Statistical Engineering

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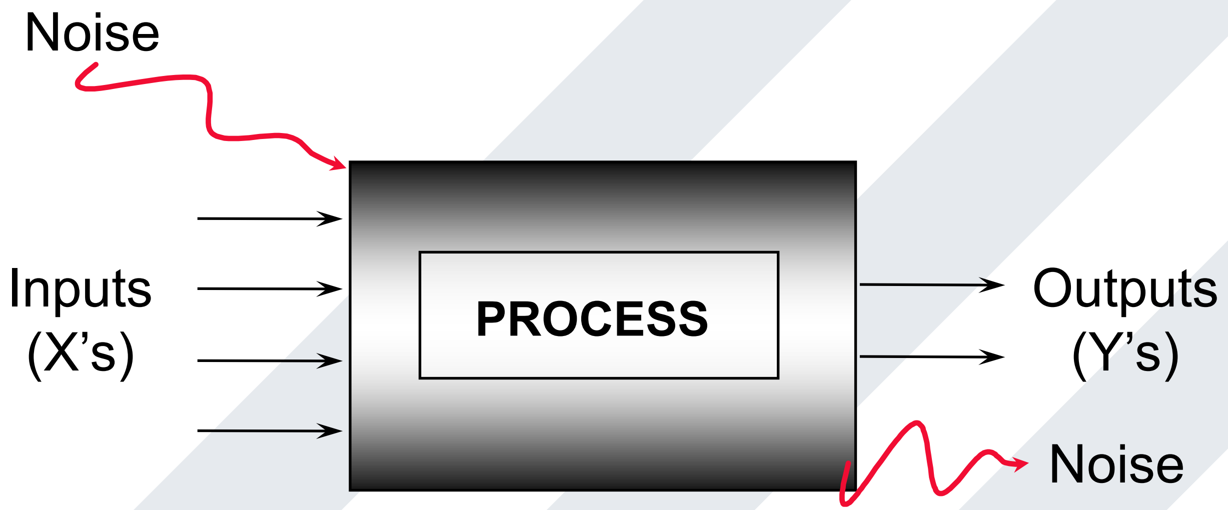
Statistical Engineering

- The discipline of *Statistical Engineering* involves systematic integration of statistical concepts, methods, and tools with all engineering and scientific disciplines.
- In many engineering groups, new integration problems are often attacked with a "best effort" or trial and error approach
  - Statistical engineering provides a comprehensive framework for effective experimentation, characterization and optimization
  - Uncertainty quantification is inherent to the methods
- Today we will focus on experiment design, empirical model building and optimization

What are statistically designed experiments ?

- Purposeful and systematic changes in the inputs (factors) to a process in order to observe corresponding changes in the outputs (responses)

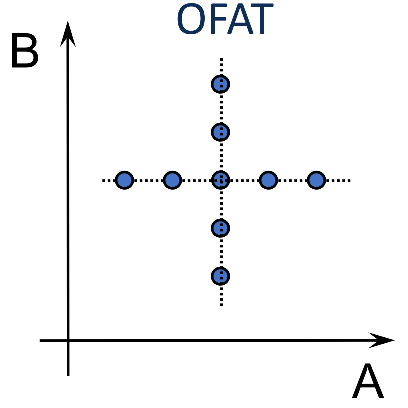
- Results in the development of a mathematical model (response surface) that predicts system responses for specified factor settings



Factorial Designs

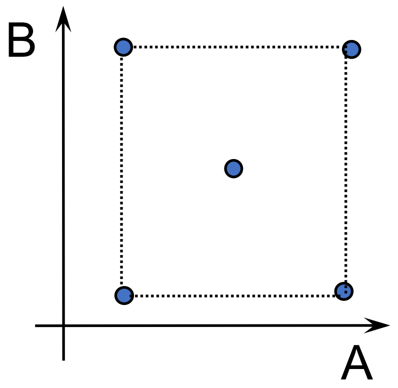
- A factorial design is one in which each trial of an experiment includes all possible combinations of factor levels
- Unearths an empirical math model for the response in terms of all the factors and includes interactions
  - Typical one-factor-at-a-time (OFAT) designs struggle to identify interactions
  - Interactions may be more important than individual factor effects
  - Factorials are run efficient

