CHAPTER III: A PHYSICAL ANALOGY FOR RESILIENCE AND VULNERABILITY

This chapter sheds lights on concepts presented in Section 2.2.1 - hysteresis. Specifically, this chapter shows how the concept of hystereis is implemented in QVA model for assessment of co-operative behavior, the tendency to resist stress and maintain system state (configuration and performance level) against driving stress.

3.1 AN ANALOGY IN HYSTERESIS

The general concesus is that any system (e.g., a energy system) will consist of *parts*, P_i, $i = 1,2, \ldots, M$, prefrebly interacting parts. Once defined, the 'parts' may be seen as individual, atomic (indivisible) components, that:

- usually come in large numbers (*M*);
- are coupled with each other with a strength that may conveniently be expressed as a generic, coupling 'energy', ε_{ij}, $i = 1,2, \ldots, M$, $j = 1,2, \ldots, M$;
- respond to external stresses, or influences ('fields'), *H*, each system part featuring an 'energy'

$\mu_i H$, of coupling with the 'field' via a coupling strength μ_i.

In the context, the notion of 'part' embraces a virtually unlimited variety of representations. For an energy system, these may include anything from mines, mills, wells, pipes, power stations, switchyards, transmission lines, distribution facilities, control rooms, dispatching centers, IT assistance facilities – their subassemblies included over entire fuel cycles, to key workers, working units, enterprises, companies, regulators, and political pressure entities. In a first, rough approximation parts either *do function* as per intent and design, or *do not function*, their state being thereby describable via a variable, *S*, that may assume two values only: $S = 1$ indicating a functional part; or $S = -1$ indicating a dysfunctional part – which accommodates systems within the *Ising mode* (Gheorghe and Vamanu, 2008), ubiquitous in Physics and well beyond. Parts may switch from a functional to a dysfunctional state, and conversely, the process being assumed to be, in the final analysis, reversible, and *probabilistic* in nature (Hopkinson and Williams, 1912).

Observant to the natural systems that are coherent enough - within their boundaries of definition - to feature a certain autonomy, or quasi-isolation of their own in respect with the remaining environment, the overall behavior of our model-system may be thought to be governed by *a variational principle*, applicable to system's total energy. According to such a principle, in a steady state of the system the individual states of the parts are such that the system 'energy,' which is given by Equation (36) is a minimum for any given *temperature*:

$$E = -(1/2) \Sigma_{ij} \, \varepsilon_{ij} \, S(i) \, S(j) - H \, \Sigma_i \, \mu_i \, S(i) \qquad (36)$$

The first term in Equation (1) denotes the total *internal* 'energy' of the system of interacting parts, whereas the second term features the total 'energy' imparted to the parts by their coupling to the external, compelling 'field' *H*.

Physicists will immediately note that, in a textbook rendering of an Ising or a Heisenberg model – that are at the origin of our analogy - the normal assumption is that both the coupling (‘exchange’) energy, $\varepsilon_{i\,j,}$ and the field-coupling constant, μ_i do *not* depend on the parts *i*, *j* – a fact that has to do with the assumption that all parts are identical (and in effect indiscriminate) to each other. In this respect, equation (36) is a generalization to a many-body system of *non-identical* parts.

In applying the notion above, note that any part-*i* state-flip (from functional, 1, to dysfunctional, -1, or vice versa) entails a change in system’s energy, of

$$\Delta E = - S(i)\ (\Sigma\,'_{j}\ \varepsilon_{i\,j}\ S(j) + \mu_i H) \tag{37}$$

where $\Sigma\,'_j$ indicates a sum that, in practice, extends over a certain, neighborhood of part *i* – while in principle it may extend over *all* the agents other than *i*.

Following the Ising model philosophy (see e.g. the discussion in references (Gheorghe and Vamanu, 2004; Gheorghe and Vamanu, 2008; Sprott, 1993), a part’s behavior is governed by the following set of rules, consistent with the assumptions above:

Rule 1: If $\Delta E <= 0$, then the part would always undergo a state-flip. (38)
Rule 2: If $\Delta E > 0$, then part flips state only with a probability,

$$P = \exp(-\Delta E/(k_B T)) \tag{39}$$

with T a ‘system temperature,’ and k_B a ‘Boltzmann constant,’ conveniently taken as 1. In practice (e.g., see Metropolis et al. (1953) and Sprott (1993)) is recommended for the implementation of *Rule 2*. It reads:

Let r be a (computer-generated) random number, $r >= 0$ and $r < 1$.
Then,
if $r <= P$ (P given by (39)) then *do* flip;
else, *do not* flip.

