

# No-core shell model for hypernuclei

Workshop on  $^{208}\text{Pb}$  (e,e'K) and neutron stars

May 11, 2020

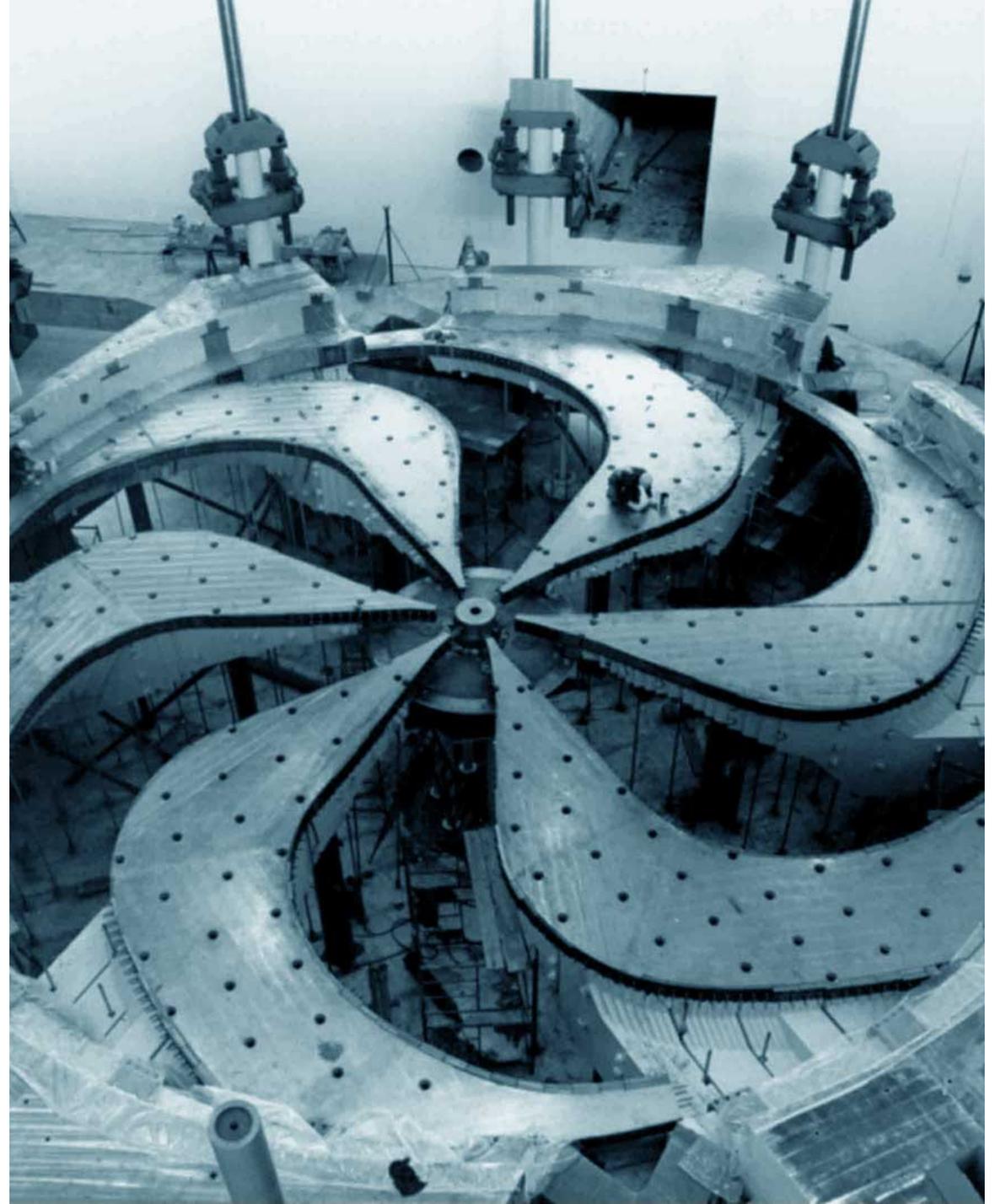
Petr Navratil

TRIUMF

Collaborators:

