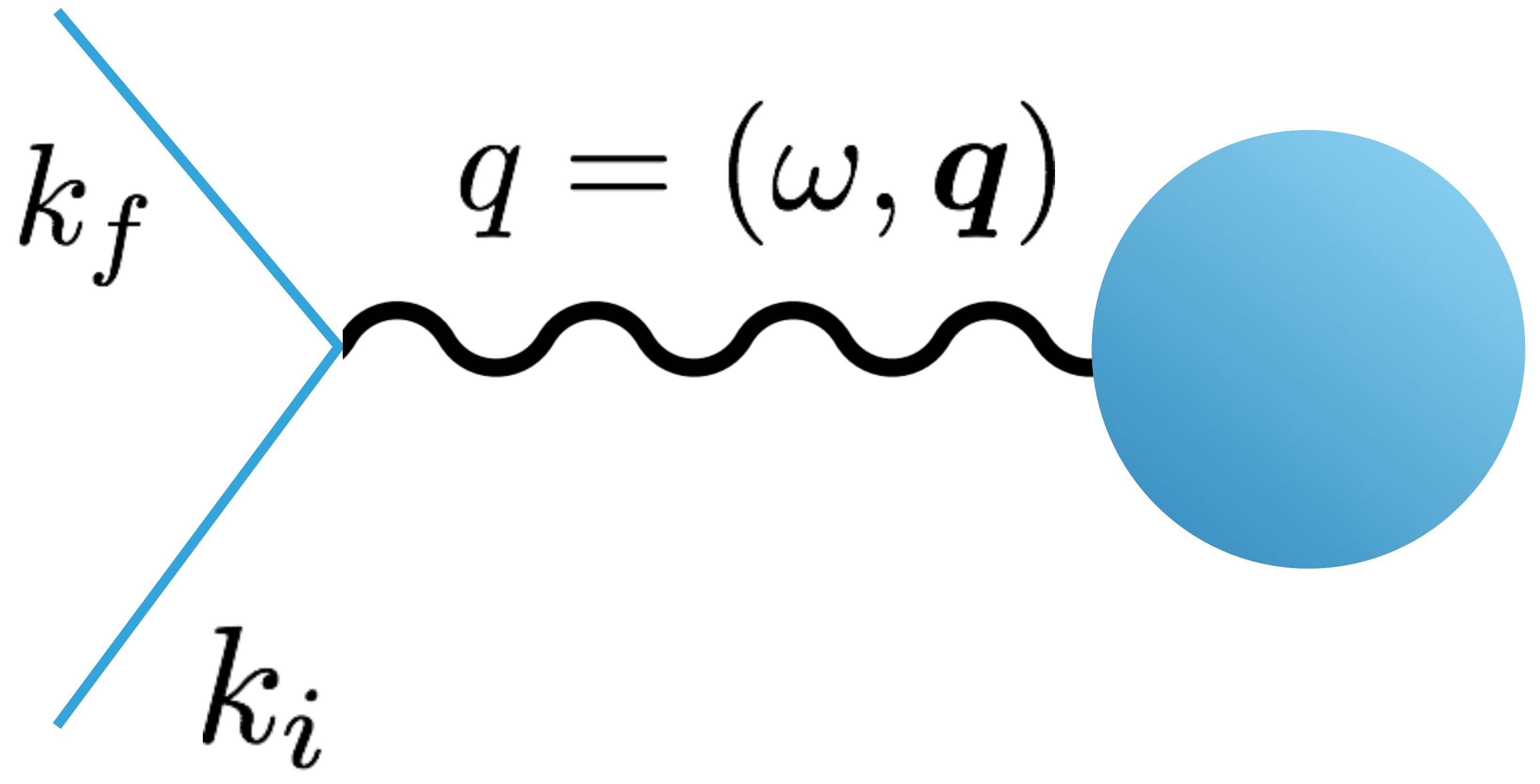
MICHAEL PAOLONE

TEMPLE UNIVERSITY

FOR THE E05-110 COLLABORATION.

THE COULOMB SUM RULE IN NUCLEI

COULOMB SUM RULE



Inclusive electron scattering cross-section:

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{\rm Mott}\left[\frac{q^4}{|\boldsymbol{q}|^4}R_L(\omega, |\boldsymbol{q}|) + \left(\frac{q^2}{2|\boldsymbol{q}|^2} + \tan^2\frac{\theta}{2}\right)R_T(\omega, |\boldsymbol{q}|)\right]$$

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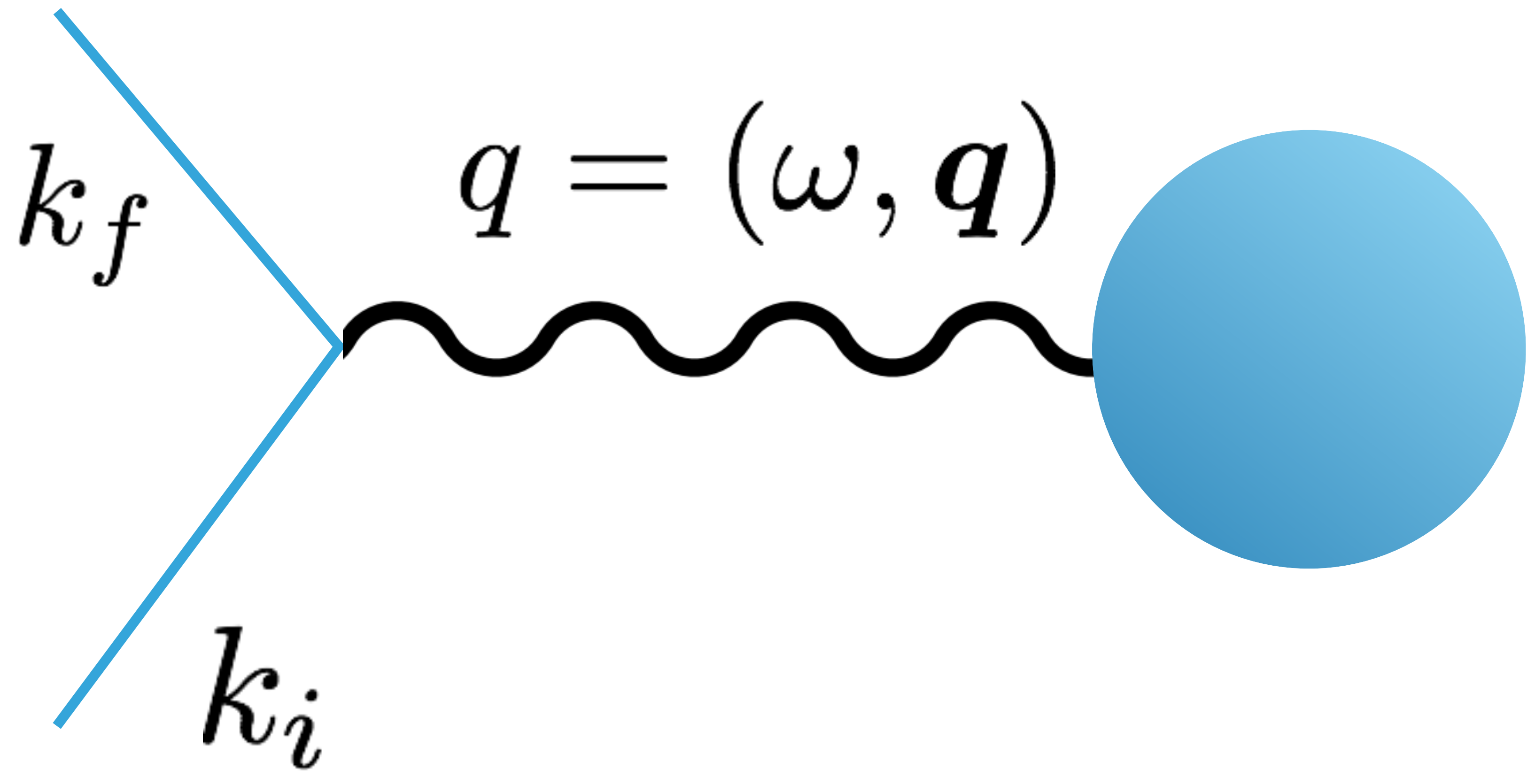

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Scattering response due to **charge** properties

Scattering response due to **magnetic** properties

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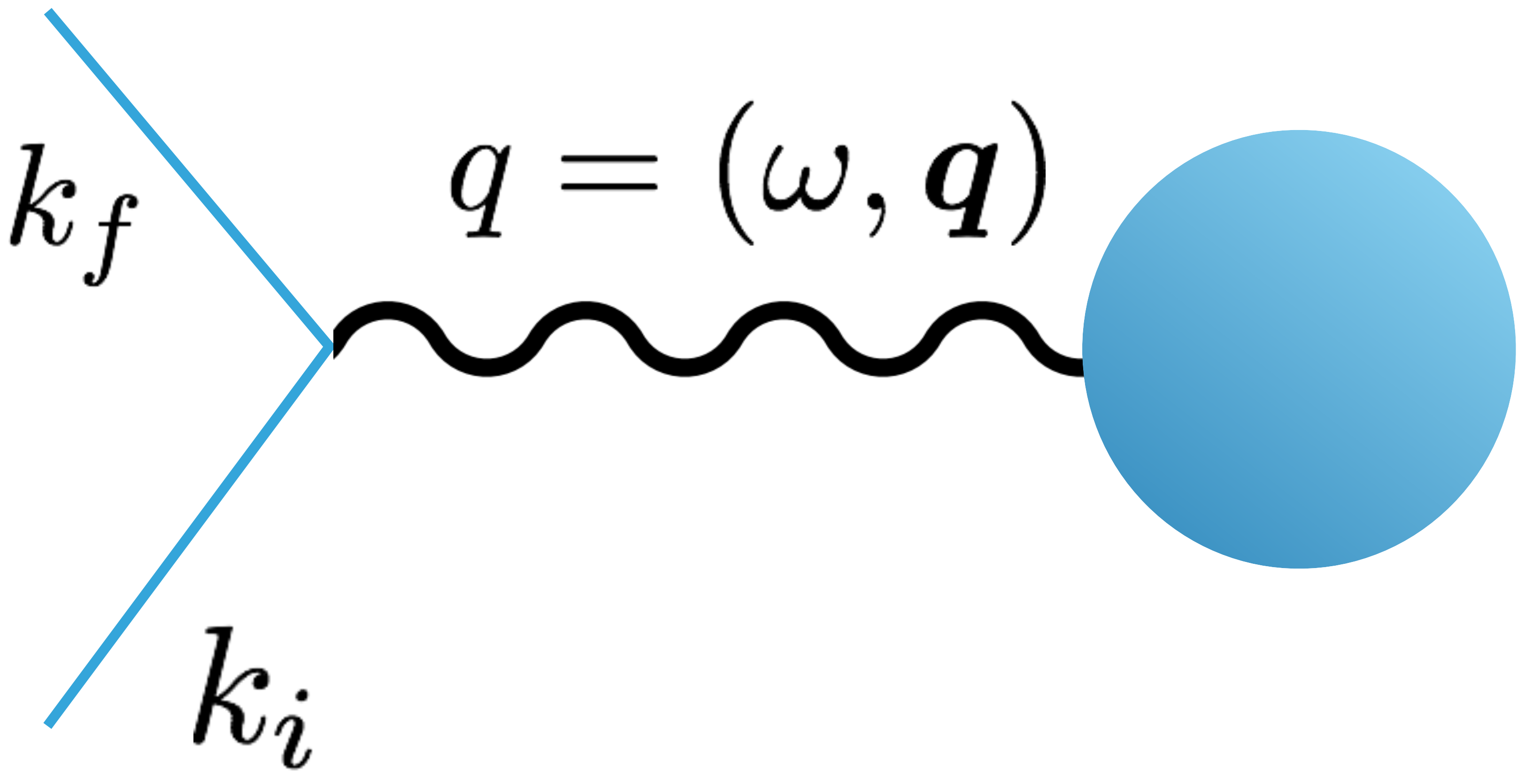

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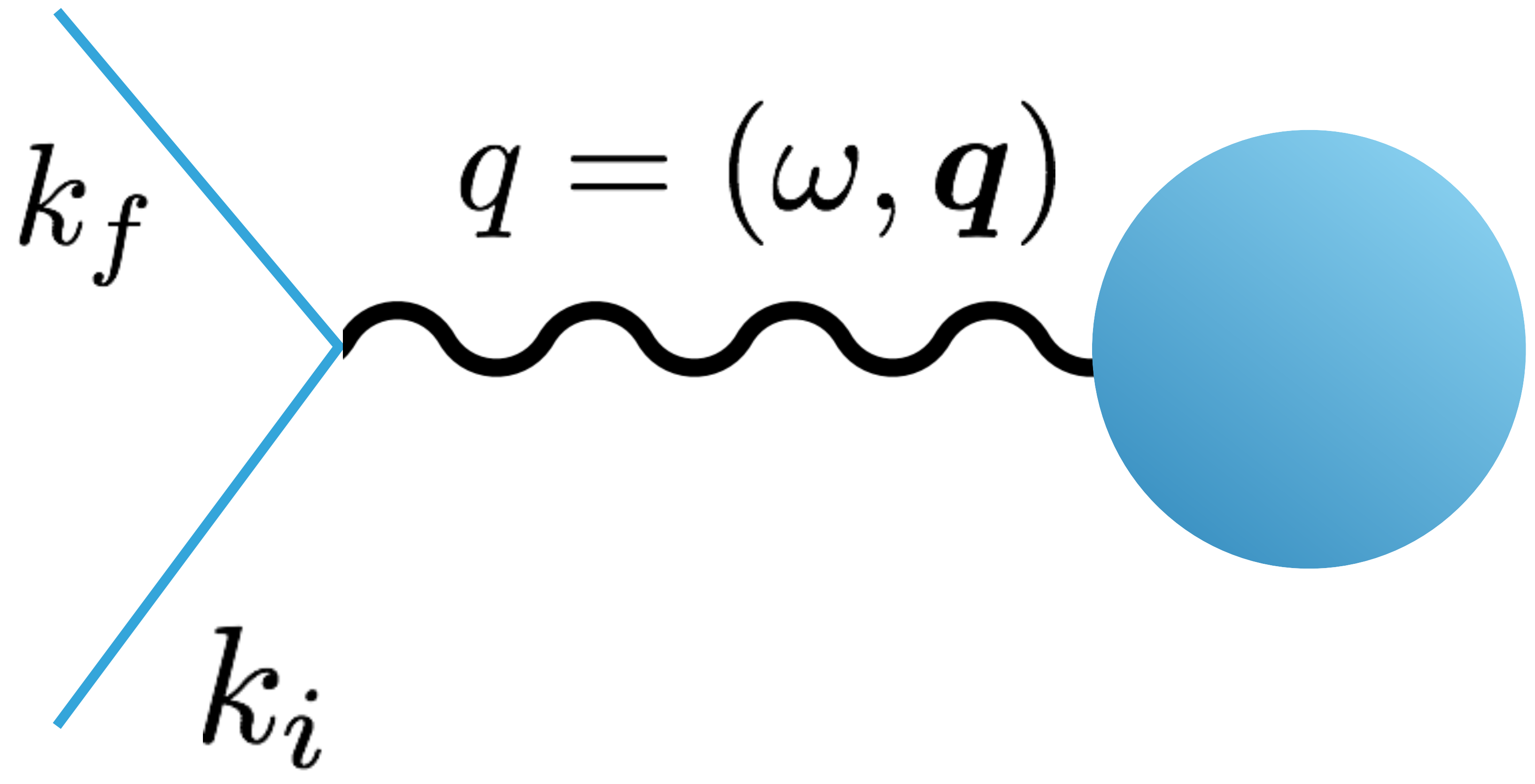
If one integrates the charge response divided by the total charge form factor over all available virtual photon energies, naively one might expect the integral to go to unity.

COULOMB SUM RULE



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At small |**q**| , S_L will deviate from unity due to long range nuclear effects, Pauli blocking. (directly calculable, well understood).

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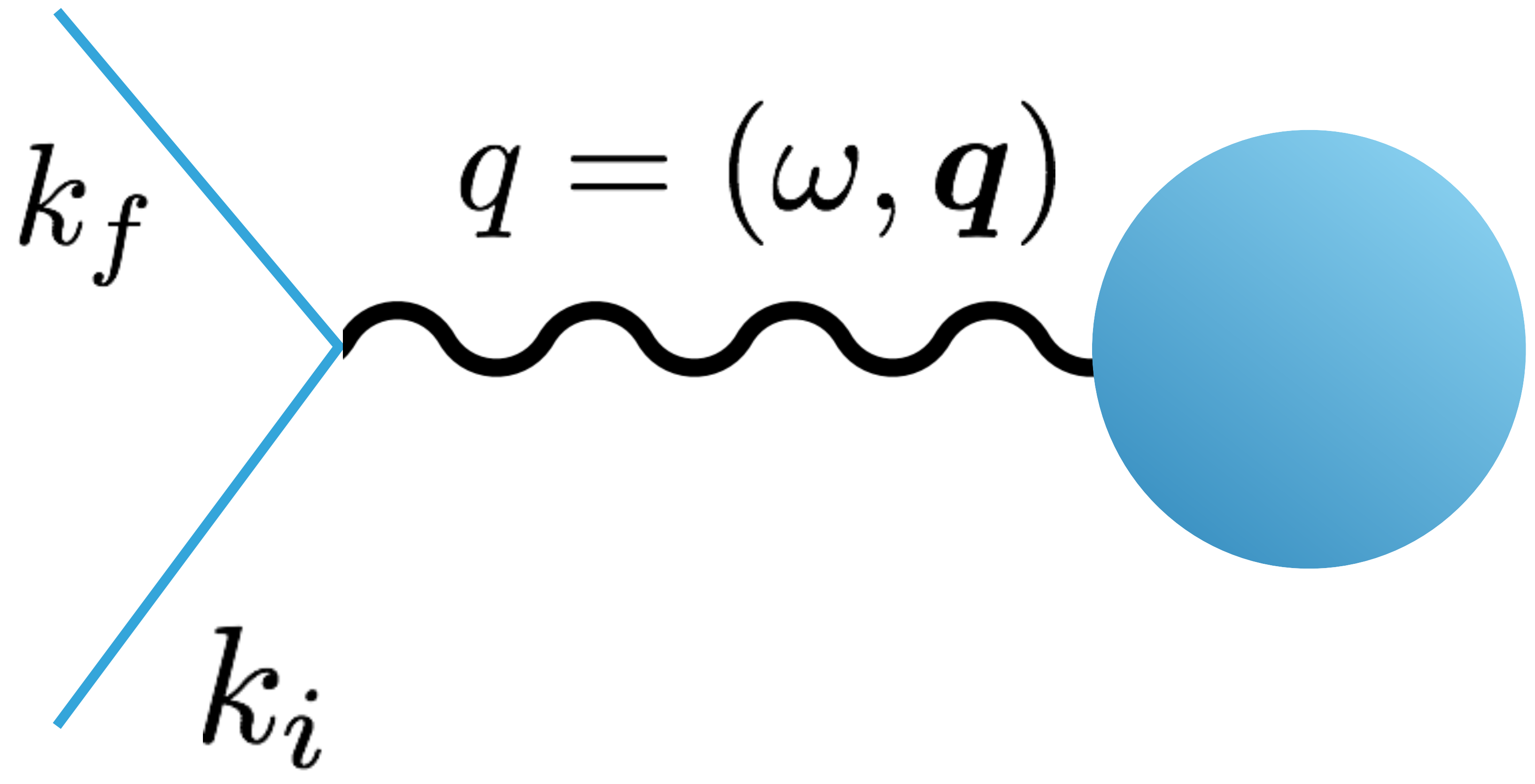

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At large |q| >> 2k_f , S_L should go to 1. Any significant* deviation from this would be an indication of relativistic or medium effects distorting the nucleon form factor!

*Short range correlations will also quench S_L, but only by < 10%

COULOMB SUM RULE

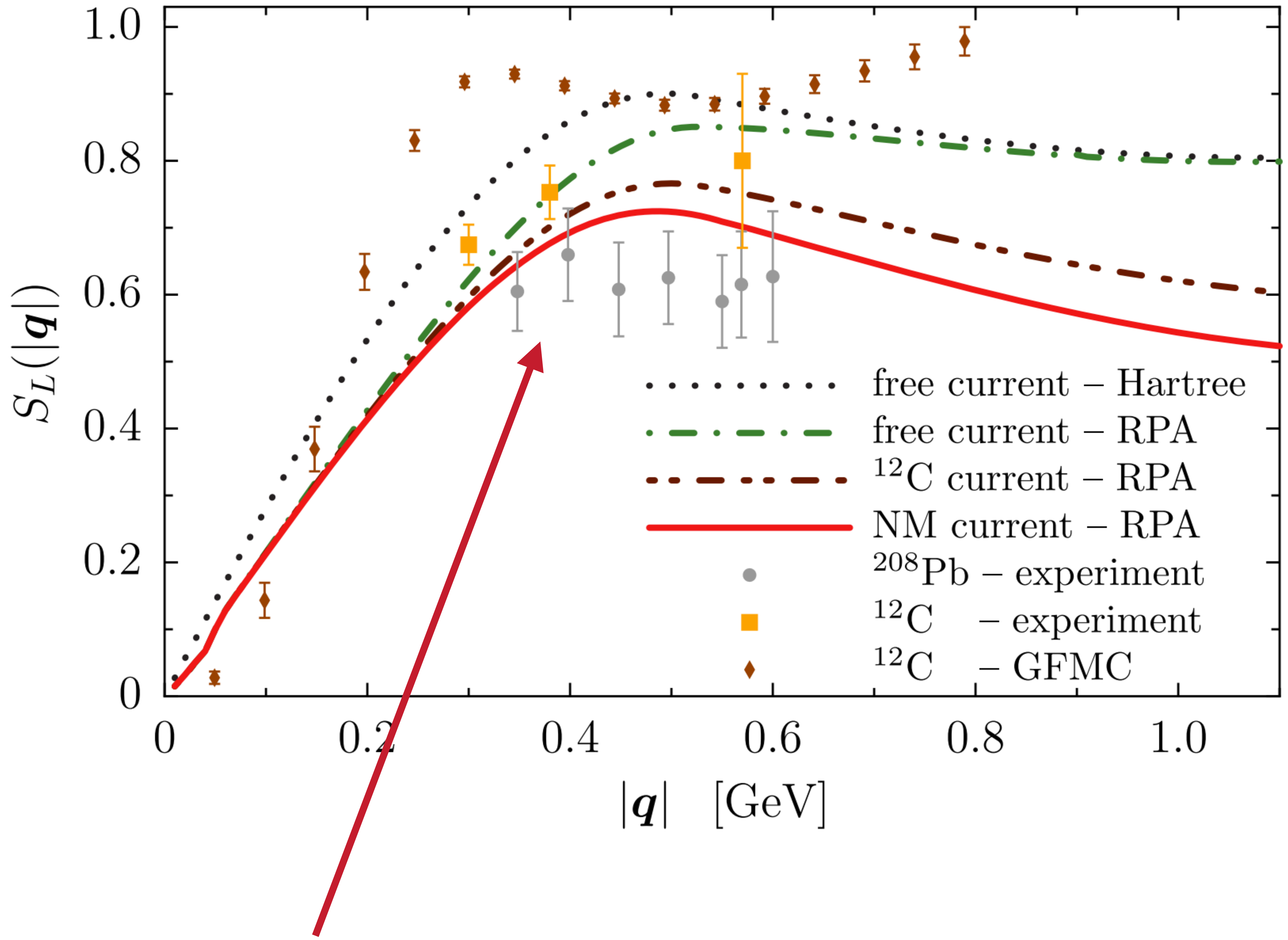
- Long standing issue with many years of theoretical interest.
- Even most state-of the-art models cannot predict existing data.
- New precise data at larger |q| would provide crucial insight and constraints to modern calculations.

$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

PRL 116, 032701 (2016) PHYSICAL REVIEW LETTERS week ending 22 JANUARY 2016

Relativistic and Nuclear Medium Effects on the Coulomb Sum Rule

Ian C. Cloët,[1] Wolfgang Bentz,[2] and Anthony W. Thomas[3]
[1]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[2]*Department of Physics, School of Science, Tokai University, Hiratsuka-shi, Kanagawa 259-1292, Japan*
[3]*CSSM and ARC Centre of Excellence for Particle Physics at the Terascale, Department of Physics, University of Adelaide, Adelaide South Australia 5005, Australia*
(Received 23 June 2015; published 19 January 2016)



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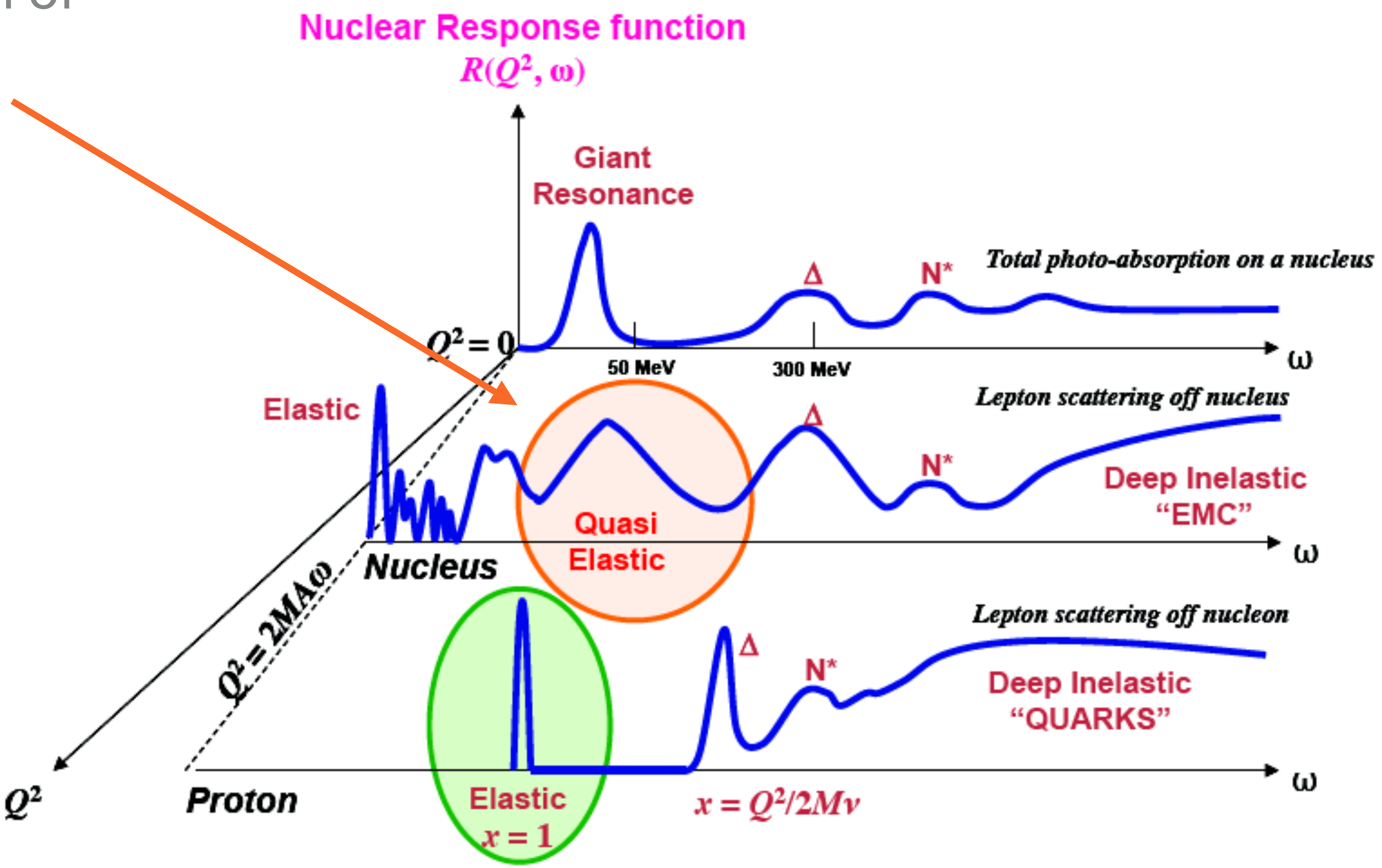
*Short range correlations will also quench S_L, but only by < 10%

QUASI-ELASTIC SCATTERING

$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}^2_{Ep}(Q^2) + N\tilde{G}^2_{En}(Q^2)}$$

We want to integrate above the coherent elastic peak:
Quasi-elastic is "elastic" scattering on constituent nucleons inside nucleus.