Under these terms, for any ‘temperature’ T there will, in principle, be M_1 system parts that would be functional and $M_2 = M – M_1$ parts that would be dysfunctional, so that one may define a *system performance fraction*, ζ as:

$$\zeta = (M_1 – M_2)/(2M) \tag{40}$$

Definition (40) places performance fraction ζ between (-0.5) and (+0.5), and favors the following assessment rule:

A system featuring $\zeta >= 0$ is *mostly functional, whereas*
A system featuring $\zeta < 0$ is *mostly dysfunctional*

And the value-judgment placed on a policy/strategy relates to *an assessment of the extent the managed system is kept mostly functional.* It is deemed that the *macroscopic* behavior of a system, normally expressed via variations in a number of indicators of definition perceived as relevant, is a result of system's *microscopic*, *co-operative* behavior, primarily characterized by the performance fraction, ζ.

The equations (36) to (40) are, in actual fact, implemented in the current application, meant to provide a graphic expression to one possible manner of characterizing *resilience and performance in systems*.

In essence, the game:

- Simulates a system made of user-specified number of parts that interact, both mutually and with external stress fields, at given strengths
- Induces in the system the afore-described microscopic process at part level, cyclically stressing the system by external fields
- Thereby obtains system performance fraction ζ as a function of the applied stress H (See Figure 23 to 26).

3.2 HYSTERESIS MODELING

The results will systematically indicate an overly important feature of large systems showing co-operative behavior: *their tendency to resist stress and maintain their state (configuration and performance level) against the driving stress applied – an effect known as hysteresis*, a common knowledge in, for instance, the theory and practice of magnetic phenomena and materials and beyond: for a starter, do a *Wikipedia* search for 'Hysteresis.' Notice that the system can have different alert states as described in the DEFense readiness CONdition (DEFCON) as used by the United States Armed Forces.

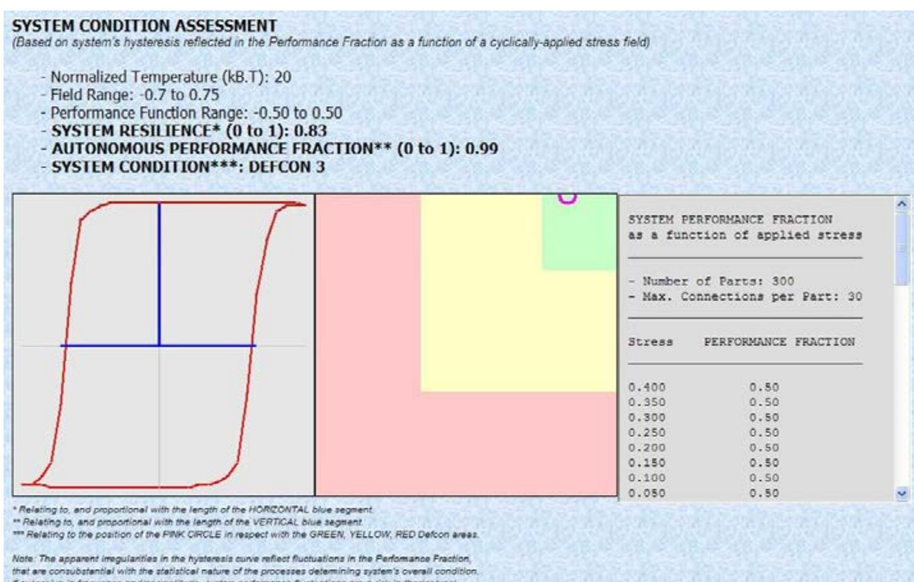

Figure 23. Hysteresis in a 300-part, 30-part links/part system; normalized temperature 20 units – DEFCON 3, adapted from Gheorghe and Vamanu (2009)

