Concepts and misconceptions about the proton mass sum rule

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2021 GHP meeting, April 16

What is a proton mass?

- Mass is the energy in the rest frame.

$$M = E_0/c^2$$

Other ways of defining are either equivalent or unacceptable.

This is how lattice QCD calculates mass!

$$M = \frac{\langle \vec{p}=0 | \hat{H}_{QCD} | \vec{p}=0 \rangle}{\langle \vec{p}=0 | \vec{p}=0 \rangle}$$

Scalar and tensor mass

- Since H is proportional to T^{00}, and the energy-momentum tensor (EMT) which can

in general be *uniquely* decomposed as

$$T^{\alpha\beta}(x) = \bar{T}^{\alpha\beta}(x) + \hat{T}^{\alpha\beta}(x) \ ,$$

with

$$\hat{T}^{\alpha\beta}(x) \equiv \frac{1}{4} g^{\alpha\beta} T_{\rho}^{\rho}(x) \ .$$

- Mass can be decomposed into scalar and tensor parts $\quad M = M_T + M_S$
- Virial theorem: $\quad M_T = 3\, M_S \qquad$ 3: space dimension

QCD energies in the nucleon

- Four different types (X. Ji, PRL, 1995)

$$H_{\rm QCD} = H_q + H_m + H_g + H_a \,.$$

$$H_q = \int d^3\vec{x}\ \bar{\psi}(-i\mathbf{D}\cdot\alpha)\psi,$$ ← Quark energy

$$H_m = \int d^3\vec{x}\ \bar{\psi}m\psi,$$ ← Quark mass

$$H_g = \int d^3\vec{x}\ \frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2),$$ ← Gluon energy

$$H_a = \int d^3\vec{x}\ \frac{9\alpha_s}{16\pi}(\mathbf{E}^2-\mathbf{B}^2).$$ ← Quantum Anomalous Energy (QAE)

Quantum anomalous energy (QAE)

- Is similar to the MIT bag model constant and dark energy in Cosmology (Ji, 1995, K.F. Liu, 2021)
- Is at the origin of the proton mass (Ji & Y. Liu, 2021)
- Can measured in threshold heavy quarkonium production (D. Kharzeev, 1996)
- Has been recently calculated in lattice QCD (Y. B. Yang et al, 2021)
- Can also be related to the momentum fractions carried by partons (Ji, 2021)

Frontiers of Physics
https://doi.org/10.1007/s11467-020-1065-x
Front. Phys.
16(6), 64601 (2021)

VIEW & PERSPECTIVE

Proton mass decomposition: Naturalness and interpretations

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I discuss the scope and naturalness of the proton mass decomposition (or sum rule) published in *phys. Rev. Lett.* 74, 1071 (1995) and answer a few criticisms that appeared recently in the literature, focusing particularly on its interpretation and the quantum anomalous energy contribution. I comment on the so-called frame-independent or invariant-mass decomposition from the trace of the energy–momentum tensor. I stress the importance of measuring the quantum anomalous energy through experiments. Finally, I point out a large discrepancy in the scalar radius of the nucleon extracted from vector-meson productions and lattice QCD calculations.

physical quantity which can be calculated on the lattice and ultimately be determined experimentally. Following and expanding the arguments given in Ref. [1], I will stress that the answer is affirmative. In doing so I will discuss in detail alternative proposals and will explain why I do not think that they are helpful to better understand the relevant physics.

In Section 2, I review the original derivation, emphasizing the key point that mass is the rest energy and there exists a complete energy basis to express the mass in QCD. In Section 3, I discuss why there is a quantum anomalous energy contribution and comment on its natural appearance in QCD Hamiltonian through time dilatation. In Section 4, I consider a well-known relation involving the matrix element of the trace of the QCD energy-momentum tensor, arguing it is not a natural frame-independent mass decomposition, but rather about scale symmetry breaking effects. In Section 5, I discuss the so-called "pressure contribution" to the mass sum rule and argue that it is based on a questionable picture. Consideration of such an effect contradicts the well-known concept of the quark mass con-

Frame-independent mass?

- Frame-independence

$$M^2 = E_P^2 - \vec{P}^2 = \left(\mathrm{M} + \frac{\mathrm{P}^2}{2\mathrm{M}} + \cdots\right)^2 - \mathrm{P}^2$$

checking the relativity is obeyed

and boosted nucleon can be well created on lattice.

- But it does not provide any additional insight about the mass itself!

A frame independent mass decomposition?

