \frac{\not{P}}{M^4} + a_3 \frac{\not{k}}{M^4} + \cdots$	$\Phi(k,P,S,\nu) = a_1 \frac{1}{M^3} + a_2 \frac{\not{P}}{M^4} + a_3 \frac{\not{k}}{M^4} + \cdots + b_1 M \frac{S\cdot\nu}{P\cdot\nu}\not{P}\gamma_5 + b_2 M \frac{S\cdot\nu}{P\cdot\nu}\not{k}\gamma_5 + \cdots$
LIR	$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}$	$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)} + \hat{g}_T(x)$

- UV divergent integrals

Aslan, Gamberg, Rogers, in prep.

$$\underbrace{\int d^2\mathbf{k}_T dk^- \left(a_4 \frac{2}{M^3} - a_8 \frac{k_T^2}{M^5}\right)}_{g_T(x)} = \underbrace{\int d^2\mathbf{k}_T dk^- \left[a_4 \frac{2}{M^3} + (a_7 - x a_8) \frac{2k^- P^+ - xM^2}{M^5}\right]}_{g_1(x)} + \frac{d}{dx} \underbrace{\int d^2\mathbf{k}_T dk^- \frac{k_T^2}{M^5}(a_7 - x a_8)}_{g_{1T}^{(1)}}$$

- Taking the transverse integrals using a fixed cut off
- Using $\overline{MS}$ renormalization scheme for the PDFs

Violation of LIRs: Treatment of the UV divergent integrals

1- Using a cut off for the UV divergent integrals

$$g_T(x) - g_1(x) - \frac{d}{dx} g_{1T}^{(1)}(x) \stackrel{?}{=} 0$$

$$a_4 = -\frac{M^4}{k^4}\delta_+[(k-P)^2 - m_s^2]$$

$$a_7 = a_8 = \frac{M^6}{k^4}\delta_+[(k-P)^2 - m_s^2]$$

Fixed transverse cutoff: μ

$$\underbrace{\int_0^{\mu} d^2\mathbf{k}_T \int dk^- \left(a_4 \frac{2}{M^3} - a_8 \frac{k_T^2}{M^5}\right)}_{g_T(x)} - \underbrace{\int_0^{\mu} d^2\mathbf{k}_T \int dk^- \left[a_4 \frac{2}{M^3} + (a_7 - x a_8)\frac{2k^- P^+ - xM^2}{M^5}\right]}_{g_1(x)} - \frac{d}{dx}\underbrace{\int_0^{\mu} d^2\mathbf{k}_T \int dk^- \frac{k_T^2}{M^5}(a_7 - x a_8)}_{g_{1T}^{(1)}} \stackrel{?}{=} 0$$

X-dependent transverse cutoff: $k_T^2 = \mu^2(x-1) + x(1-x)M^2 - x m_s^2$

$$\underbrace{\int_0^{k_T^2} d^2\mathbf{k}_T \int dk^- \left(a_4 \frac{2}{M^3} - a_8 \frac{k_T^2}{M^5}\right)}_{g_T(x)} - \underbrace{\int_0^{k_T^2} d^2\mathbf{k}_T \int dk^- \left[a_4 \frac{2}{M^3} + (a_7 - x a_8)\frac{2k^- P^+ - xM^2}{M^5}\right]}_{g_1(x)} - \frac{d}{dx}\underbrace{\int_0^{k_T^2} d^2\mathbf{k}_T \int dk^- \frac{k_T^2}{M^5}(a_7 - x a_8)}_{g_{1T}^{(1)}} \stackrel{?}{=} 0$$

Violation of LIRs: Treatment of the UV divergent integrals

1- Using a cut off for the UV divergent integrals

Transverse cut off	LIR | DGLAP
Fixed	✗ | ✓
x-dependent	✓ | ✗

$$\underbrace{g_T(x)}_{\substack{Fixed\\cut-off}} - \underbrace{g_1(x)}_{\substack{Fixed\\cut-off}} - \frac{d}{dx}\underbrace{g_{1T}^{(1)}}_{\substack{Fixed\\cut-off}} \overset{\mu\to\infty}{=} -\pi\frac{(1+x)}{2}$$ *→ LIR is violated*

$$\underbrace{g_T(x)}_{\substack{x-dependent\\cut-off}} - \underbrace{g_1(x)}_{\substack{x-dependent\\cut-off}} - \frac{d}{dx}\underbrace{g_{1T}^{(1)}}_{\substack{x-dependent\\cut-off}} = 0$$ *→ LIR is satisfied*

Assuming that DGLAP equations hold for all collinear functions, the divergent integrals over k_T are defined by implementing a fixed cutoff on the large transverse momentum, which is independent of the momentum fraction x. Such cutoffs result in a violation of LIRs even in the limit that the cut off is taken to infinity.