OFAT



$Y = K_A A + K_B B$

Factorial



$Y = K_A \cdot A + K_B \cdot B + K_{AB} \cdot A \cdot B$

Sequential Experimentation

- Factorial designs support model adequacy testing
- The factorial design is easily augmented to include higher order terms
- Sequential investment in resources
- Efficient route to complex system understanding with many factors

Factorial

Augmented to Second Order Design

$Y = K_A \cdot A + K_B \cdot B + K_{AB} \cdot A \cdot B$

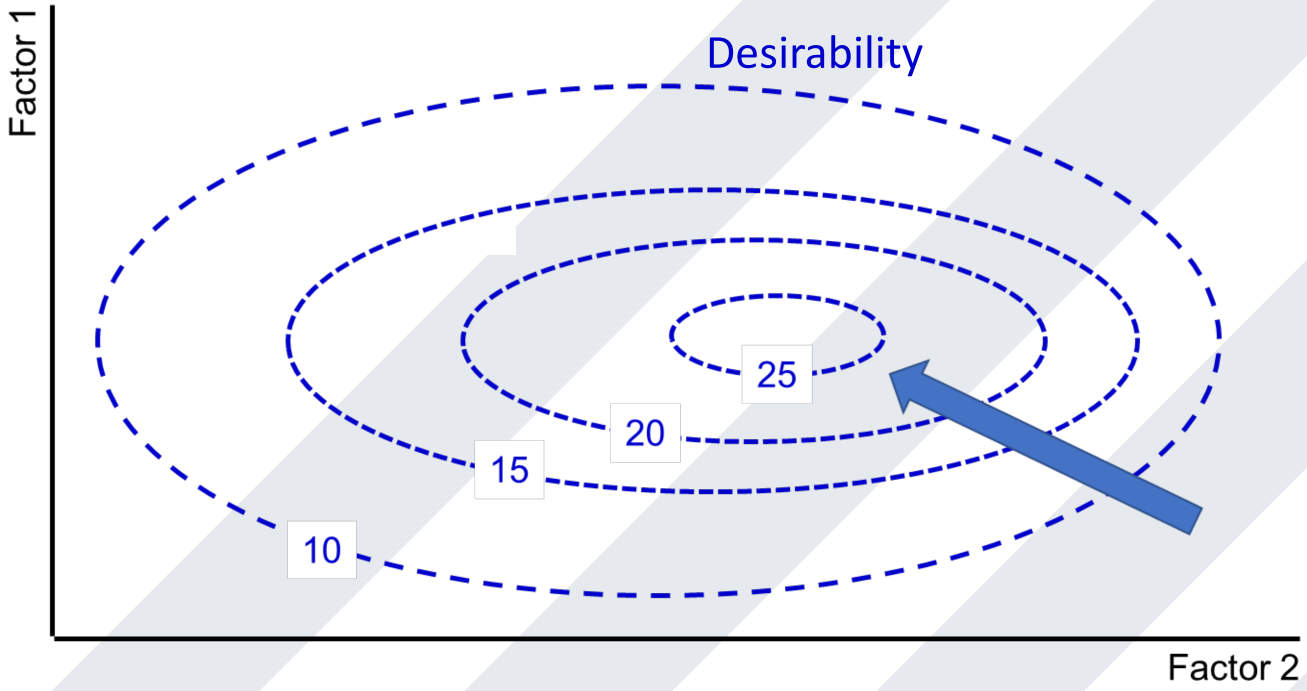
$Y = K_A \cdot A + K_B \cdot B + K_{AB} \cdot A \cdot B + K_{AA} \cdot A^2 + K_{BB} \cdot B^2$