D. Gazda (NPI Rez), R. Roth (TU Darmstadt), R. Wirth (MSU),  
Takayuki Miyagi (TRIUMF)

2020-05-12

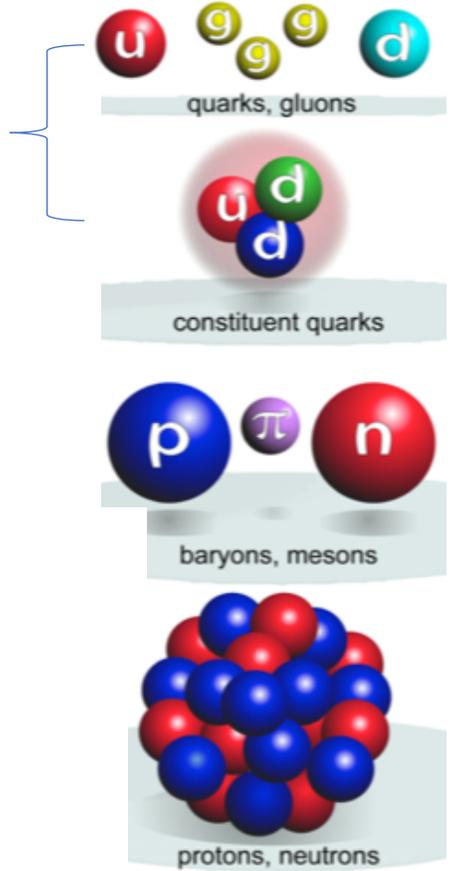


# First principles or *ab initio* nuclear theory

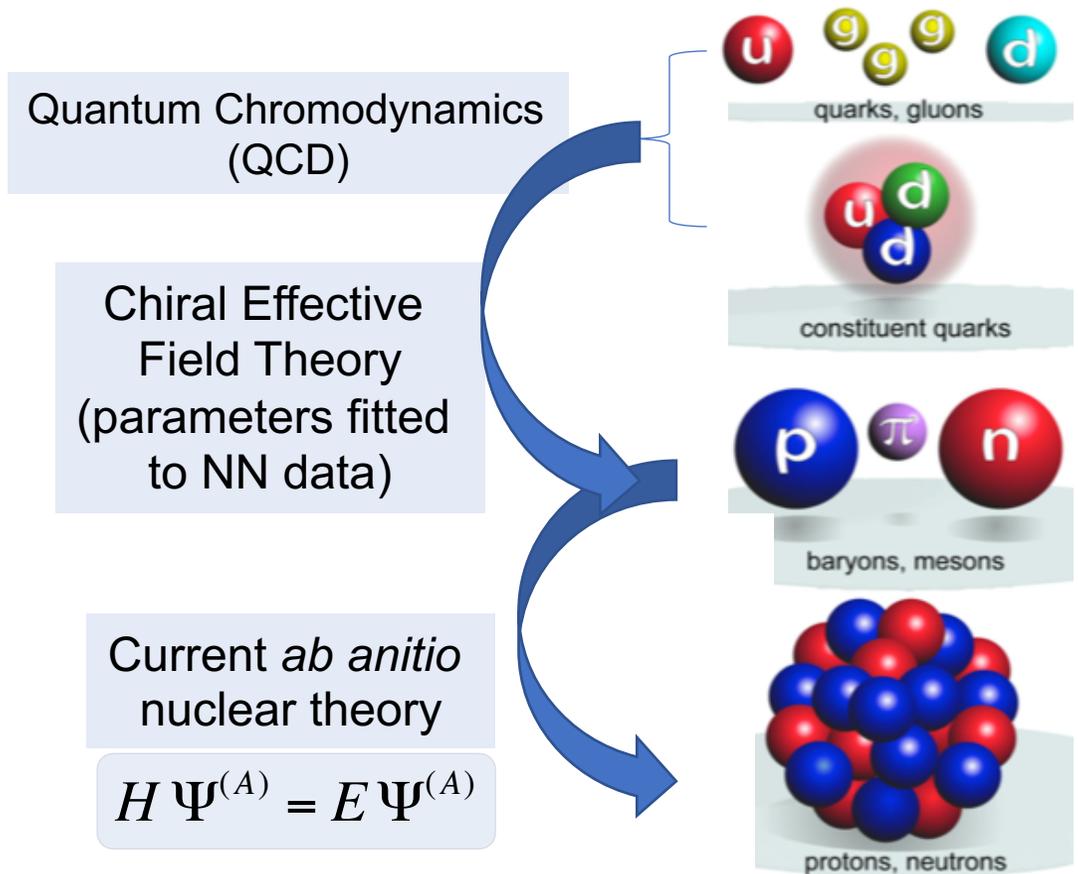
Quantum Chromodynamics  
(QCD)