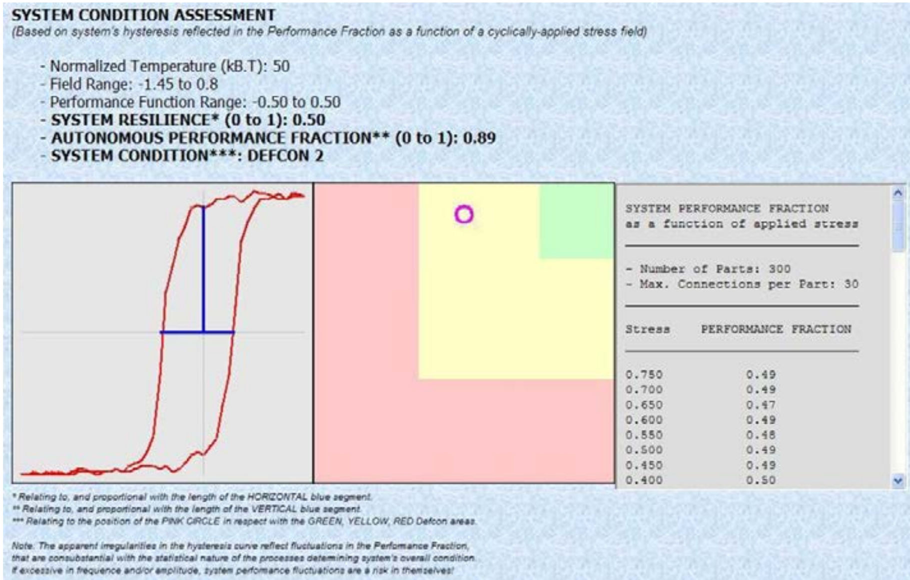

Figure 24. Hysteresis in a 300-part, 30-part links/part system; normalized temperature 50 units – DEFCON 2, adapted from Gheorghe and Vamanu (2009)

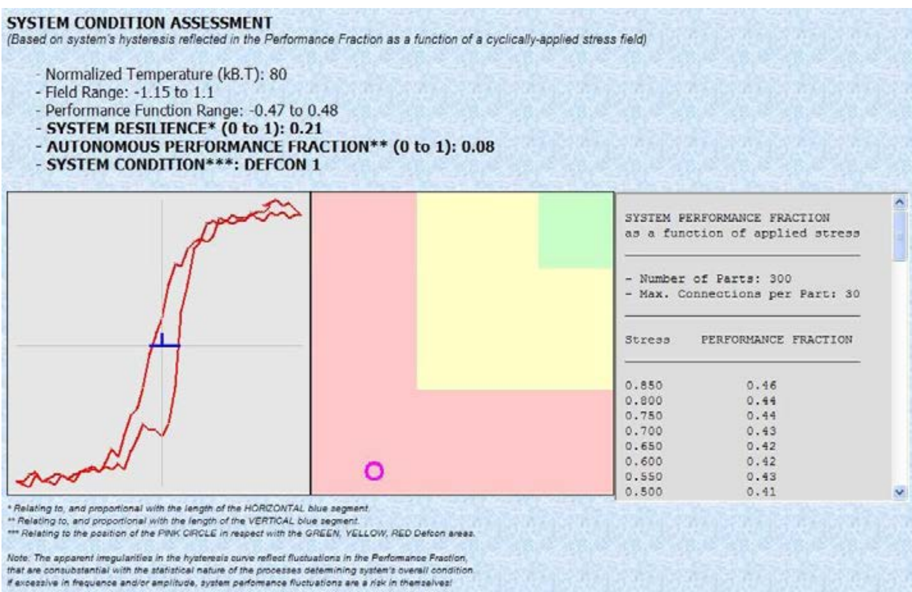

Figure 25. Hysteresis in a 300-part, 30-part links/part system; normalized temperature 50 units – DEFCON 1, adapted from Gheorghe and Vamanu (2009)

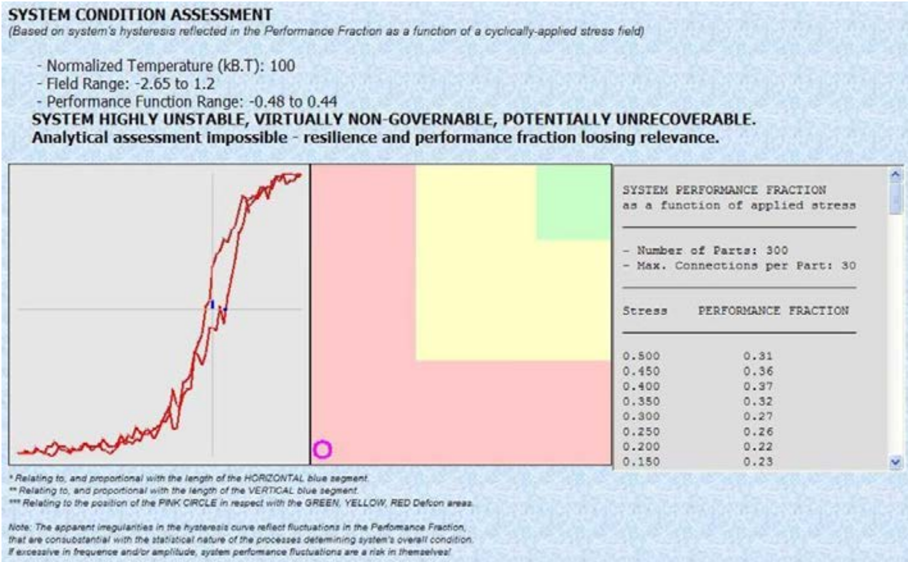

Figure 26. Hysteresis in a 300-part, 30-part links/part system; normalized temperature 100 units, adapted from Gheorghe and Vamanu (2009)