Response Surfaces and Optimization

- Empirical regression models are called response surfaces
- The math model of system behavior provides a framework for optimization
- Multi-Factor/Multi-Response optimization using Desirabilty Approach

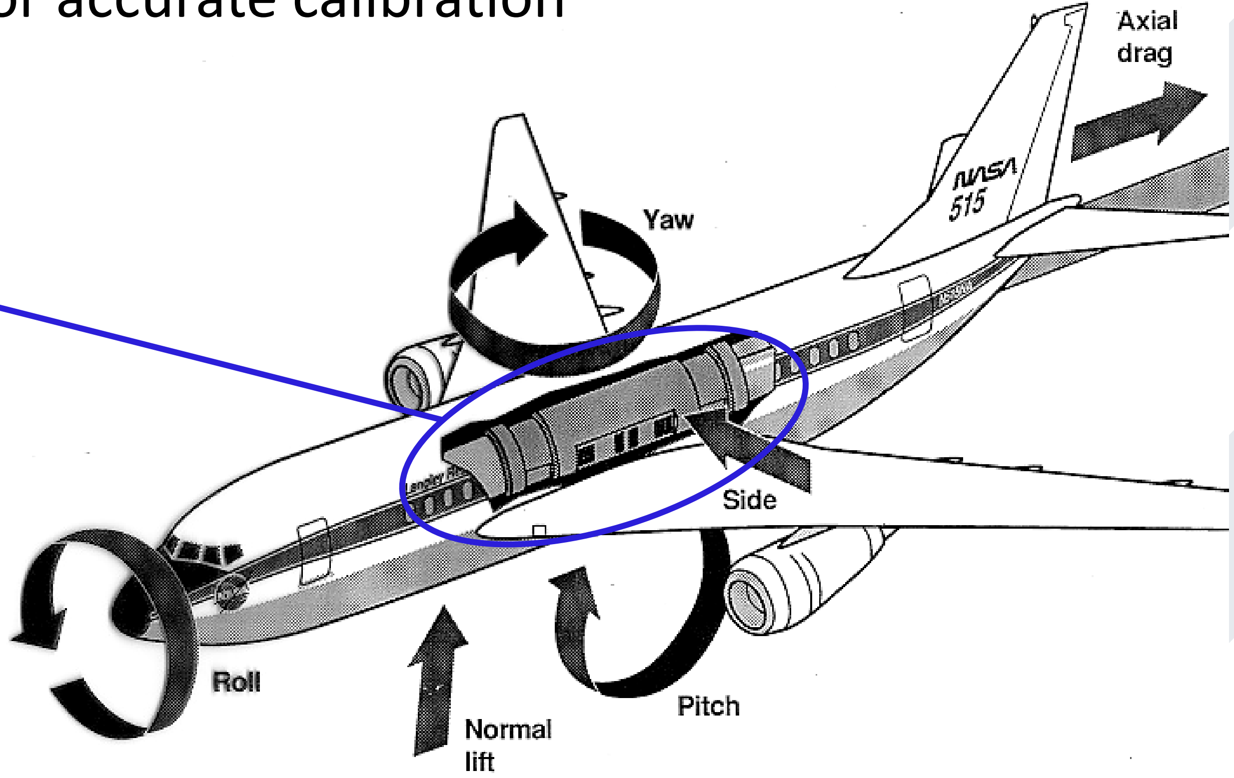
Example: 3 objectives with
objective 1 the most important
and objective 3 the least important
D is overall desirability

$$D = (d_1^{r_1} d_2^{r_2} d_3^{r_3})^{\frac{1}{r_1+r_2+r_3}} = (d_1^5 d_2^3 d_3^1)^{\frac{1}{9}}$$