Genuine *Ab Initio*



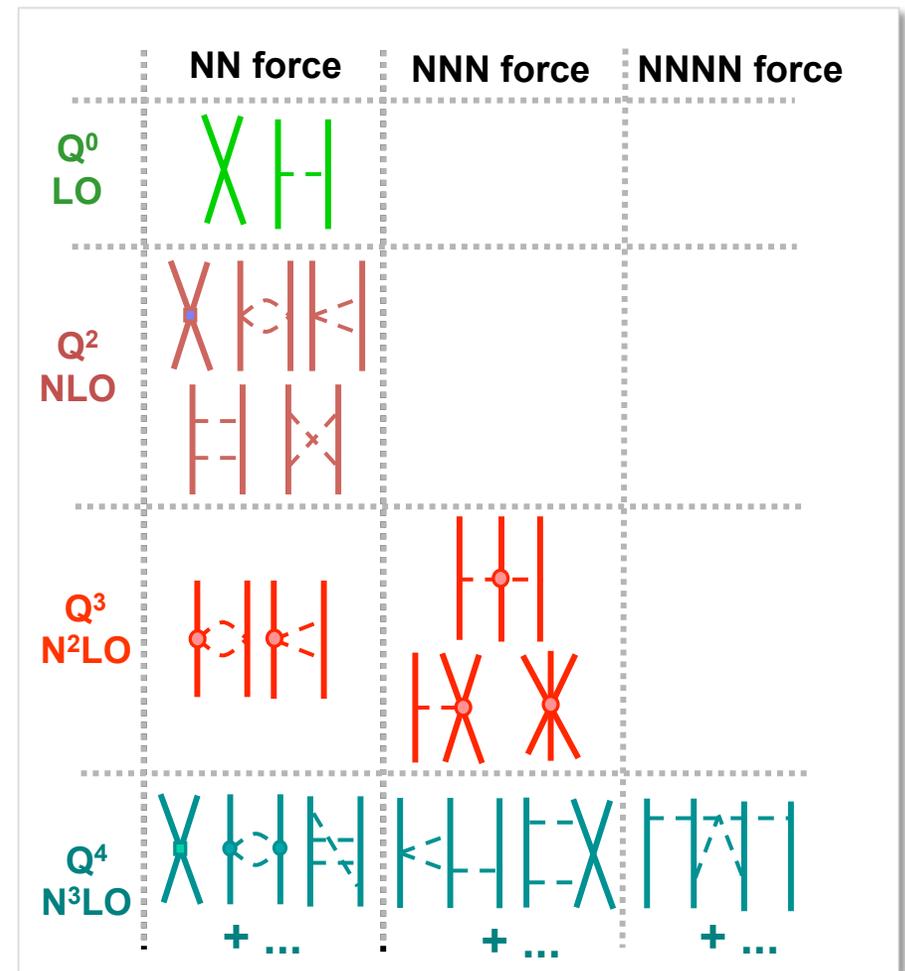
# First principles or *ab initio* nuclear theory – what we do at present



- *Ab initio*
  - ✧ Degrees of freedom: Nucleons
  - ✧ All nucleons are active
  - ✧ Exact Pauli principle
  - ✧ Realistic inter-nucleon interactions
    - ✧ Accurate description of NN (and 3N) data
  - ✧ Controllable approximations

## Chiral Effective Field Theory

- Inter-nucleon forces from chiral effective field theory
  - Based on the symmetries of QCD
    - Chiral symmetry of QCD ( $m_u \approx m_d \approx 0$ ), spontaneously broken with pion as the Goldstone boson
    - Degrees of freedom: nucleons + pions
  - Systematic low-momentum expansion to a given order ( $Q/\Lambda_\chi$ )
  - Hierarchy
  - Consistency
  - Low energy constants (LEC)
    - Fitted to data
    - Can be calculated by lattice QCD



$\Lambda_\chi \sim 1 \text{ GeV}$  :  
Chiral symmetry breaking scale

# Nucleon-Hyperon Interaction in chiral EFT

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)

ScienceDirect

Nuclear Physics A 779 (2006) 244–266

ELSEVIER

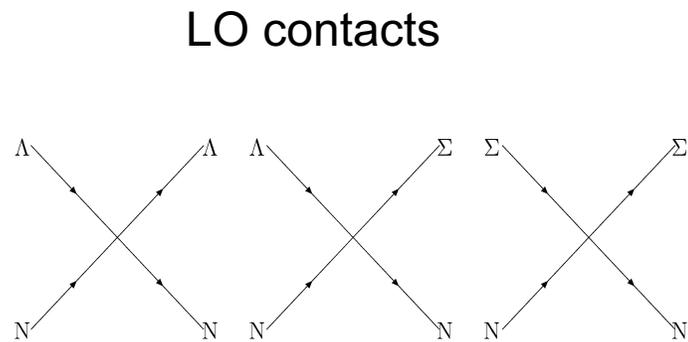
NUCLEAR PHYSICS A

Hyperon–nucleon interactions—a chiral effective field theory approach

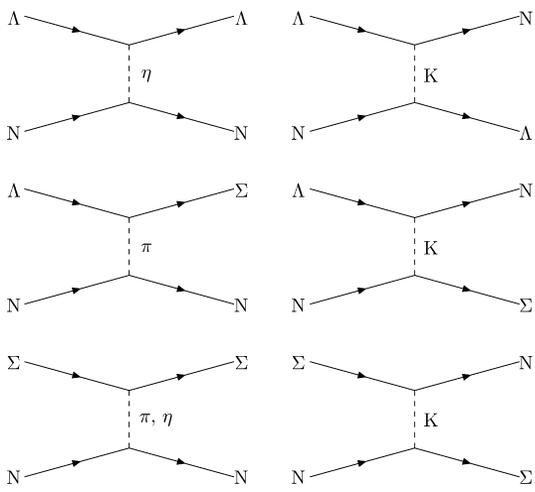
Henk Polinder<sup>a,\*</sup>, Johann Haidenbauer<sup>a</sup>, Ulf-G. Meißner<sup>a,b</sup>