Notice that in Figure 26, the system is ‘unstable’ and virtually ‘non-governable’ and potentially ‘unrecoverable.’

Additionally, and in plain terms:

- If a system is dominantly functional, then it tends to maintain its level of functionality (performance) despite applied stresses threatening to make parts dysfunctional
- If a system is dominantly dysfunctional, then it tends to maintain low levels of functionality (performance) despite the applied stresses (i.e., efforts) attempting to make parts functional again; and, perhaps more strikingly,
- The transition from a dominantly functional to a dominantly dysfunctional system, and the other way around, tends to be abrupt (as opposed to gradual) and essentially depends on system's ‘temperature.’

It has been suggested that ‘the reluctance to changes in the level of performance under applied stress, of large systems featuring cooperative, statistical phenomena that animate their interconnected parts is *resilience* (Gheorghe et al., 2011). A natural measure of the resilience, in the present discussion could be described as *the distance of the intersections of the hysteresis cycle with the abscissa* (see Figure 23). Expressed in units of the applied stress (field), this quantity may be termed - by analogy with the Theory of Magnetism - a *‘Coercive Force,’ or ‘Coercivity*.’ Further along the analogy, the maximum value of the performance function ζ, measured on the ordinate axis for a nil-stress may be termed *Remnant Performance Level* as opposed to *remnant magnetization* or *remanence*. An alternative, and perhaps a more appropriate term in the context may be *Autonomous Performance Fraction* (APF), indicating a desirable feature of complex systems: their capability to sustain operations even when most of the 'positive stress' (financial, logistic, etc.) required to set the system in motion has been tempered, or withdrawn (Gheorghe et al., 2011).

In such terms, a system deemed '*in good order*', or '*condition*' should display both,

- a high *resilience* - indicating a good resistance to the effects of negative stresses; and
- a high *autonomous performance fraction* - indicating an acceptable level of performance even in the absence of a positive stress to maintain it.

These finding leaves one with the need to employ in the representation of the system condition *the Cartesian product* of the said quantities in an X-Y plane, one choice being to place the resilience on the X-axis and the APF on the Y-axis. Tis manner of visualizing/monitoring a systems' condition would immediately call to mind the defense drill, that deals with readiness for appropriate response in threatening conditions in terms of 'DEFCONs.' In the context, one may, for instance, leave to the gamer the definition of boundaries between, say, three 'DEFCONs' of incremental degree of severity, the most severe featuring *the lowest system resilience*, OR *the lowest autonomous performance fraction* (APF).

The current module of the ROSTREC Arcade platform (See Muresan, 2010) plays with some basic parameters defining a system, namely:

- the *number* of parts and their *susceptibility* (reactivity) to applied stress, assumed to, generally, differ from part to part;
- the *number* of links of every part in the system with other parts, in either physical and/or logical a sense, and the strength of the respective links - that also may differ from one link to the other while remaining however reciprocal for any given pair of parts; and
- the 'temperature' of the system - the net effect of which, in a purely algebraic sense is to diminish in bulk, by the same factor, all part susceptibilities and link strengths, which turns out to result in quite dramatic effects on resilience and performance fraction.

As the gamer stretches these parameters within the allowed limits (essentially resulting from the computing power of the average-price desk-/lap/tops) he/she will get:

- a variety of *hysteresis loops*, each providing an indeed graphic expression of the system condition via system's resilience and APF; and
- an X-Y map of APF vs. resilience, for a comparative analysis in terms of DEFCONs, of the consequences of different choices, or evolutions with system's parameters

REMARKS

After some enduring exercises, one might end up with a 'feeling' of how large systems behave. To these 'feelings,' present authors suggest that on one hand:

- Large and internally coherent systems tend to show a higher level of resilience and APF. Contrastingly, the level of resilience for small and poorly-coherent systems tends to be low. Thin of the characteristics of stable and fluctuation-free operations and regimes.
- Systems that are subject to poor, negligent, lax management and governance in terms of, among others, maintenance, monitoring, updating, corporate spirit, truthful self-assessment, and ethics, which translates as 'disorder,' or 'higher temperatures' tend to show degraded resilience and/or performance fractions, down to complete collapse.