Example : NASA Wind Tunnel Instrument Calibration

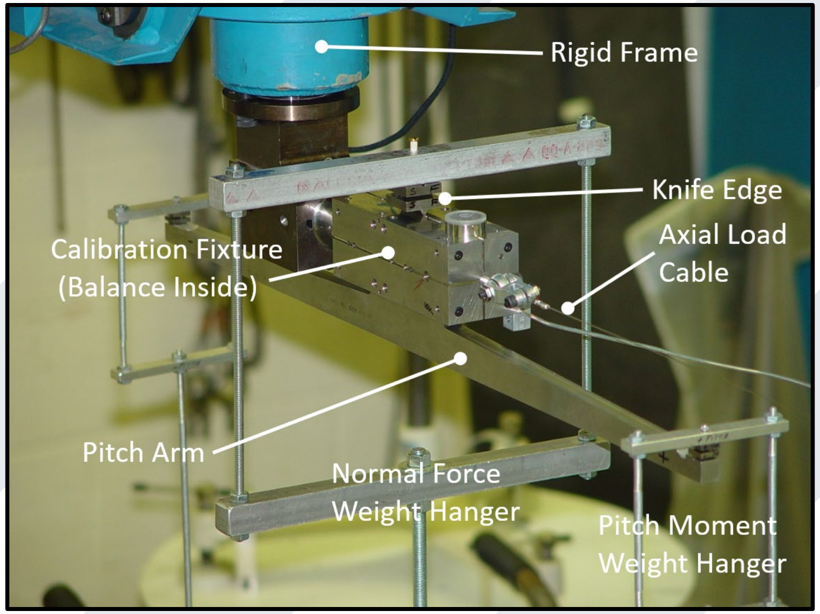
- Forces and moments measured by strain gage transducers
  - Beam elements bend – strain gages mounted to sensitive areas
  - Electrical response for each axis
  - Interactions must be quantified for accurate calibration



NASA Wind Tunnel Instrument Calibration

- Factors: 6 forces and moment loads (x)
- Responses: 6 voltages, one for each component (y)
- Experiment to apply combinations of loads to excite all six responses
- We build a mathematical model for each response (y) as a function of all factors (x) including factor interactions and pure quadratics (27 terms)

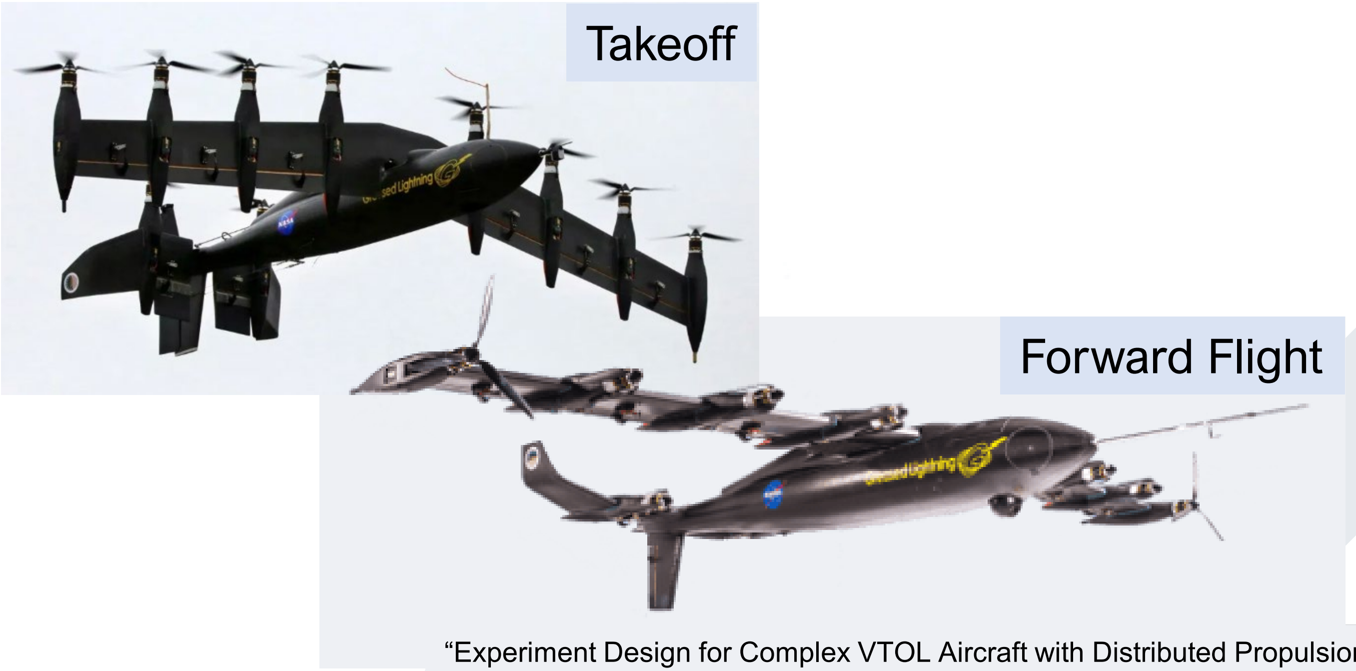
$$y = \beta_0 + \sum_{i=1}^{6} \beta_i x_i + \sum_{i=1}^{6} \beta_{ii} x_i^2 + \sum_{i<j} \sum \beta_{ij} x_i x_j + \varepsilon$$