Hyperon–nucleon interaction at next-to-leading order in chiral effective field theory

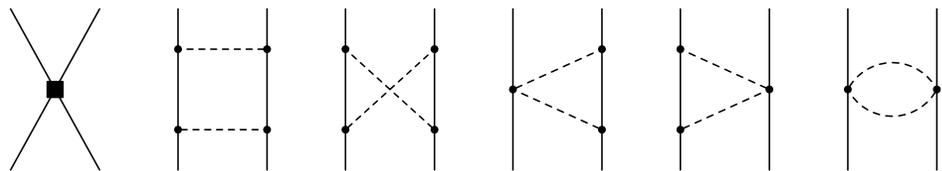
J. Haidenbauer<sup>a</sup>, S. Petschauer<sup>b</sup>, N. Kaiser<sup>b</sup>, U.-G. Meißner<sup>c,a,\*</sup>, A. Nogga<sup>a</sup>, W. Weise<sup>b,d</sup>



## LO one-boson exchange



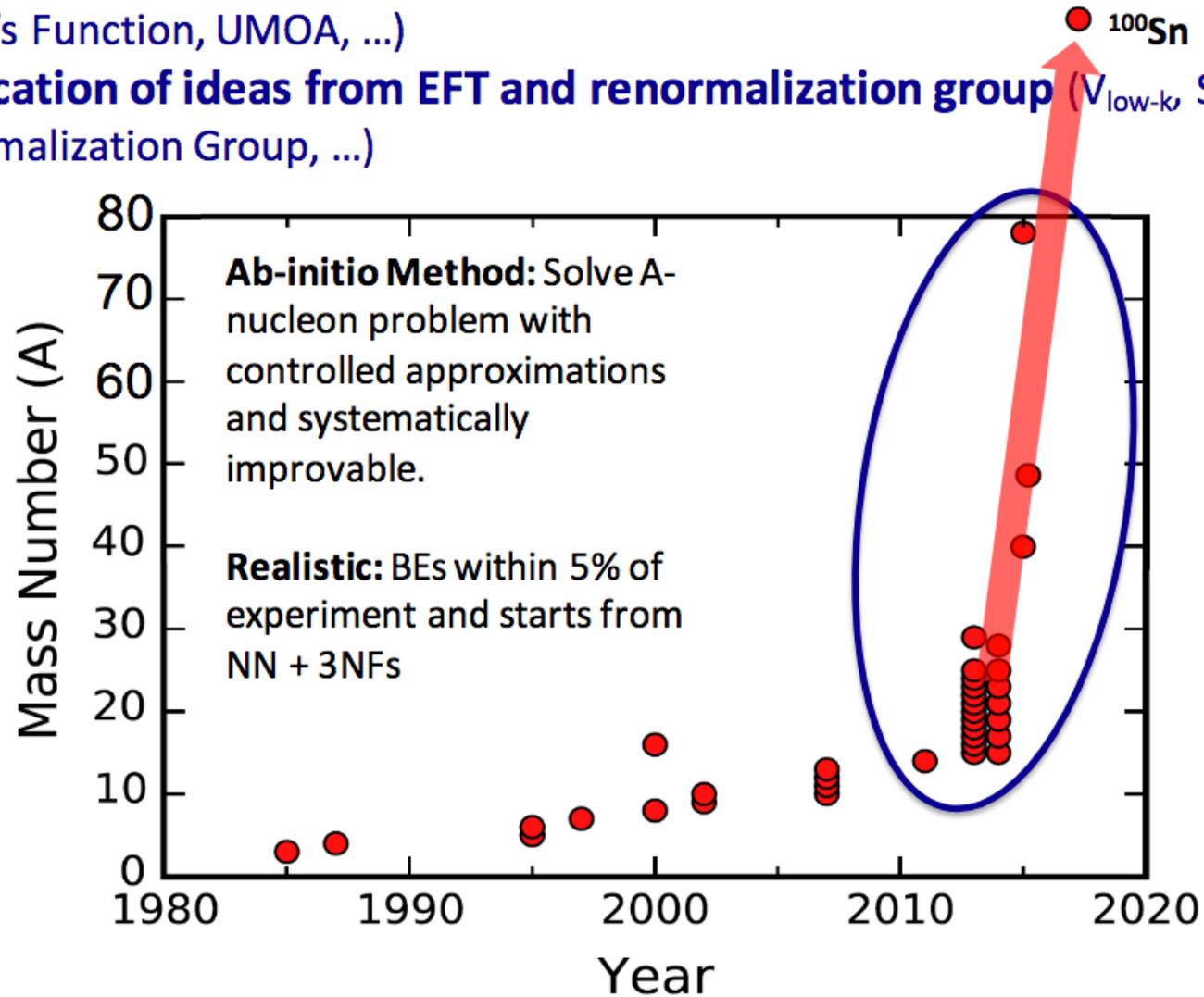
