Monte Carlo Calculations of Hypernuclear Properties

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In collaboration with:

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- ✓ S. Gandolfi @ LANL

Workshop on ^{208}Pb $(e,e'K)$ and neutron stars - May 11, 2020

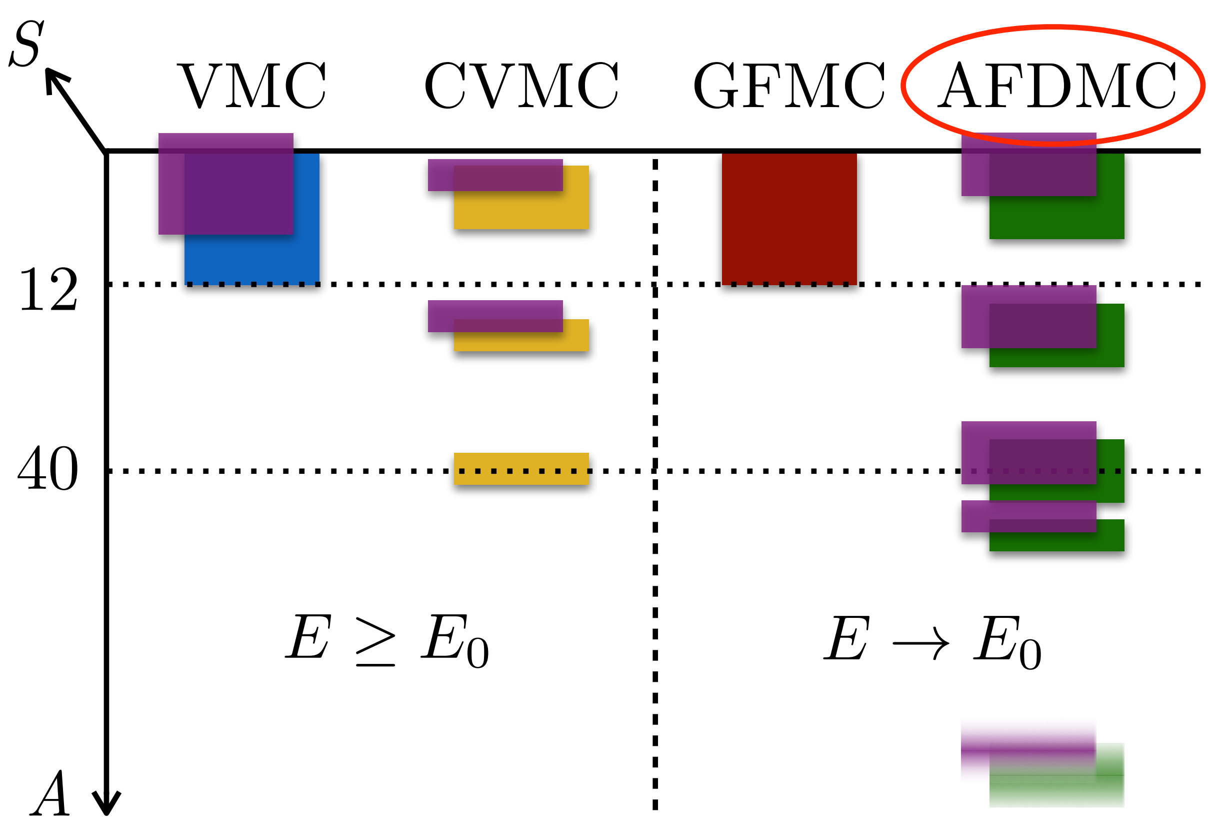
Quantum Monte Carlo

Method: why Quantum Monte Carlo?

i. Solve the many-body problem for strongly correlated systems in a non-perturbative fashion (exact, ab-initio)

ii. Bare interactions + controlled approximations + uncertainty quantification

iii. Accurate description of ground-state properties



(C)VMC GFMC AFDMC	light systems	$A \leq 12$
CVMC AFDMC	light to medium-mass nuclei	$A \sim 50$
AFDMC	nuclear matter	$A = \infty$

$$\tau = \frac{it}{\hbar} : \; -\frac{\partial}{\partial \tau}|\psi(\tau)\rangle = (H - E_0)|\psi(\tau)\rangle \quad \longrightarrow \quad |\psi(\tau)\rangle = e^{-(H-E_0)\tau}|\psi(0)\rangle \overset{\tau\to\infty}{\longrightarrow} |\psi_0\rangle$$

Nuclear Hamiltonians

Model*:** non-relativistic ***nucleons interacting with an effective two-body (NN) and three-body (NNN) nucleon force

p, n

$$H = -\frac{\hbar^2}{2m_N}\sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk} + \dots$$

v_{ij} fit to NN scattering data & deuteron
v_{ijk} fit to properties of nuclei

$\longmapsto$ many data

Hypernuclear Hamiltonians

Model: non-relativistic ***baryons*** interacting with an effective two-body (BB) and three-body (BBB) baryon force

$p,\ n$

$$H = -\frac{\hbar^2}{2m_N}\sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk} + \ldots$$

Λ

$$+ -\frac{\hbar^2}{2m_\Lambda}\sum_\lambda {\nabla_\lambda}^2 + \sum_{\lambda<\mu} v_{\lambda\mu} + \sum_{\lambda<\mu<\nu} v_{\lambda\mu\nu} + \ldots$$

$$+ \sum_{\lambda i} v_{\lambda i} + \sum_{\lambda,i<j} v_{\lambda ij} + \sum_{\lambda<\mu,i} v_{\lambda\mu i} + \ldots$$

$\Sigma^-, \Sigma^0, \Sigma^+$

$$+ -\frac{\hbar^2}{2m_\Sigma}\sum_\epsilon {\nabla_\epsilon}^2 + \ldots$$

v_{ij}	fit to NN scattering data & deuteron	→	many data
v_{ijk}	fit to properties of nuclei		
$v_{\lambda i}$	fit to ΛN scattering data	→	not many data: new info?
$v_{\lambda ij}$	fit to properties of Λ-hypernuclei		
$v_{\lambda\mu}, v_{\lambda\mu i}$	fit to properties of ΛΛ-hypernuclei		
…	fit to …???		

Hypernuclear Hamiltonians

Idea: revisit the problem of the hyperon-nucleon interaction using Quantum Monte Carlo

Λ-hypernuclei

$$H = -\frac{\hbar^2}{2m_N}\sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

phenomenological interactions

$$+ -\frac{\hbar^2}{2m_\Lambda}\sum_\lambda \nabla_\lambda{}^2 + \sum_{\lambda i} v_{\lambda i} + \sum_{\lambda, i<j} v_{\lambda ij}$$

↓

fit to Λp scattering data

T. Schimmert *et al.*,
Nucl. Phys. A **343** (1980) 429-434

Hypernuclear Hamiltonians

Idea: revisit the problem of the hyperon-nucleon interaction using Quantum Monte Carlo

Λ-hypernuclei

$$H = -\frac{\hbar^2}{2m_N}\sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

phenomenological interactions

$$+ -\frac{\hbar^2}{2m_\Lambda}\sum_\lambda \nabla_\lambda^{\,2} + \sum_{\lambda i} v_{\lambda i} + \sum_{\lambda, i<j} v_{\lambda ij}$$