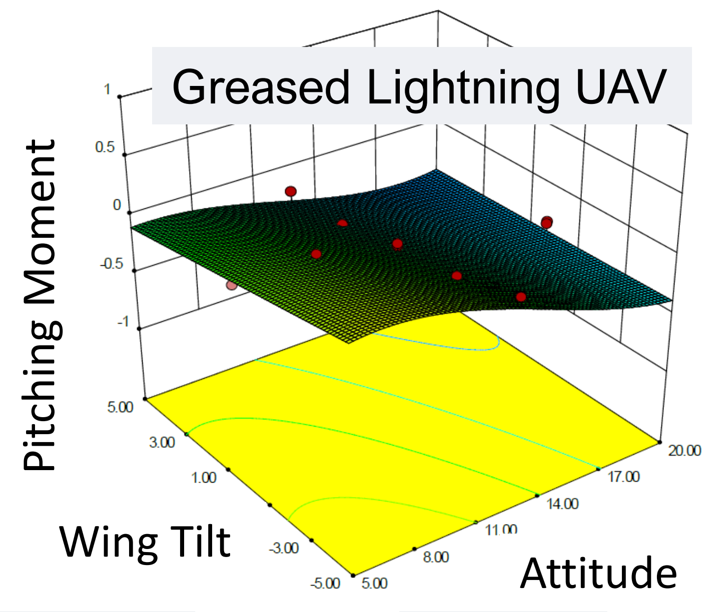


- Uncertainty estimates

Example UAV Aerodynamic Characterization

- Consider a UAV design with distributed propulsion and tilt-wing
  - Over 20 factors: 10 motor speeds, 10 control surfaces, wing tilt, tail tilt, attitude
  - Hover, transition to forward flight and forward flight modes
  - Aerodynamic force and moment response surface model required for optimizing motor and control surface settings