## NLO



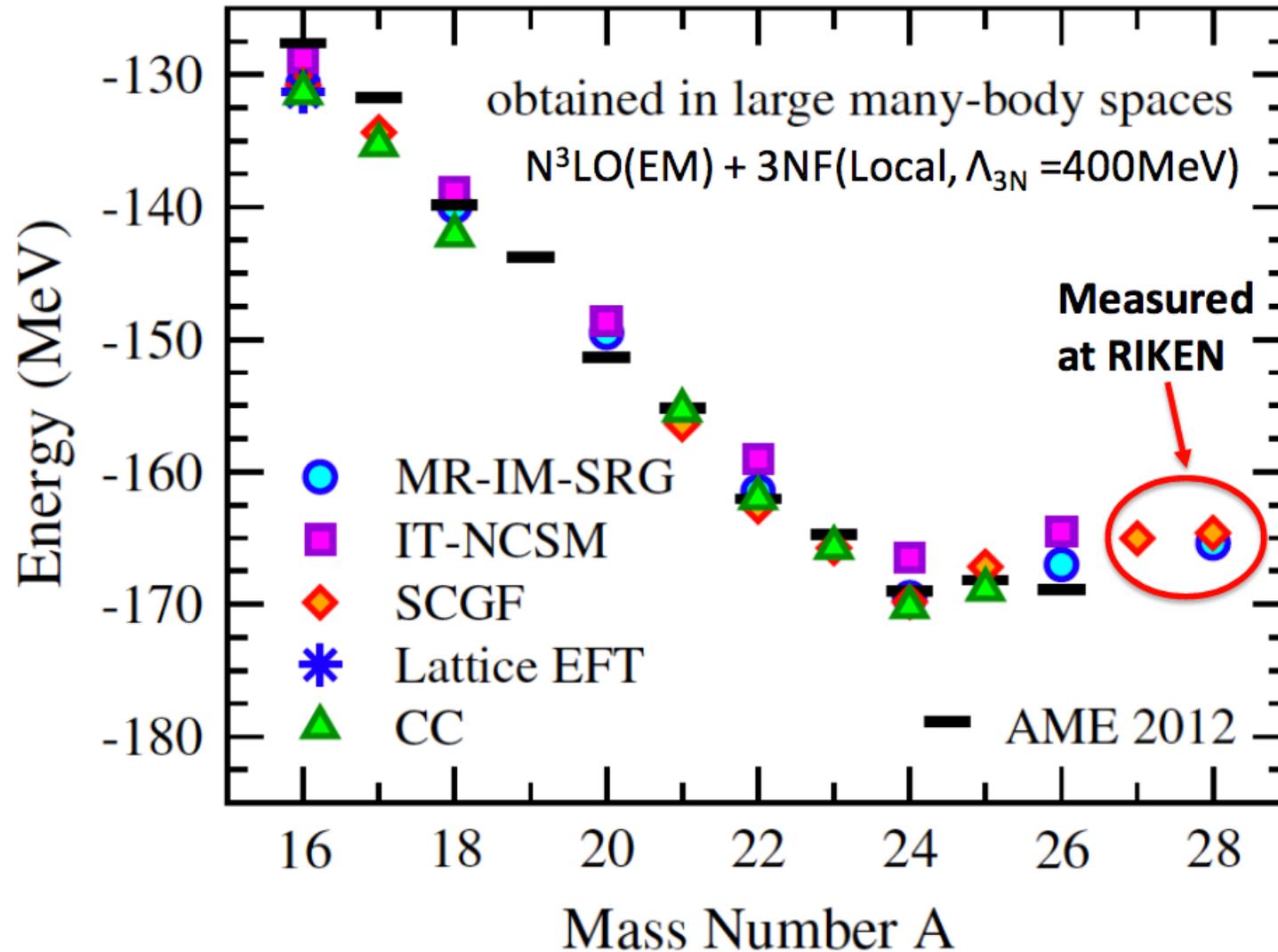
## Rapid developments in nuclear *ab initio* calculations

**Explosion of many-body methods** (Coupled clusters, Green's function Monte Carlo, In-Medium SRG, Lattice EFT, MCSM, No-Core Shell Model, Self-Consistent Green's Function, UMOA, ...)