On the other hand, the following remarks are also suggested:

- *Highly resilient systems – systems that have a high-grade tend to be... highly vulnerable*! Their vulnerability relates to the near-ideal shape of their hysteresis cycle: quasirectangular and covering a large expanse in the performance versus stress in the form of an X-Y plan. This remark is based in in the fact that such a shape may encourage a feeling that 'things are all right.' Regardless of the cause of shape, be it negligence, or external circumstances, a prolonged recession, for example, could be seen as 'normal.'

Interestingly, residual positive stress, normally known as 'production and maintenance costs' (e.g., financial, logistic, intelligence) can more the system into the negative-stress realm. The system could find itself into dangerously-close to the edge, that if reached by a mere further, apparently insignificant decrement or fluctuation, *will take the entire system down into a full-fledged collapse*. Oddly enough, what we have referred to as a 'feeling' that all things are all right as it related to 'systems theory' concept of punctuated equilibrium (Gould and Eldredge, 1986) where the long periods of stasis as suggested in Katina (2015) could create a false sense of 'safeness.' Unfortunately, the feeling of safeness, tends to lead to system designs that:

- Lack virtuallyany complete and credible early-warning systems. Thus, a system might stay stable at a high(est) level of performance although its environment is clearly deteriorating.
- The brutality of the collapse (the steep slope of the hysteresis) that would dramatize the entire scenario; and - perhaps more importantly.
- The remarkably long and costly way to a full system recovery (see the length of the lower hysteresis cycle plateau)

However, all is not bad. The examined analogy suggests a need to create recovery points in design of complex systems. However, that remains a point of further research as to how to establish recovery points based on hysteresis. Moreover, literature suggestes that there can be types of hysteresis (Mayagoitia, 1991). There remains an issue of implications of such types on man-made systems.

REFERENCES

Hopkinson, B., & Williams, G. T. (1912). The elastic hysteresis of steel. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *87*(598), 502–511. https://doi.org/10.1098/rspa.1912.0104

Gheorghe, A., Muresan, L., Celac, S., Caceu, S., Degeratu, C., Lenes, L., Kanovits, C., Bores, R., Prebenses, C., and Refvem, T. (2011). An energy security strategy for Romania: Promoting energy efficiency and renewable energy sources. In A. V. Gheorghe & L. Muresan (Eds.), *Energy Security: International and local issues, perspectives, and critical energy infrastructures* (pp. 337–412). New York: Springer Science & Business Media.

Gheorghe, A. V., & Vamanu, D. V. (2004). Towards QVA – Quantitative Vulnerability Assessment: A generic practical model. *Journal of Risk Research*, *7*(6), 613–628.

Gheorghe, A. V., & Vamanu, D. V. (2008). Mining intelligence data in the benefit of critical infrastructures security: Vulnerability modelling, simulation and assessment, system of systems engineering. *International Journal of System of Systems Engineering*, *1*(1), 189–221.

Gheorghe, A. V., & Vamanu, D. V. (2009). Resilience and vulnerability in critical infrastructure systems - A physical analogy. *International Journal of Critical Infrastructures*, *5*(4), 389–397.

Gould, S. J., & Eldredge, N. (1986). Punctuated equilibrium at the third stage. *Systematic Zoology*, *35*(1), 143–148.

Katina, P. F. (2015). *Systems theory-based construct for identifying metasystem pathologies for complex system governance* (Ph.D.). Old Dominion University, United States -- Virginia.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., & Teller, E. (1953). Equation of state calculations by fast computing machines. *The Journal of Chemical Physics*, *21*(6), 1087–1092.

Mayagoitia, V. (1991). The five types of porous structures and their hysteresis loops. In *Studies in Surface Science and Catalysis* (Vol. 62, pp. 51–60). Elsevier. Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0167299108613087

Muresan, L. (2010). *Energy security and critical infrastructure protection strategy for Romania and the regional perspective*. Odessa: EURISC Foundation - Romania. Retrieved from http://www.energycharter.org/fileadmin/DocumentsMedia/Events/20100727-PESAIITBSATROMC_S2_LMuresan.pdf

Sprott, J. C. (1993). *Strange attractors: Creating patterns in chaos*. New York: M&T Books.