- Mass relation

$$2M^2 = \left\langle P \left| (1+\gamma_m) m\bar{\psi}\psi + \frac{\beta(g)}{2g} F^2 \right| P \right\rangle$$

M^2 = quark + gluon contributions

- Why $m\bar{\psi}\psi$, F^2 are related to mass-squared?
- A correct way to look at this

$$M = \left\langle P \left| (1+\gamma_m) m\bar{\psi}\psi + \frac{\beta}{2g} F^2 \right| P \right\rangle / 2M$$

scalar part of the Hamiltonian!

= $4 \times M_s$ (rest frame)

Boosting mass components

- Frame-independent mass

$$M = \langle \gamma (H - \vec{\beta} \cdot \vec{P}) \rangle \ ,$$

- When H and P are separated into different pieces

$$M = \langle \gamma (H_q - \vec{\beta} \cdot \vec{P}_q) \rangle + \langle \gamma (H_g - \vec{\beta} \cdot \vec{P}_g) \rangle + \gamma \langle H_a \rangle$$

- Individual contributions are frame independent!

$$\begin{aligned} \langle \gamma (H_q - \vec{\beta} \cdot \vec{P}_q) \rangle &= \langle x \rangle_q M \left[\gamma (\gamma - 1/(4\gamma)) - \gamma^2 \beta^2 \right. \\ &= \frac{3}{4} \langle x \rangle_q M \qquad (\end{aligned}$$

Gluon contributions to mass

- There are two terms:

$\frac{1}{2}\left(E^2 + B^2\right)$ as defined from traceless part of the gluon EMT

$\frac{\beta}{8g}F^2$ anomaly contribution

- They can be mixed under renormalization in certain Lorentz-symmetry-breaking renormalization scheme, like in DIM-REG, O(d-1,1) is not O(3,1)
- Due to anomaly, order ε mixing will generate a finite term: the 2nd order tensor is mixed with scalar!

Mass decomposition using GFF separation of mass

- Quark and gluon part of gravitational form factors

$$\left\langle P' \left| T^{\mu\nu}_{q,g} \right| P \right\rangle = \bar{u}\left(P'\right) \left[A_{q,g}(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(t)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M_N} + C_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M_N} + \bar{C}_{q,g}(t)M_N g^{\mu\nu} \right] u(P) \ .$$

Then T^{00} matrix element is related A & $\bar{C}$ (ji96)

- Decomposition of mass into quark and gluon contributions

$$M = M_q + M_g$$

$$\mathrm{M_q} = A_q + \bar{C}_q; \quad \mathrm{M_g} = A_g + \bar{C}_g$$

Splitting the scalar contribution

- One needs to split the scale-independent scalar contribution into quark's and gluon's

$$\langle P|(1+\gamma_m)m\bar{\psi}\psi + (\beta/2g)F^2|P\rangle$$

- Such a separation is most natural in terms of the above, with both terms nearly scale-independent
- Separation in terms of EMT in the literature is highly scheme-dependent! Even in DIM-REG, it depends on the detailed procedure.
- The value of the scheme-dependent splitting for scalar contribution is limited.

There is no pressure effect in mass decomposition

- It has been claimed separating the energy operator into trace and traceless part will introduce the pressure effects!

$$T^{00} = \frac{1}{4}\left(3T^{00} + T^{ii}\right) + \frac{1}{4}\left(T^{00} - T^{ii}\right)$$

- Not a problem: $T^{\mu\nu} = \bar{\psi}\gamma^{\mu} i D^{\nu}\psi.$

  the pressure $T^{ii} = \psi^{+}\alpha \cdot D\psi$ which is part of the Hamiltonian

$$H = \psi^{\dagger}(i\vec{\alpha}\cdot\vec{D} + m\beta)\psi \ ,$$

Mass or scalar radius

- Form factors

$$\langle P' | T^{\mu\nu} | P \rangle = \bar{u}(P') \Big[A(Q^2)\, \gamma^{(\mu} \bar{P}^{\nu)} + B(Q^2)\, \bar{P}^{(\mu} i\sigma^{\nu)\alpha} q_\alpha / 2M + C(Q^2)\, (q^\mu q^\nu - g^{\mu\nu} q^2) / M \Big] u(P) \,,$$

- Scalar and mass radius

$$\langle r^2 \rangle_{s,m} = -6 \frac{dG_{s,m}(Q^2)/M}{dQ^2} \,,$$

$$\langle r^2 \rangle_s - \langle r^2 \rangle_m = -12 \frac{C(0)}{M^2} \,,$$

$$\langle r^2 \rangle_s = -6 \frac{dA(Q^2)}{dQ^2} - 18 \frac{C(0)}{M^2} \,,$$

$$\langle r^2 \rangle_m = -6 \frac{dA(Q^2)}{dQ^2} - 6 \frac{C(0)}{M^2} \,,$$

Conclusion

- Much confusions exist in the literature about the proton mass.
- More papers and ideas but less discussions
- Hope things will improve after COVID-19