**Application of ideas from EFT and renormalization group** ( $V_{\text{low-}k}$  Similarity Renormalization Group, ...)



## Oxygen chain with interactions from chiral EFT

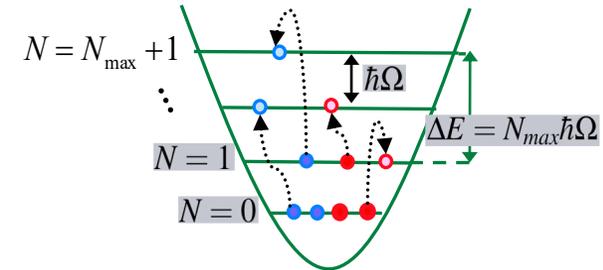


## Conceptually simplest *ab initio* method: No-Core Shell Model (NCSM)



NCSM

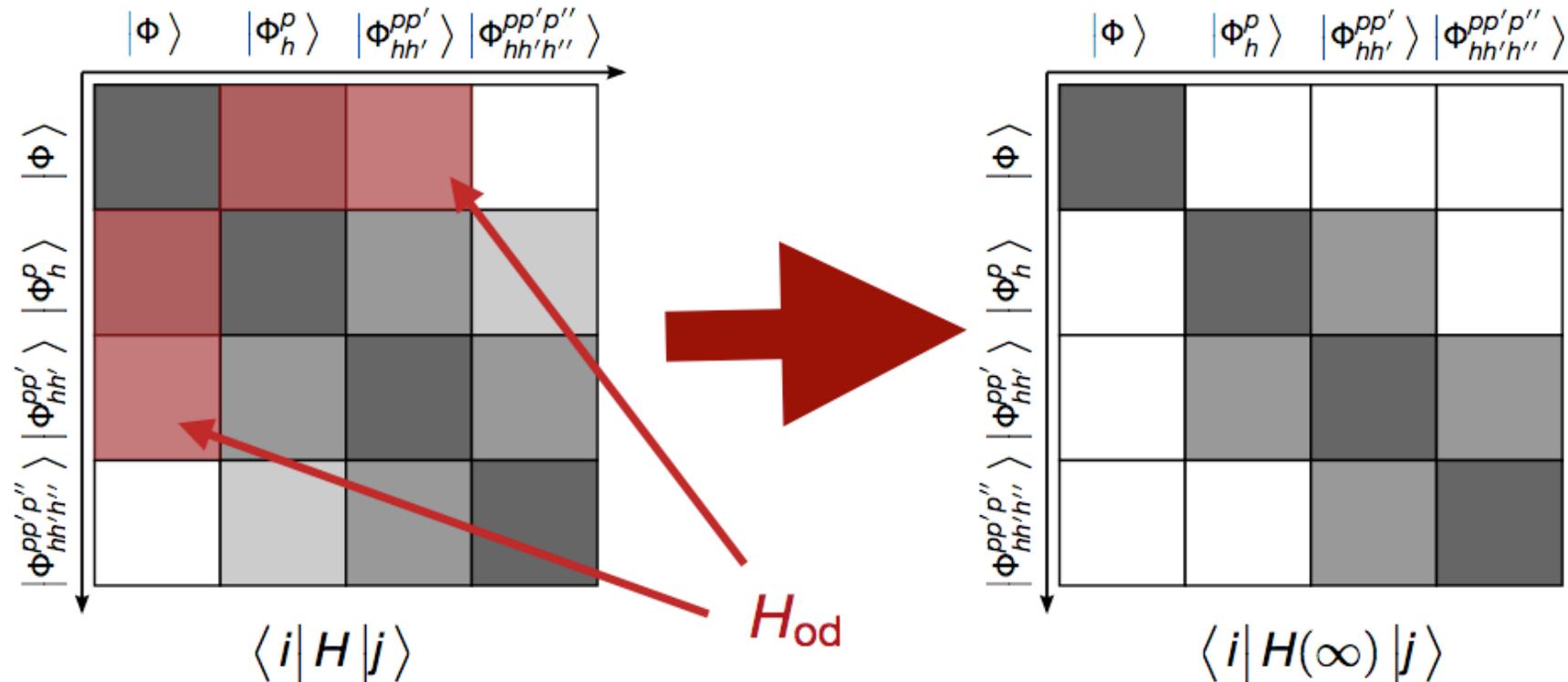
- Basis expansion method
  - Harmonic oscillator (HO) basis truncated in a particular way ( $N_{\max}$ )
  - Why HO basis?
    - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 –  $^4\text{He}$ ,  $^{16}\text{O}$ ,  $^{40}\text{Ca}$ )
    - **Equivalent description in relative(Jacobi)-coordinate and Slater determinant basis**
- Short- and medium range correlations
- Bound-states, narrow resonances



$${}^{(A)} \Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^{HO}(\vec{\eta}_1, \vec{\eta}_2, \dots, \vec{\eta}_{A-1})$$

$${}^{(A)} \Psi_{SD}^A = \sum_{N=0}^{N_{\max}} \sum_j c_{Nj}^{SD} \Phi_{SDNj}^{HO}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{CM})$$

# In-Medium Similarity Renormalization Group – Flow equation



$$\frac{d}{ds} H(s) = [\eta(s), H(s)], \quad \text{e.g.,} \quad \eta(s) \equiv [H_d(s), H_{od}(s)]$$

*Annual Review of Nuclear and Particle Science*  
**Nonempirical Interactions  
 for the Nuclear Shell Model:  
 An Update**

S. Ragnar Stroberg,<sup>1,2,3</sup> Heiko Hergert,<sup>4</sup>  
 Scott K. Bogner,<sup>4</sup> and Jason D. Holt<sup>1</sup>

Annu. Rev. Nucl. Part. Sci. 2019. 69:307–62

The Hamiltonian matrix is never constructed explicitly!