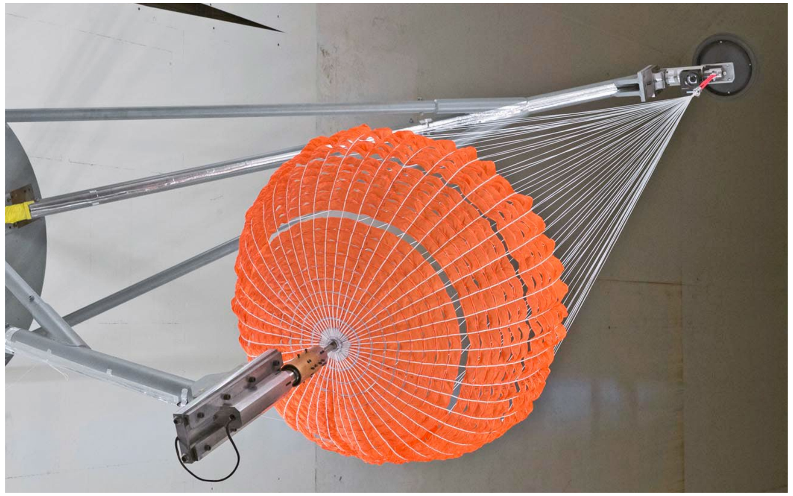


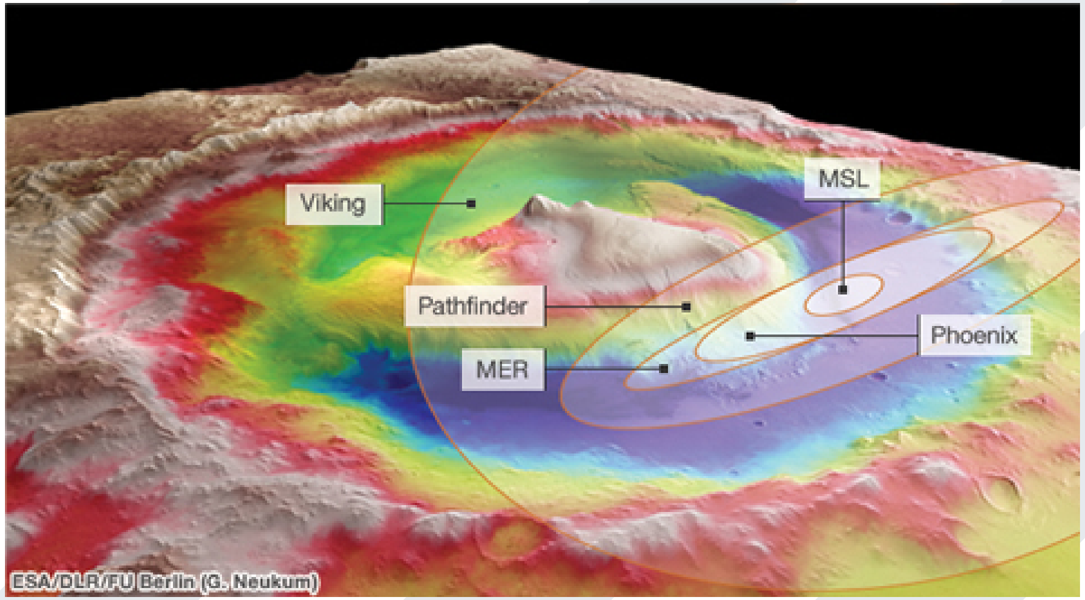


"Experiment Design for Complex VTOL Aircraft with Distributed Propulsion and Tilt Wing," Murphy, P., Landman, D., AIAA paper AIAA 2015-0017, presented at the AIAA Atmospheric Flight Mechanics Conference, Kissimmee, FL, January 5-9, 2015, DOI: 10.2514/6.2015-0017.

