A High Precision Measurement of the Proton Charge Radius at JLab

Weizhi Xiong
Syracuse University
for the PRad Collaboration

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Outline

• Introduction and the proton charge radius puzzle
• PRad experiment and apparatus
• Analysis and results
• Future improvements
• Summary
Proton Charge Radius Puzzle in 2018

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- **Pohl 2010 (μH spect.)**
- **Antognini 2013 (μH spect.)**
- **Beyer 2017 (H spect.)**
- **Bernauer 2010 (ep scatt.)**
- **Zhan 2011 (ep scatt.)**
- **CODATA-2014 (ep scatt.)**
- **CODATA-2014 (H spect.)**
- **CODATA-2014**
- **Fleurbaey 2018 (H spect.)**

Proton charge radius $r_p$ [fm]

5.6 $\sigma$
Elastic ep scattering, in the limit of Born approximation (one photon exchange):

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left( \frac{E'}{E} \right) \frac{1}{1 + \tau} \left( G_{E}^{p} (Q^2) + \frac{\tau}{\alpha} G_{M}^{p} (Q^2) \right) \]

\[ Q^2 = 4EE' \sin^2 \frac{\theta}{2} \quad \tau = \frac{Q^2}{4M_p^2} \quad \alpha = \left[ 1 + 2(1 + \tau) \tan^2 \frac{\theta}{2} \right]^{-1} \]

- Structure-less proton:

\[ \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \frac{\alpha^2 \left[ 1 - \beta^2 \sin^2 \frac{\theta}{2} \right]}{4k^2 \sin^4 \frac{\theta}{2}} \]

- \( G_E \) and \( G_M \) can be extracted using Rosenbluth separation

- For PRad, cross section dominated by \( G_E \)

\[ G_E^p (Q^2) = 1 - \frac{Q^2}{6} \langle r^2 \rangle + \frac{Q^4}{120} \langle r^4 \rangle + ... \]

Derivative at low \( Q^2 \) limit:

\[ \langle r^2 \rangle = -6 \left. \frac{dG_E^p (Q^2)}{dQ^2} \right|_{Q^2=0} \]
PRad Experiment Overview

- PRad goal: Measuring proton charge radius using ep elastic scattering

- Covers two orders of magnitude in low $Q^2$ with the same detector setting
  - $\sim 2 \times 10^{-4} - 6 \times 10^{-2}$ GeV$^2$

- Unprecedented low $Q^2$ ($\sim 2 \times 10^{-4}$ GeV$^2$)
  - Fill in very low $Q^2$ region

- Normalize to the simultaneously measured Møller scattering process
  - best known control of systematics

- Windowless H$_2$ gas flow target removes major background source

- Extract the radius with precision from sub-percent cross section measurement
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Bernauer data for lowest spectrometer setting

K. Griffioen et al.
PRC 93, 065207, 2016
PRad Experimental Apparatus

Introduction | PRad and Apparatus | Analysis and results | Future Improvements
Major Steps in Analysis

- Calibration
- Event Selection
- Background Subtraction
- Getting $ep/ee$ Ratio
- Radiative Correction
- Elastic $ep$ cross section
- Proton Electric Form Factor
- Fitting for the Radius
Event selection method

1. For all events, require hit matching between GEMs and HyCal

2. For $ep$ and $ee$ events, apply angle dependent energy cut based on kinematics
   1. Cut size depend on local detector resolution

3. For $ee$, requiring double-arm events, apply additional cuts
   1. Elasticity
   2. Co-planarity
   3. Vertex $z$
**Analysis – Background Subtraction (2.2 GeV)**

- *ep* background rate ~ 10% at forward angle (<1.1 deg, dominated by upstream beam halo blocker), less than 2% otherwise
- *ee* background rate ~ 0.8% at all angles

**Residual hydrogen gas:** hydrogen gas filled during background runs

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**Graph:**

- (b)
- (c)
- (d)
- (b) - (c)

- **2.2 GeV ep data**

**Diagrams:**

- **(a)**
- **(b)**
- **(c)**
- **(d)**
Analysis – Background Subtraction (2.2 GeV)

- $ep$ background rate $\sim 10\%$ at forward angle ($<1.1 \text{ deg}$, dominated by upstream beam halo blocker), less than $2\%$ otherwise
- $ee$ background rate $\sim 0.8\%$ at all angles

Residual hydrogen gas: hydrogen gas filled during background runs
Analysis – Inelastic ep Contribution

- Using Christy 2018 empirical fit to study inelastic ep contribution
- Good agreement between data and simulation
- Negligible for the PbWO$_4$ region ($<3.5^\circ$), less than 0.2%(2.0%) for 1.1GeV(2.2GeV) in the Lead glass region