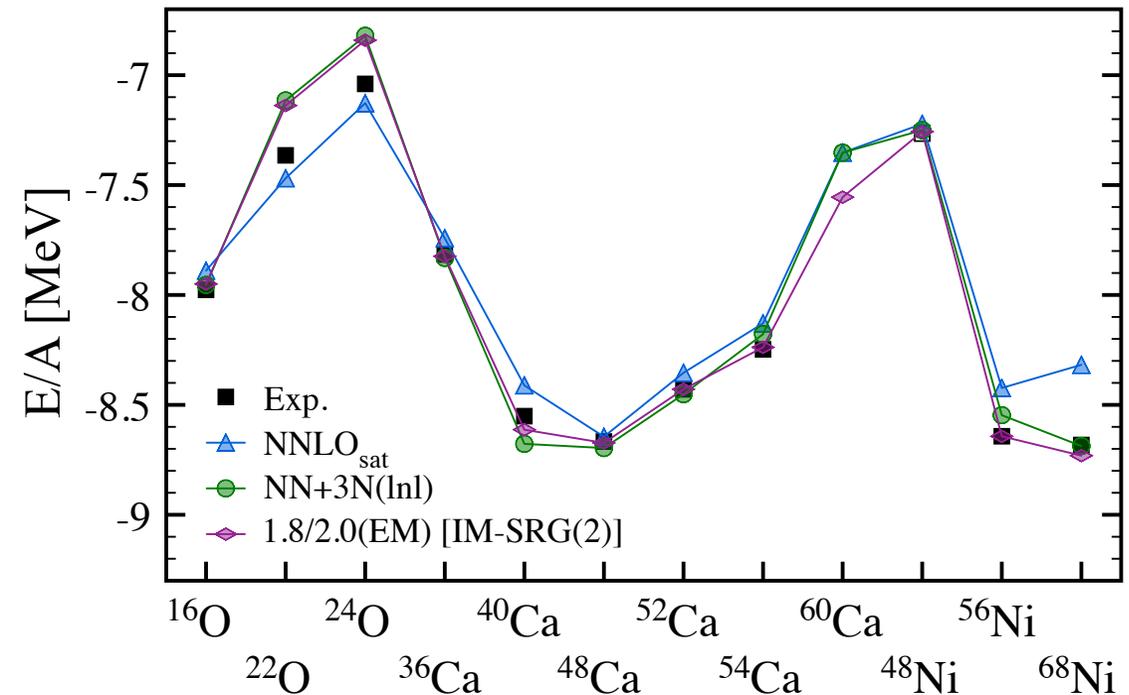
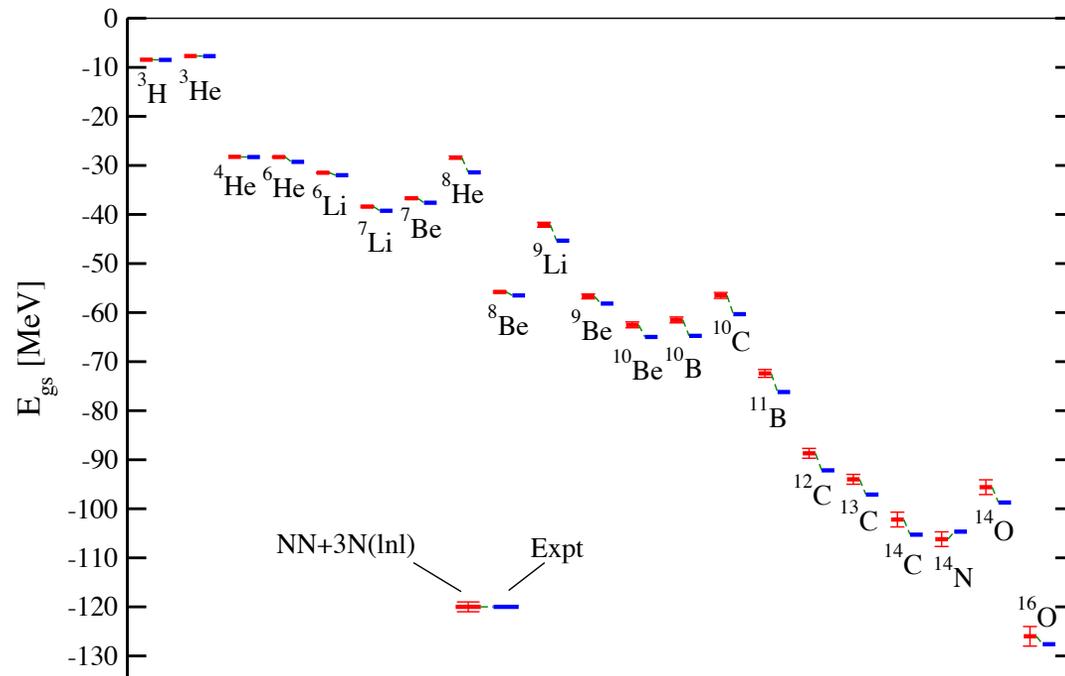