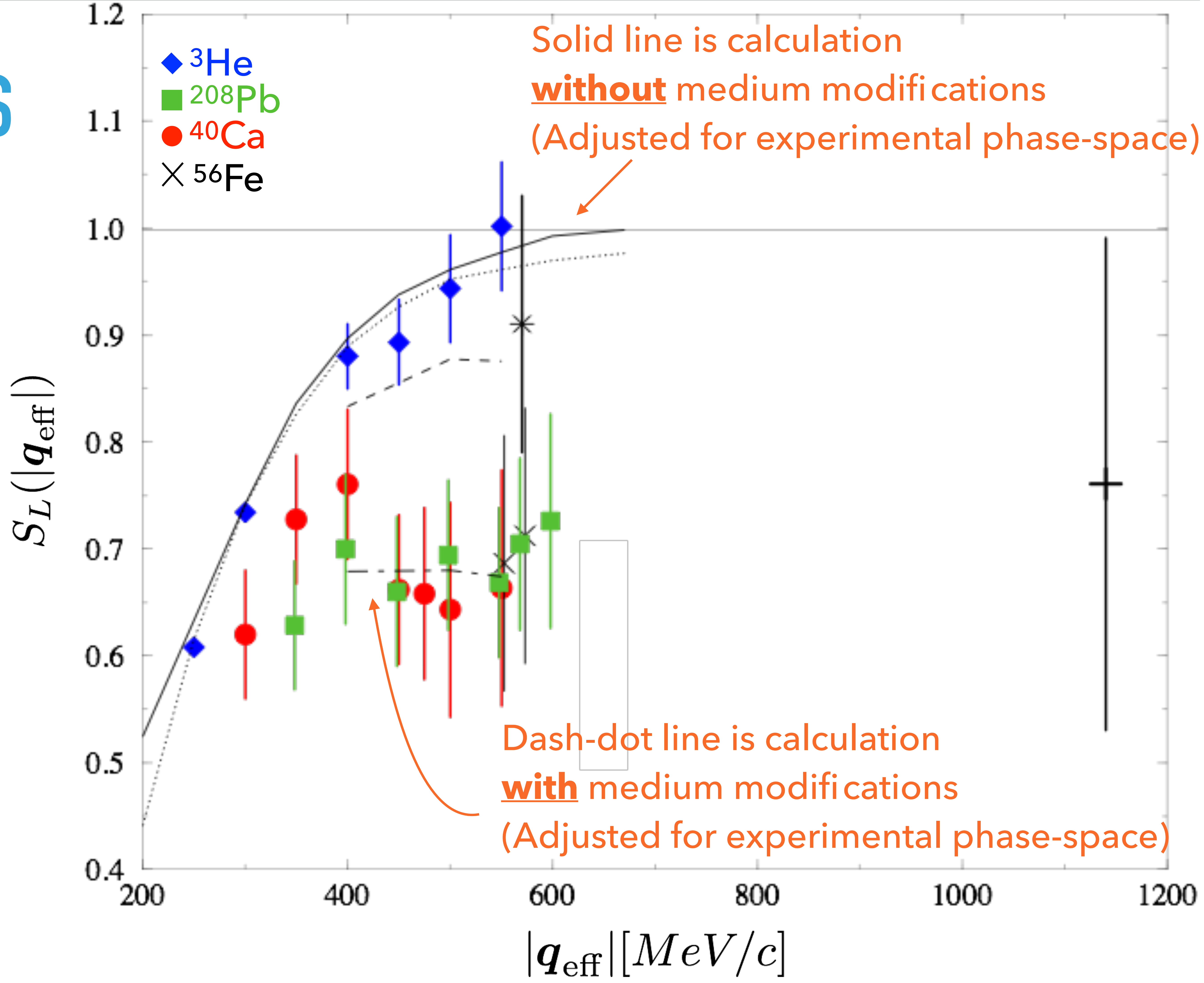
- Quasi-elastic scattering at intermediate Q^2 is the region of interest for our experiment:
  - Nuclei investigated:
    - ^{4}He
    - ^{12}C
    - ^{56}Fe
    - ^{208}Pb



PUBLISHED EXPERIMENTAL RESULTS

- First group of experiments from Saclay, Bates, and SLAC show a quenching of S_L consistent with medium modified form-factors.

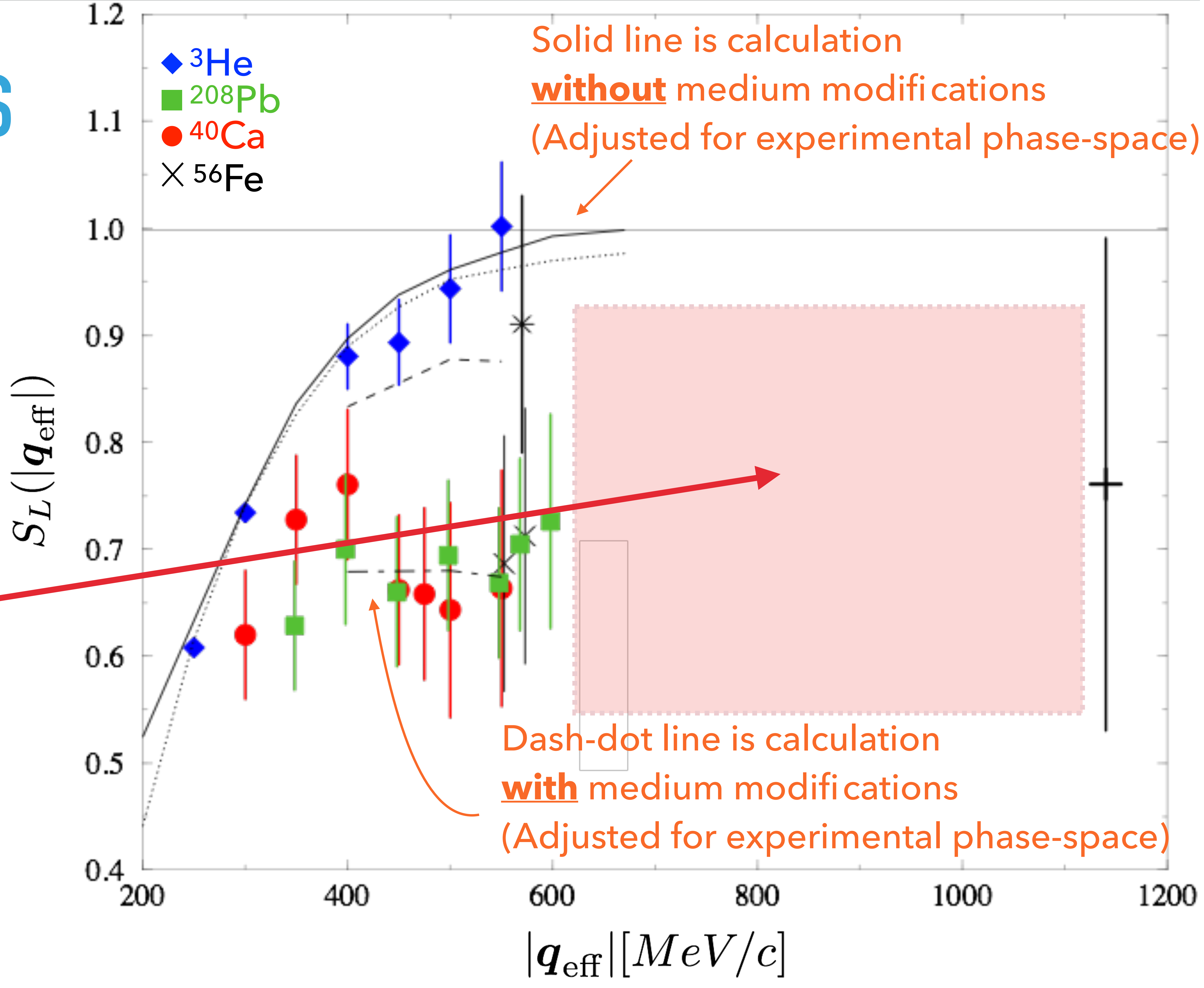
$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}^2_{Ep}(Q^2) + N\tilde{G}^2_{En}(Q^2)}$$



$|\boldsymbol{q}_{\text{eff}}|$ is $|\boldsymbol{q}|$ corrected for a nuclei dependent mean coulomb potential. Methodology agreed on by Andreas Aste, Steve Wallace and John Tjon.

PUBLISHED EXPERIMENTAL RESULTS

- First group of experiments from Saclay, Bates, and SLAC show a quenching of S_L consistent with medium modified form-factors.
- Very little data above |**q**| of 600 MeV/c, where the cleanest signal of medium effects should exist!
  - Saclay, Bates limited in beam energy reach up to 800 MeV.
  - SLAC limited in kinematic coverage of scattered electron at |q| below 1150 MeV/c.



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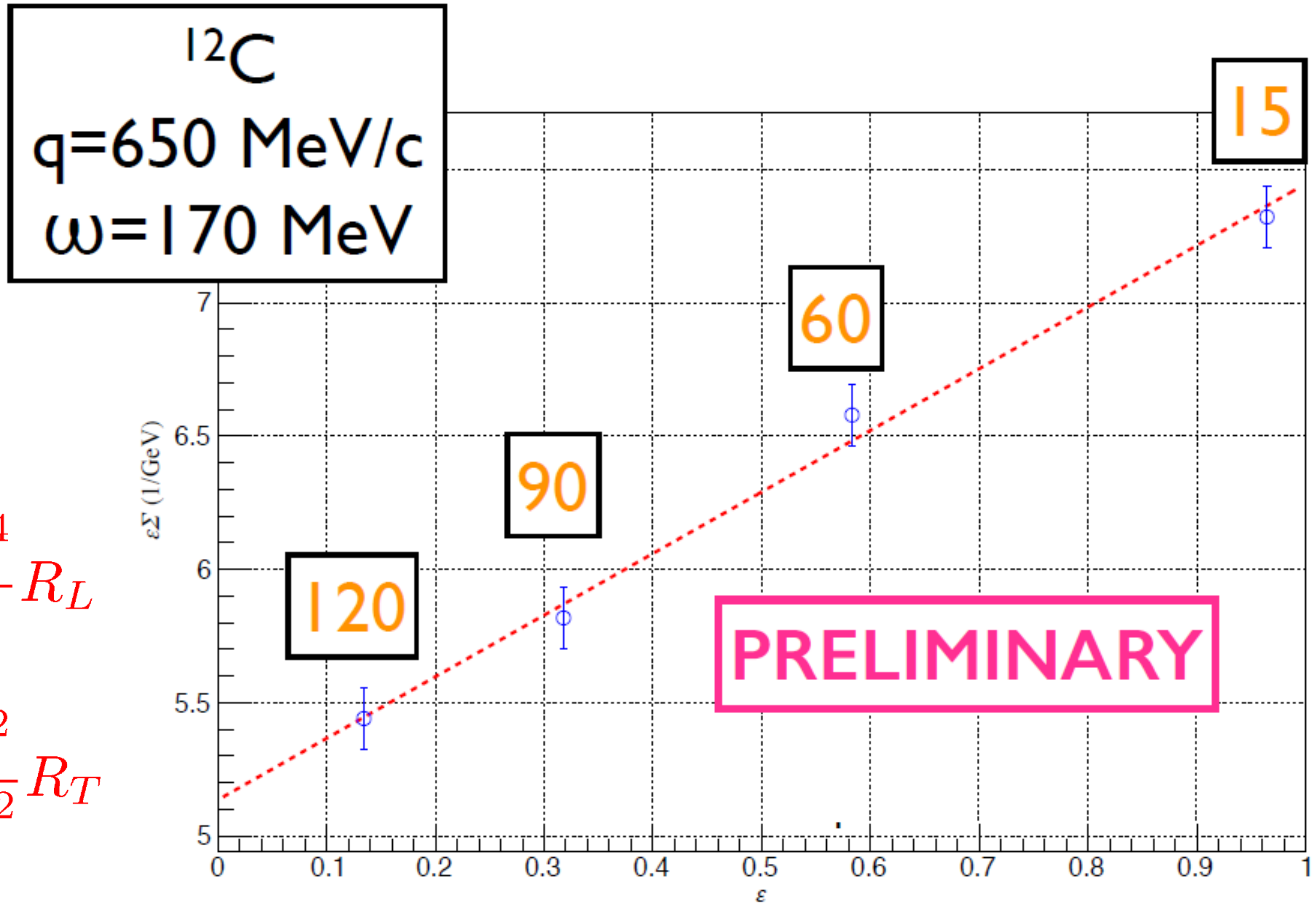
EXPERIMENTAL DESIGN

- Need R_L ⟶ Use Rosenbluth separation!

$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

Slope = $\frac{Q^4}{\vec{q}^4} R_L$

Intercept = $\frac{Q^2}{2\vec{q}^2} R_T$



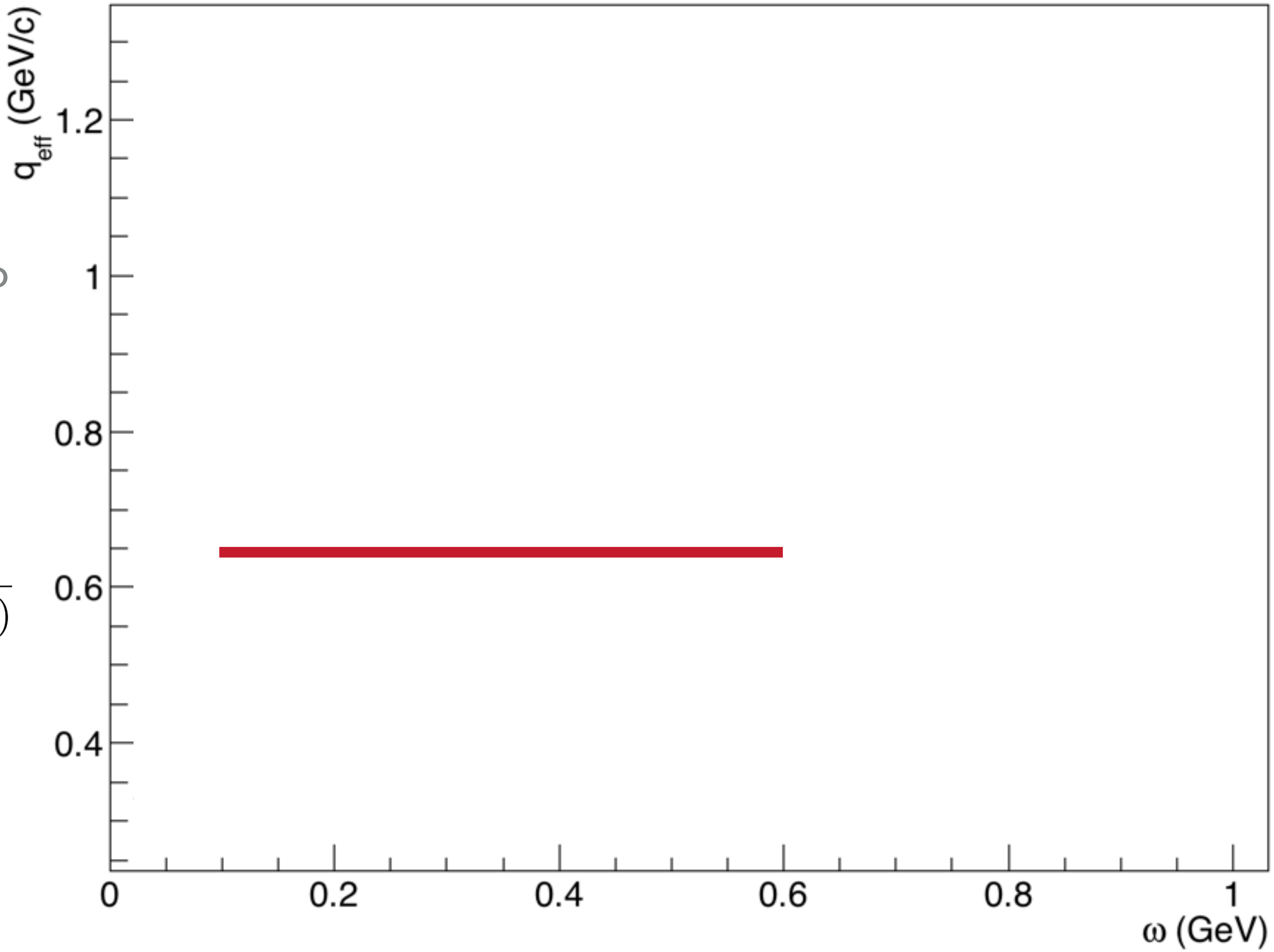
  - Experiment run at 4 angles per target: 15, 60, 90, 120 degs. Very large lever arm for precise calculation of R_L!
- Need data for each angle at a constant |q| over an ω range starting above the elastic peak up to |q|.
  - When running a single arm experiment with fixed beam energy and scattering angle, |q| is NOT constant over your momentum acceptance.
    - Need to take data at varying beam energies, and “map-out” |q| and ω space.

EXPERIMENTAL DESIGN

- If one wants to measure from 100 to 600 MeV ω at constant |q| = 650 MeV/c

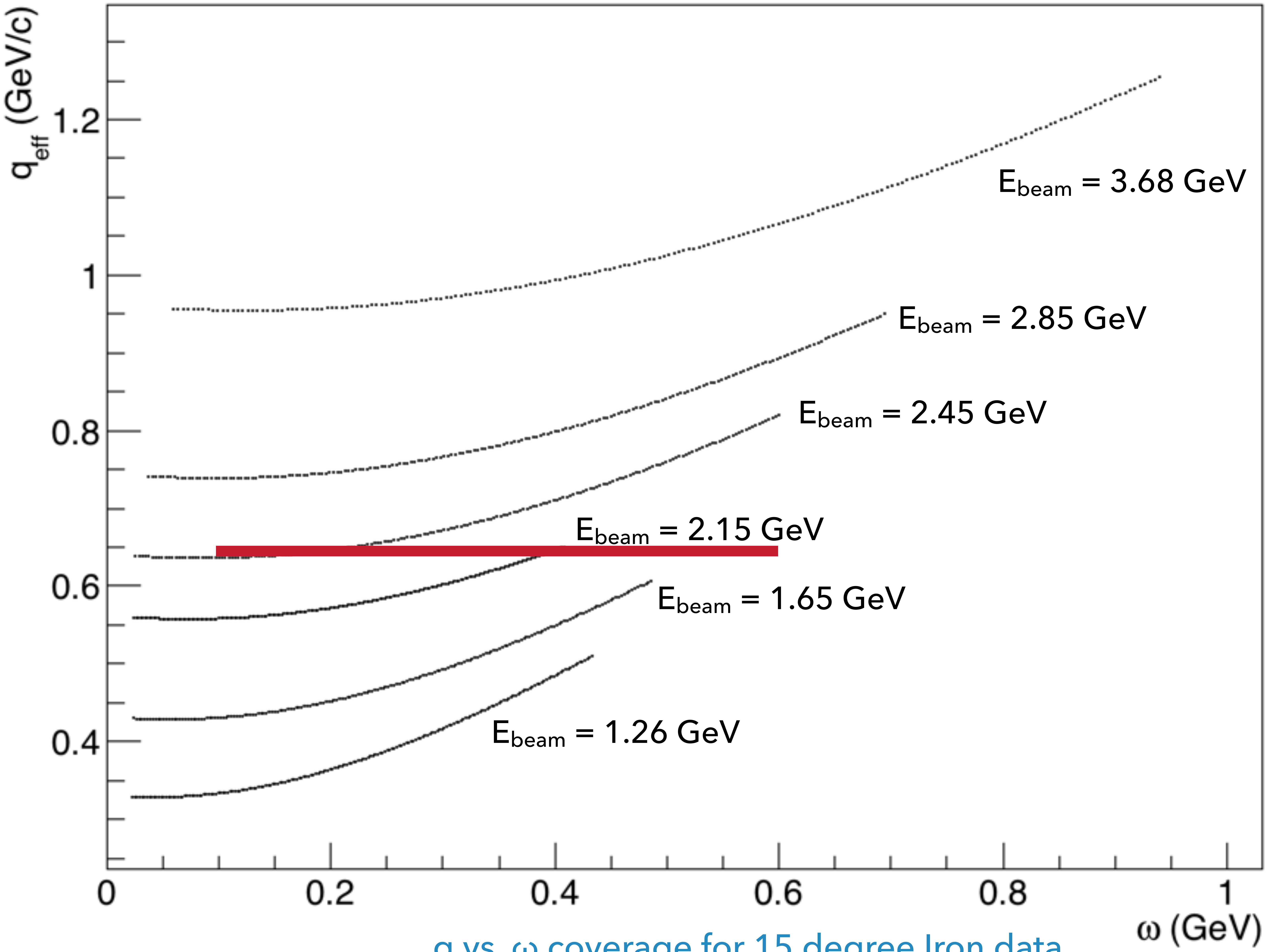
CSR calculated at constant |**q**| !!

$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}^2_{Ep}(Q^2) + N\tilde{G}^2_{En}(Q^2)}$$



EXPERIMENTAL DESIGN

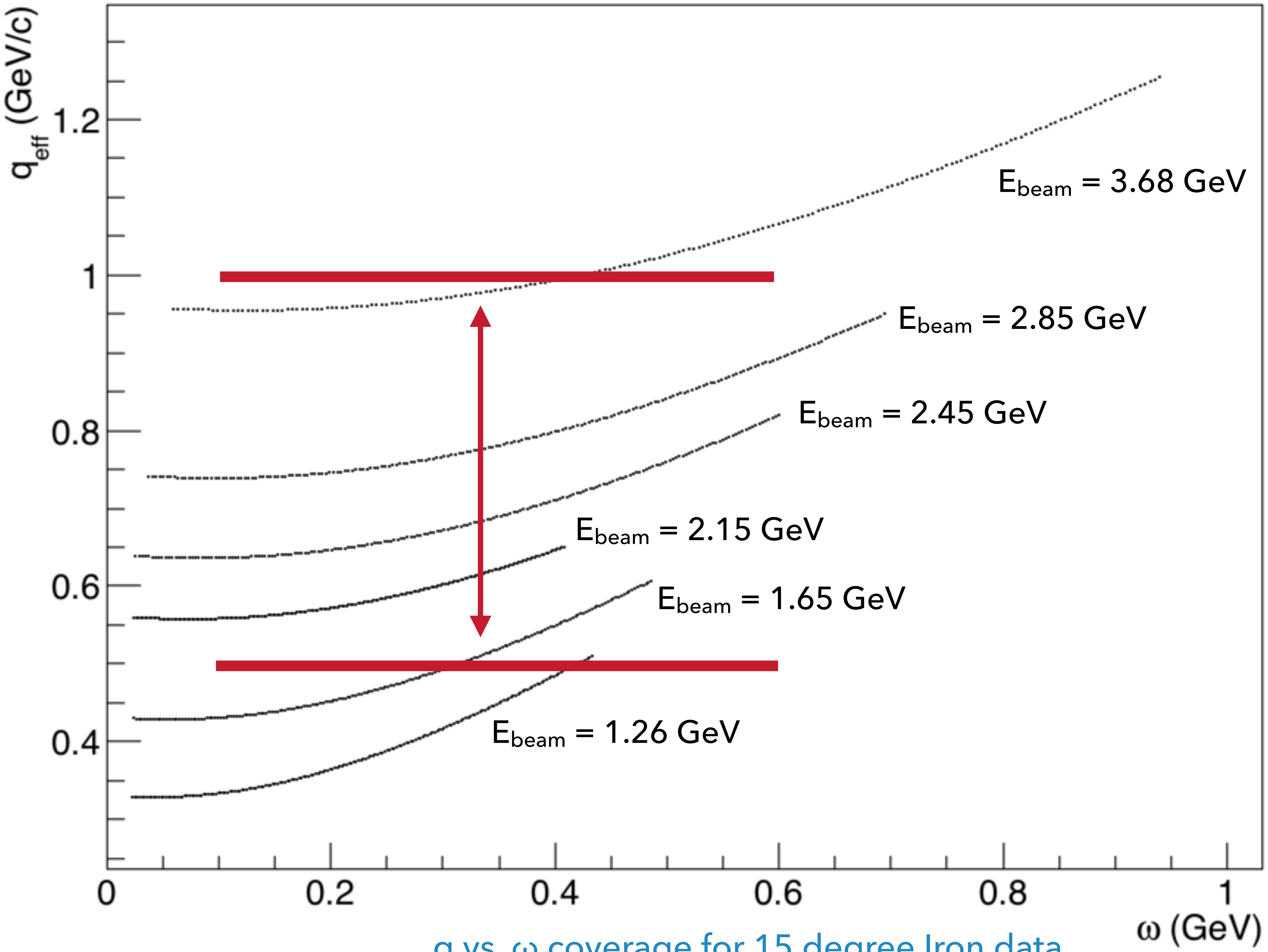
- If one wants to measure from 100 to 600 MeV ω at constant |q| = 650 MeV/c
  - Take data at different beam energies, and interpolate to determine cross-section at constant |q|.



q vs. ω coverage for 15 degree Iron data

EXPERIMENTAL DESIGN

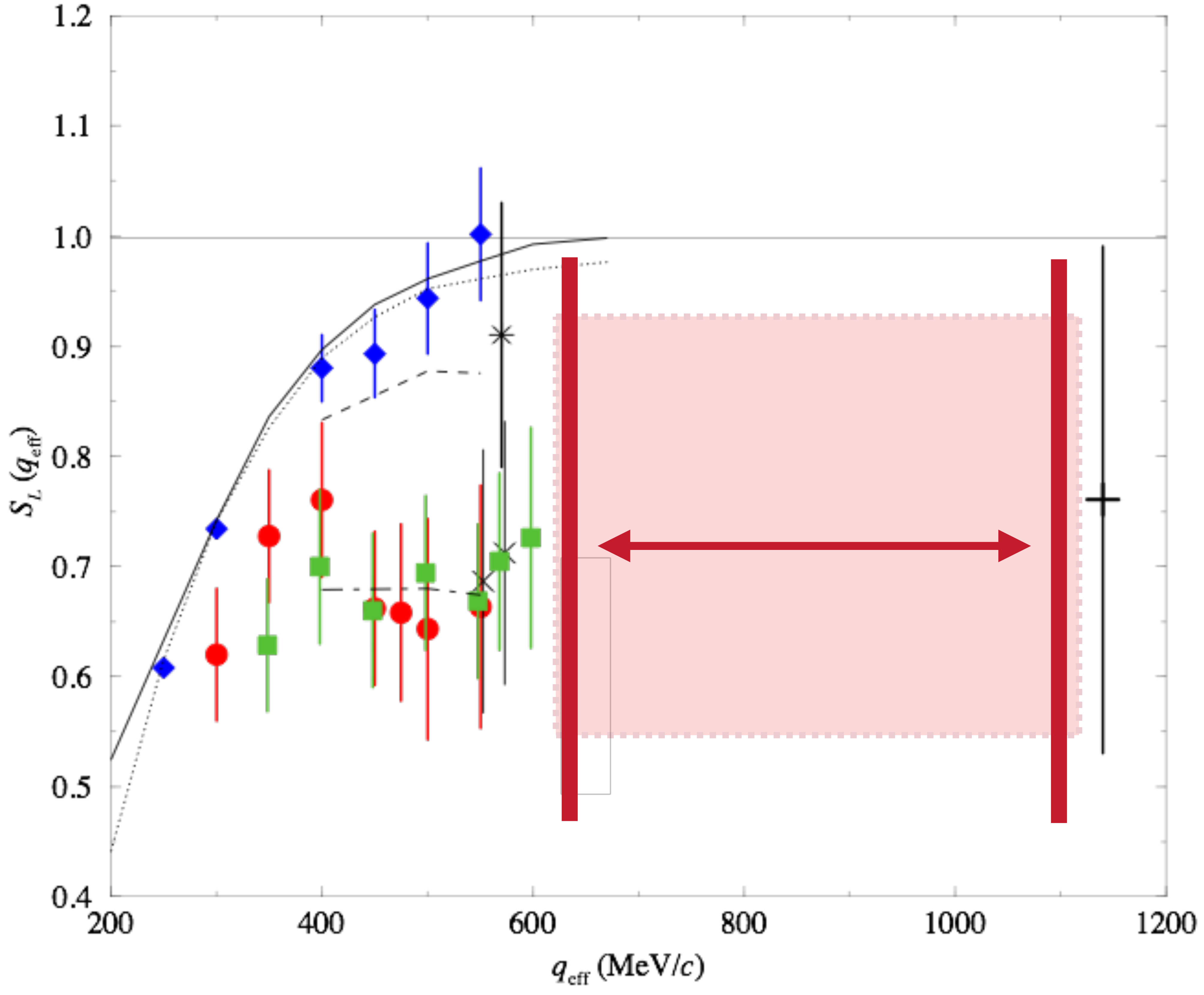
- If one wants to measure from 100 to 600 MeV ω at constant |q| = 650 MeV/c
    - Take data at different beam energies, and interpolate to determine cross-section at constant |q|.
    - |q| can be selected between 550 and 1000 MeV/c

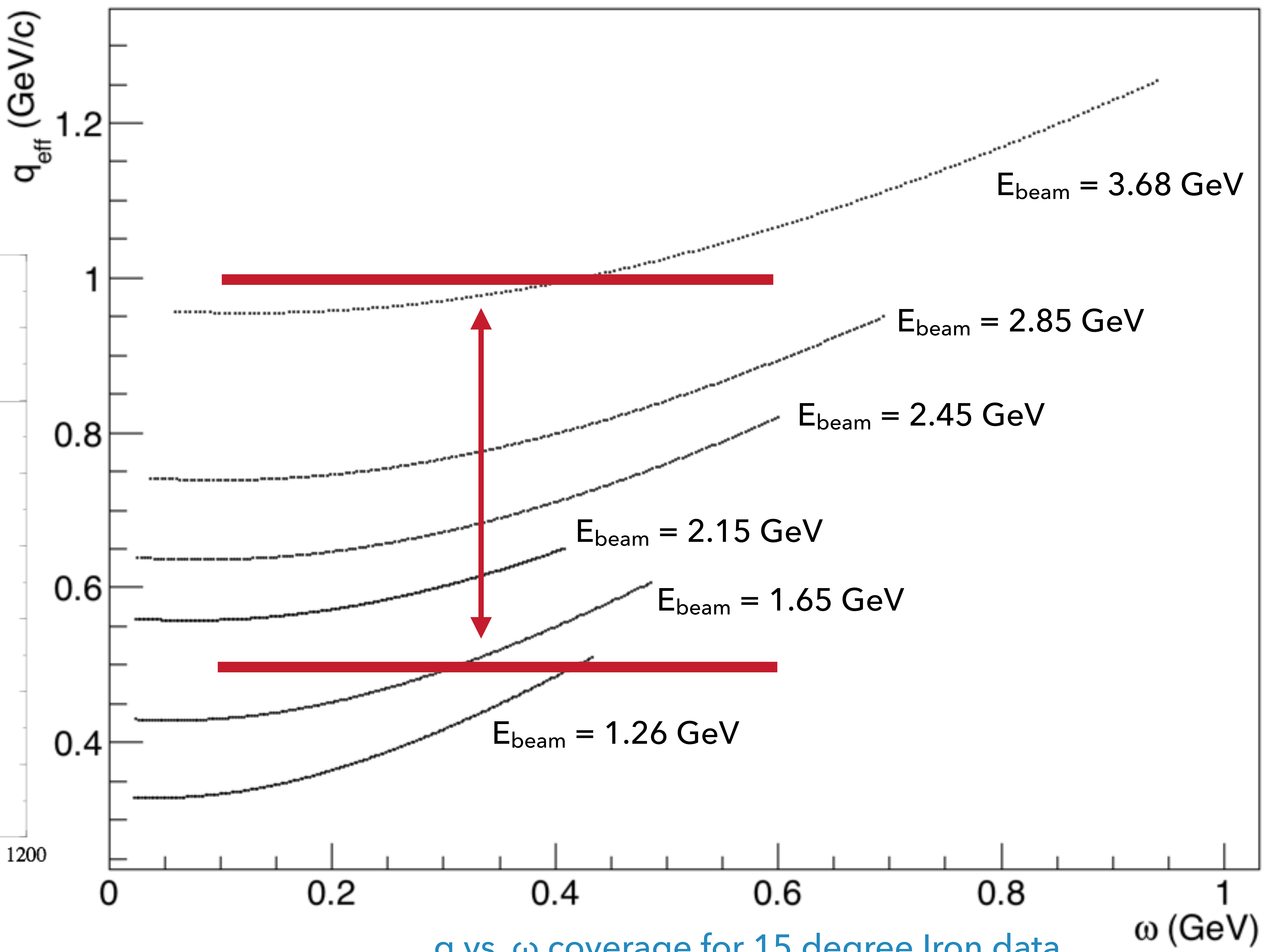


q vs. ω coverage for 15 degree Iron data

Repeat this "mapping" for 60, 90, and 120 degree spectrometer central angles.