i. low number of parameters

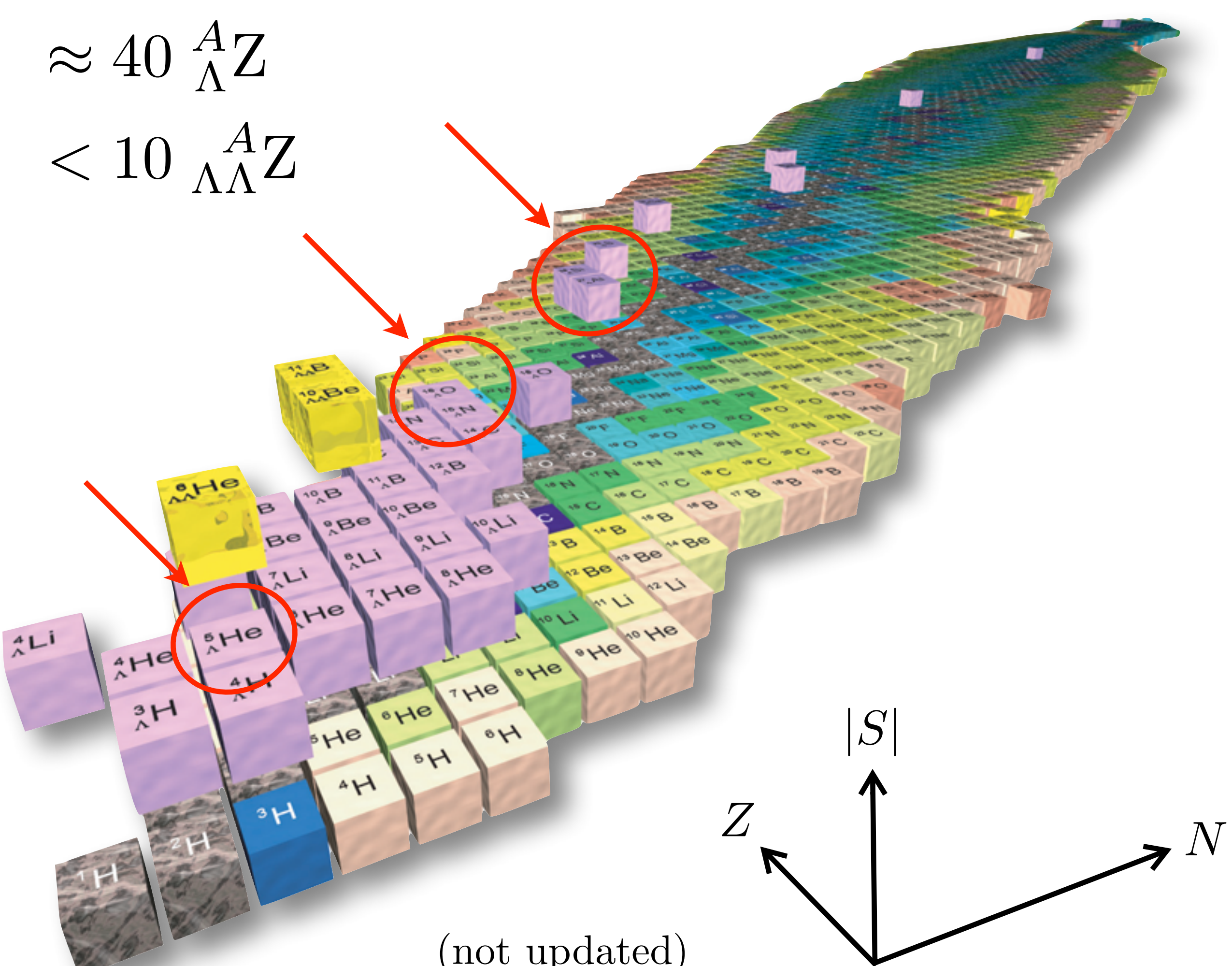
ii. fit to available exp data

$$B_\Lambda = E\left({}^{A-1}\mathrm{Z}\right) - E\left({}^{A}_{\Lambda}\mathrm{Z}\right)$$

core nucleus — hypernucleus

$\approx 40\ {}^{A}_{\Lambda}\mathrm{Z}$

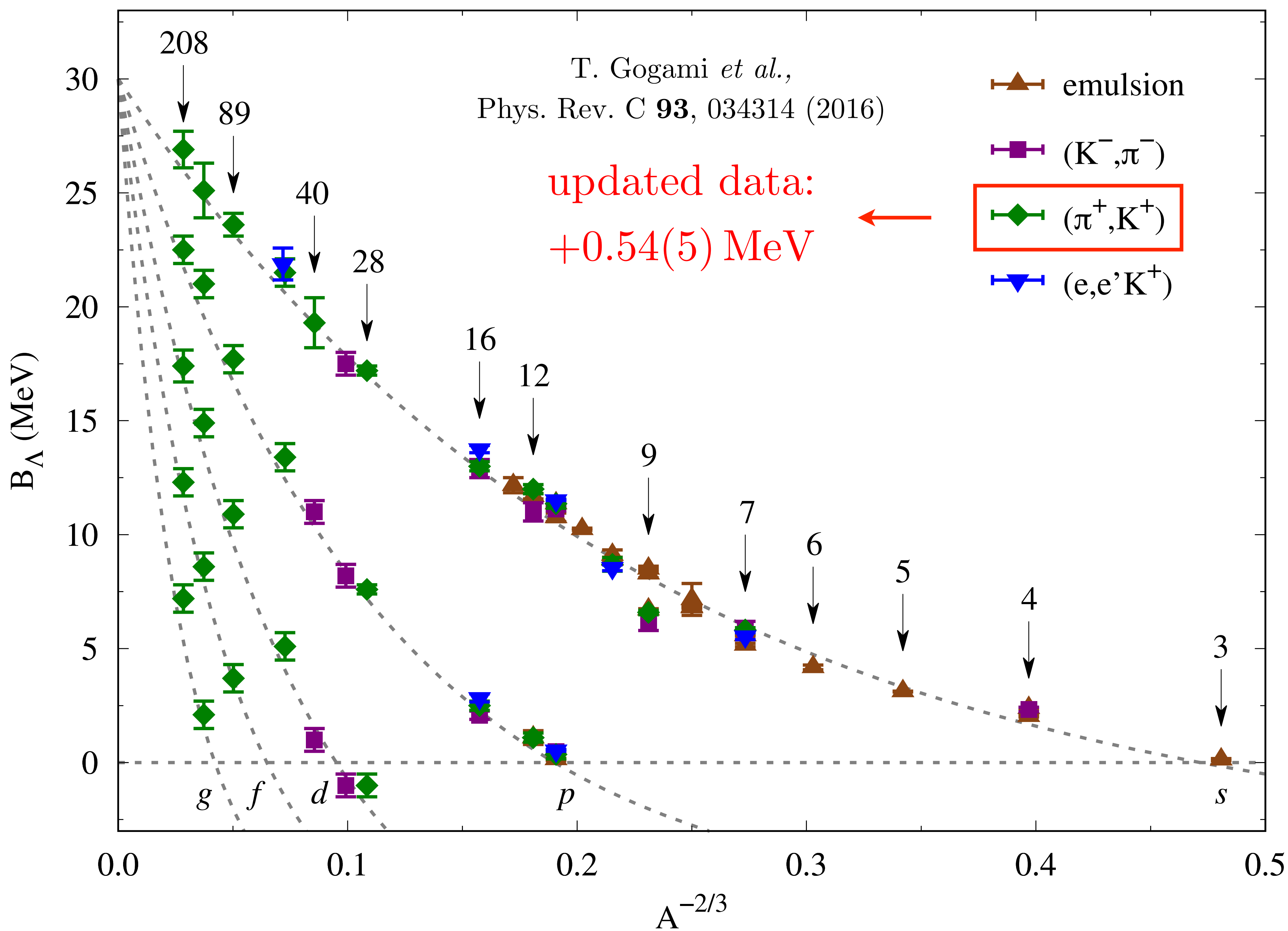
$< 10\ {}^{A}_{\Lambda\Lambda}\mathrm{Z}$



(not updated)