Example Mars Parachute Uncertainty

- Mars probe trajectory analysis relies on large Monte Carlo simulations
  - Probability based models that are fed uncertainties from hundreds of sources
  - Minimizing uncertainty in all sources is paramount to landing within a small area
- A designed experiment for the parachute testing provided required uncertainty bounds for simulation for Mars 2020 mission

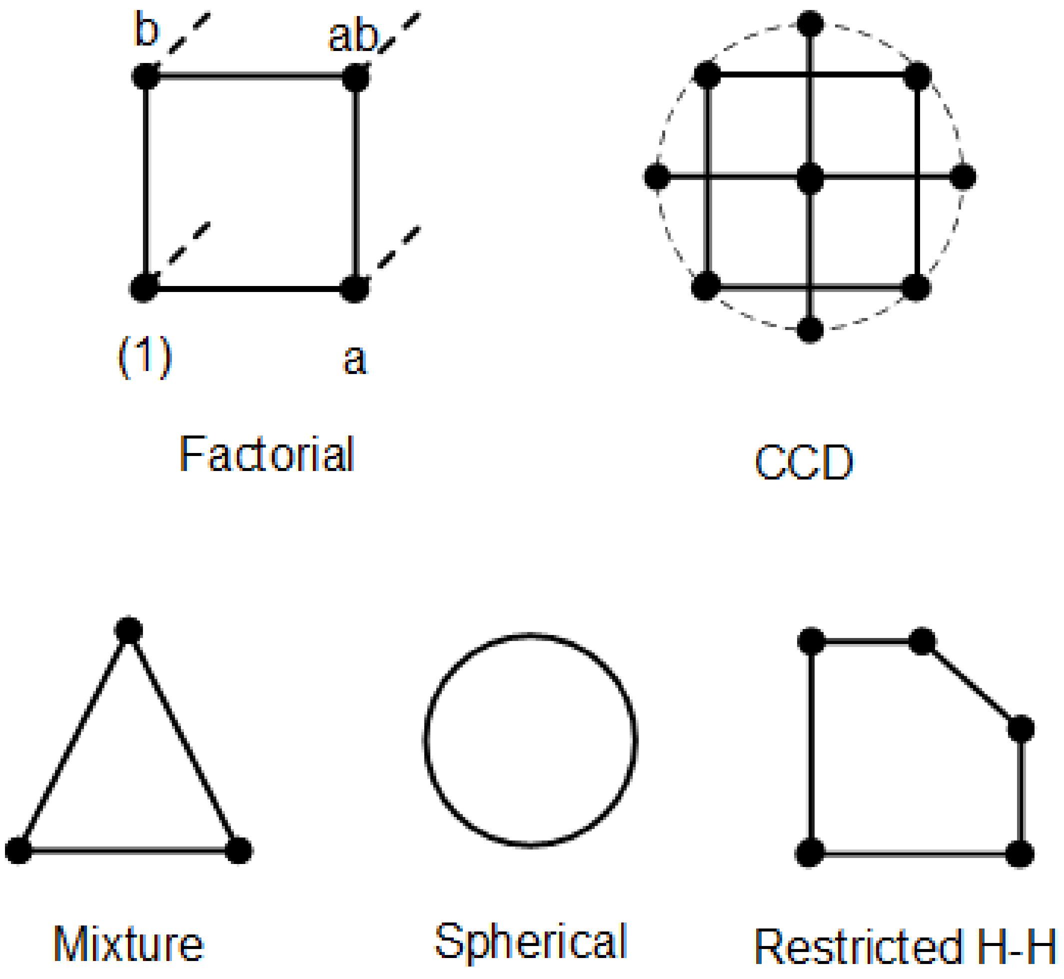




“Statistical Engineering for Wind Tunnel Testing of Mars Parachute Designs,” Landman, D., *AIAA Journal of Aircraft*, May 2020, https://doi.org/10.2514/1.C035913

Optimum Experimental Designs

- Irregular restricted experimental regions
- Nonstandard models
- Unbalanced sample sizes
- Controlled experimental design induced error
- Optimum Bayesian designs
- Optimum Monte Carlo designs
- Discrimination among competing designs



Optimum Experimental Design Regions

Training

- Short Courses in Design of Experiments and Response Surface Methods
  - Clients include NASA, USAF, Airbus, DOD, Institute for Defense Analysis
- Lean Six Sigma Black Belt Training
  - Clients include ODU-TAC, LifeNet Health

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