Spectrum for $3.0^\circ < \theta < 3.3^\circ$ ($Q^2 \sim 0.014$ GeV$^2$)

Spectrum for $6.0^\circ < \theta < 7.0^\circ$ ($Q^2 \sim 0.059$ GeV$^2$)

M. E. Christy and P. E. Bosted, PRC 81, 055213 (2010)
Extraction of $ep$ Elastic Scattering Cross Section

- To reduce the systematic uncertainty, the $ep$ cross section is normalized to the Møller cross section:

$$
\left( \frac{d\sigma}{d\Omega} \right)_{ep} = \left[ \frac{N_{\text{exp}}(ep \rightarrow ep \text{ in } \theta_i \pm \Delta \theta_i)}{N_{\text{exp}}(ee \rightarrow ee)} \cdot \frac{\varepsilon_{ee}^{\text{geom}}}{\varepsilon_{ep}^{\text{geom}}} \cdot \frac{\varepsilon_{det}^{ee}}{\varepsilon_{det}^{ep}} \right] \left( \frac{d\sigma}{d\Omega} \right)_{ee}
$$

- Method 1: bin by bin method – taking $ep/ee$ counts from the same angle bin
  - Cancellation of energy independent part of the efficiency and acceptance
  - Limited converge due to double arm Møller acceptance

- Method 2: integrated Møller method – integrate Møller in a fixed angle range and use it as common normalization for all angle bins

- Luminosity cancelled from both methods
Radiative Correction

• Radiative effects corrected by Monte-Carlo method:

1. Geant4 simulation package with full geometry setup

2. Event generators with complete calculations of radiative corrections\(^1,2\), include emission of radiative photons

3. Consistent results between generators

4. Include TPE effect\(^3\), less than 0.2% for \( ep \) in PRad kinematic range

5. Iterative procedure applied for radiative correction

\[
\sigma_{ep}^{\text{Born}(exp)} = \left( \frac{\sigma_{ep}}{\sigma_{ee}} \right)^{\text{exp}} / \left( \frac{\sigma_{ep}}{\sigma_{ee}} \right)^{\text{sim}} \cdot \left( \frac{\sigma_{ep}}{\sigma_{ee}} \right)^{\text{Born}(model)} \cdot \sigma_{ee}^{\text{Born}(model)}
\]

References:
Systematic Uncertainties

• For PRad, systematic uncertainties may come from:
  1. Event selection (elasticity cuts, co-planarity cuts…)
  2. Radiative correction
  3. Detector efficiencies (GEM and HyCal)
  4. Beam-line background (Halo hitting collimator, residual gas…)
  5. HyCal energy calibration
  6. Detector position
  7. Beam energy
  8. Inelastic $ep$ contribution
  9. Assumed magnetic form factors during the $G_E$ extraction
  10. …
Systematic Uncertainties
(Example of Event Selection)

• Changing elasticity cut at the radiative tail and obtain different sets of cross section results

• Sensitivity on cross section: typically within +/- 0.15%

• Mostly due to non-uniformity of HyCal modules
Checking Systematics – Azimuthal Symmetry

1.1 GeV data

Reconstructed scattering angle [deg]

- Quadrant 1
- Quadrant 2
- Quadrant 3
- Quadrant 4

Data/Signal ratio (ep/ee)_{data/num}
Checking Systematics – Different methods of Forming $ep/ee$ ratio

- Method 1: bin-by-bin method – taking $ep/ee$ counts from the same angular bin
- Method 2: integrated Møller method – integrate Møller in a fixed angular range and use it as common normalization for all angle bins
- Luminosity cancelled in both methods

1.1 GeV data

![Graph showing $ep/ee$ ratio for 1.1 GeV data]

2.2 GeV data

![Graph showing $ep/ee$ ratio for 2.2 GeV data]
Differential Cross Sections

- Differential cross section v.s. $Q^2$, with 2.2 and 1.1 GeV data
- Statistical uncertainties: ~0.15% for 2.2 GeV, ~0.2% for 1.1 GeV per point
- Systematic uncertainties: 0.3% ~ 1.1% for 2.2 GeV, 0.3% ~0.5% for 1.1 GeV

![Graph showing differential cross sections](image)
Searching the Robust fitters

- Rational (1,1), 2$^{\text{nd}}$ order $z$ transformation and 2$^{\text{nd}}$ order continuous fraction are identified as robust fitters with also reasonable uncertainties
- Typically a floating parameter $n$ is included to take care normalization uncertainties

\[ f(Q^2) = n \, G_E^P(Q^2) \]

### Rational (1,1)

\[ \frac{1 + p_1 Q^2}{1 + p_2 Q^2} \]