Note: try to use both light & medium-mass systems $A \sim 5, 16, 40$

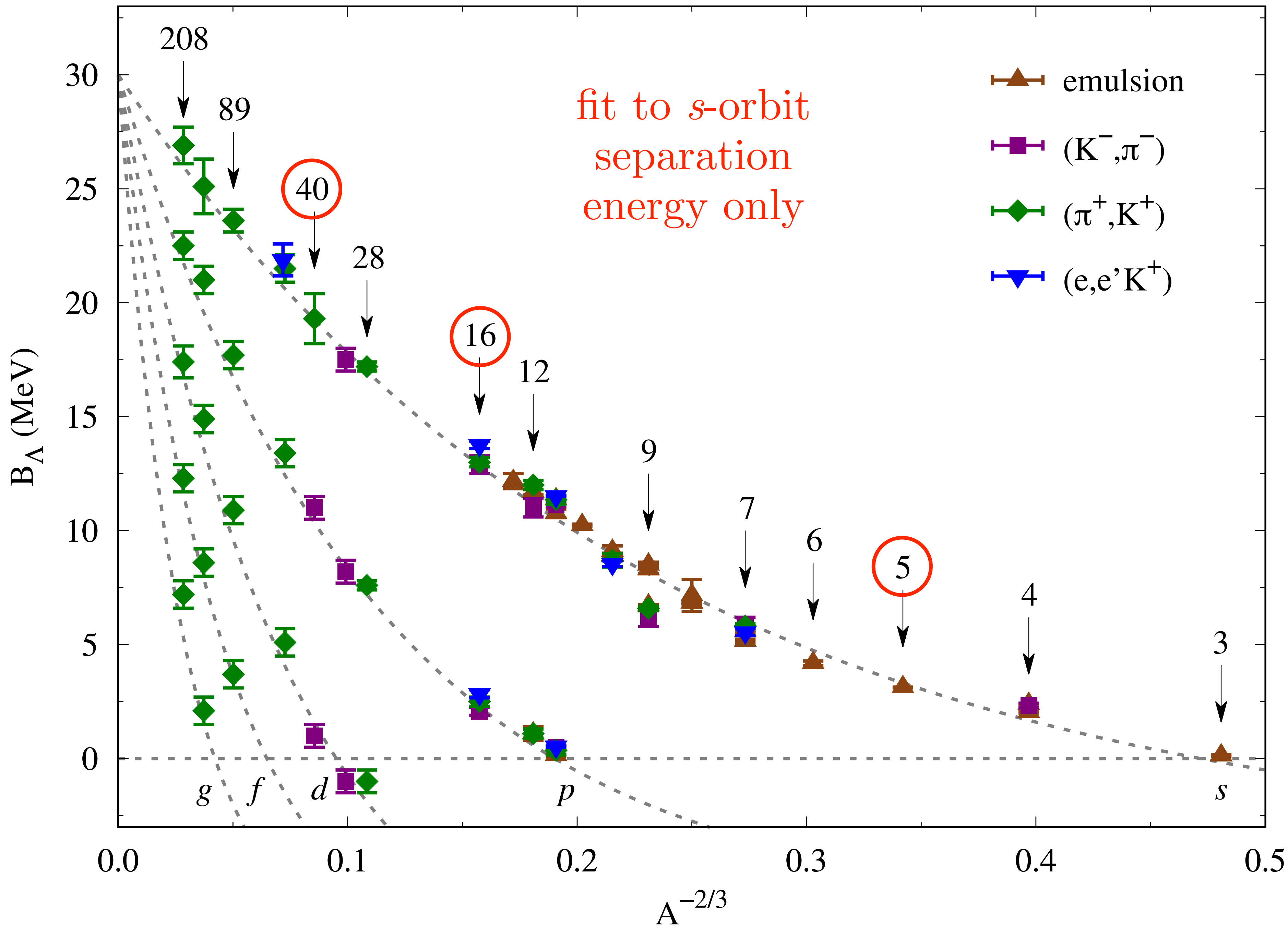
***AFDMC*:** Λ-hypernuclei



D. L. and F. Pederiva, arXiv:1711.07521

Results: Hypernuclei

***AFDMC*:** Λ-hypernuclei



D. L. and F. Pederiva, arXiv:1711.07521

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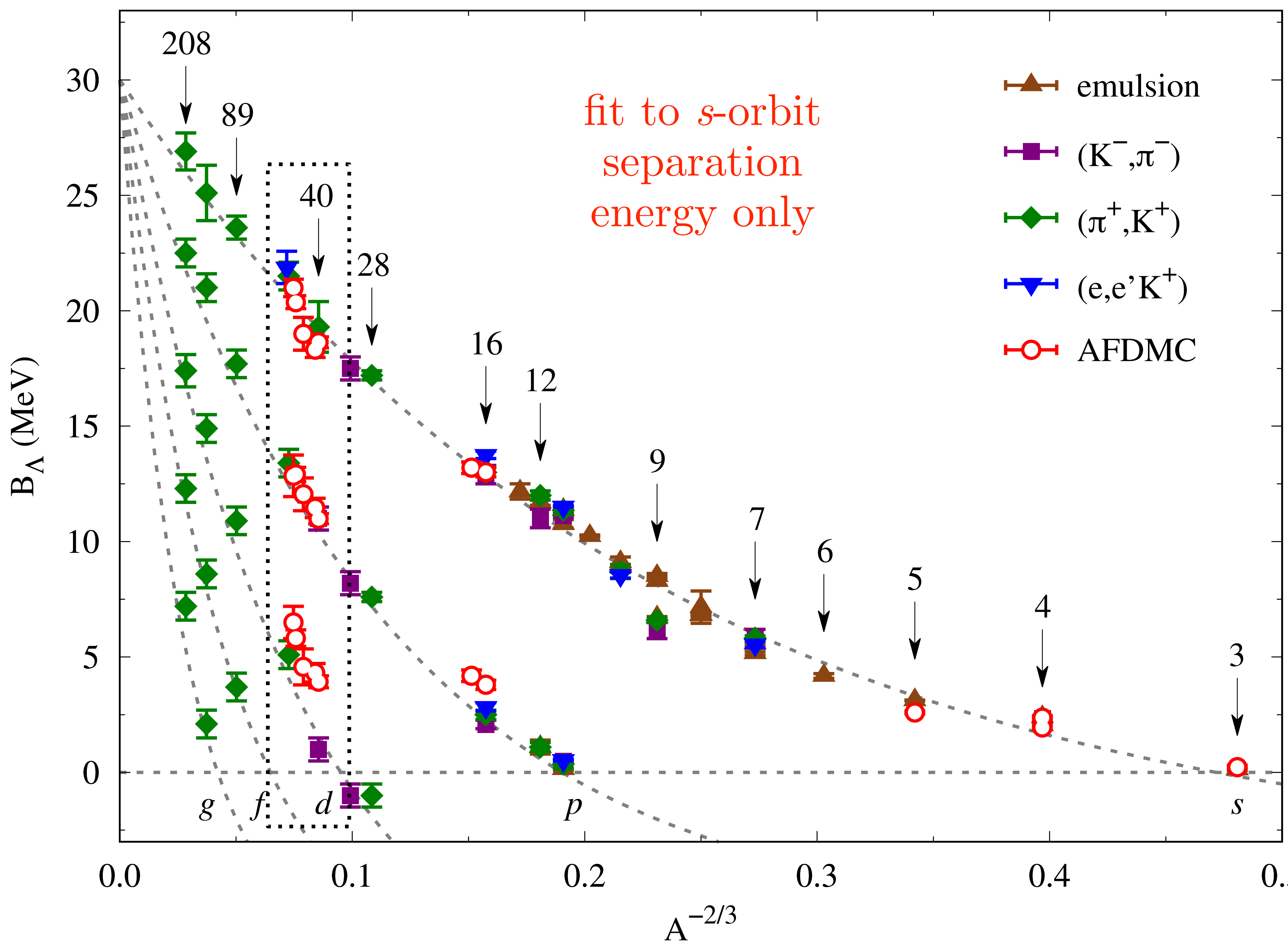