Violation of LIRs: Treatment of the UV divergent integrals

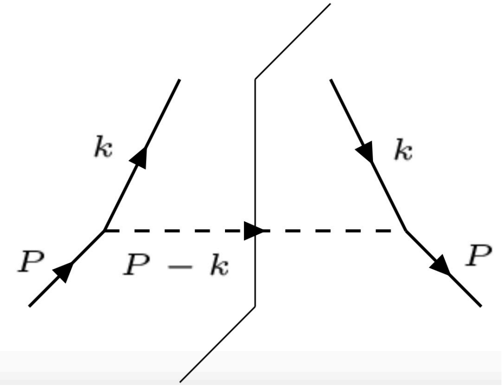
2- Using $\overline{MS}$ renormalization scheme for the PDFs

Distributions	**Transverse integrals**
Collinear PDFs	$\overline{MS}$
TMD PDFs	Cut off

$$\underbrace{g_T(x)}_{\overline{MS}} - \underbrace{g_1(x)}_{\overline{MS}} - \frac{d}{dx} \underbrace{g_{1T}^{(1)}}_{\substack{x-dependent \\ cut-off}} \overset{\mu\to\infty}{=} \pi x \ln(1-x)$$

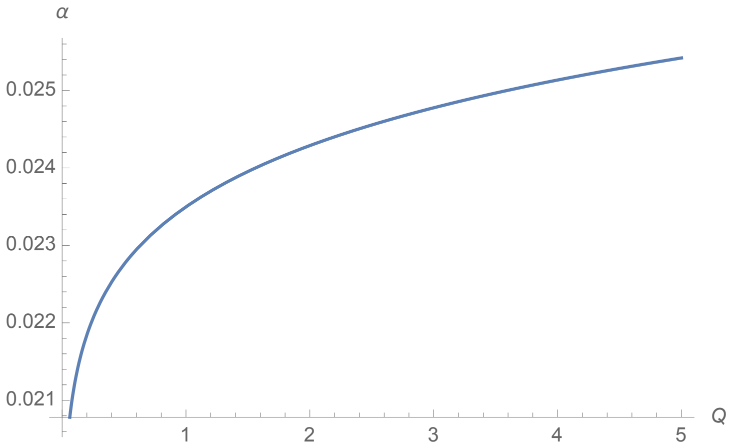
John C. Collins, What exactly is a parton density? (2003)

Violation of LIRs in a renormalizable theory: Scalar Yukawa model



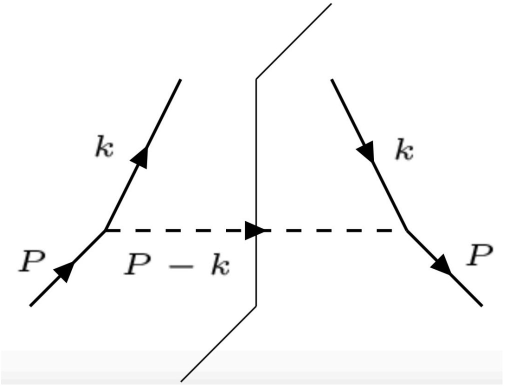
Yukawa coupling

$$\alpha(\mu) \sim \frac{\alpha(\mu_0)}{(1-\alpha(\mu_0)\log(\mu^2/\Lambda^2))} :$$