EXPERIMENTAL DESIGN





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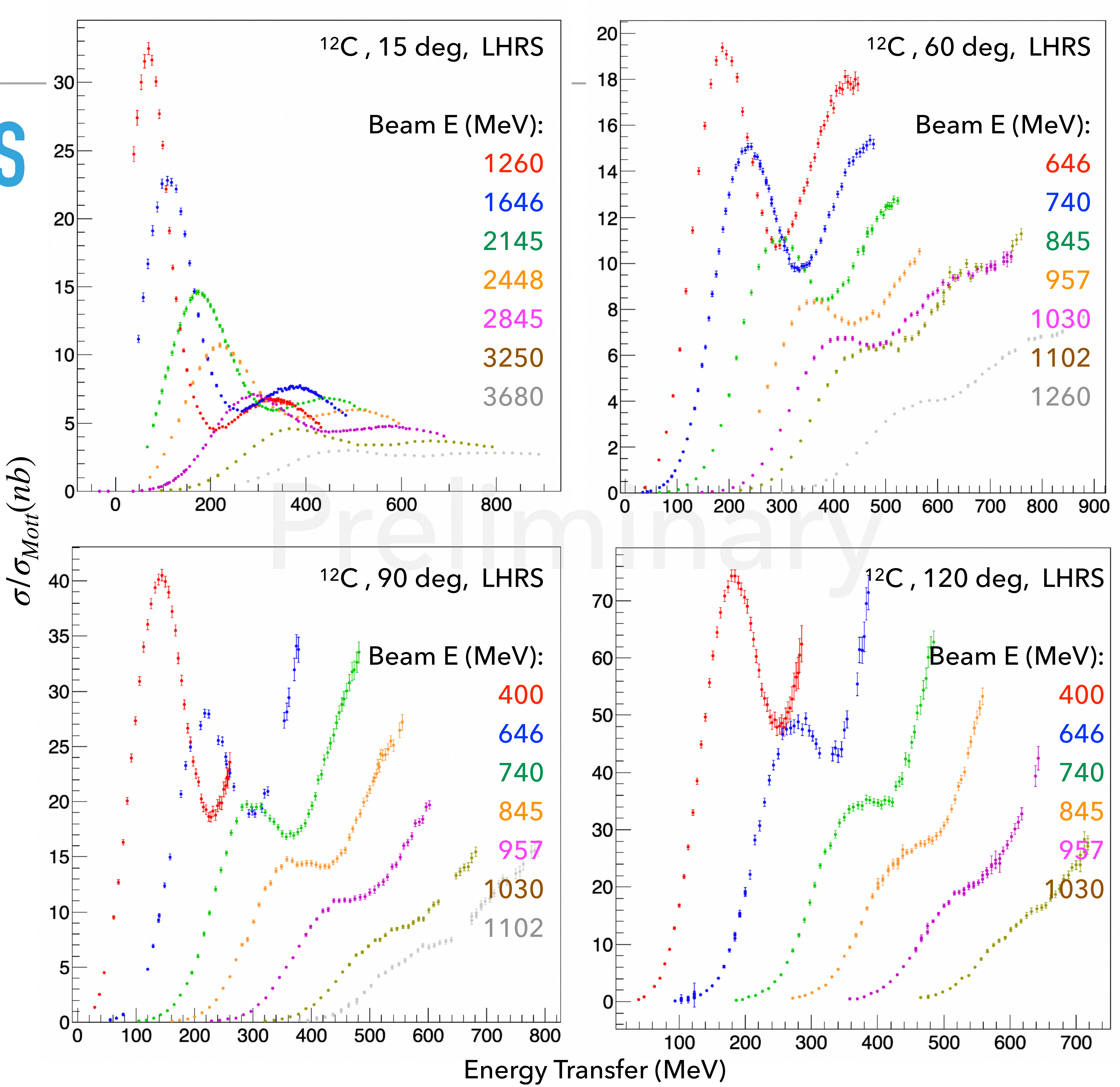
q vs. ω coverage for 15 degree Iron data

EXPERIMENTAL SPECIFICS

- E05-110:
  - Data taken from October 23rd 2007 to January 16th 2008
  - 4 central angle settings: 15, 60, 90, 120 degs.
  - Many beam energy settings: 0.4 to 4.0 GeV
  - Many central momentum settings: 0.1 to 4.0 GeV
  - LHRS and RHRS independent (redundant) measurements for most settings
  - 4 targets: ^{4}He, ^{12}C, ^{56}Fe, ^{208}Pb.



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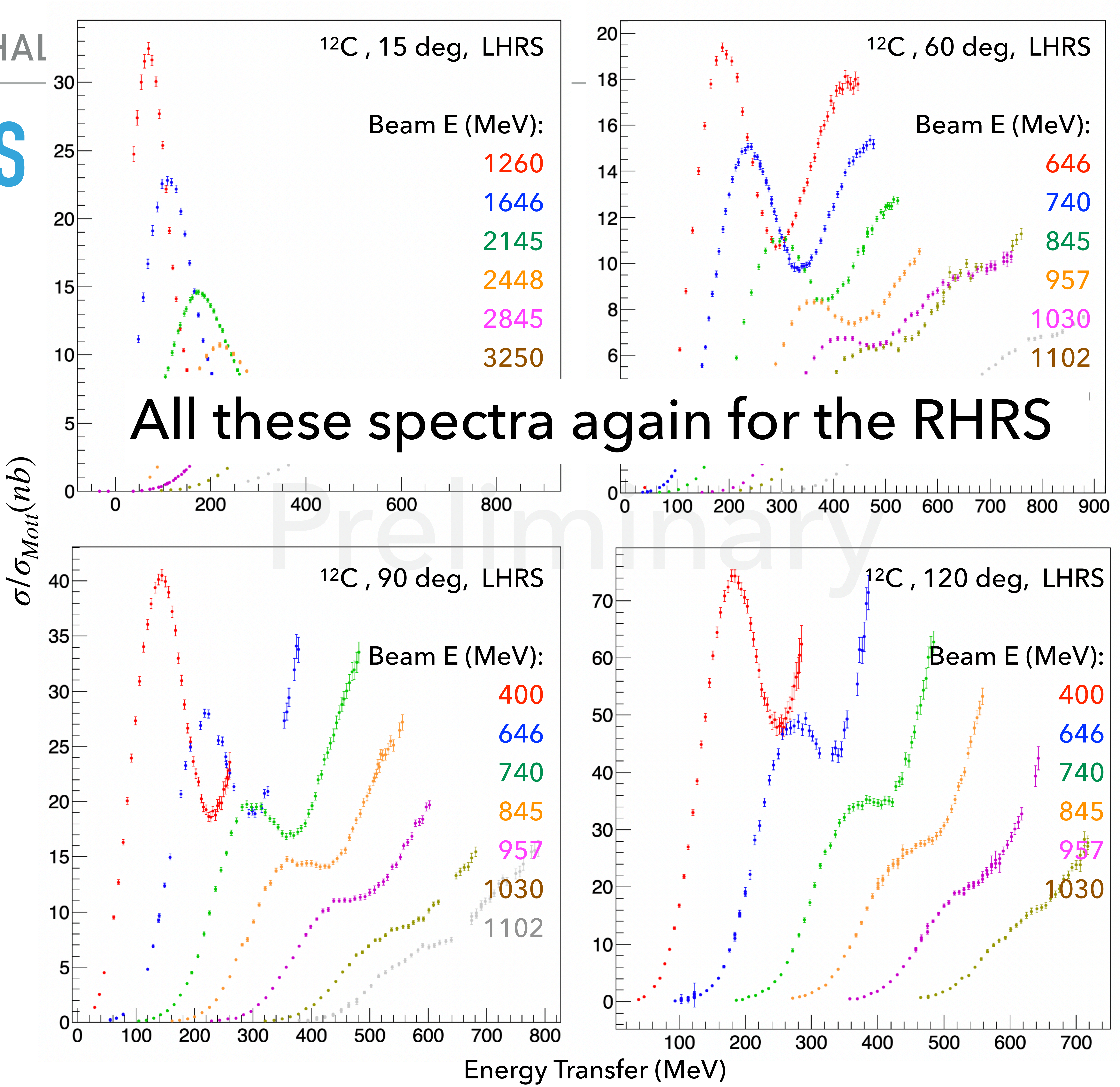

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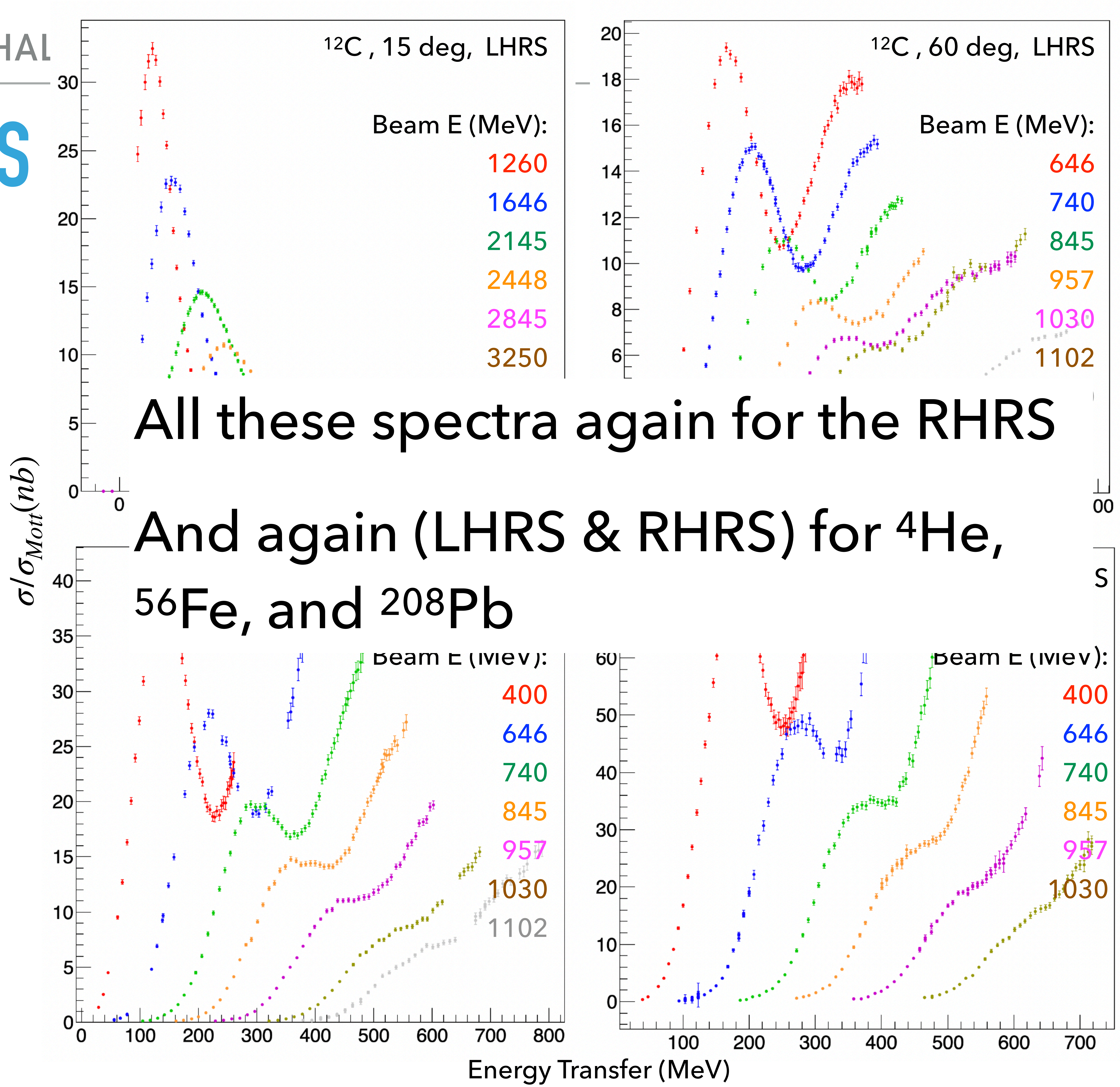

All these spectra again for the RHRS

And again (LHRS & RHRS) for ^{4}He, ^{56}Fe, and ^{208}Pb

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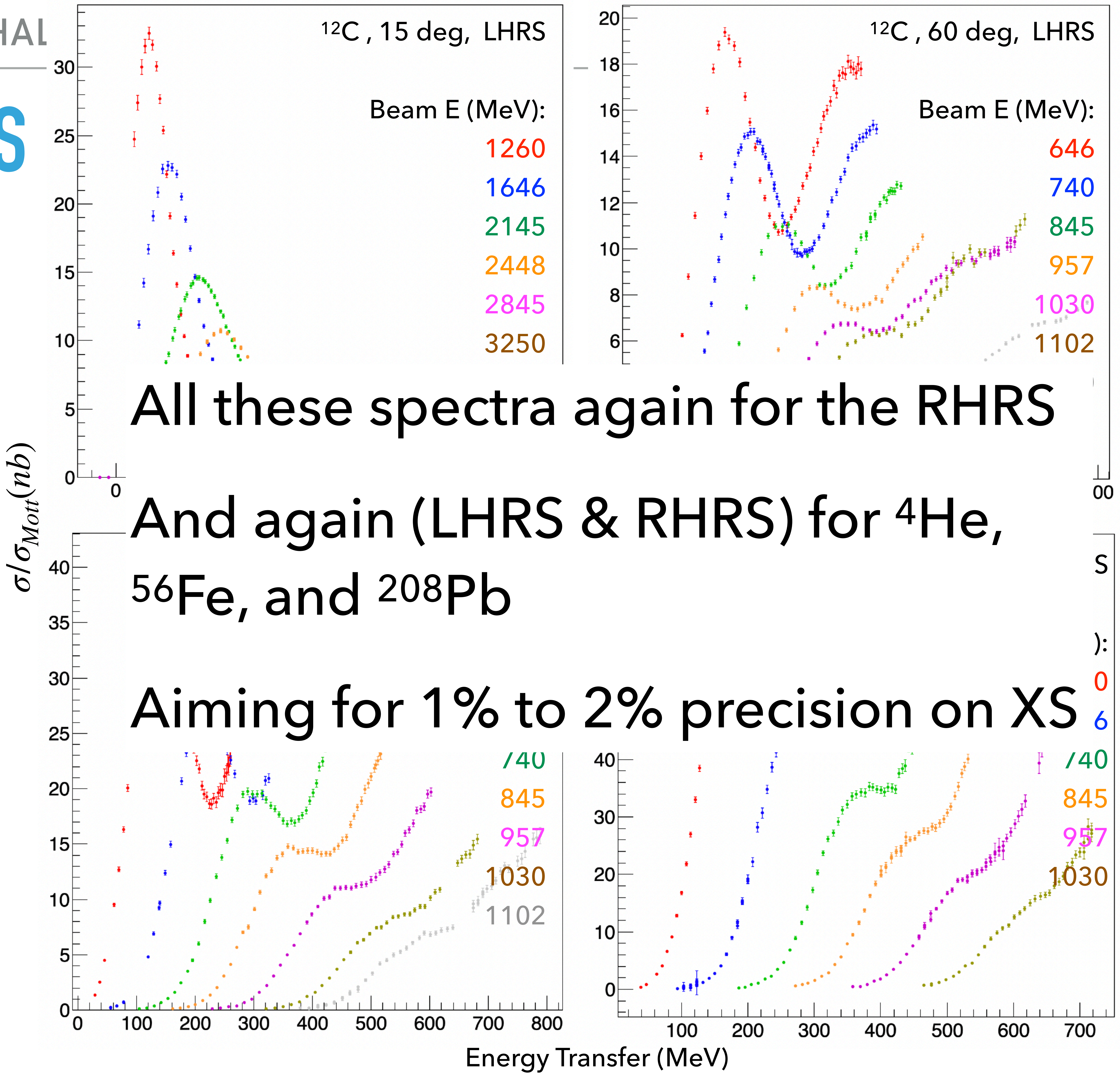

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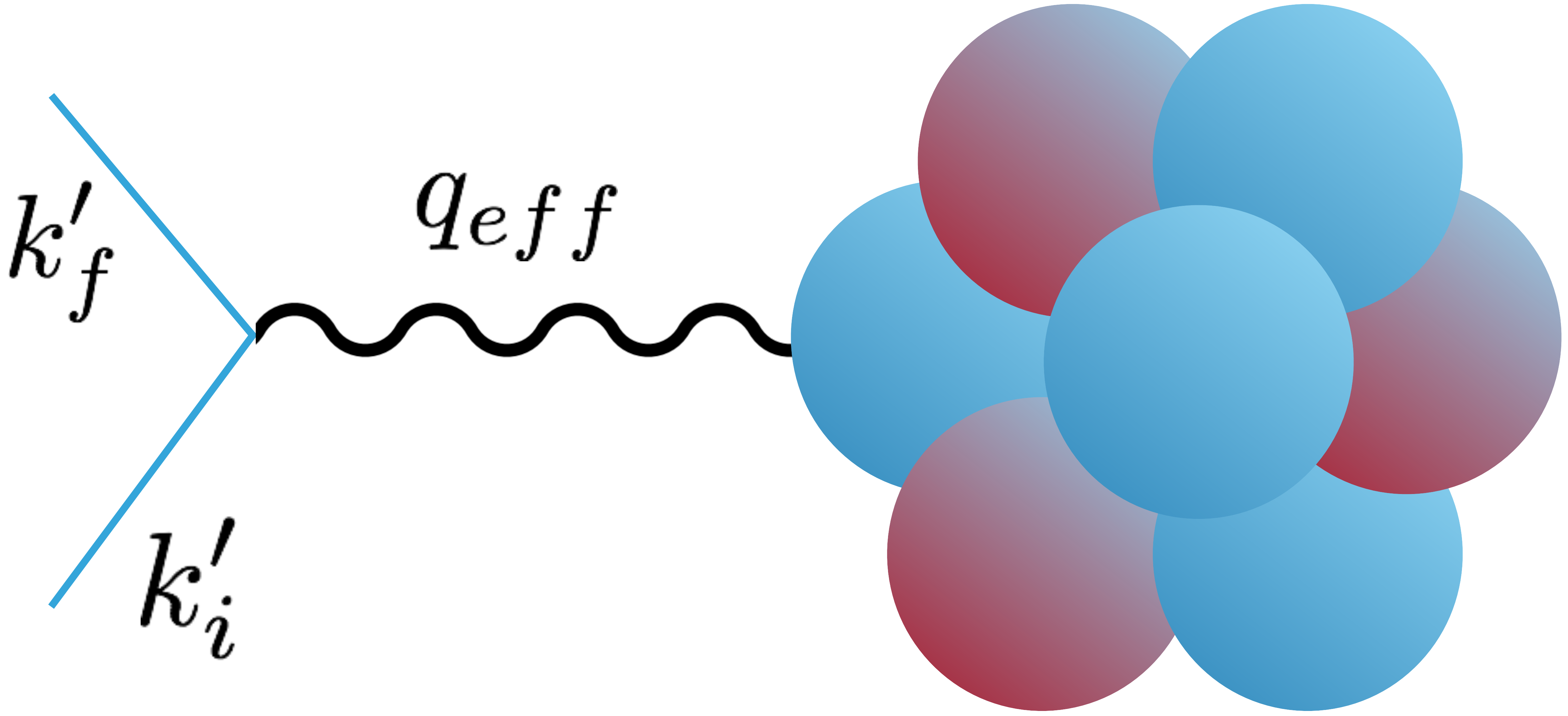
Aiming for 1% to 2% precision on XS

MEAN COULOMB POTENTIAL AND EMA

- An effective momentum approximation (EMA) takes into account the mean field potential of the target nucleus during quasi-elastic scattering.



$$k'_i = k_i - \kappa_A \frac{V_0}{c} \qquad k'_f = k_f - \kappa_A \frac{V_0}{c}$$

~ 0.75 to 0.8

$$V_0 = \frac{3\alpha Z}{2r_c}$$

$$\omega' = (k'_i - k'_f) = (k_i - k_f) = \omega$$

Nucleus	V_0 (MeV)
^{12}C	3.46 +/- 0.11
^{56}Fe	9.80 +/- 0.32
^{208}Pb	20.57 +/- 0.66

$$Q'^2 = 4(k'_i)(k'_f)\sin^2\theta/2$$

$$q_{eff} = \sqrt{\omega^2 + Q'^2}$$

A. Aste, D. Trautmann, EPJ A33 (2007), and A. Aste. Nucl.Phys.A806 (2008)

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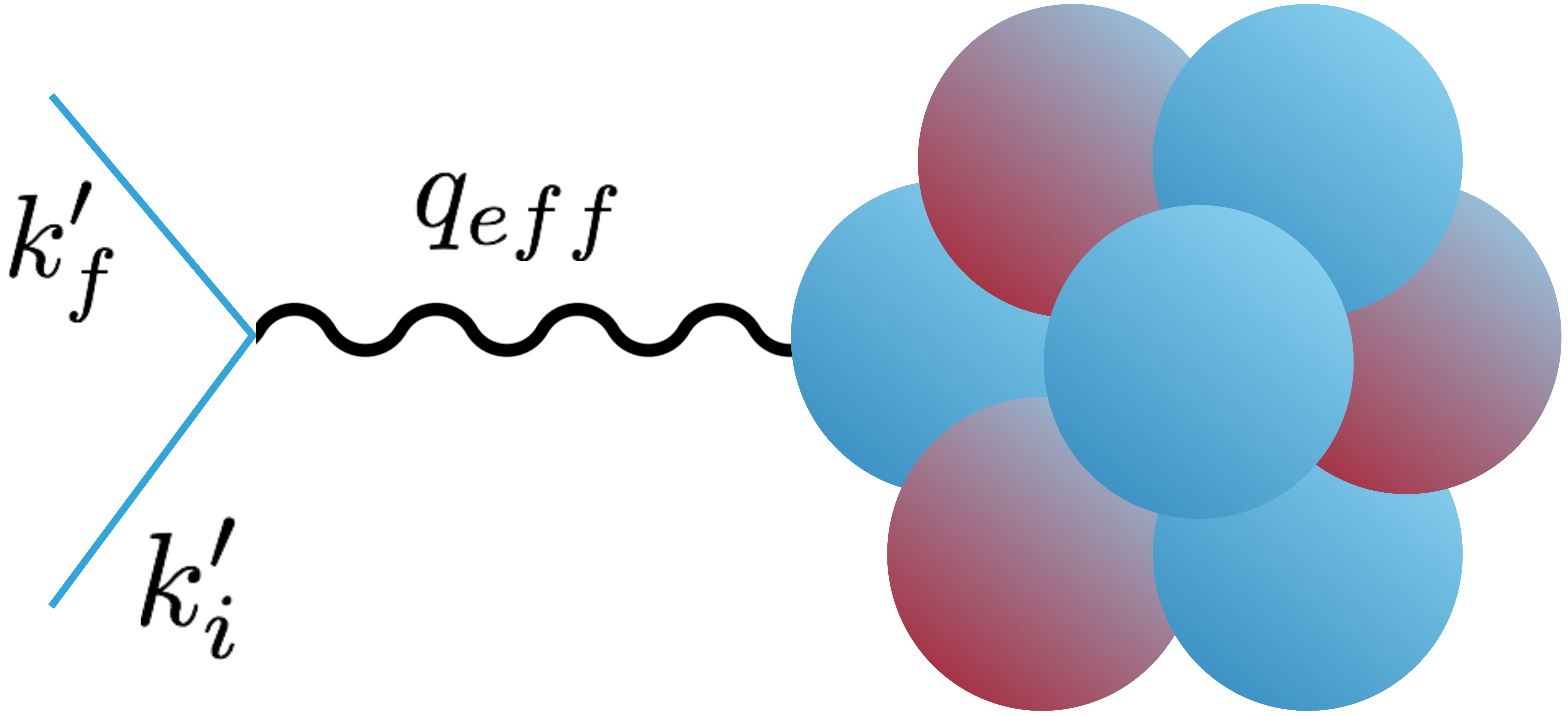

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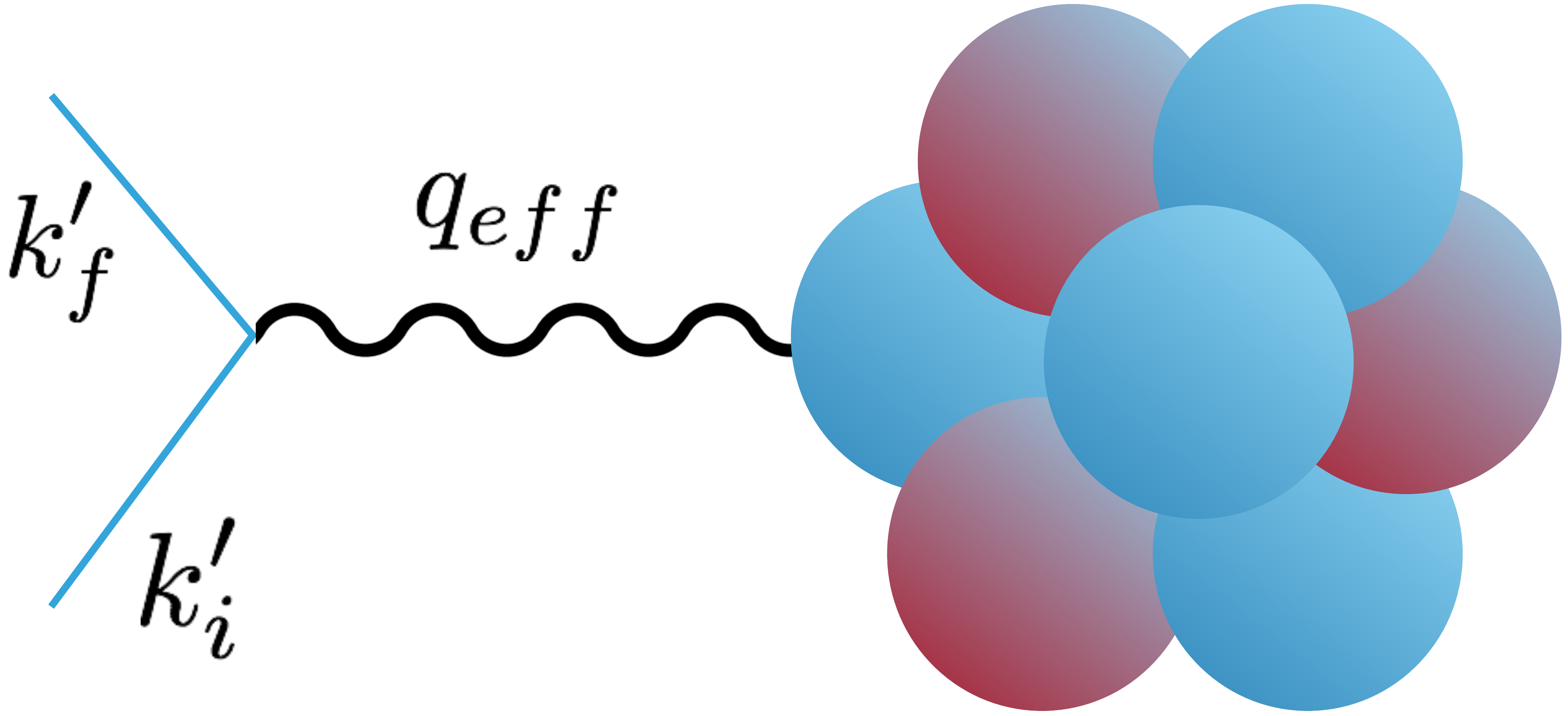