D. L. and F. Pederiva, arXiv:1711.07521

Results: Hypernuclei

***AFDMC*:** Λ-hypernuclei

	${}^{A}_{\Lambda}\mathrm{Z}\ (J^\pi, T)$	s-orbit	p-orbit	d-orbit
AFDMC	${}^{40}_{\Lambda}\mathrm{K}$	$(2^+, \frac{1}{2})$ 18.6(3)	$(3^-, \frac{1}{2})$ 11.0(3)	$(4^+, \frac{1}{2})$ 3.9(3)
	${}^{41}_{\Lambda}\mathrm{Ca}$	$\left(\frac{1}{2}^+, 0\right)$ 18.3(4)	$\left(\frac{3}{2}^-, 0\right)$ 11.5(4)	$\left(\frac{5}{2}^+, 0\right)$ 4.3(4)
	${}^{45}_{\Lambda}\mathrm{Ca}$	$\left(\frac{1}{2}^+, 2\right)$ 19.0(8)	$\left(\frac{3}{2}^-, 2\right)$ 12.1(8)	$\left(\frac{5}{2}^+, 2\right)$ 4.6(8)
	${}^{48}_{\Lambda}\mathrm{K}$	$(1^+, \frac{9}{2})$ 20.4(4)	$(2^-, \frac{9}{2})$ 12.9(4)	$(3^+, \frac{9}{2})$ 5.8(4)
	${}^{49}_{\Lambda}\mathrm{Ca}$	$\left(\frac{1}{2}^+, 4\right)$ 21.0(5)	$\left(\frac{3}{2}^-, 4\right)$ 12.9(9)	$\left(\frac{5}{2}^+, 4\right)$ 6.5(7)
Exp	${}^{40}_{\Lambda}\mathrm{Ca}$	$(2^+, \frac{1}{2})$ 19.3(1.1)	$(3^-, \frac{1}{2})$ 11.0(5)	$(4^+, \frac{1}{2})$ 1.0(5)
	${}^{51}_{\Lambda}\mathrm{V}$	(?, 2) 21.5(6)	(?, 2) 13.4(6)	(?, 2) 5.1(6)
	${}^{52}_{\Lambda}\mathrm{V}$	$(3^-, \frac{5}{2})$ 21.9(7)	—	—
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D. L. and F. Pederiva, arXiv:1711.07521

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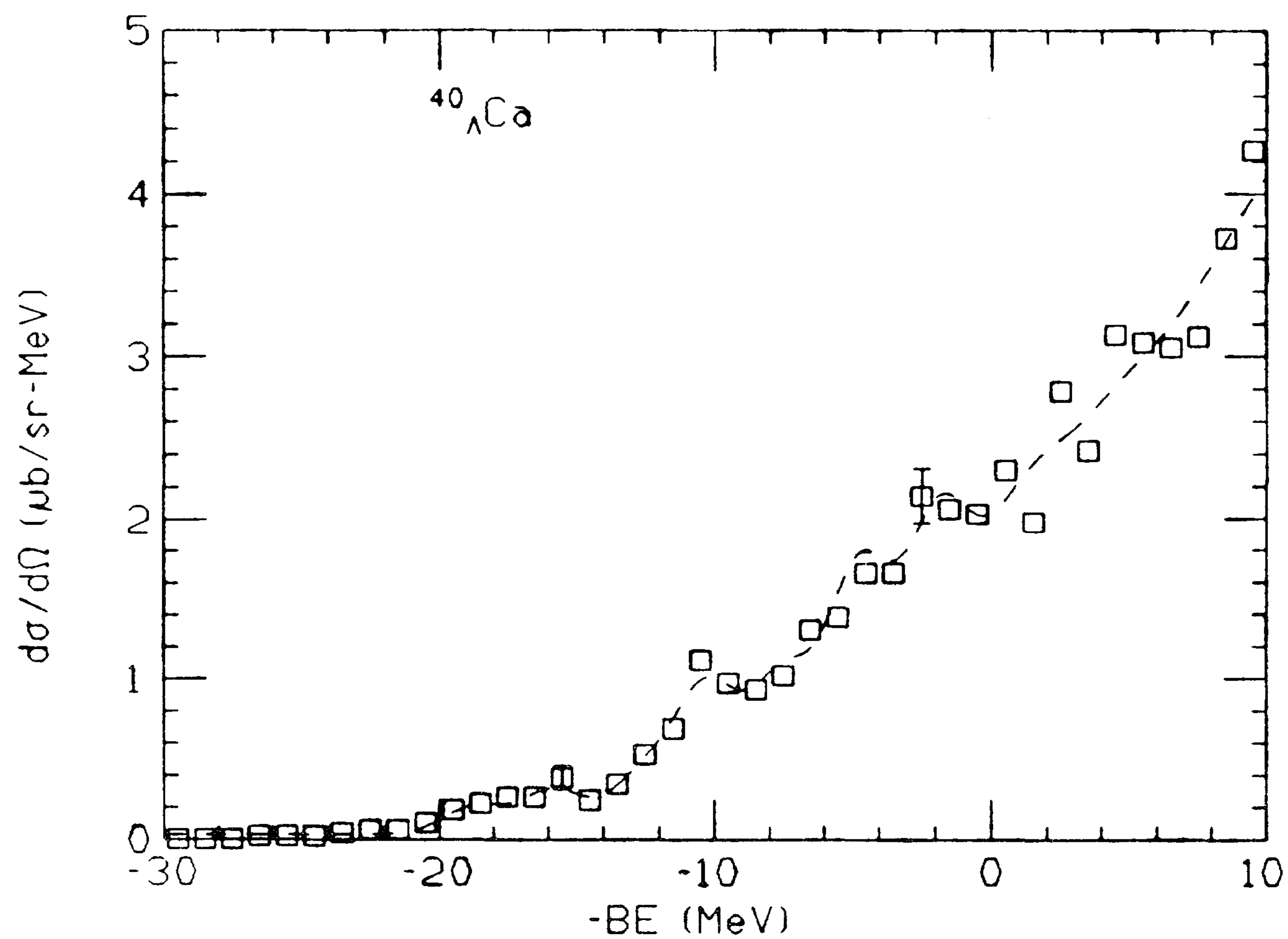

P. H. Pile *et al.*, Phys. Rev. Lett **66**, 2585 (1991)

D. L. and F. Pederiva, arXiv:1711.07521

Results: Hypernuclei

AFDMC: Λ-hypernuclei

${}^{A}_{\Lambda}\mathrm{Z}\ (J^\pi, T)$	s-orbit	p-orbit	d-orbit
${}^{40}_{\Lambda}\mathrm{K}$	$(2^+, \frac{1}{2})$ 18.6(3)	$(3^-, \frac{1}{2})$ 11.0(3)	$(4^+, \frac{1}{2})$ 3.9(3)
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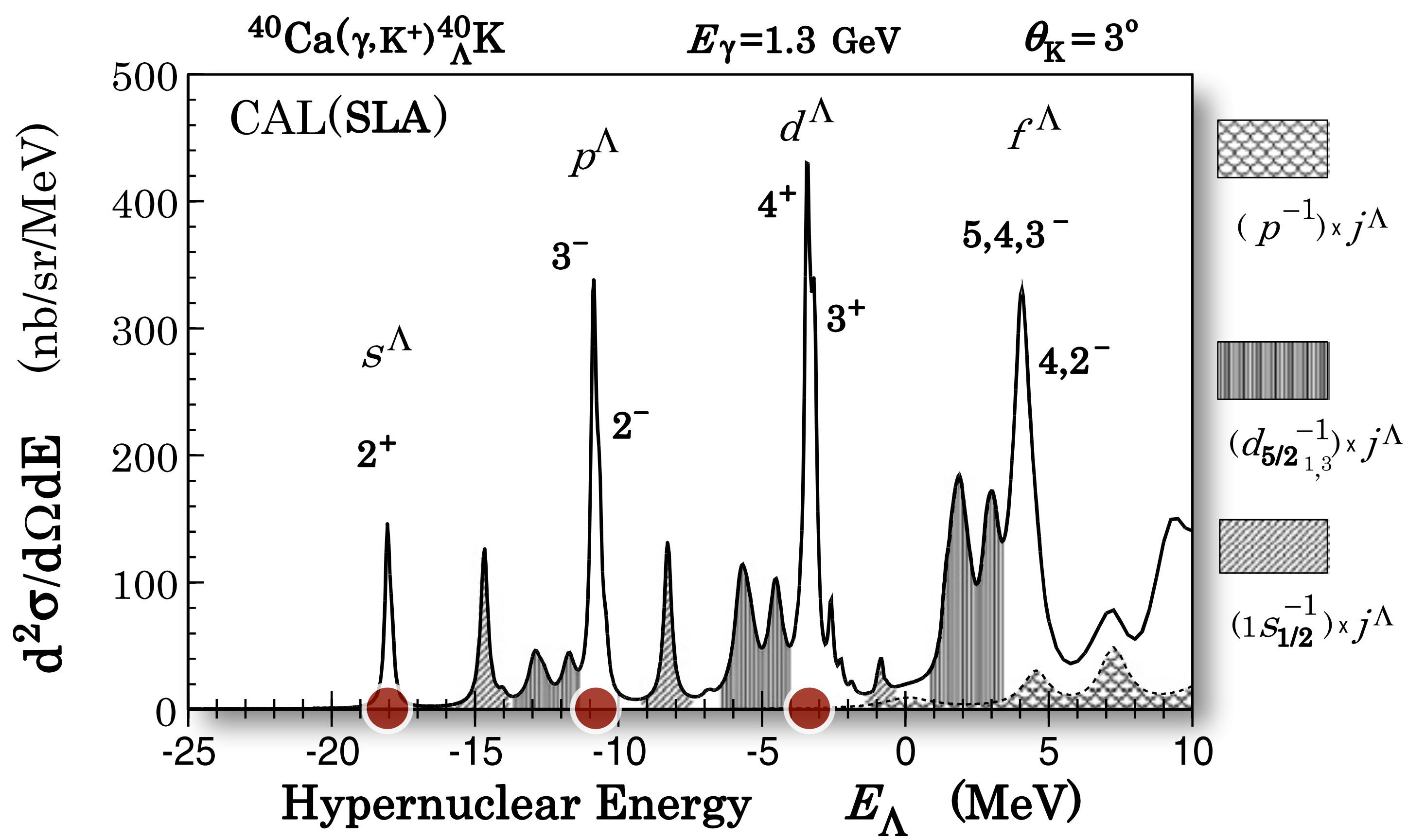
			$(4^+, \frac{1}{2})$ 1.0(5)
			$(?, 2)$ 5.1(6)
			—
			—