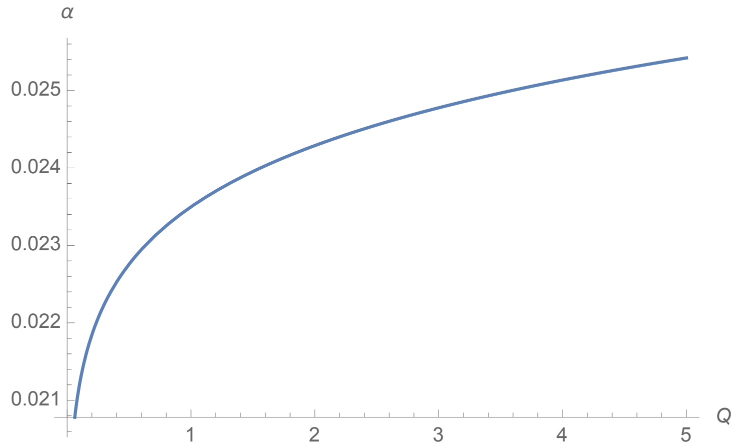
$$\underbrace{g_T(x)}_{\overline{MS}} - \underbrace{g_1(x)}_{\overline{MS}} - \frac{d}{dx}\underbrace{g_{1T}^{(1)}}_{\substack{Fixed\\ cut-off}} = -\frac{g^2(2x+\frac{m_q}{M}-1)}{16\pi^2} + \frac{g^2(x+\frac{m_q}{M})[M^2(x-1)(3x-1)-2m_q^2(x-1)+m_s^2(2x-1)]}{16\pi^2\mu^2} + \mathcal{O}(\frac{m^4}{\mu^4})$$

Violation of LIRs in a renormalizable theory: Scalar Yukawa model

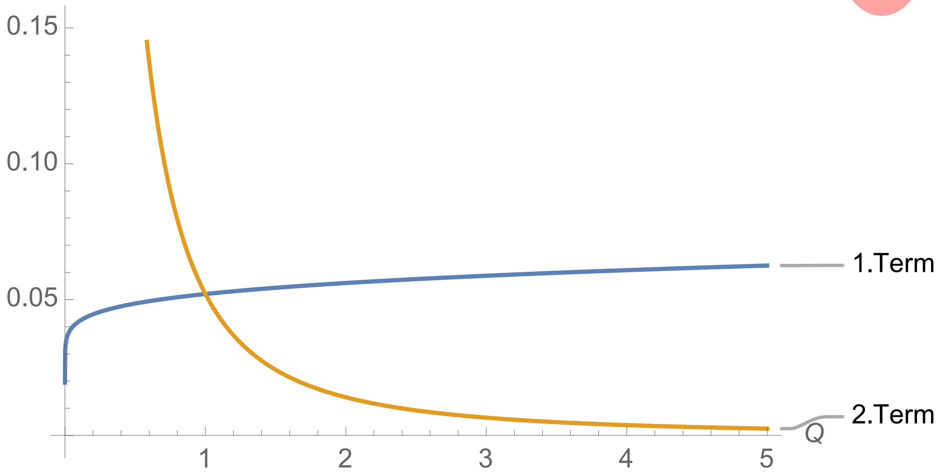


Yukawa coupling

$$\alpha(\mu) \sim \frac{\alpha(\mu_0)}{(1 - \alpha(\mu_0)\log(\mu^2/\Lambda^2))}$$