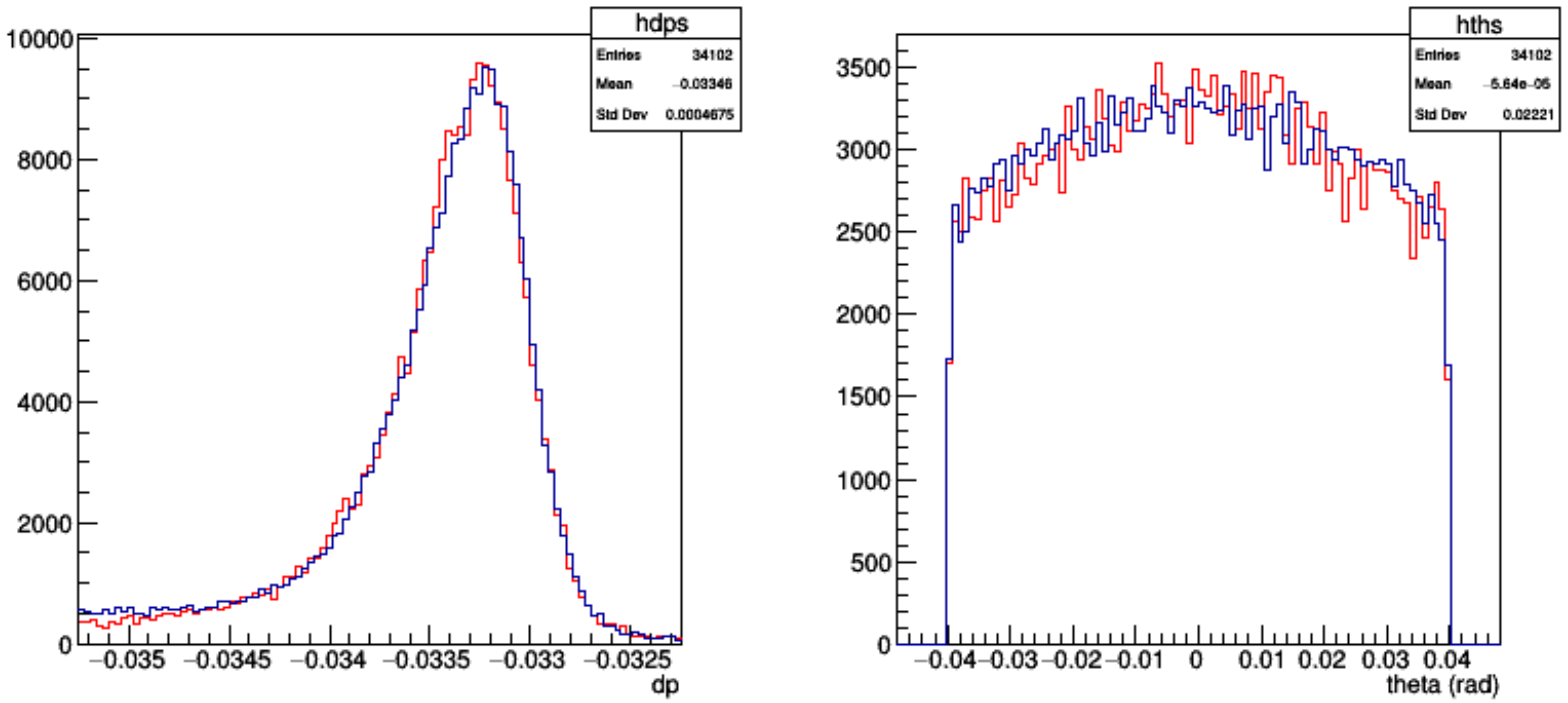
$$V_0 = \frac{3\alpha Z}{2r_c}$$

When scattering with positrons, we effectively change the sign of the mean potential

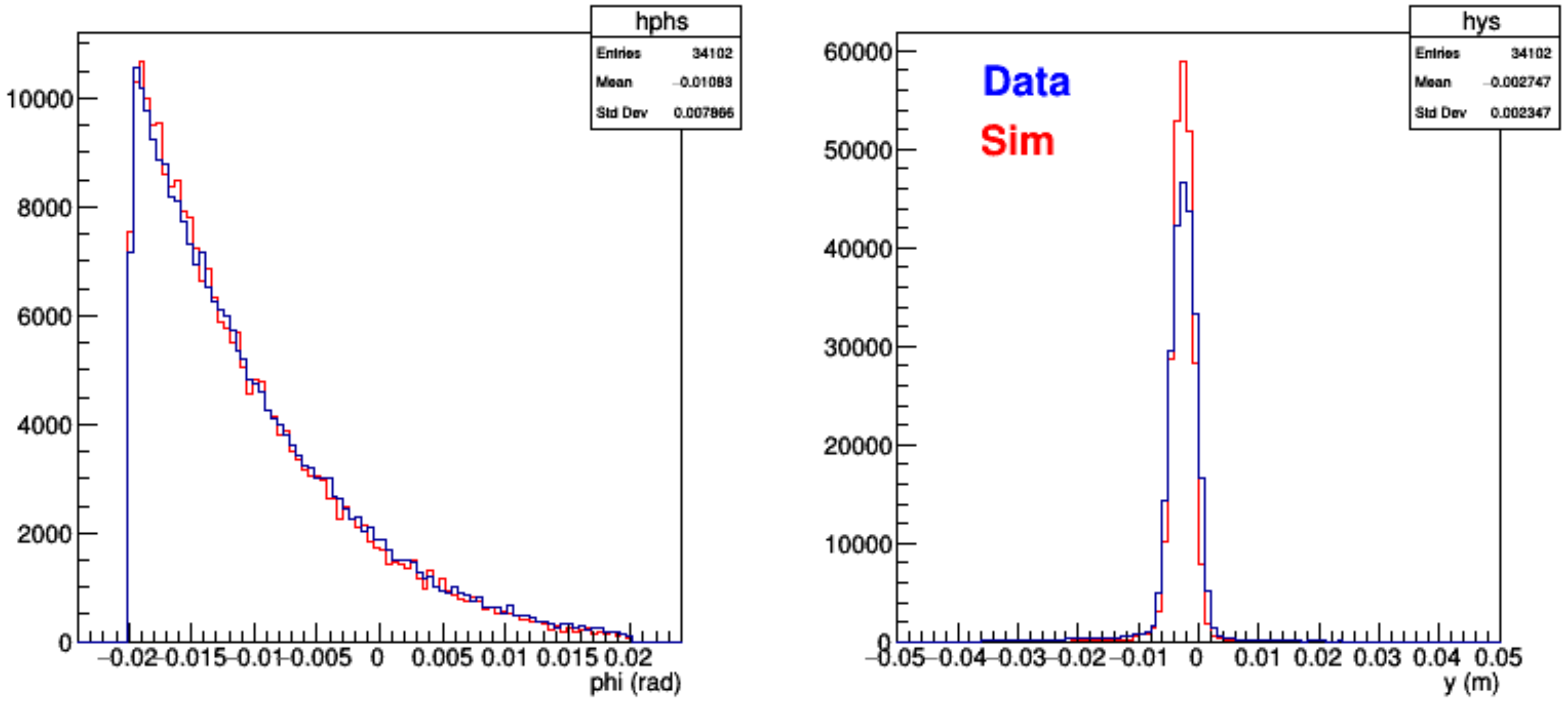
An e⁺ beam at JLab would allow a very detailed study of coulomb corrections!

A. Aste, D. Trautmann, EPJ A33 (2007), and A. Aste. Nucl.Phys.A806 (2008)

ELASTIC XS CALCULATIONS, AND ELASTIC TAIL CORRECTIONS

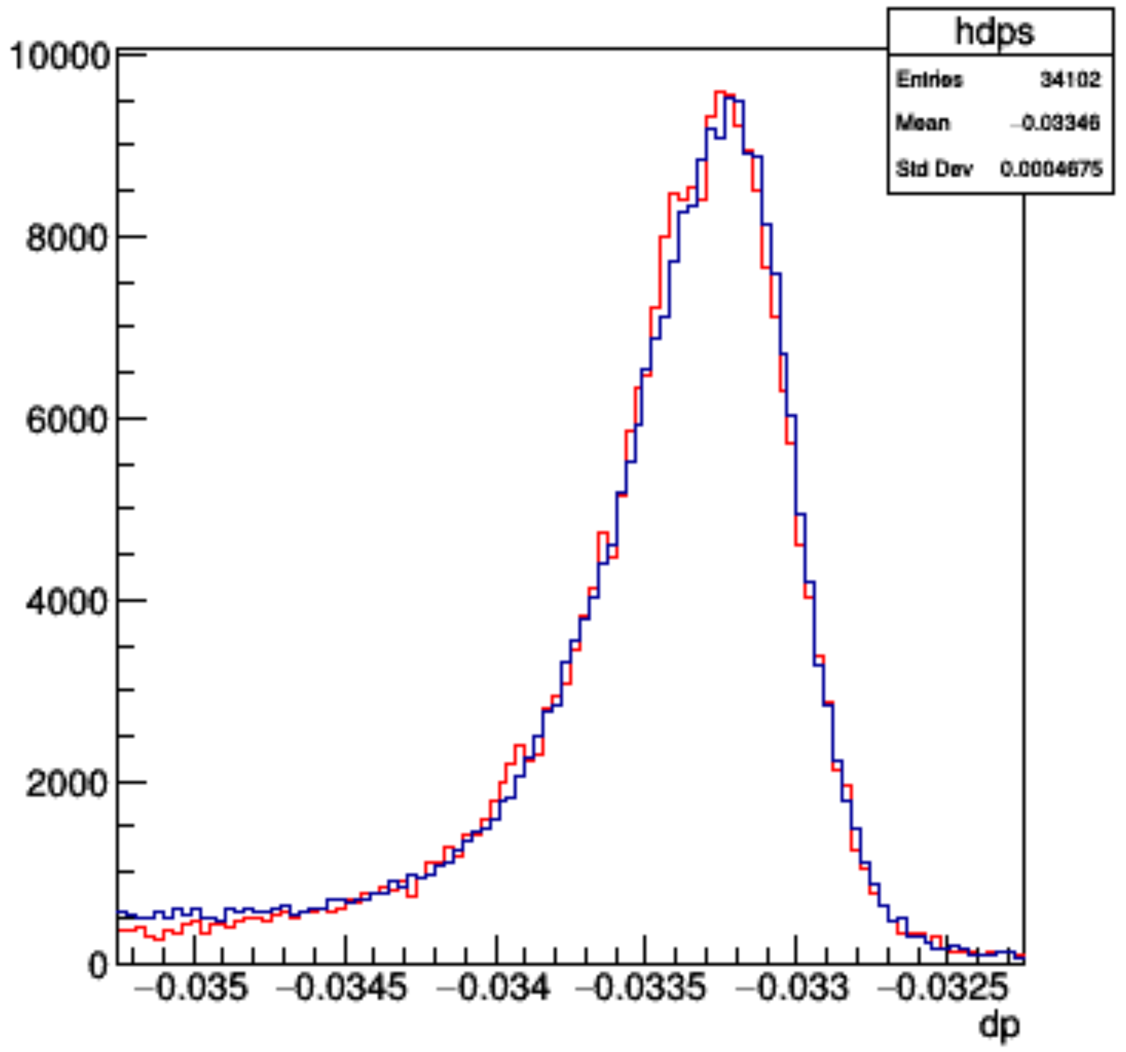


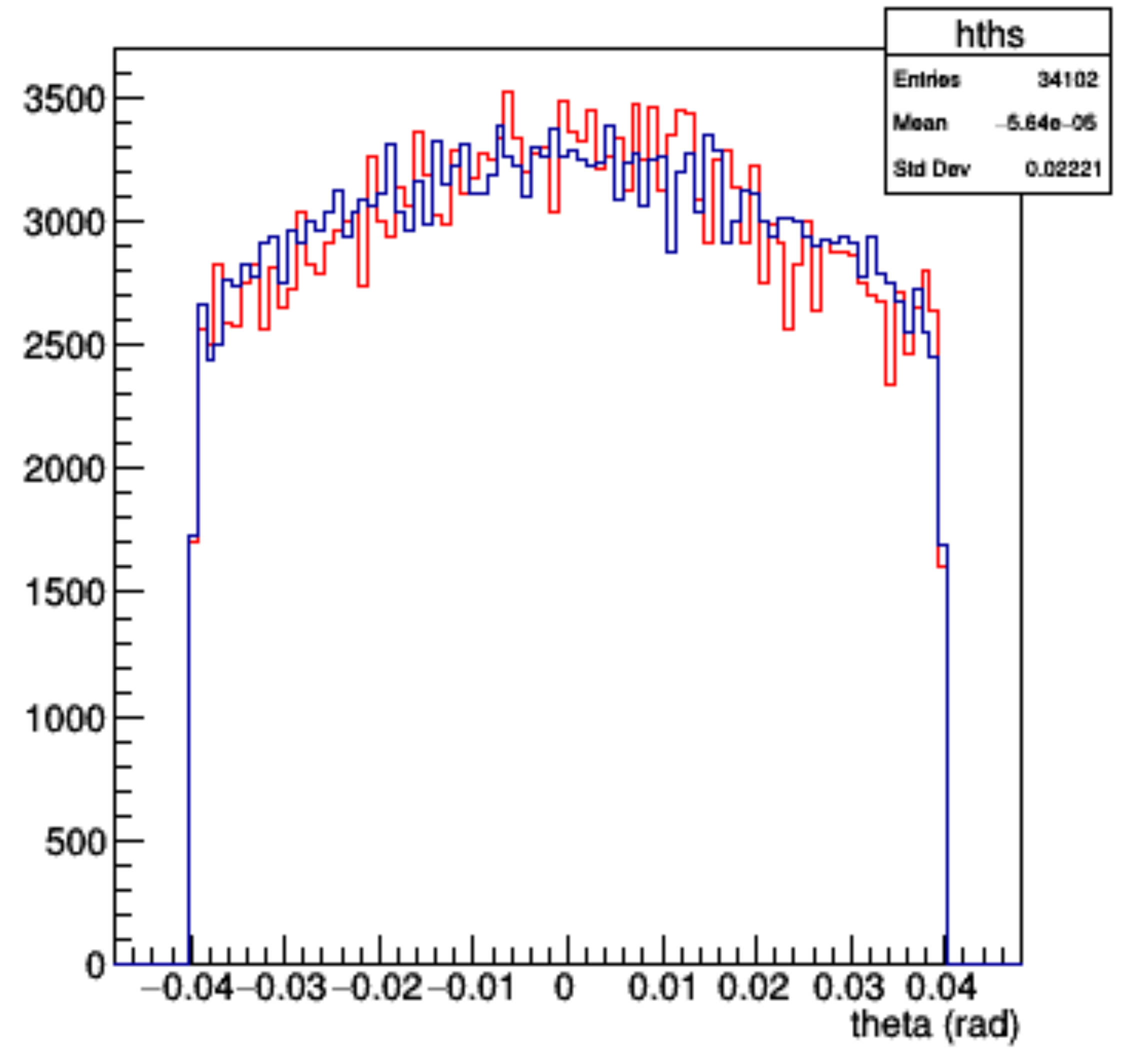
^{12}C elastic XS at 1260 MeV, 15 degrees



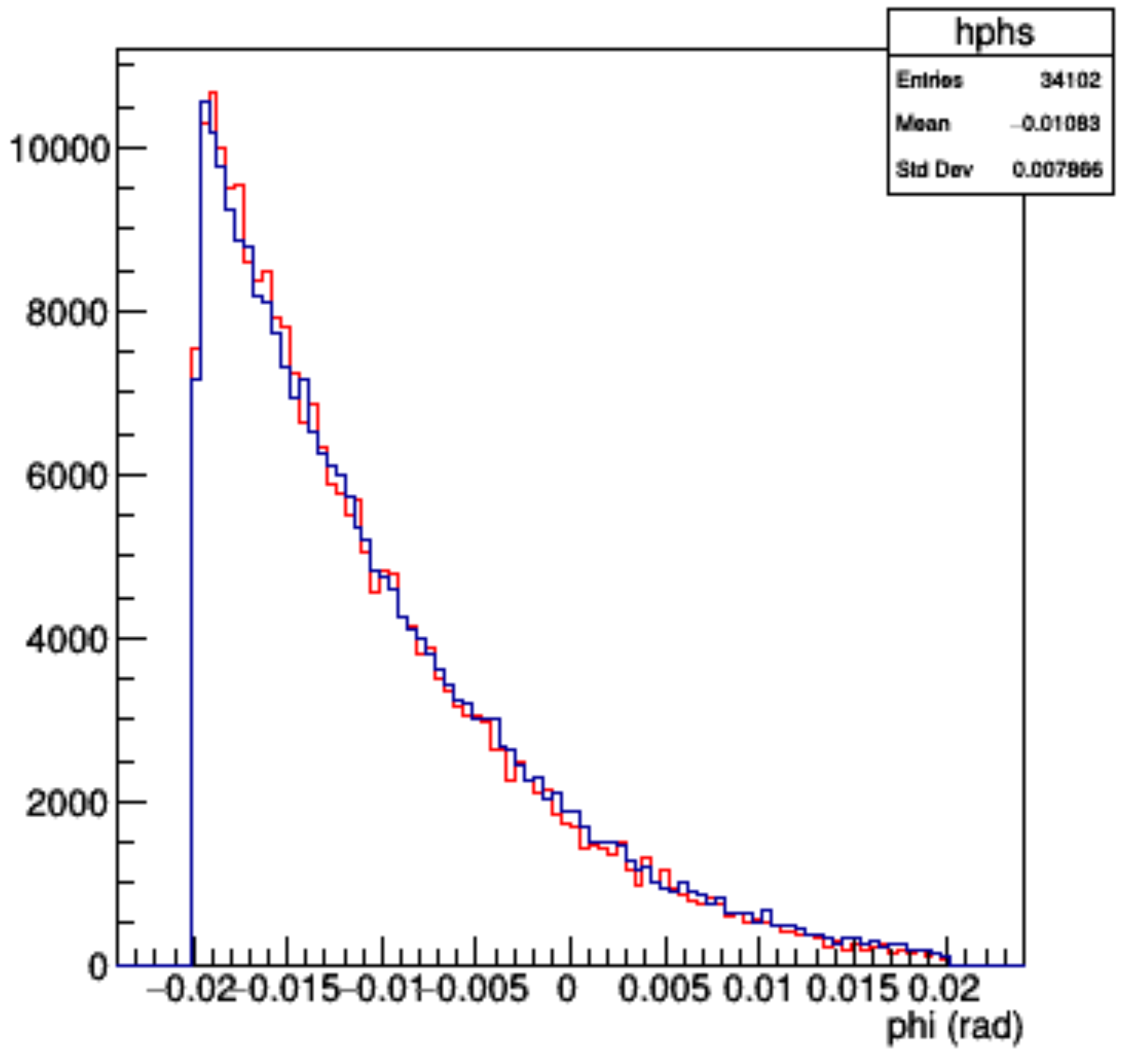
- Blue histograms are reconstructed data.
- Red histograms are monte-carlo:
  - Event sample generated from expected XS calculations (Fourier-Bessel fit to world data)
  - Radiative effects (internal, external, vertex) are handled, including exact bremsstrahlung distributions.
  - Resolution effects are applied by calculating the expected material effects of tracks passing through the VDC chamber materials.