P. Bydžovský *et al.*, Nucl. Phys. A **881** (2012) 199-217

$B^s_\Lambda \simeq 18.0\,\mathrm{MeV} \quad \mathrm{B}^p_\Lambda \simeq 10.7\,\mathrm{MeV} \quad \mathrm{B}^d_\Lambda \simeq 3.3\,\mathrm{MeV}$

Results: Hypernuclei

***AFDMC*:** Λ-hypernuclei

	$^{A}_{\Lambda}\mathrm{Z}\ (J^\pi, T)$	s-orbit	p-orbit	d-orbit
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Jefferson Lab

E12-15-008

$^{40}\mathrm{Ca}\,(e, e'K^+)\,^{40}_{\Lambda}\mathrm{K}$

$\delta = 0.025$

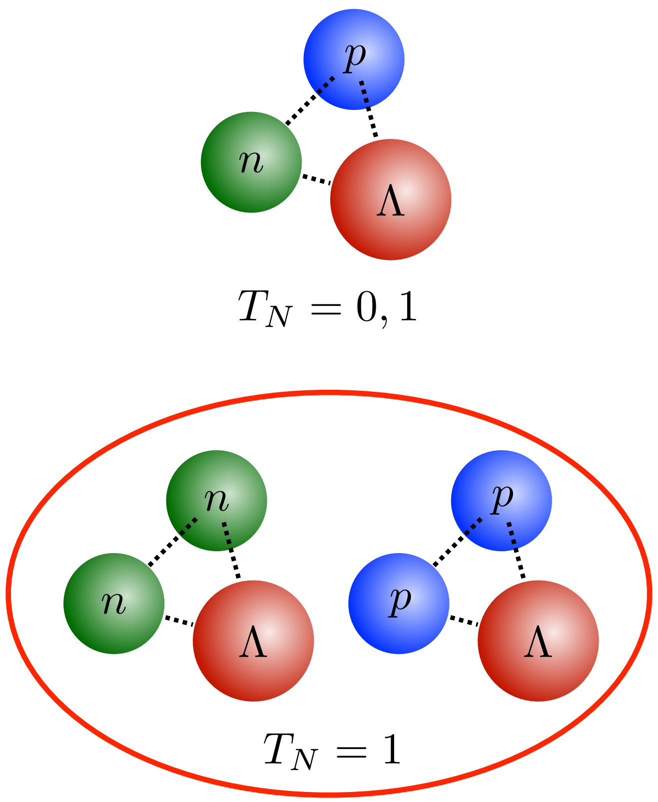
$^{48}\mathrm{Ca}\,(e, e'K^+)\,^{48}_{\Lambda}\mathrm{K}$