### 2$^{\text{nd}}$ order $z$ transformation

\[ 1 + p_1 z + p_2 z^2, \quad z = \frac{\sqrt{T_c + Q^2} - \sqrt{T_c - T_0}}{\sqrt{T_c + Q^2} + \sqrt{T_c - T_0}} \]

### 2$^{\text{nd}}$ order continuous fraction

\[ \frac{1}{1 + \frac{p_1 Q^2}{1 + p_2 Q^2}} \]
Proton Electric Form Factor $G_E^p$

- $n_1$ and $n_2$ obtained by fitting PRad electric form factors to $f(Q^2) = \begin{cases} n_1 G_E^p(Q^2), & \text{for } 1.1 \text{ GeV data} \\ n_2 G_E^p(Q^2), & \text{for } 2.2 \text{ GeV data} \end{cases}$

- $G_E^p$ as normalized electric form factor: $\begin{cases} f(Q^2)/n_1, & \text{for } 1.1 \text{ GeV data} \\ f(Q^2)/n_2, & \text{for } 2.2 \text{ GeV data} \end{cases}$

- $G_E^p(Q^2) = \frac{1+p_1Q^2}{1+p_2Q^2}$, the rational $(1,1)$, a robust fitter based on X. Yan et al. Phys. Rev. C98, 025204 (2018)
Proton Electric Form Factor $G_E^p$

$r_p = 0.831 +/- 0.007 \text{ (stat.)} +/- 0.012 \text{ (syst.)} \text{ fm}$

$n_1 = 1.0002 +/- 0.0002 \text{(stat.)} +/- 0.0020 \text{ (syst.)}, \quad n_2 = 0.9983 +/- 0.0002 \text{(stat.)} +/- 0.0013 \text{ (syst.)}$
# Systematic Uncertainties

<table>
<thead>
<tr>
<th>Item</th>
<th>$r_p$ uncertainty [fm]</th>
<th>$n_1$ uncertainty</th>
<th>$n_2$ uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event selection</td>
<td>0.0070</td>
<td>0.0002</td>
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<tr>
<td>Radiative correction</td>
<td>0.0069</td>
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<td>Detector efficiency</td>
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<tr>
<td>Beam background</td>
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<td>HyCal response</td>
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<tr>
<td>Acceptance</td>
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<td>Beam energy</td>
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<td>Inelastic $ep$</td>
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<td>$G_M^p$ parameterization</td>
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<tr>
<td>Total</td>
<td>0.0115</td>
<td>0.0020</td>
<td>0.0013</td>
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PRad Proton Charge Radius

PRad result: \( r_p = 0.831 +/- 0.007 \text{ (stat.)} +/- 0.012 \text{ (syst.)} \text{ fm} \)

- Pohl 2010 (\( \mu \)H spect.)
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- Bezginov 2019 (H spect.)
- PRad exp. (ep scatt.)
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- CODATA-2014 (ep scatt.)
- CODATA-2014 (H spect.)
- CODATA-2014
- Fleurbaey 2018 (H spect.)
- Mihovilovic 2019 (ep scatt.)
- Grinin 2020 (H spect.)

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PRad-II Experiment

- JLab PAC 48 approved PRad-II (PR12-20-004) with the highest scientific rating “A”
- Goal: reach ultra-high precision (~4 times smaller total uncertainty), resolve tension with modern $e-p$ scattering results
- The new proposal includes:
  1. Adding tracking capacity (second GEM plane)

![Diagram of PRad-II experiment setup with GEM and HyCal components]
• The new proposal includes:
  1. Adding tracking capacity (second GEM plane)
  2. Upgraded HyCal with all high resolution PbWO₄ modules
  3. Convert to FADC based readout for HyCal
  4. Four times smaller stat. uncertainty
  5. Better RC calculating including NNLO diagrams
Expected total uncertainty: 0.0036 fm
Projected result with full detector upgrades
Summary

• The PRad collaboration carried out the first electron scattering experiment using a non-magnetic spectrometer approach – calorimeter and GEMs
  1. Covers two orders of magnitude in low $Q^2$ with the same detector setting
  2. Unprecedented low $Q^2$ data set ($\sim 2 \times 10^{-4}$ GeV$^2$) has been collected in $e-p$ elastic scattering experiment
  3. Novel use of a window-less cryogenically cooled hydrogen gas target
  4. Simultaneous measurements of $e+p$ and $e+e$ scattering to reduce systematics

• The PRad result: $r_p = 0.831 \pm 0.007$ (stat.) $\pm 0.012$ (syst.) fm

• Planning on follow-up experiment, aim for $\sim 4$ times of improvement on total $r_p$ uncertainty

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