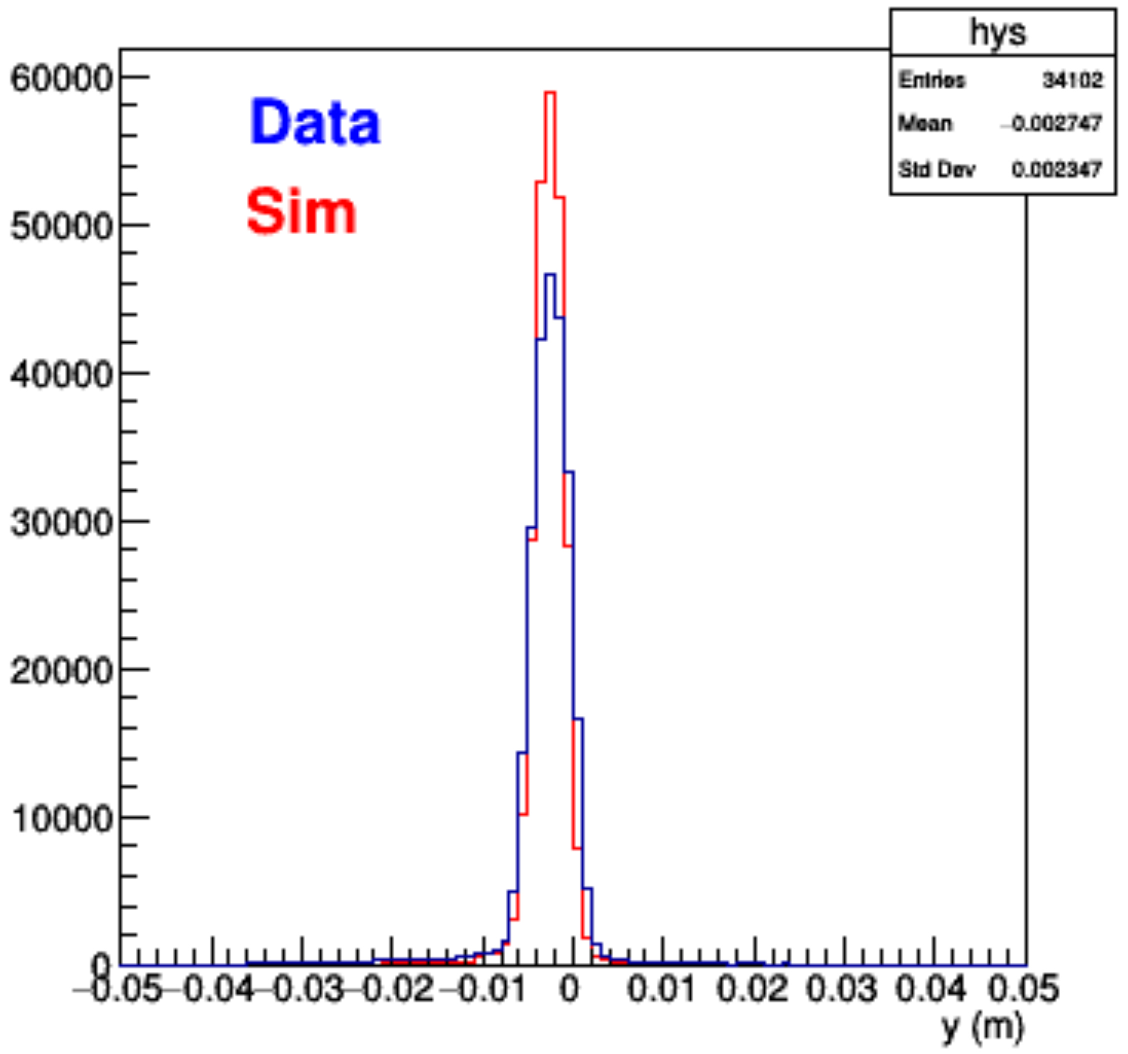
ELASTIC XS CALCULATIONS, AND ELASTIC TAIL CORRECTIONS

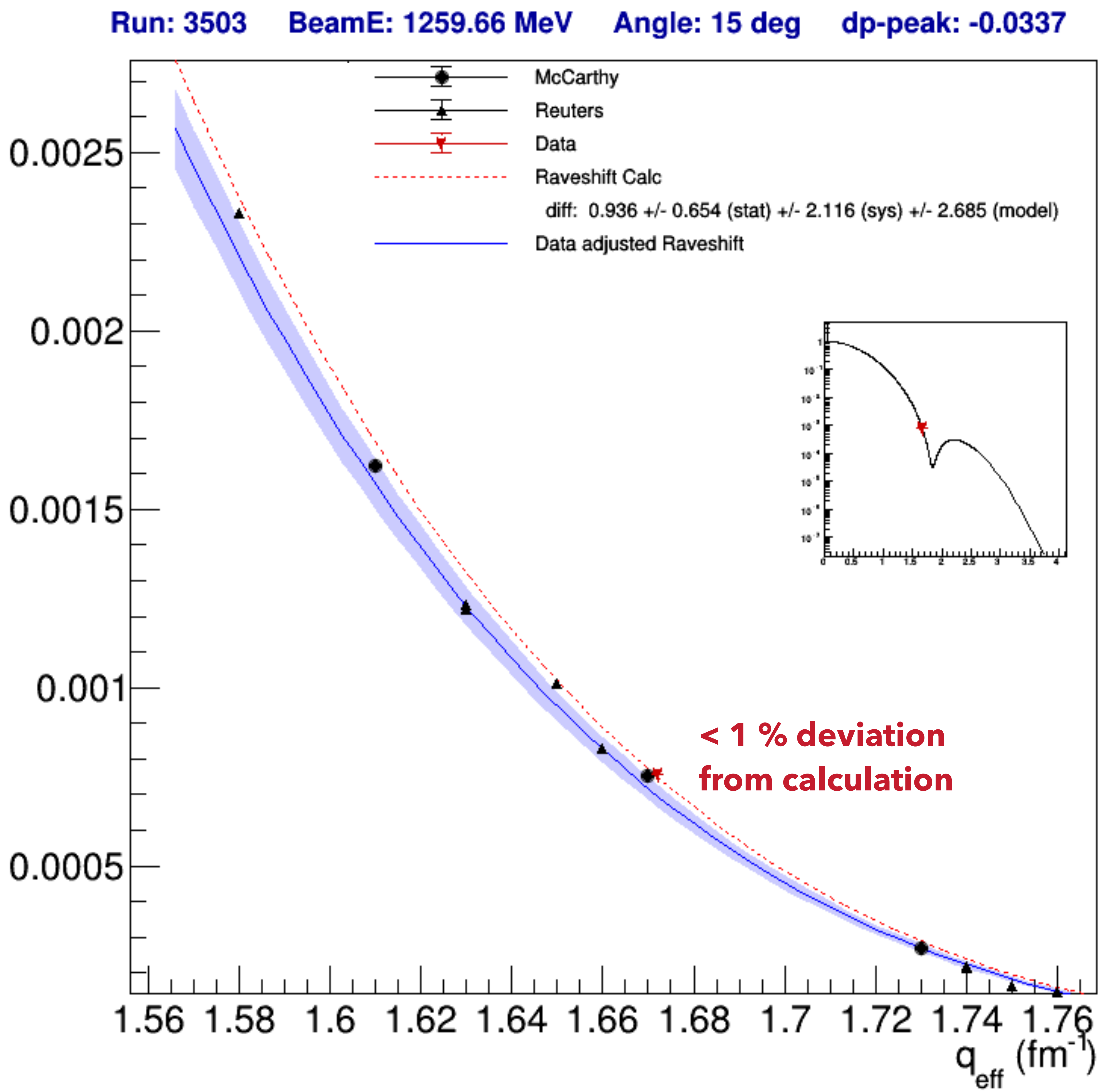




^{12}C **elastic XS at 1260 MeV, 15 degrees**







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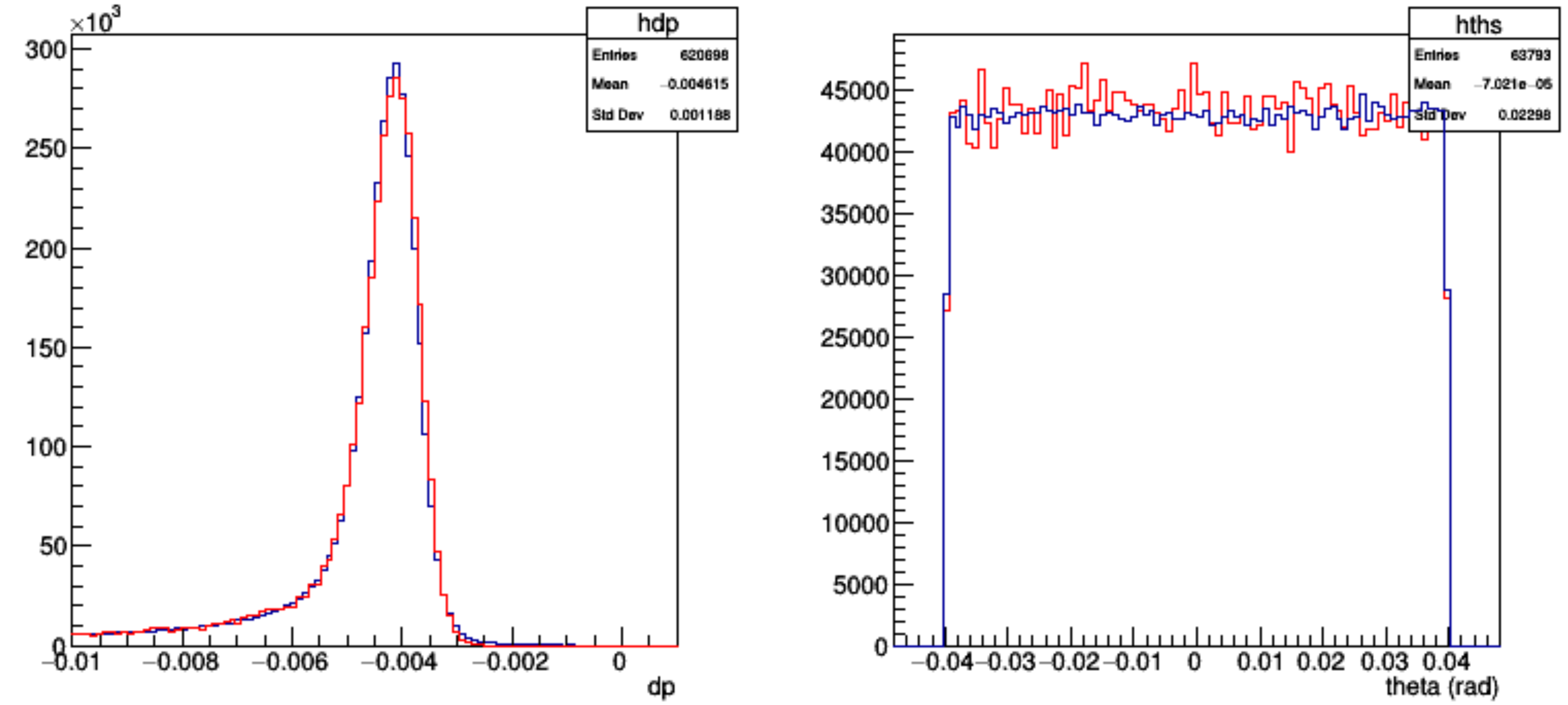
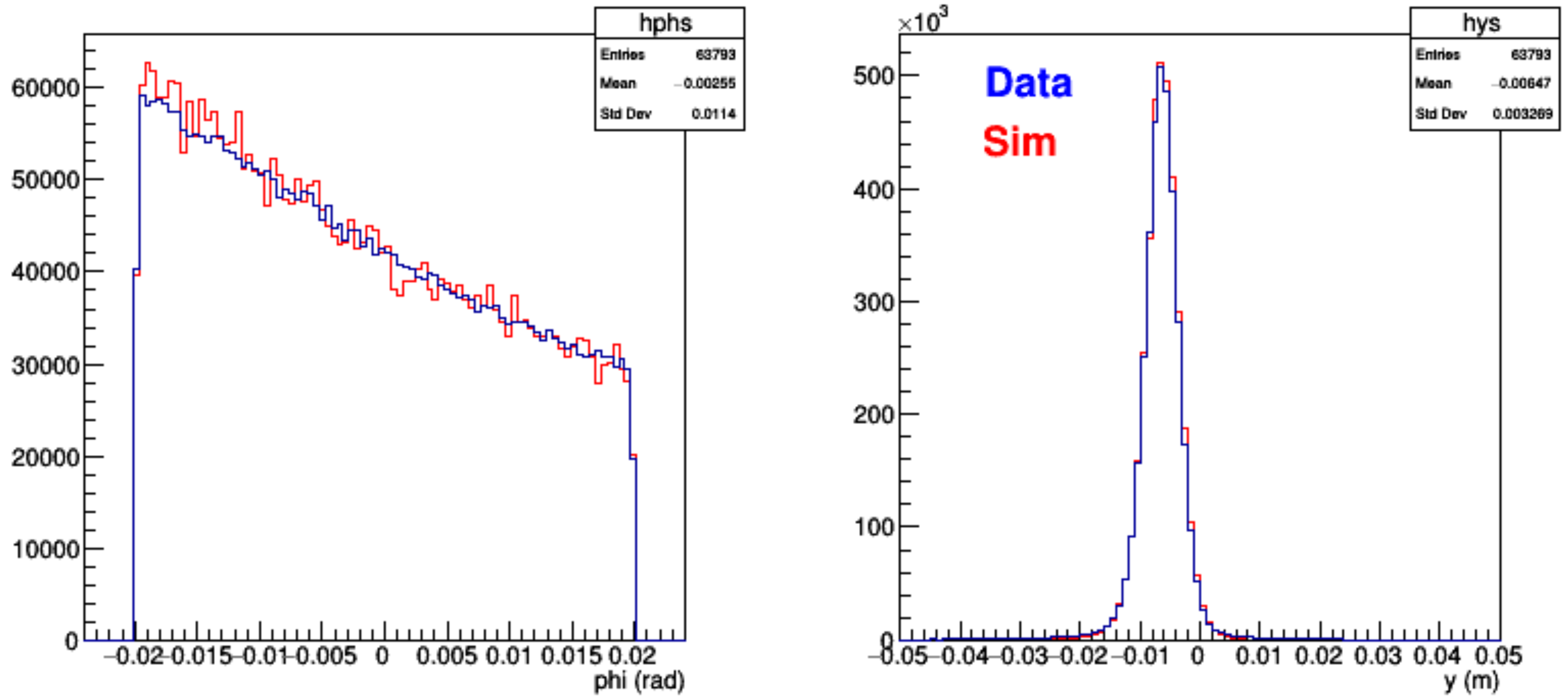
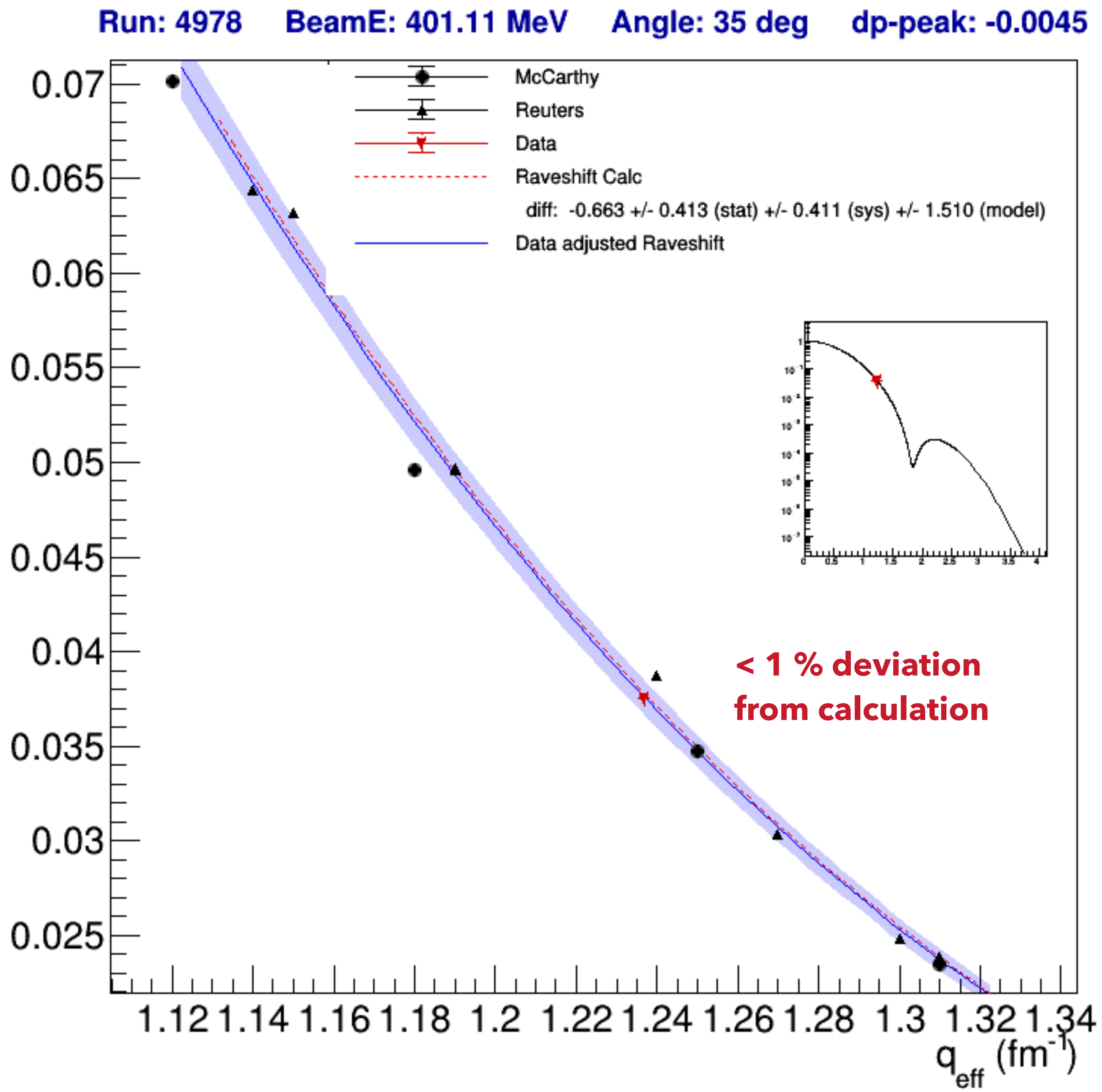
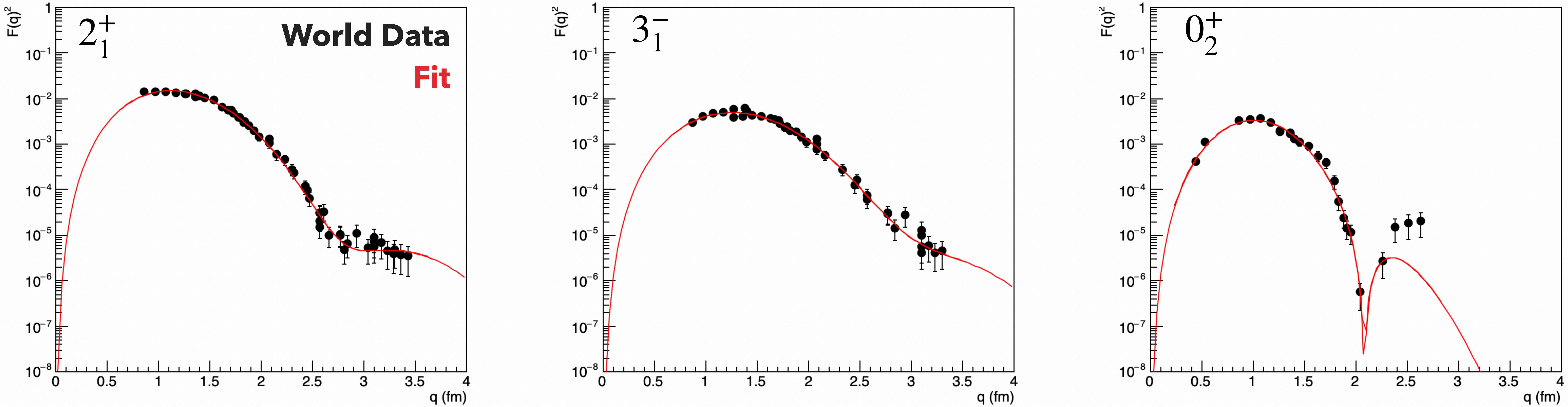


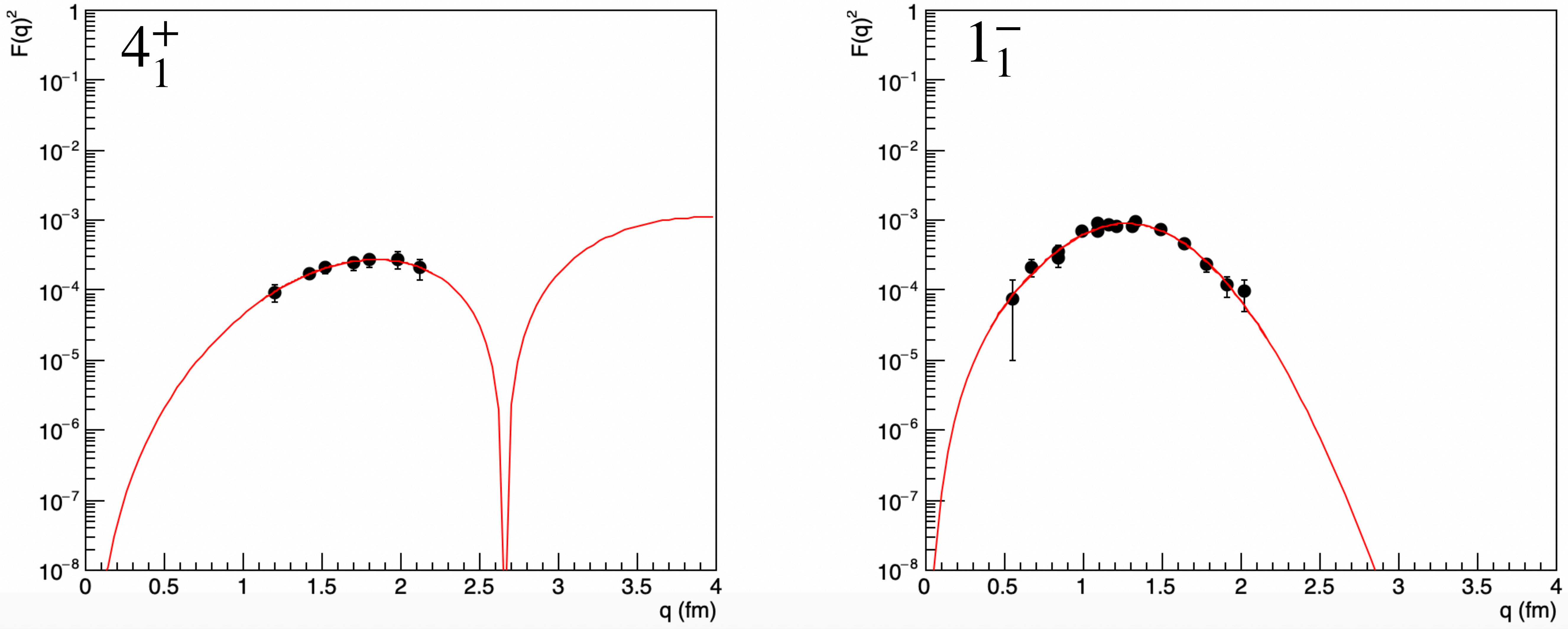
^{12}C elastic XS at 400 MeV, 35 degrees





EXCITED ELASTIC STATES





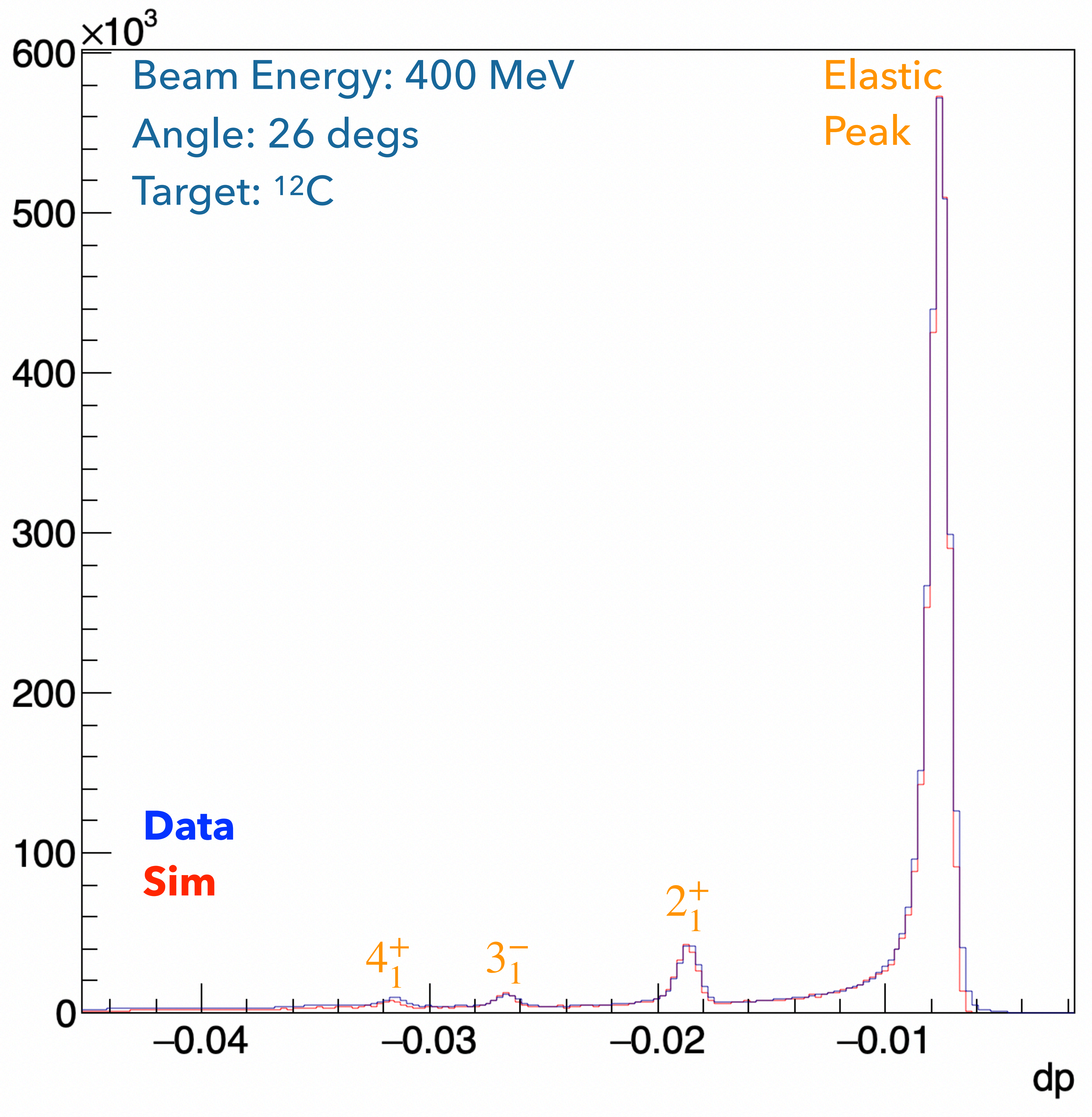
Extractions of excited elastic states based on fit of transition form-factors to world data.

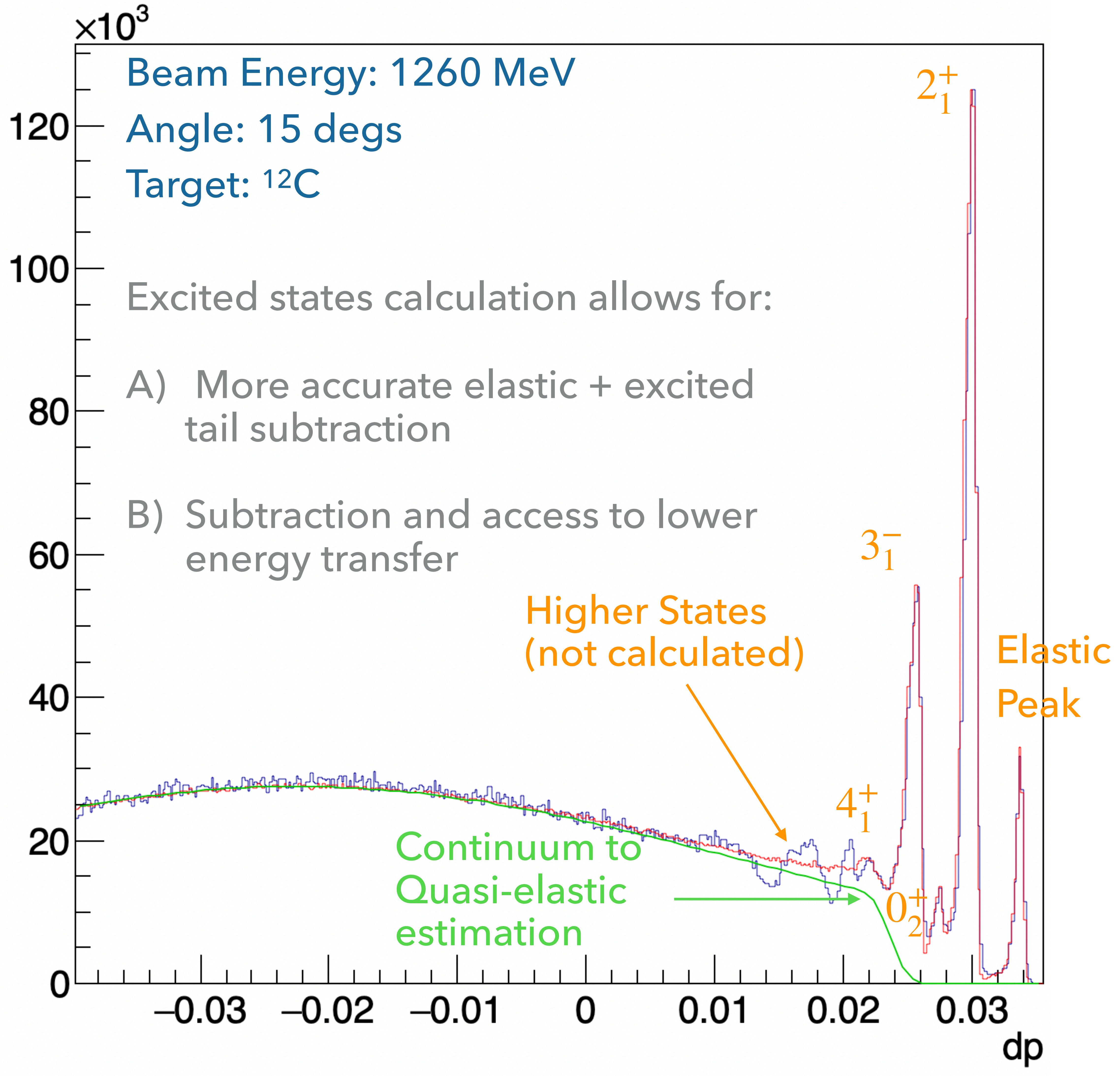
Functional form follows an analytic, global, and model-independent analysis introduced recently* (mostly in the study of the 0_2^+ "Hoyle" state)

$$F(q) = \frac{1}{Z} e^{-\frac{1}{2}(bq)^2} \sum_{n=1}^{n_{\max}} c_n (bq)^{2n}$$

* M. Chernykh, *et al*. Phys. Rev. Lett. 105

EXCITED ELASTIC STATES



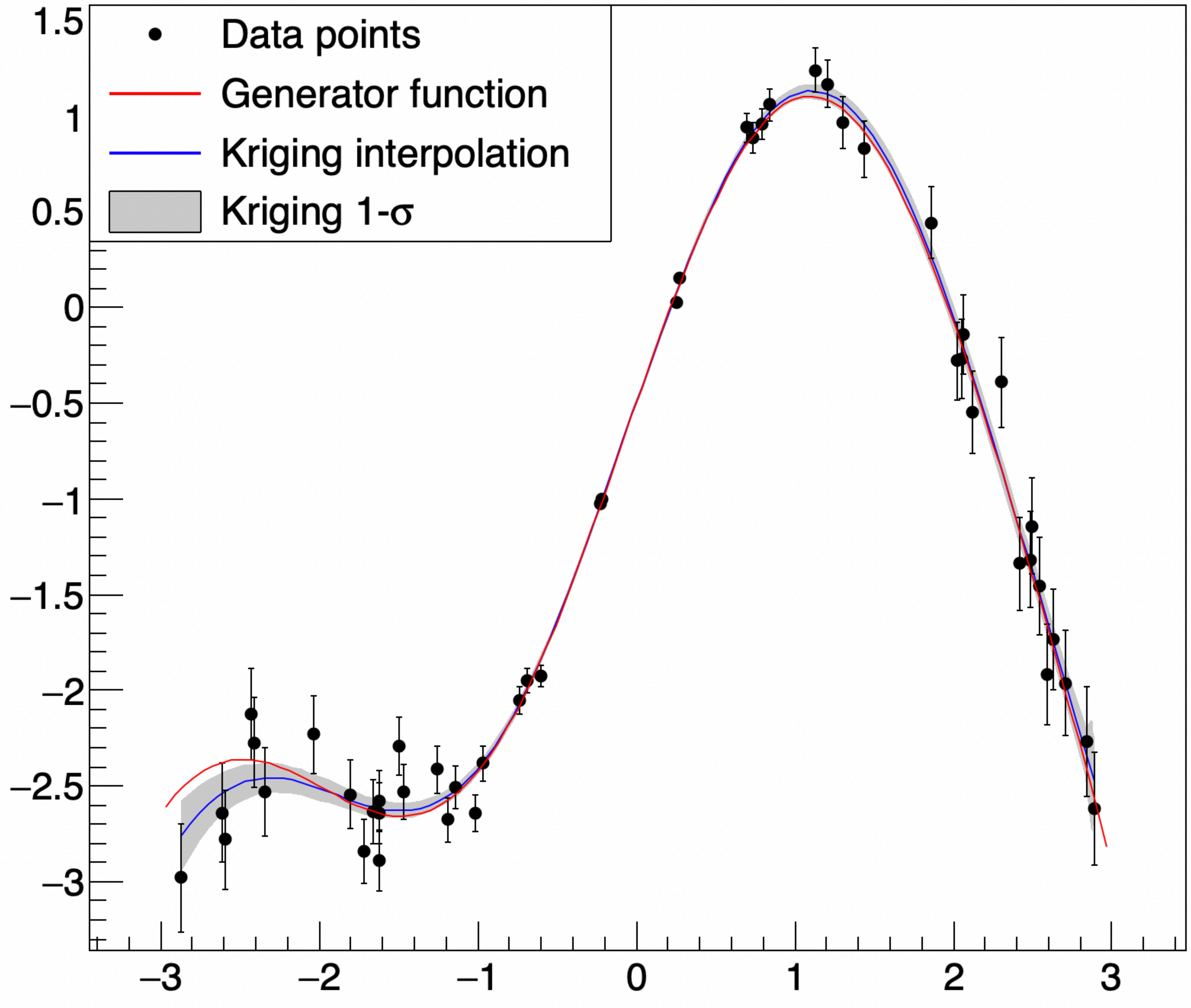


* M. Chernykh, *et al*. Phys. Rev. Lett. 105

INTERPOLATION TECHNIQUE: GAUSSIAN PROCESS REGRESSION

- Utilizes the supervised machine learning technique "Gaussian Process Regression"
  - Well documented and utilized process:
    - Gaussian Processes for Machine Learning, Carl Edward Rasmussen and Chris Williams, the MIT Press, 2006*
    - Used in packages like scikit-learn (python) and Weka (Java)
  - I've written a similar package in C++ with root integration.
    - Recent updates:
      - Full N-Dimensional interpolation (not available in scikit-learn, Weka, or anywhere else I've seen)
      - Hyper-parameter tuning methods (using both Log-marginal likelihood and psuedo-log-likelihood.

Generic 1D example



*Documentation: www.gaussianprocess.org/gpml/chapters/

INTERPOLATION TECHNIQUE: GAUSSIAN PROCESS REGRESSION

- Utilizes the supervised machine learning technique "Gaussian Process Regression"
  - Well documented and utilized process:
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    - Used in packages like scikit-learn (python) and Weka (Java)
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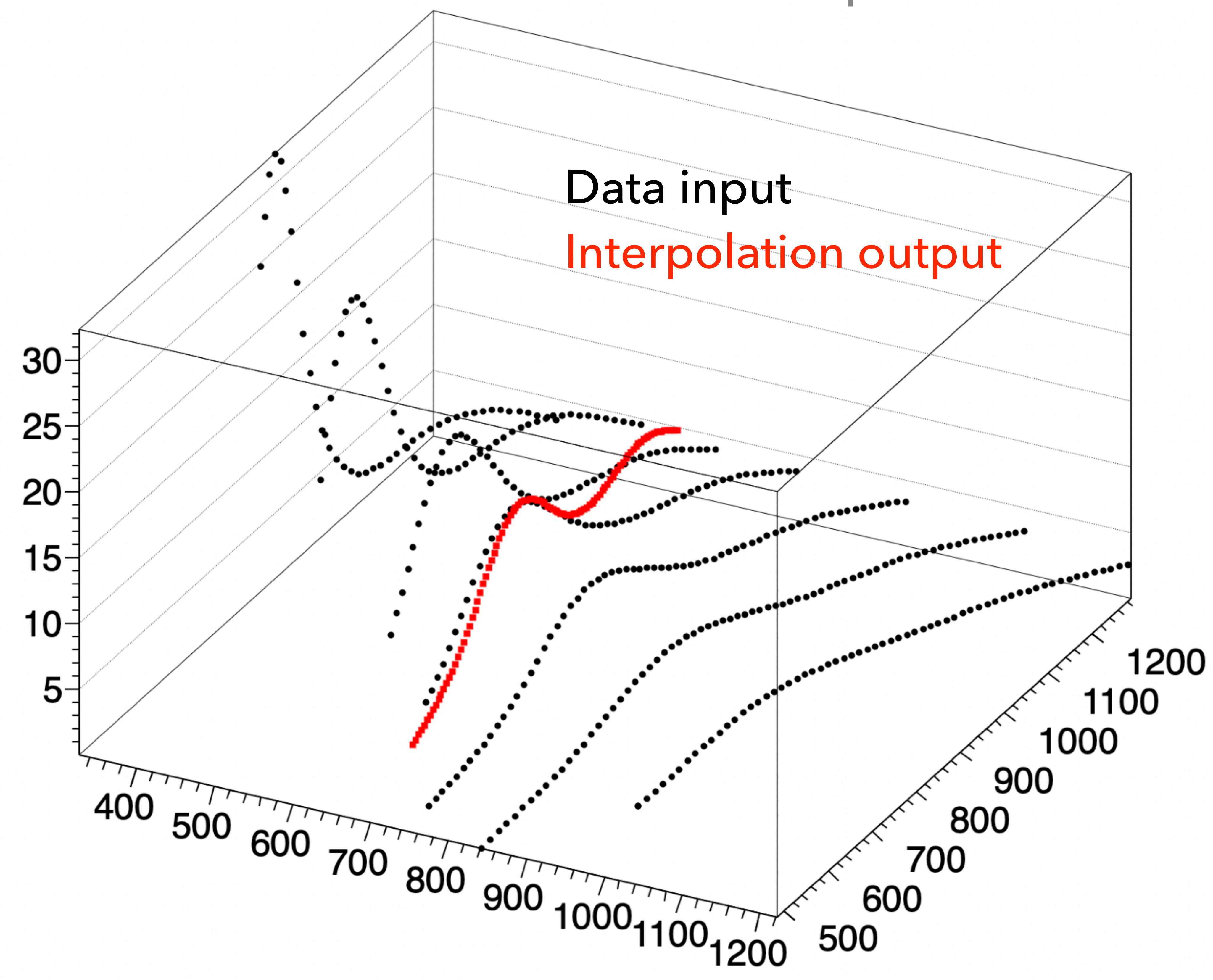
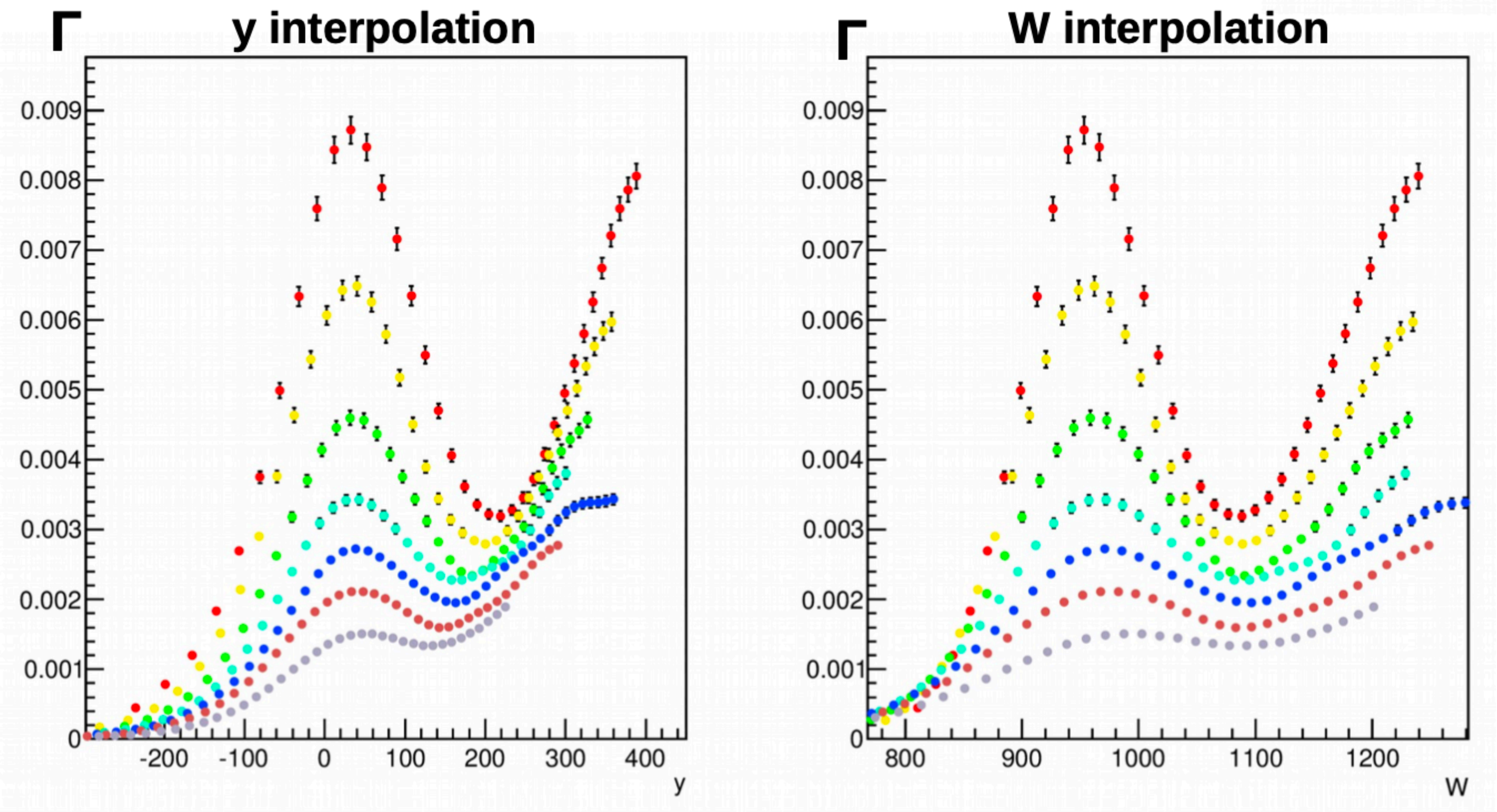
      - Full N-Dimensional interpolation (not available in scikit-learn, Weka, or anywhere else I've seen)
      - Hyper-parameter tuning methods (using both Log-marginal likelihood and psuedo-log-likelihood.