$\delta = 0.188$

nucleon-isospin asymmetry: $\delta = \dfrac{N - Z}{A}$

D. L. and F. Pederiva, arXiv:1711.07521

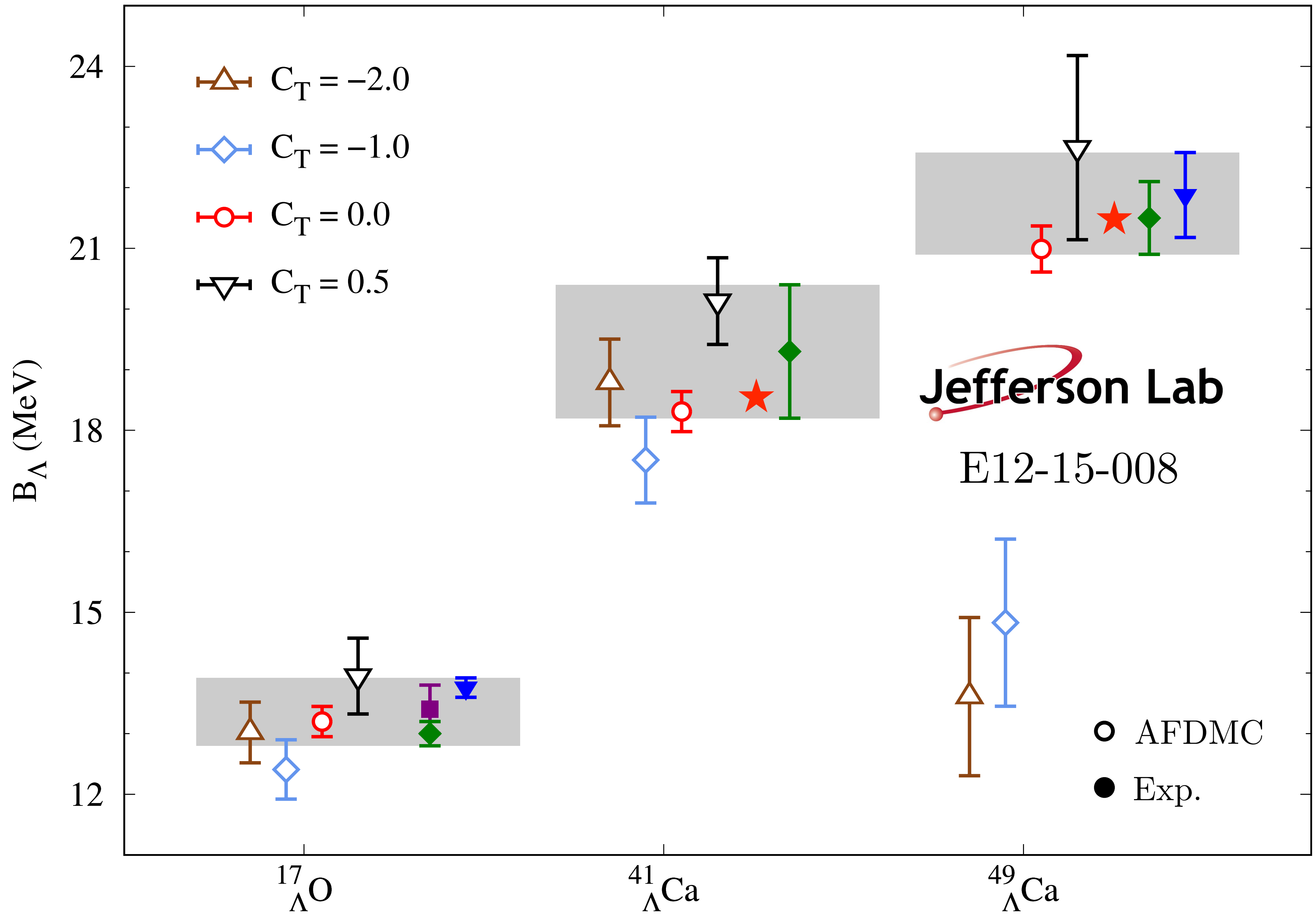
AFDMC: Λ-hypernuclei



↓

relevant in NS

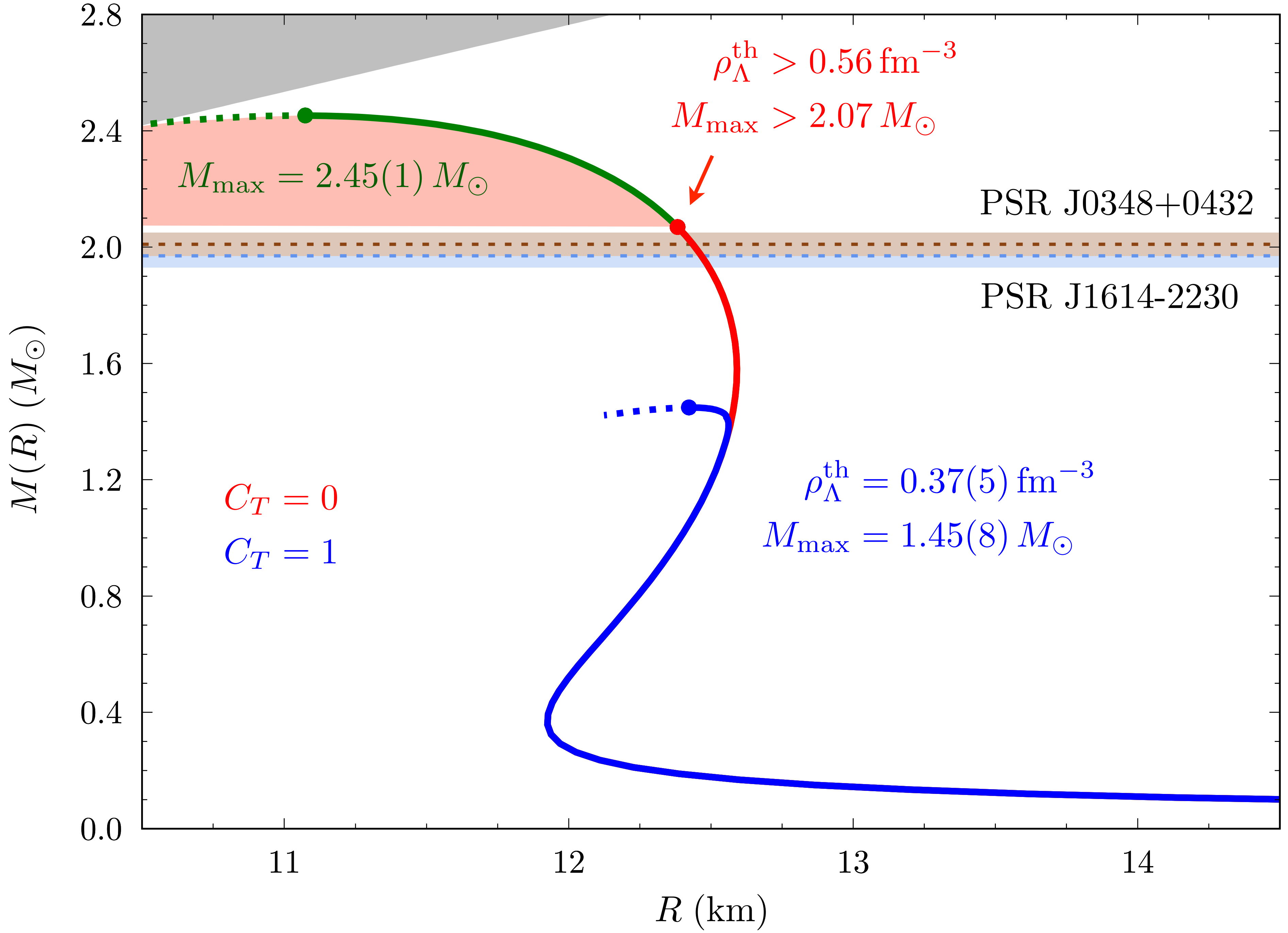
control parameter C_T



is AFDMC sensitive to a nucleon-isospin dependence of the ΛNN force?

D. L. and F. Pederiva, arXiv:1711.07521

***AFDMC*:** Λ-hypermatter



C_T=0: D. L. *et al.*, Phys. Rev. Lett. **114**, 092301 (2015)

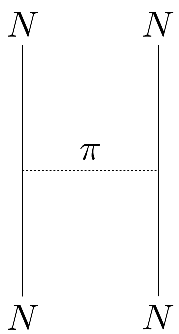
C_T=1: preliminary results!