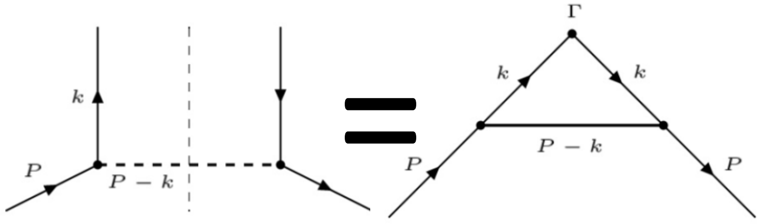


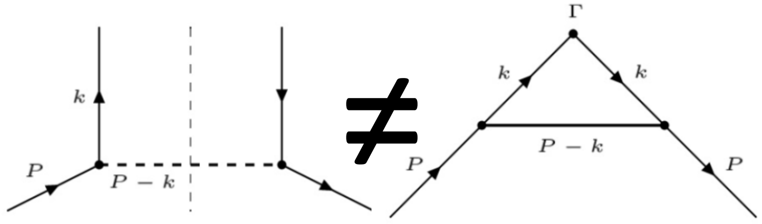
$$\underbrace{g_T(x)}_{\overline{MS}} - \underbrace{g_1(x)}_{\overline{MS}} - \frac{d}{dx}\underbrace{g_{1T}^{(1)}}_{\substack{Fixed\\ cut-off}} = -\frac{g^2(2x + \frac{m_q}{M} - 1)}{16\pi^2} + \frac{g^2(x + \frac{m_q}{M})[M^2(x-1)(3x-1) - 2m_q^2(x-1) + m_s^2(2x-1)]}{16\pi^2\mu^2} + \mathcal{O}(\frac{m^4}{\mu^4})$$



Another source of violation: The zero modes in twist-3 distributions

Twist-2 pdfs	Twist-3 pdfs





Twist-2 PDF	*Is there a $\delta(x)$ in Yukawa model?*
f_1	No
g_1	No
h_1	No

Twist-3 PDF	*Is there a $\delta(x)$ in Yukawa model?*
e	Yes
h_L	Yes
g_T	Yes

$$\underbrace{g_T(x)}_{\substack{twist-3\\ pdf}} = \underbrace{g_1(x)}_{\substack{twist-2\\ pdf}} + \frac{d}{dx}\underbrace{g_{1T}^{(1)}(x)}_{\substack{twist-2\\ tmd}}$$

Aslan, Burkardt, Singularities in Twist-3 Quark Distributions, 2018

Summary and Conclusion

- The LIRs are valid when the transverse integrals are convergent.

- Treatment of UV divergent integrals leads to violation of LIRs in two ways:

Distributions	Transverse integrals
Collinear PDFs	$\overline{MS}$
TMD PDFs	Cut off

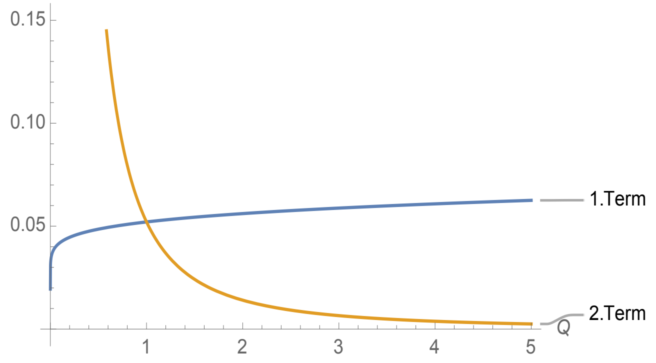
- Taking the transverse integrals using a fixed cut off

$$\underbrace{g_T(x)}_{Fixed\ cut-off} - \underbrace{g_1(x)}_{Fixed\ cut-off} - \frac{d}{dx}\underbrace{g_{1T}^{(1)}}_{Fixed\ cut-off} \overset{\mu\to\infty}{=} -\pi\frac{(1+x)}{2}$$

Transverse cut off	LIR	DGLAP
Fixed	✗	✓
x-dependent	✓	✗

- Using $\overline{MS}$ renormalization scheme for the PDFs

$$\underbrace{g_T(x)}_{\overline{MS}} - \underbrace{g_1(x)}_{\overline{MS}} - \frac{d}{dx}\underbrace{g_{1T}^{(1)}}_{x-dependent\ cut-off} \overset{\mu\to\infty}{=} \pi x\ln(1-x)$$



- In scalar Yukawa model the violation of LIRs increase with increasing energies

Outlook

- Studying the treatments which lead to the violation of LIRs in QCD

- **EoM** relations potentially have the **same problems:** Checking how the treatment of UV divergent integrals affect the EoM relations

$$e(x) = \tilde{e}(x) + \frac{m}{Mx} f_1(x)$$

$$h_L(x) = \tilde{h}_L(x) - \frac{2}{x} h_{1L}^{\perp(1)} + \frac{m}{Mx} g_1(x)$$

$$g_T(x) = \tilde{g}_T(x) + \frac{1}{x} g_{1T}^{(1)} + \frac{m}{Mx} h_1(x)$$

- Considering which UV treatment is optimal, given the problems that arise when using standard renormalization treatments

THANK YOU