CSR 2D example



PROGRESS ON ^{4}HE (KAI JIN)



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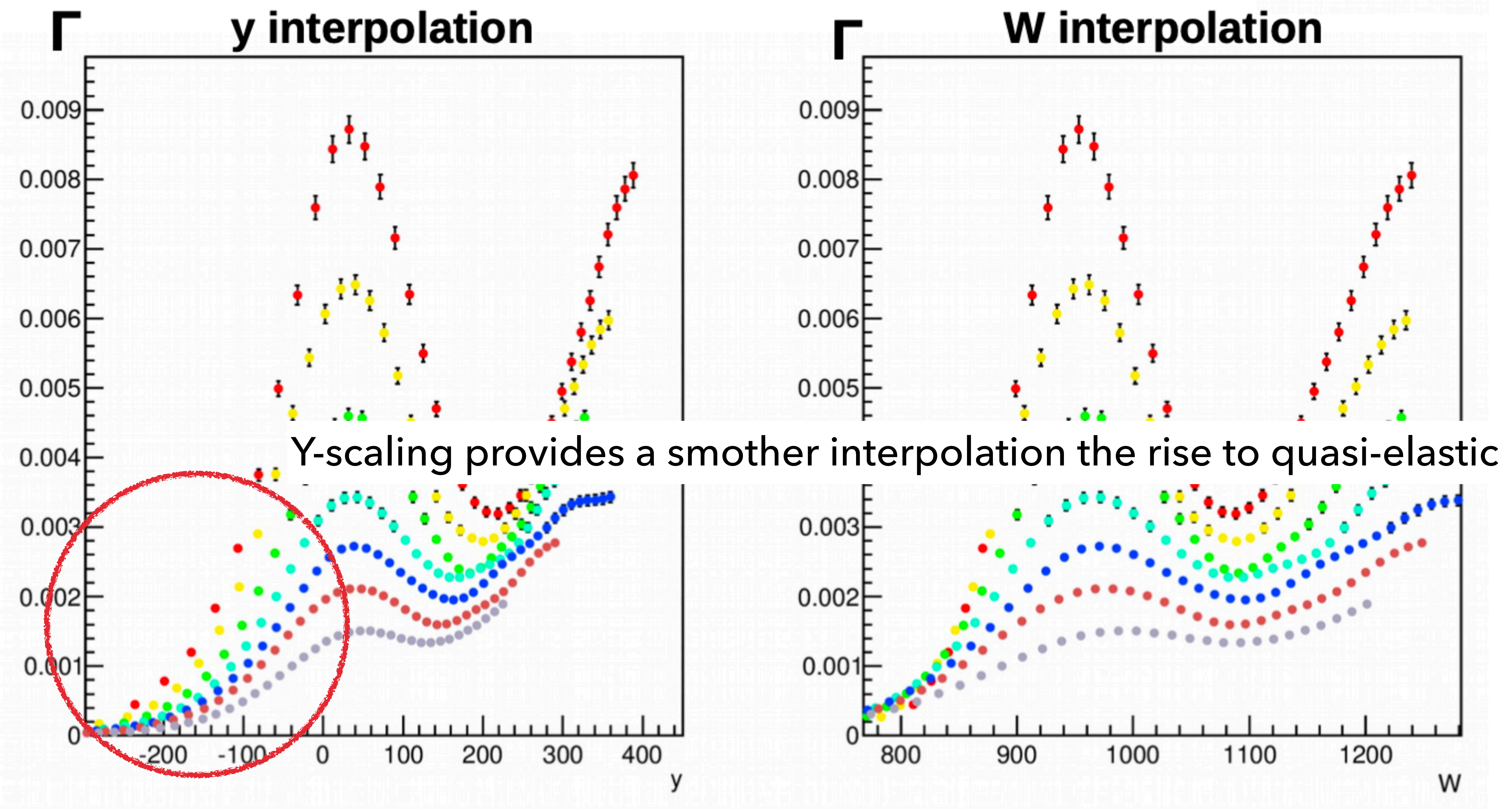
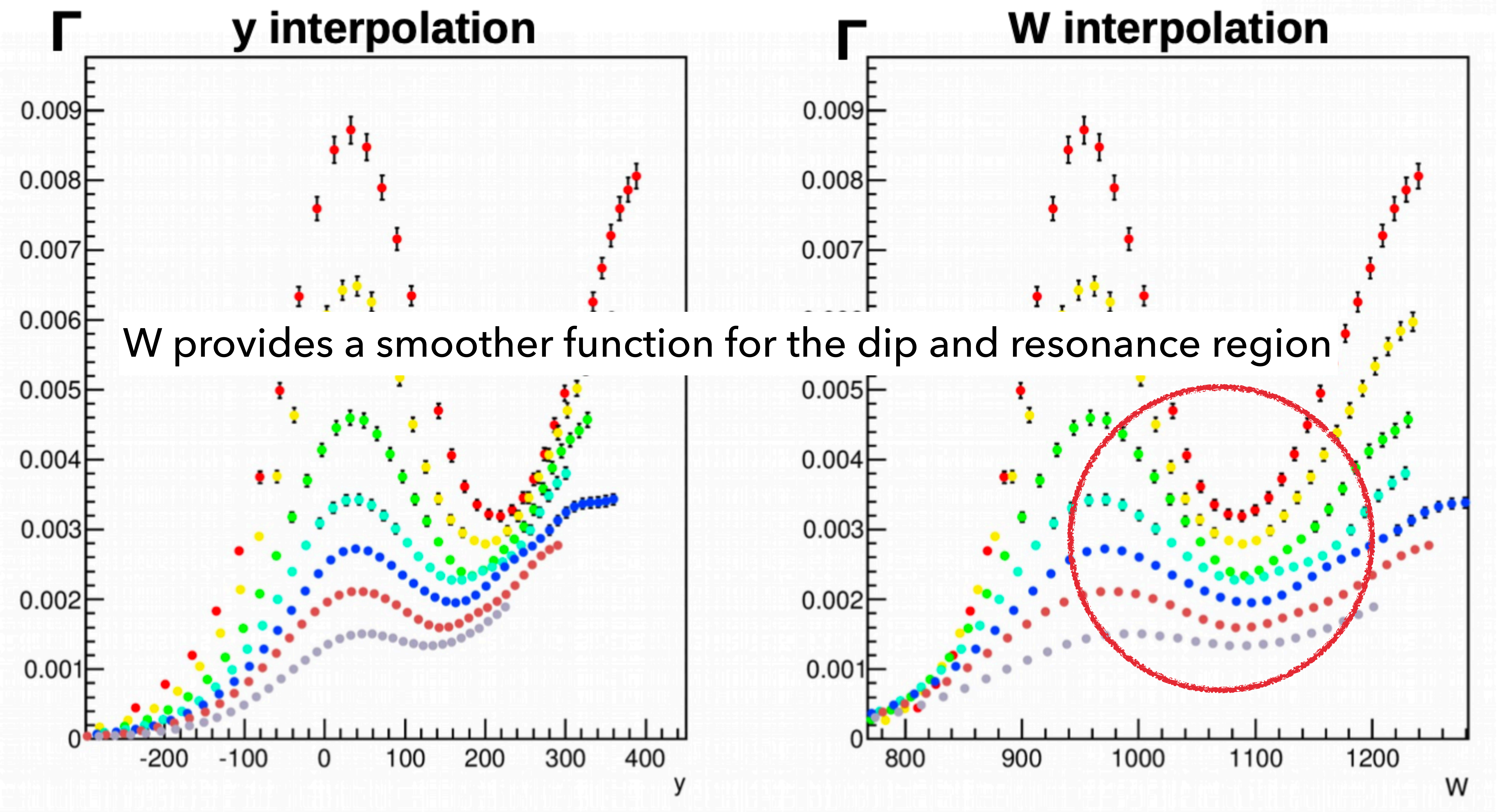


Y-scaling provides a smother interpolation the rise to quasi-elastic peak

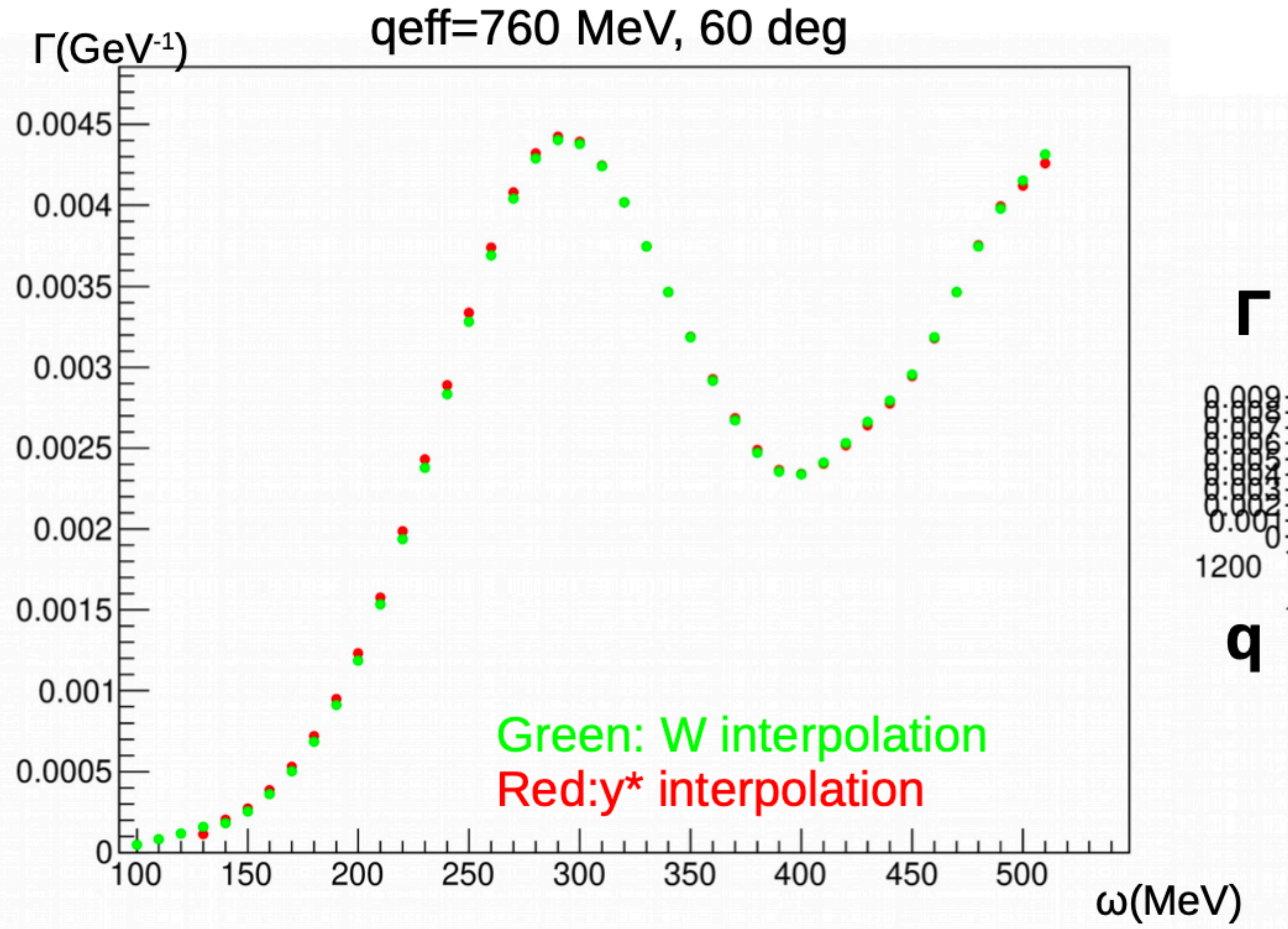
PROGRESS ON ^{4}HE (KAI JIN)

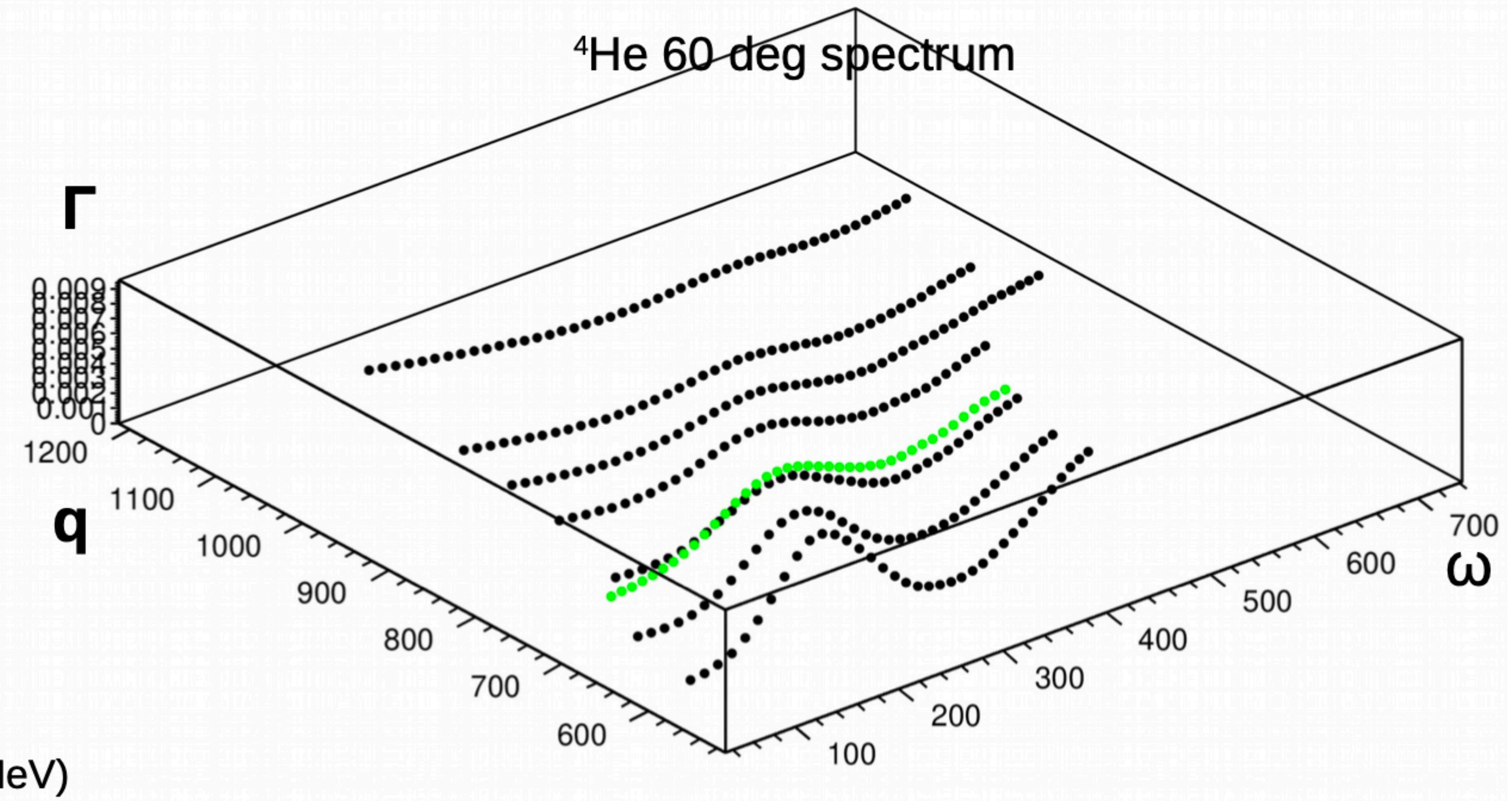


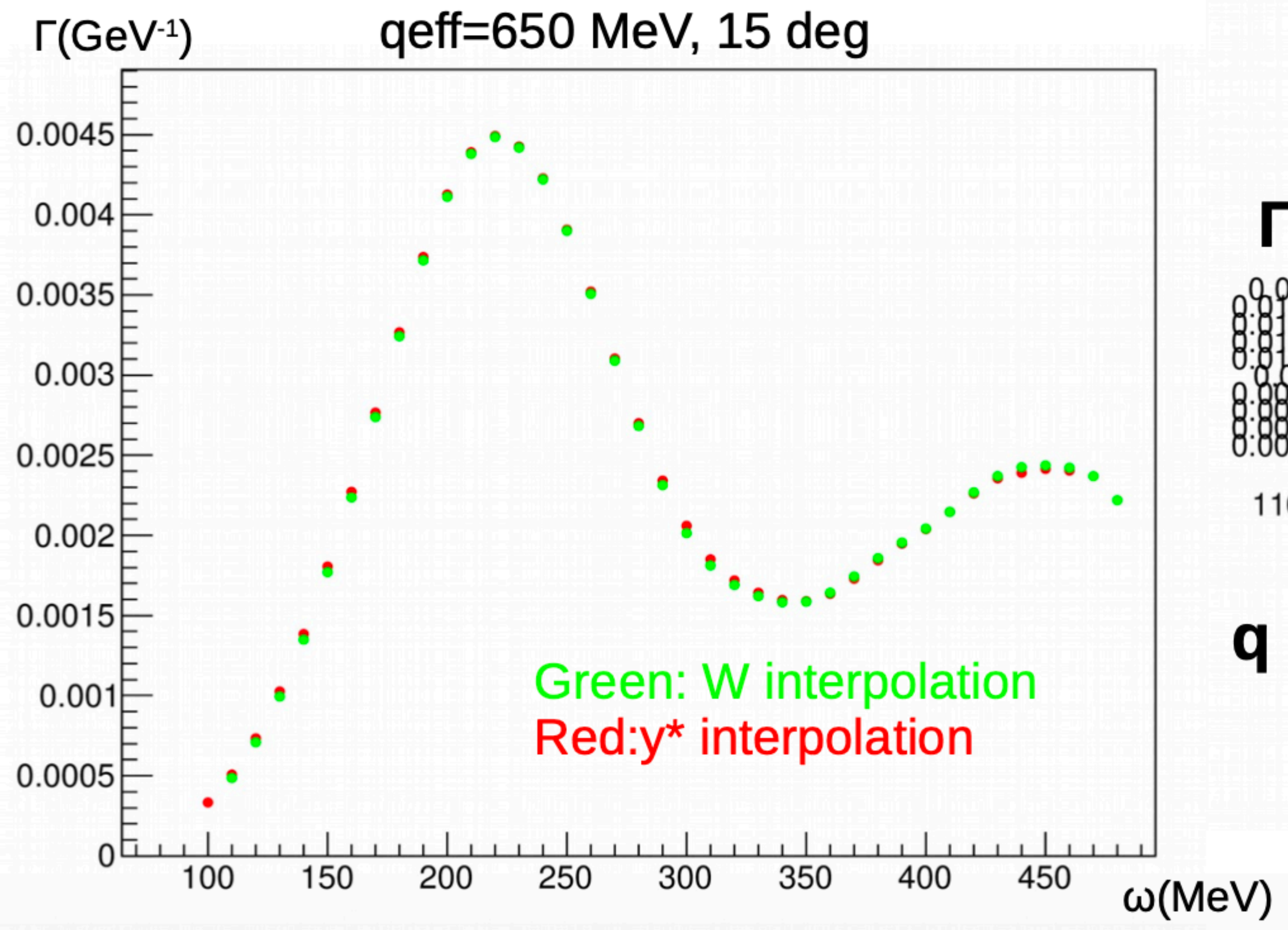
W provides a smoother function for the dip and resonance region

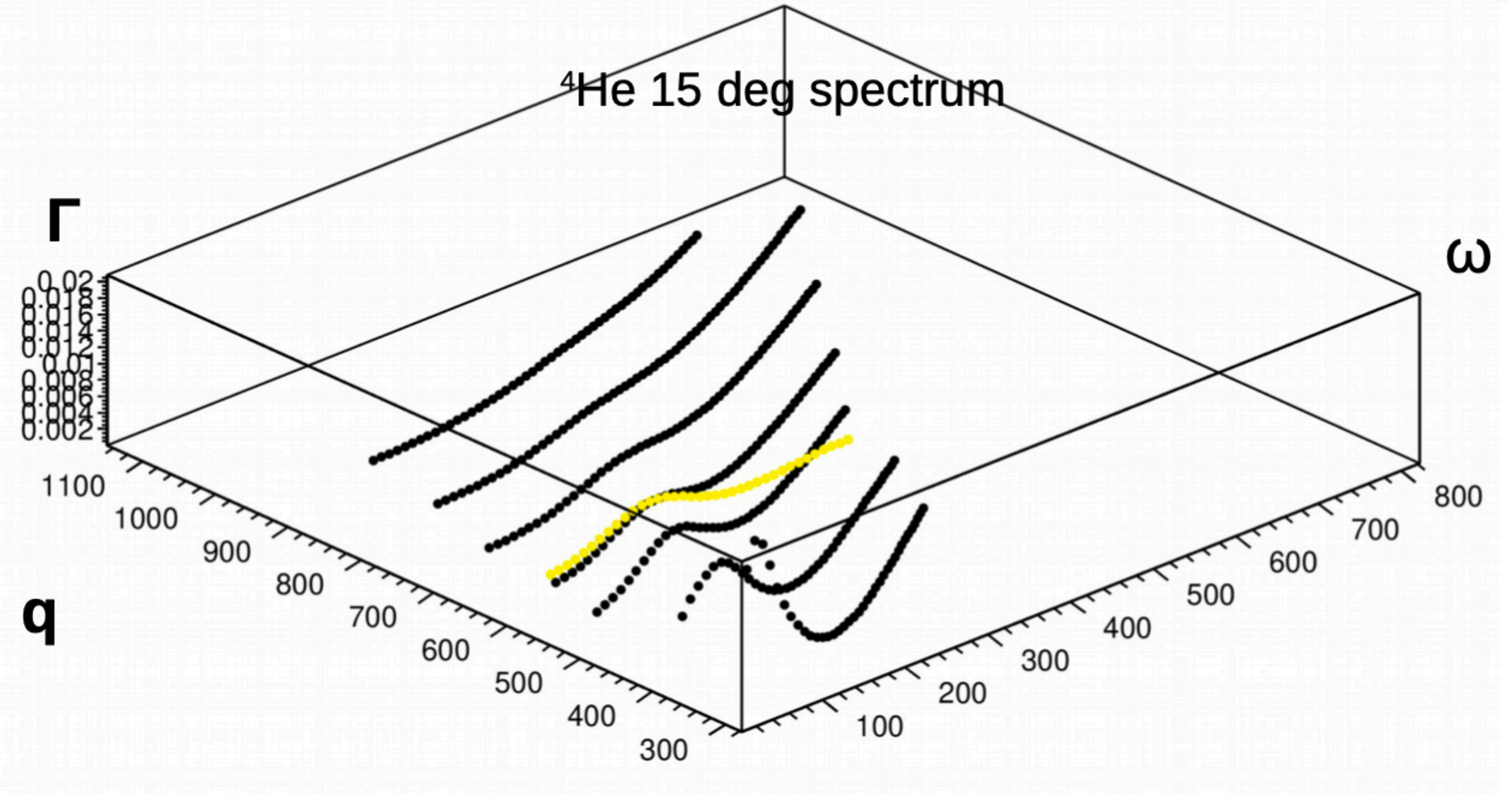
PROGRESS ON 4HE (KAI JIN)