***Note*:** the energy contribution of a many-body force is scheme (and scale) dependent

phenomenological potentials

nucleons & pion exchange
AV18 + UIX

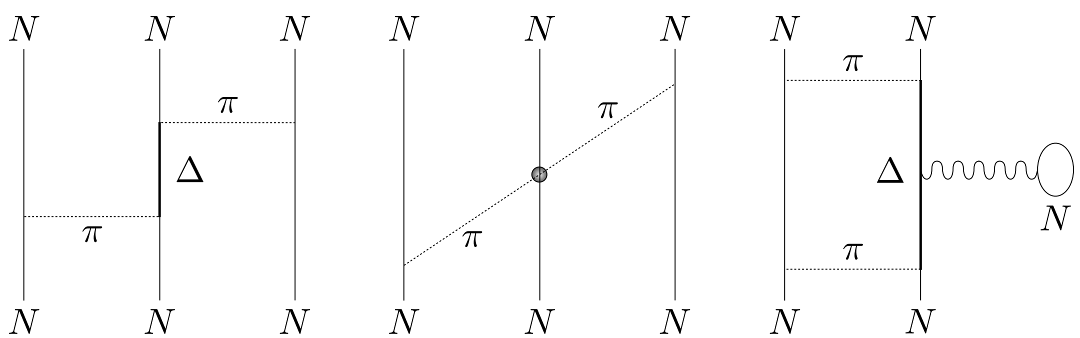
2b



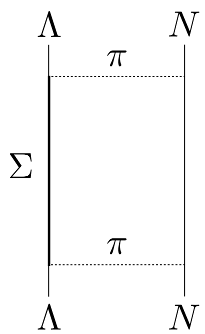
1π – exchange

2π – exchange

3b

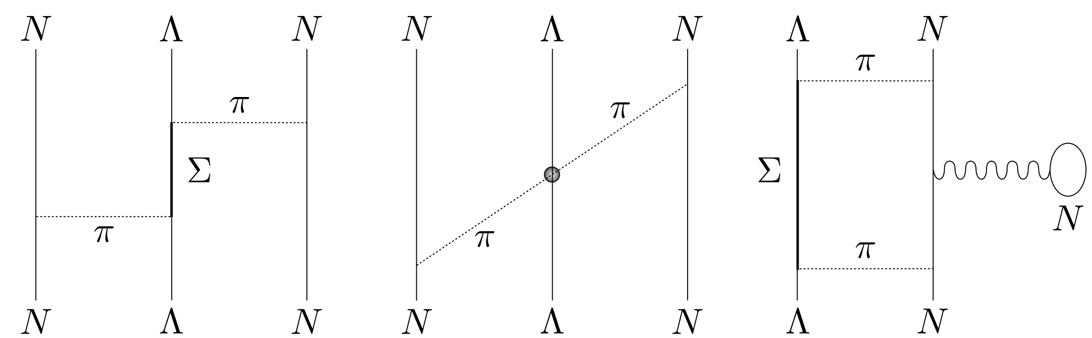


nucleons/lambda & pion exchange
Bodmer/Usmani



~~1π – exchange~~

2π – exchange

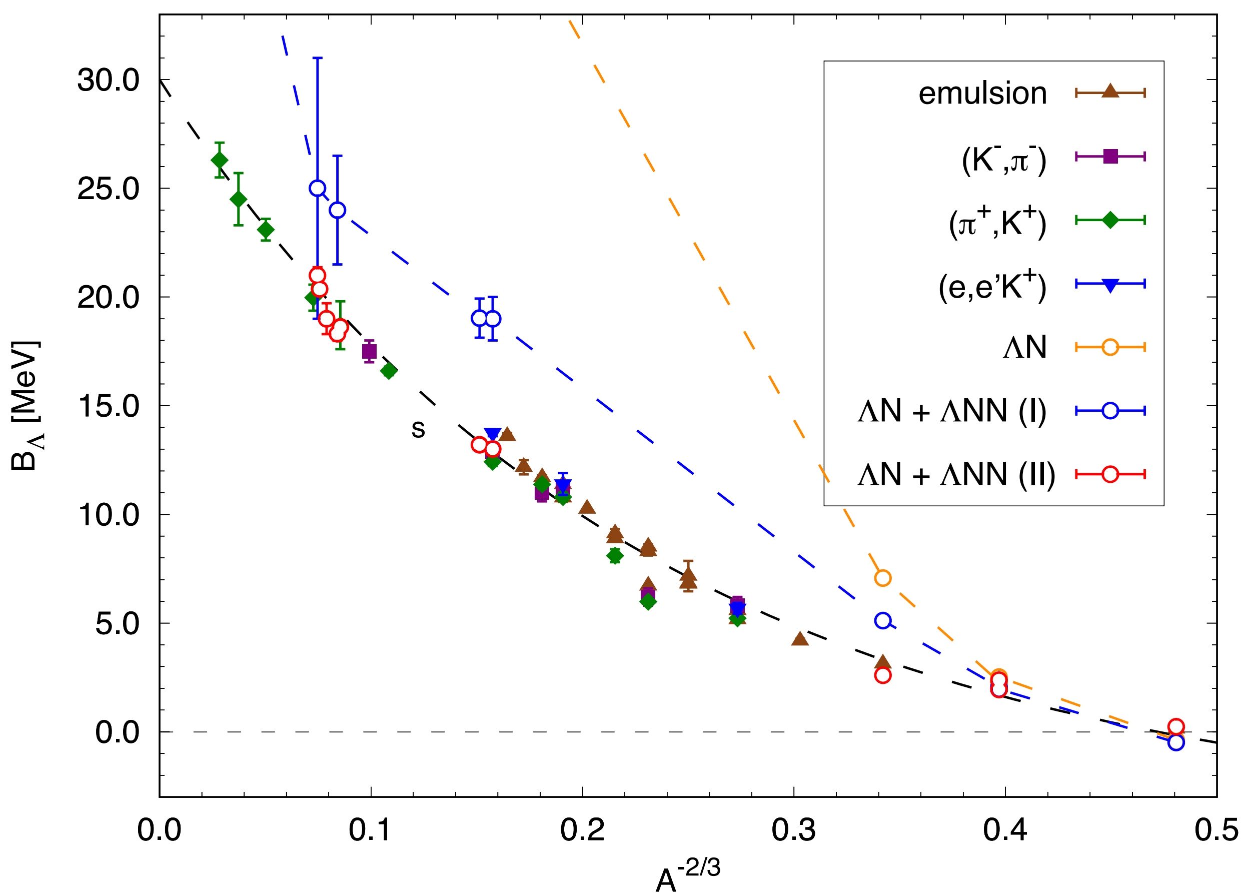


this scheme: $S = 0 : \langle V_{2b} \rangle \gg \langle V_{3b} \rangle$ $S = -1 : \langle V_{2b} \rangle \gtrsim \langle V_{3b} \rangle$