Interpolation

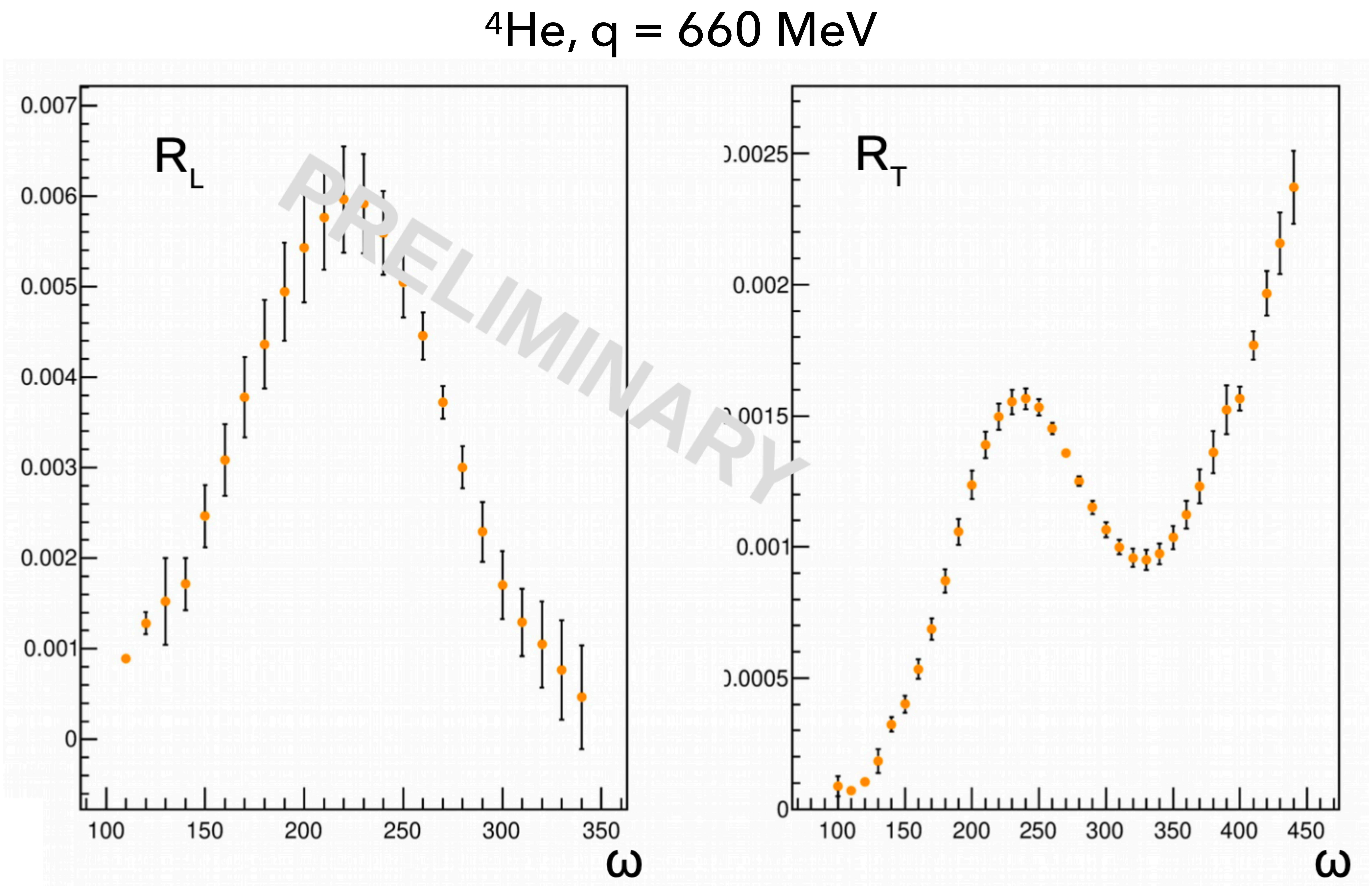






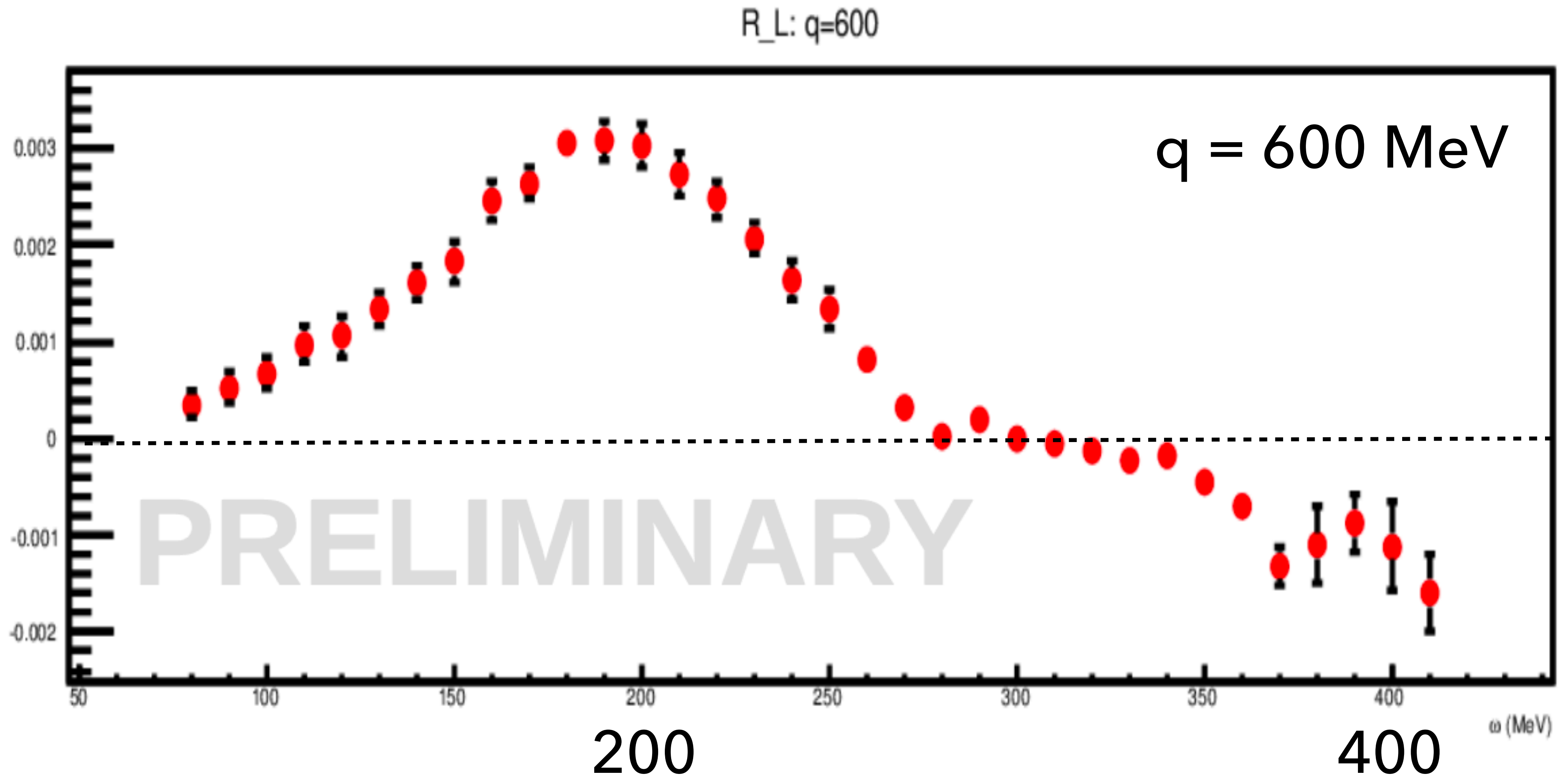


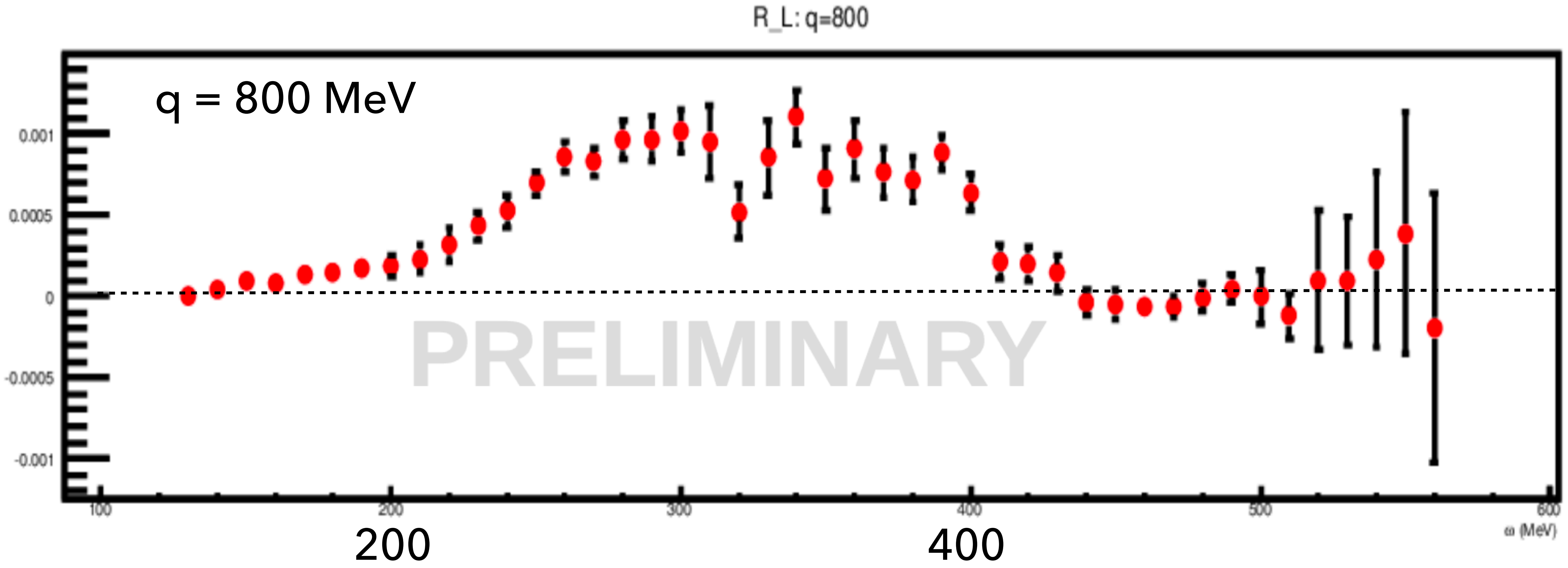
PROGRESS ON 4HE (KAI JIN)

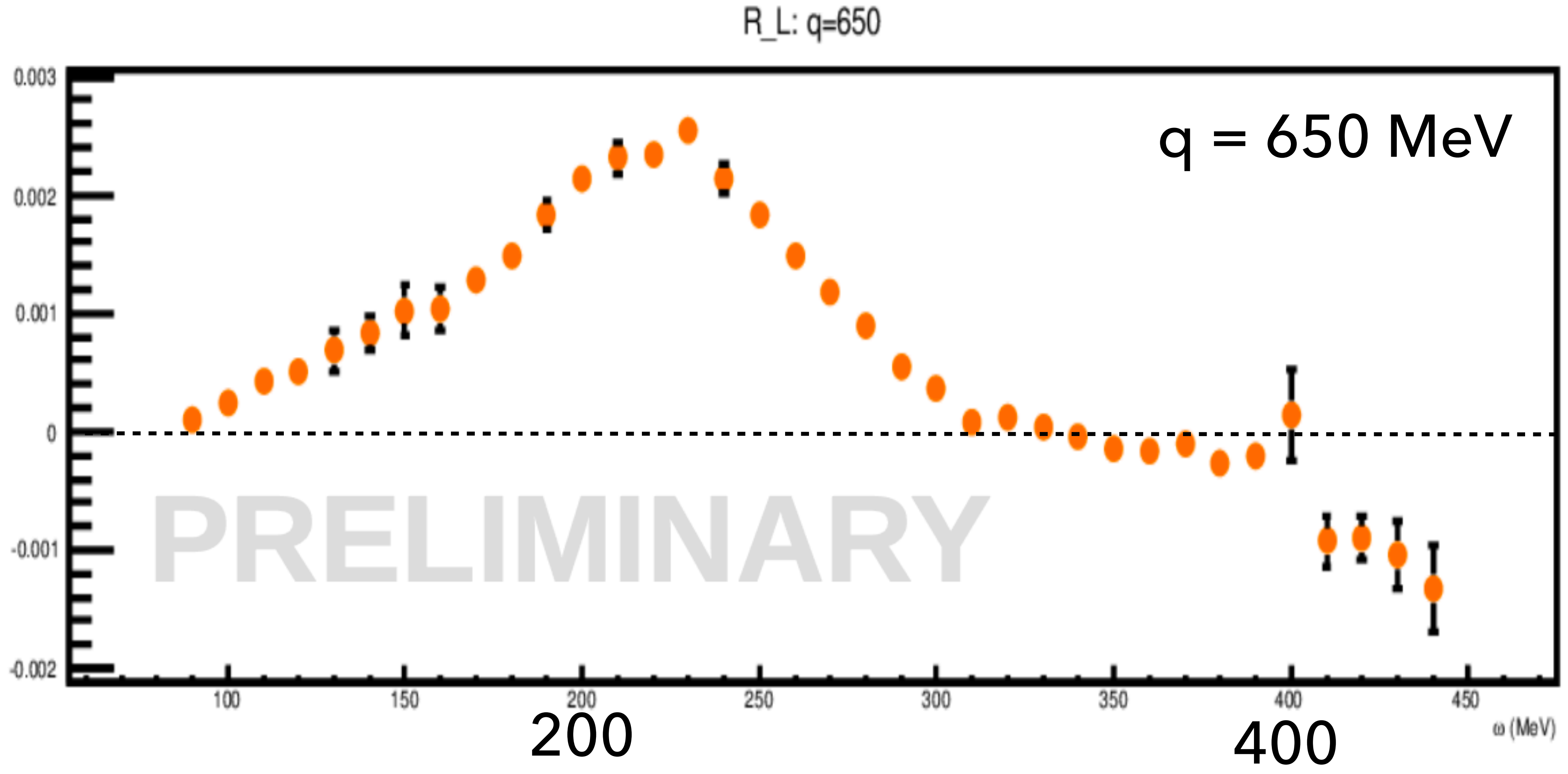


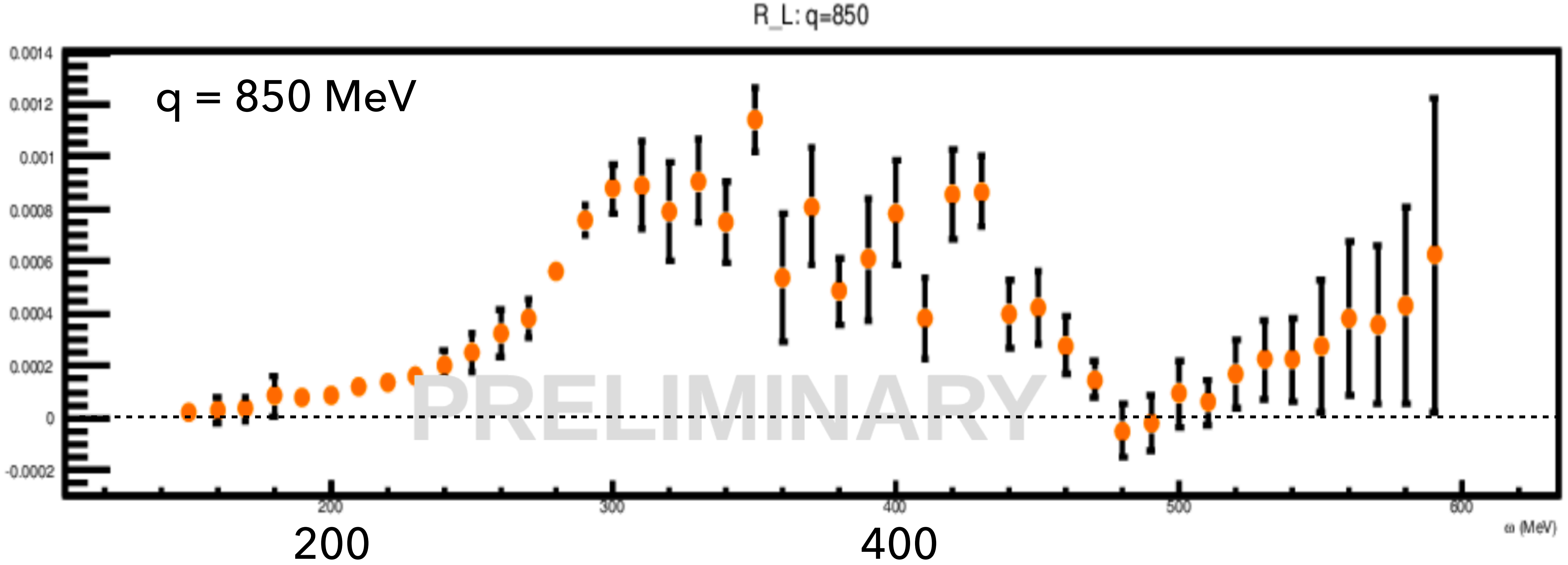
PROGRESS ON ^{4}HE (KAI JIN)

R_L vs Energy Transfer (MeV)

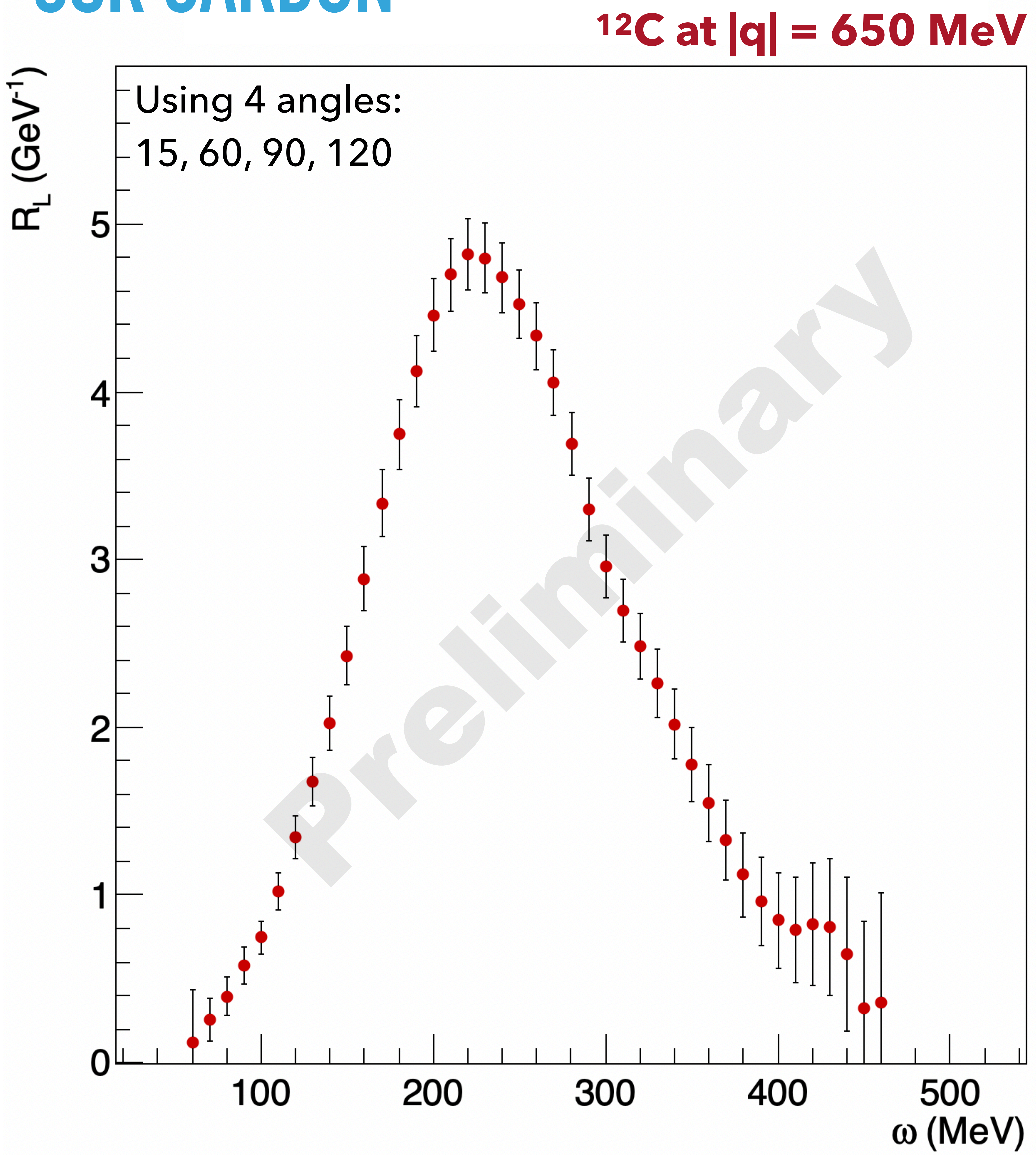






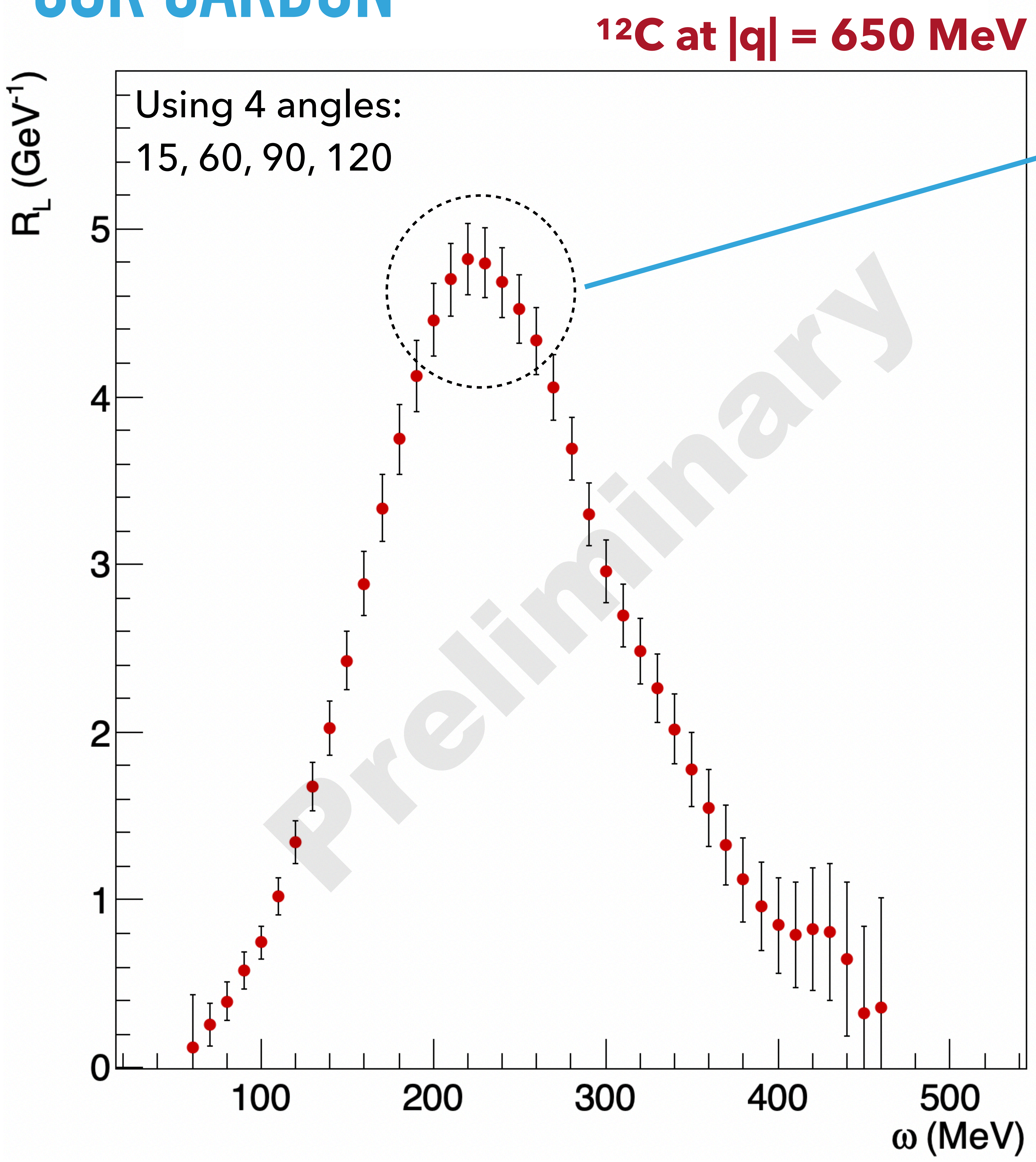


CSR CARBON

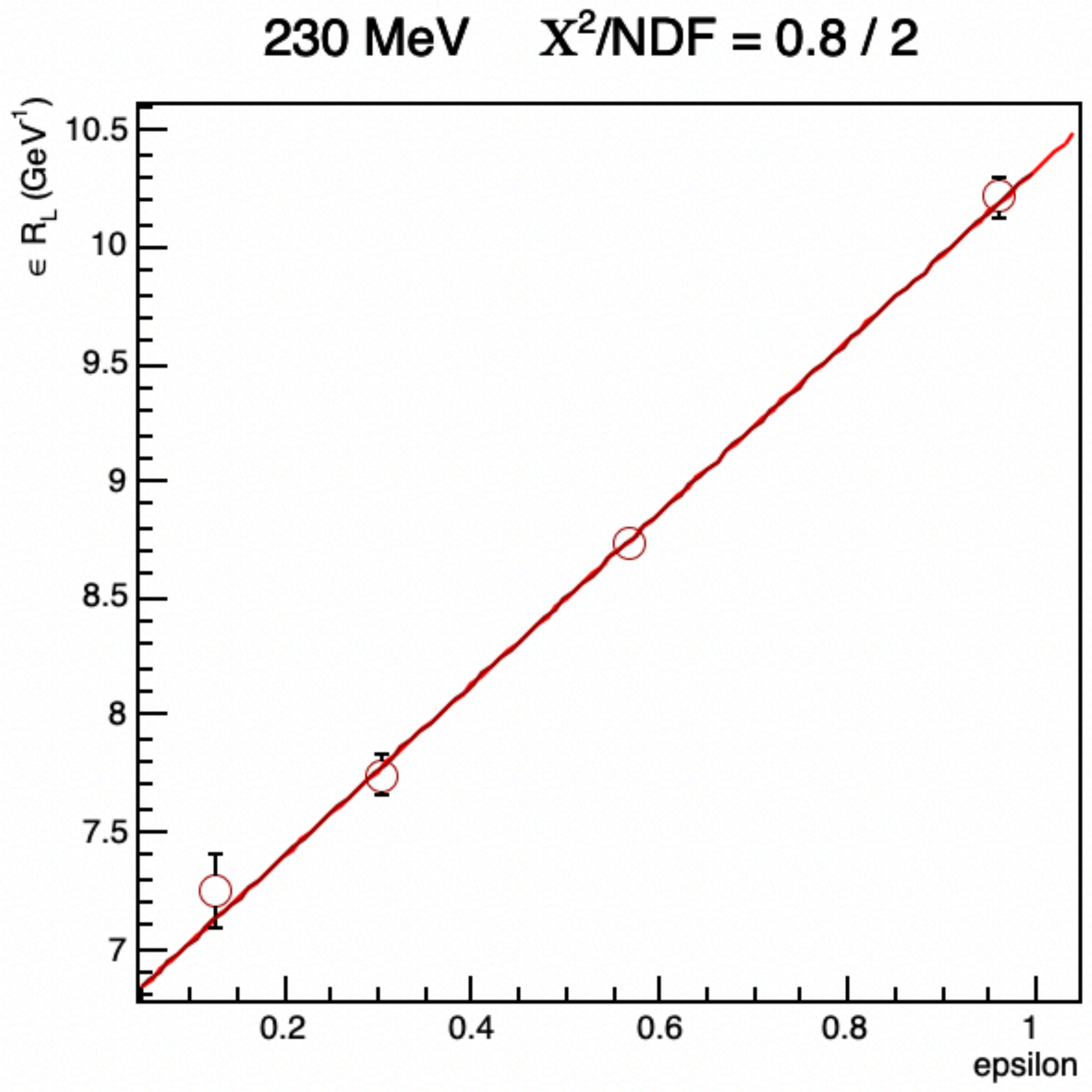


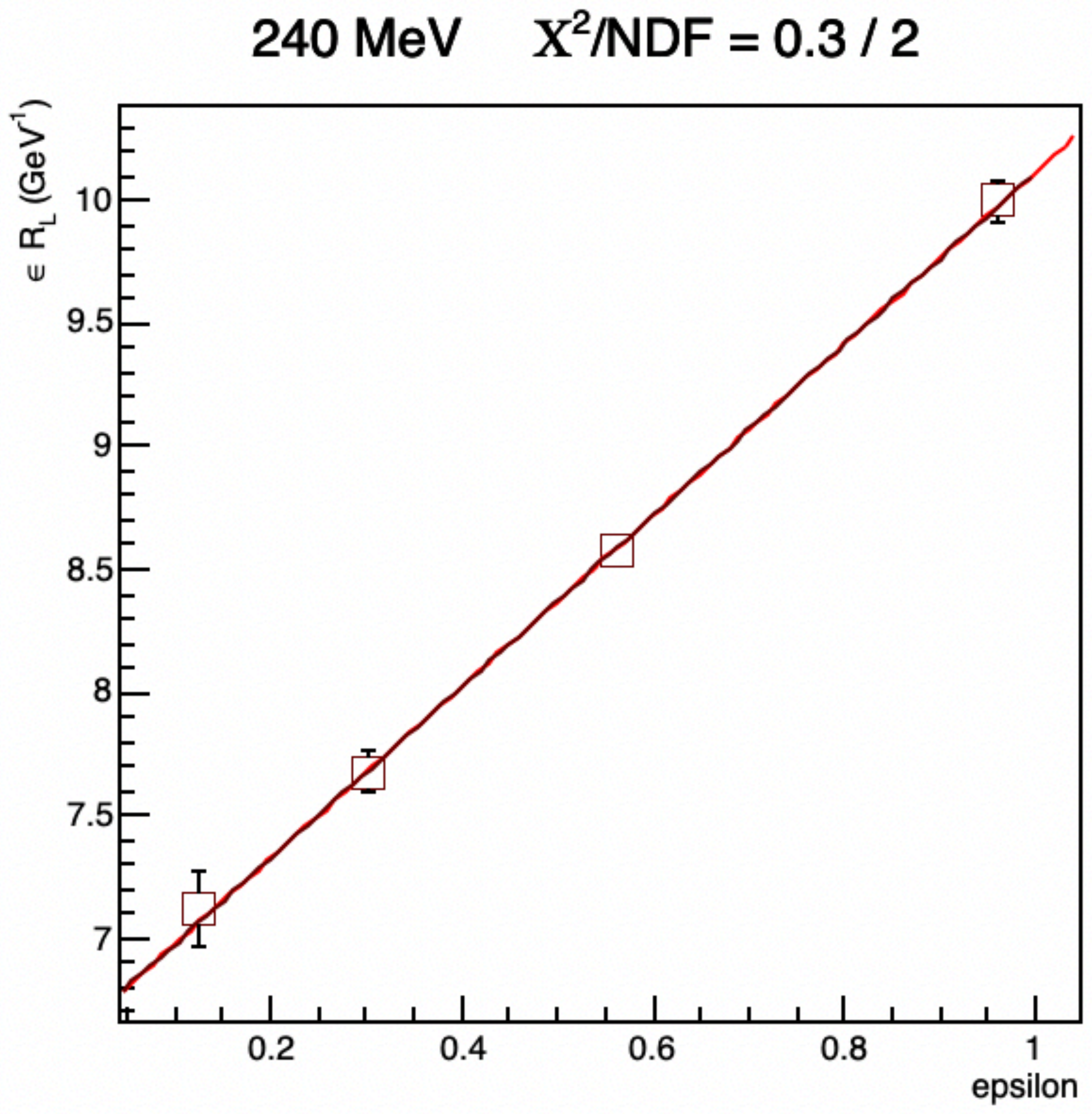
^{12}C at |q| = 650 MeV
Using 4 angles:
15, 60, 90, 120
R_L (GeV^{-1})
0
1
2
3
4
5
100
200
300
400
500
ω (MeV)
Preliminary

CSR CARBON



- Goodness-of-fit for Rosenbluth can indicated where a more careful study is needed.
  - Quasi-elastic peak region is well under control





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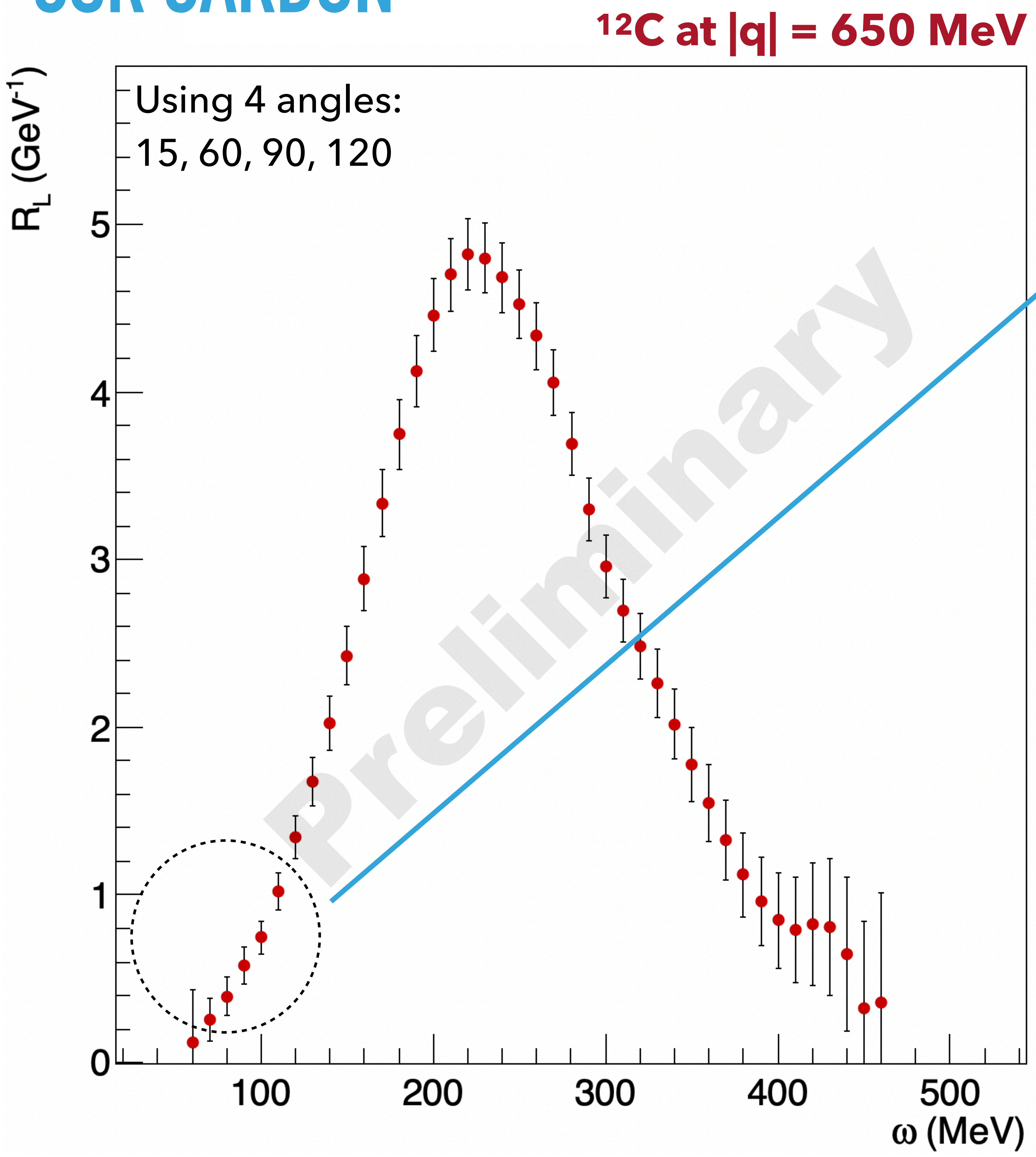
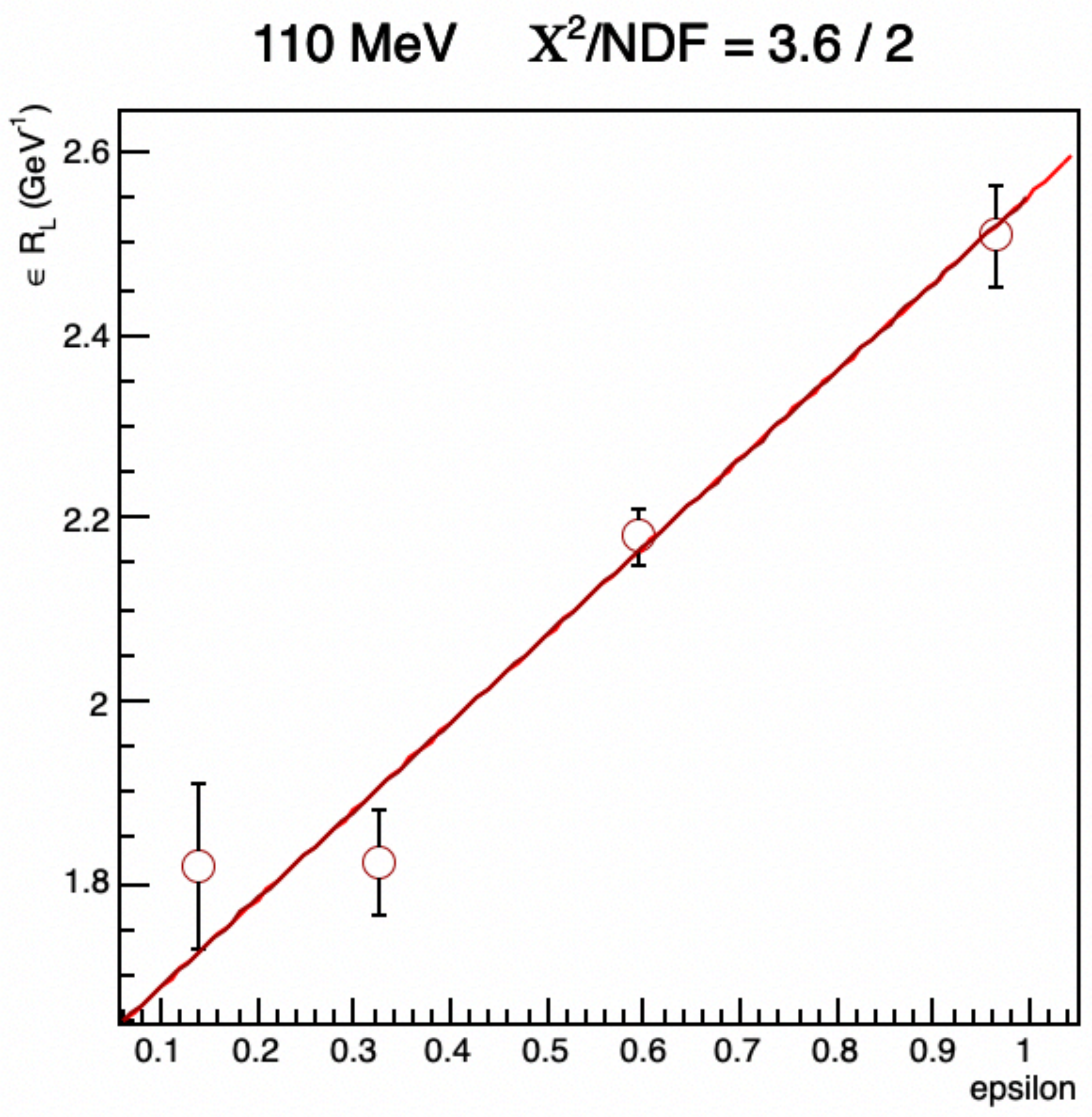
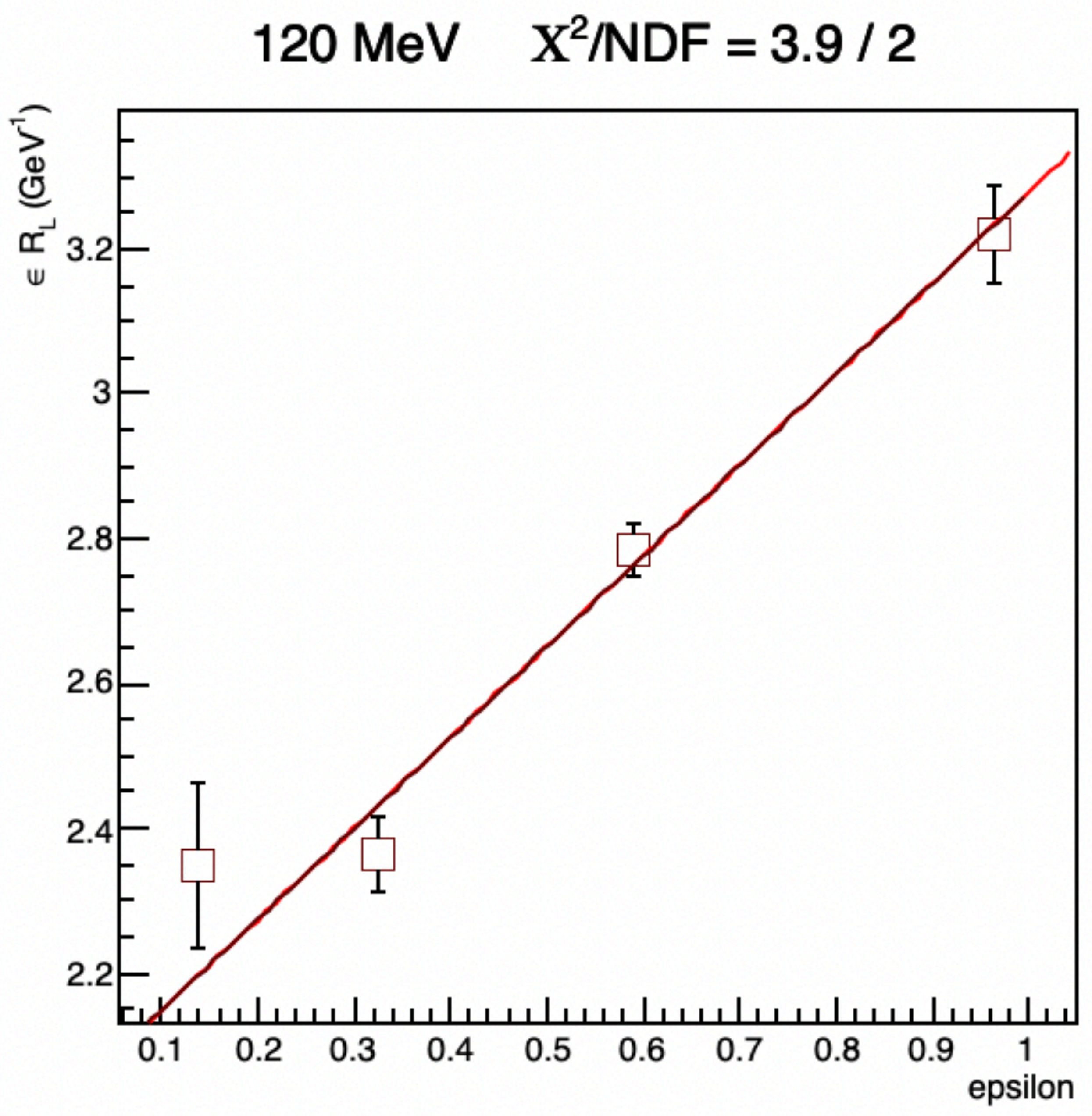
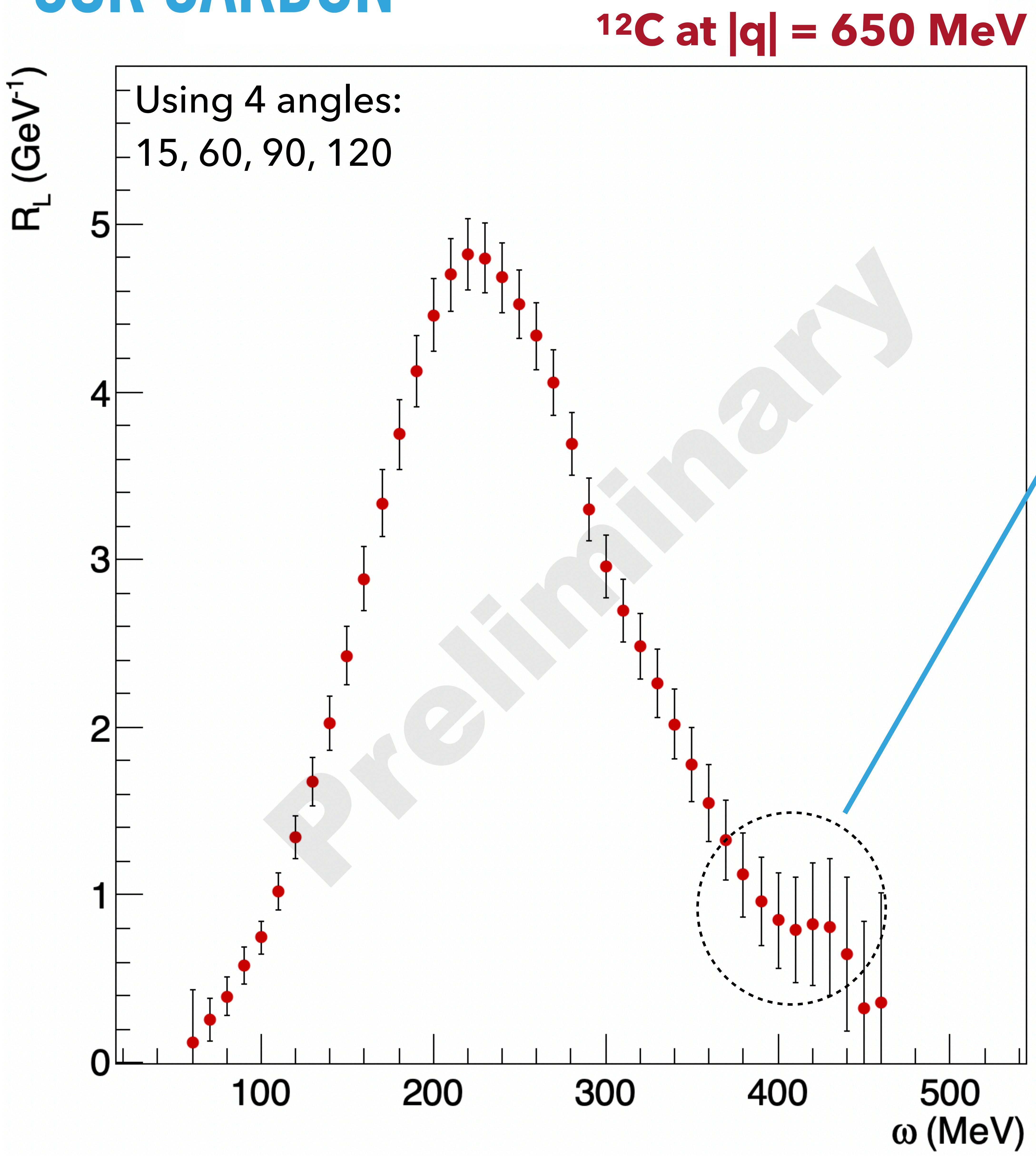


- Goodness-of-fit for Rosenbluth can indicated where a more careful study is needed.
  - Edges of phase-space are more difficult to pin down.
    - Lower energy transfer region is sensitive to interpolation process, acceptance and bin centering corrections.

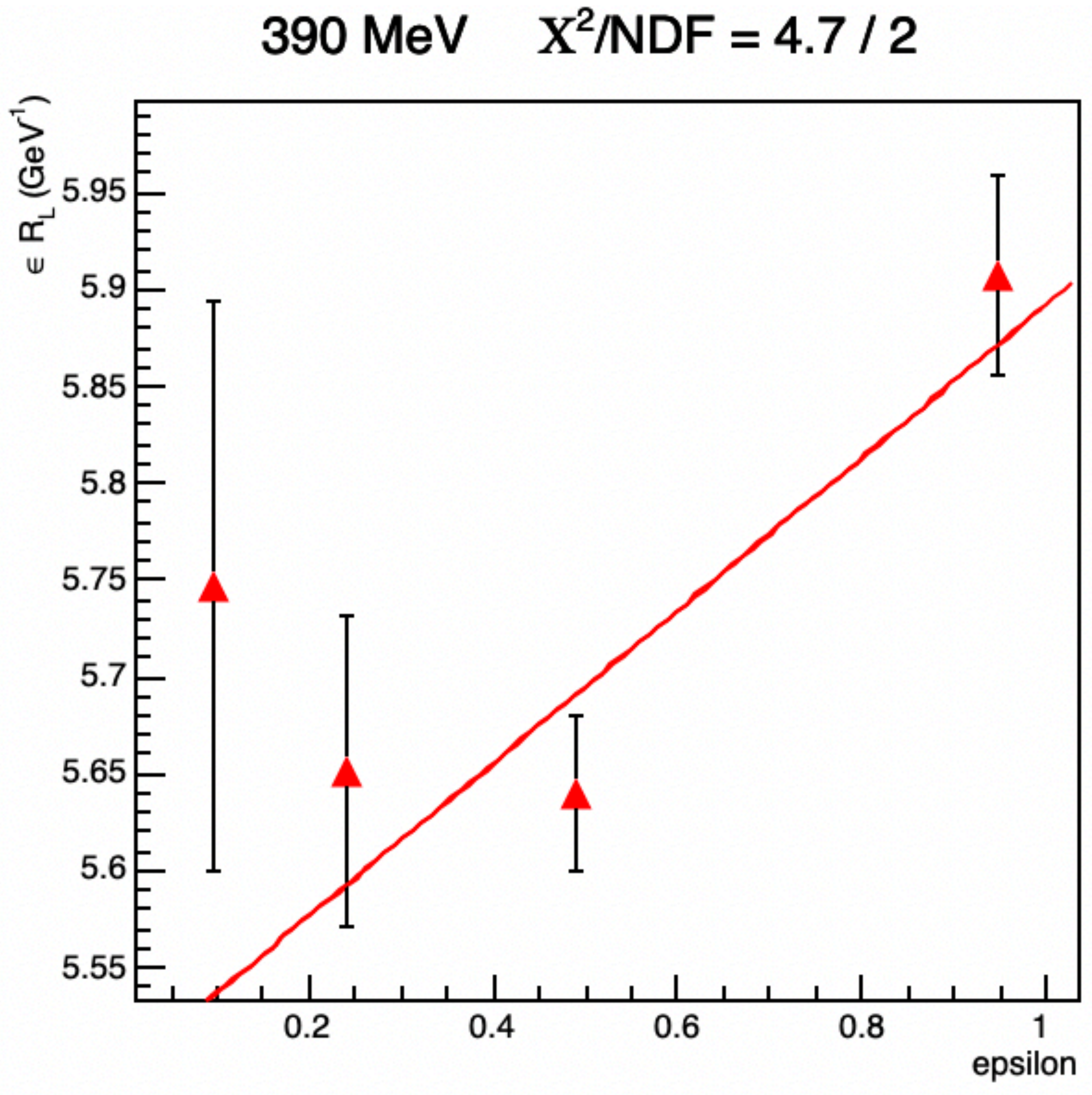


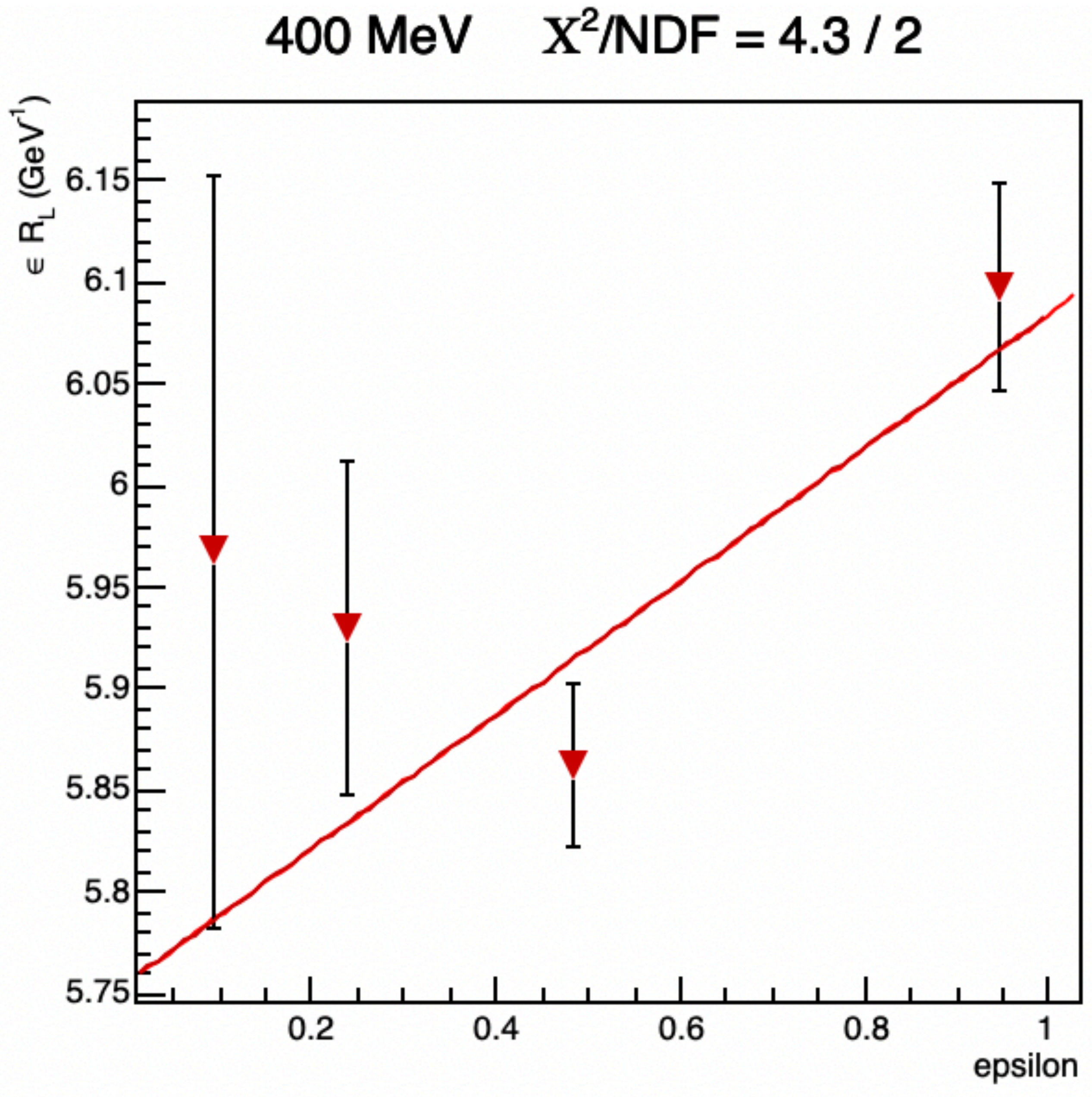


CSR CARBON

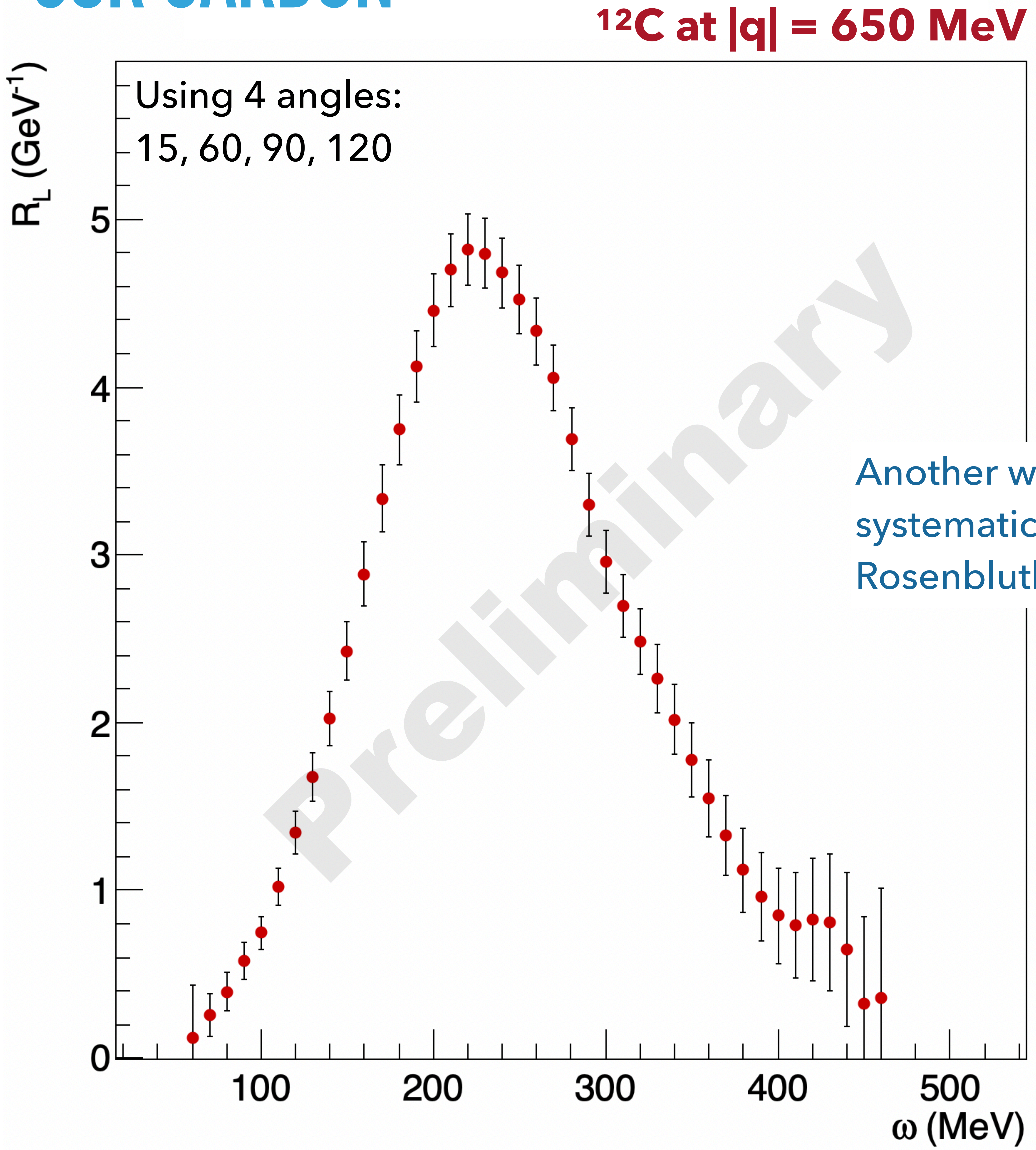


- Goodness-of-fit for Rosenbluth can indicated where a more careful study is needed.
  - Edges of phase-space are more difficult to pin down.
    - Higher energy transfer region is sensitive to interpolation process, elastic tail corrections, systematics of spectrometer low central momentum, and radiative corrections.

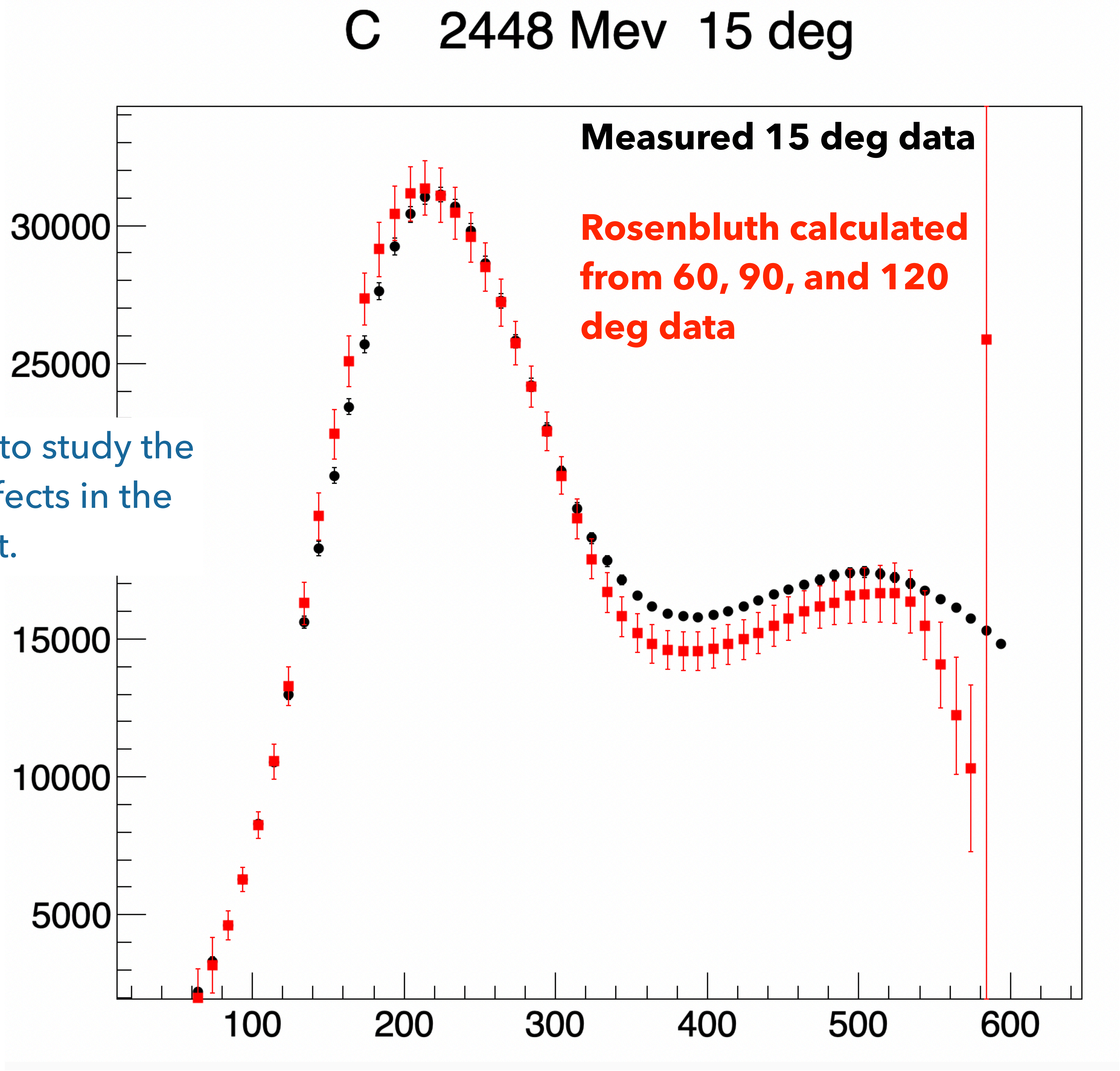




CSR CARBON



Another way to study the systematic effects in the Rosenbluth fit.



SUMMARY / LOOKING AHEAD

- Recent efforts:
  - Full treatment of elastic excited states in Carbon
  - Updated the interpolation technique.
  - Much progress with the extended Helium target
  - A more detailed study of low and energy transfer and the Rosenbluth sensitivity in this region.
- Looking ahead:
  - The Iron, Carbon and now Helium CSR is very close to completion (expected this year).
    - Lead target analysis -> Need coulomb correction calculations beyond the EMA.

THANK YOU!!!

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PEOPLE

and
Hall-A collaboration

PhD Students **Spokespersons** **Run Coordinators**