Results: Hypernuclei & Neutron Star Matter

***Note*:** the energy contribution of a many-body force is scheme (and scale) dependent

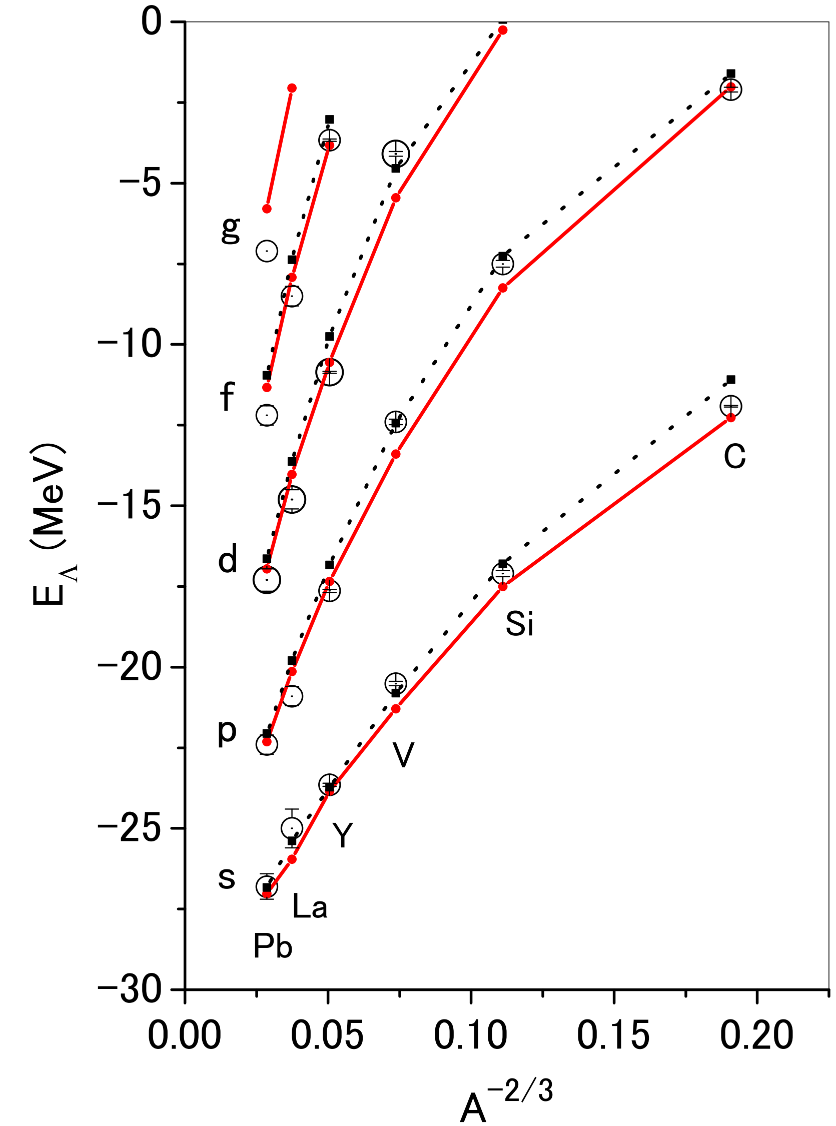
AFDMC: Argonne + Urbana



D. L. *et al.*, Phys. Rev. C **89**, 014314 (2014)

D. L. and F. Pederiva, arXiv:1711.07521

G-Matrix: ESC08 + MPa



Y. Yamamoto *et al.*, Phys. Rev. C **90**, 045805 (2014)

Results: Neutron Star Matter

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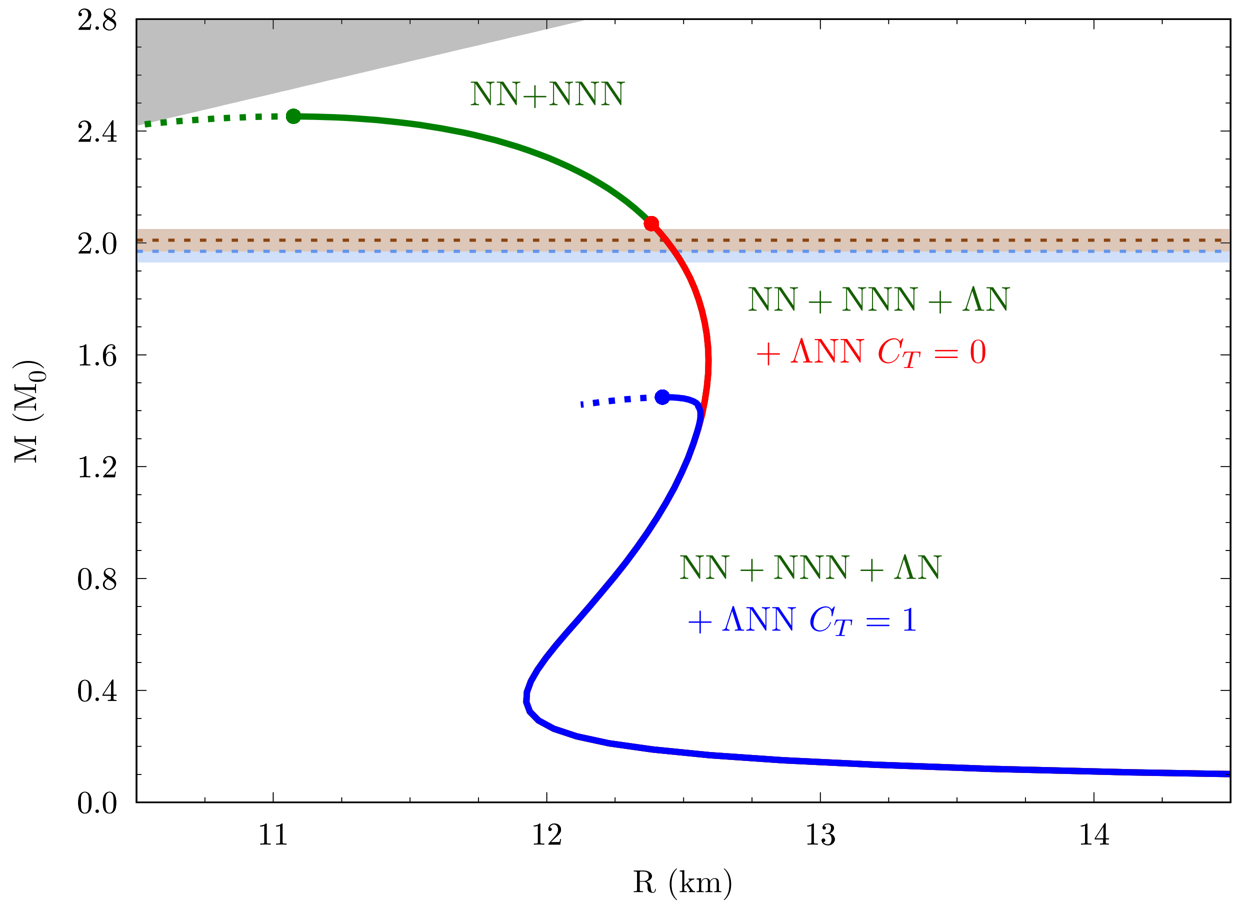

D. L. *et al.*, Phys. Rev. Lett. **114**, 092301 (2015) & preliminary

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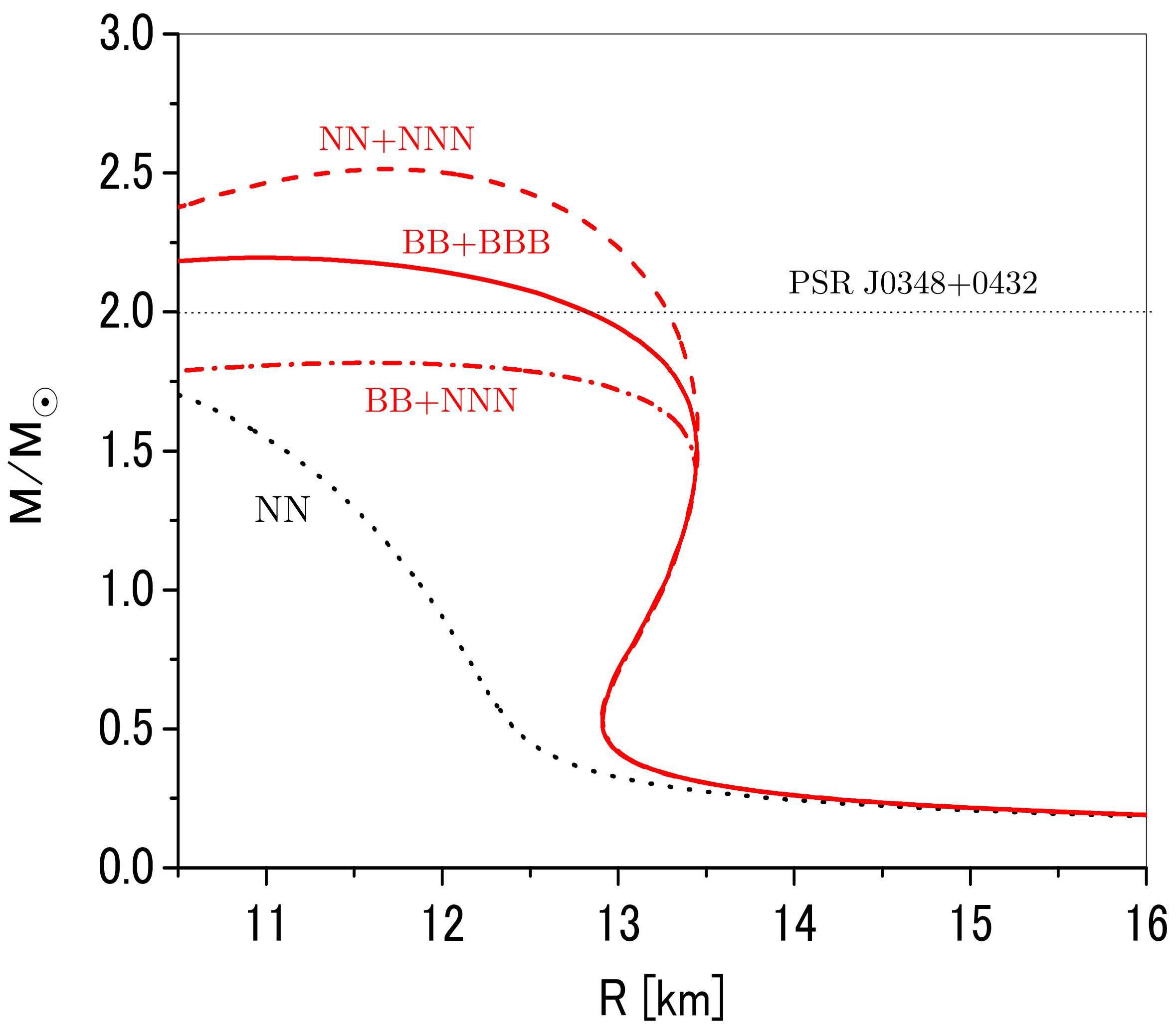
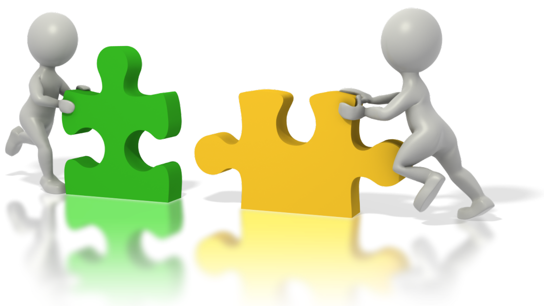


Y. Yamamoto *et al.*, Phys. Rev. C **90**, 045805 (2014)

Summary

- Quantum Monte Carlo calculations are feasible for medium-mass hypernuclei, and the accuracy of numerical predictions is comparable to that of currently available experimental data.
- Phenomenological potentials developed in a pion exchange model with nucleons and lambdas as explicit degrees of freedom can describe the ground-state physics of hypernuclei in a wide mass range and for different hyperon orbits.
- A sensitivity study suggests the possibility to investigate the nucleon-isospin dependence of the three-body hyperon-nucleon-nucleon force in the medium-mass region of the hypernuclear chart using QMC methods.
- Future hypernuclear experiments on medium-mass targets (and others) will provide the opportunity to develop more accurate many-body hypernuclear interactions, crucial for the prediction of neutron star properties and for the solution of the hyperon puzzle.



Thank you!!