## Binding energies of light and selected medium mass nuclei from chiral NN+3N forces

11

- Quite reasonable description of binding energies across the nuclear charts becomes feasible
  - The Hamiltonian fully determined in  $A=2$  and  $A=3,4$  systems**
    - Nucleon–nucleon scattering, deuteron properties,  $^3\text{H}$  and  $^4\text{He}$  binding energy,  $^3\text{H}$  half life
  - Light nuclei – NCSM
  - Medium mass nuclei – Self-Consistent Green’s Function method

NN N<sup>3</sup>LO (Entem-Machleidt 2003)  
3N N<sup>2</sup>LO w local/non-local regulator

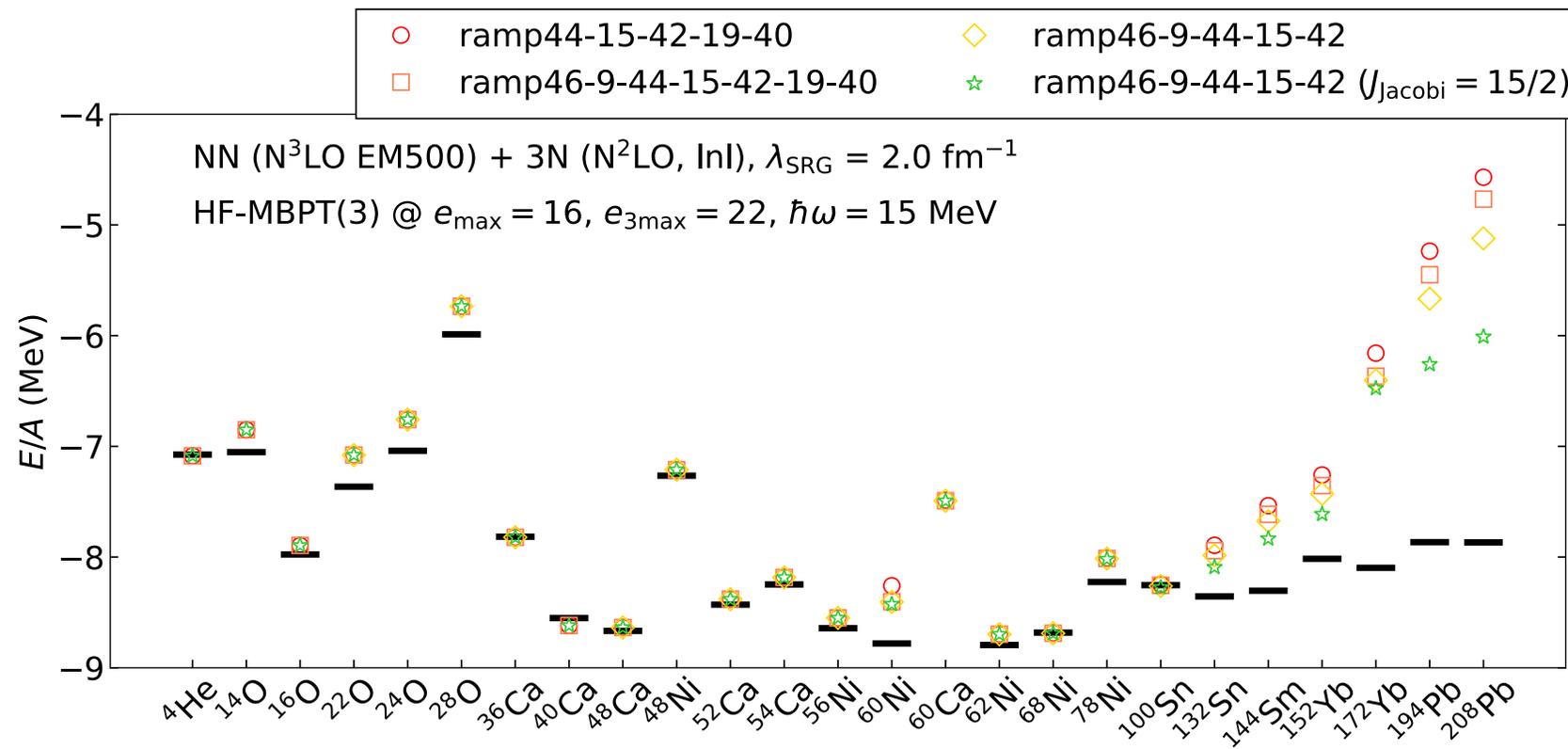


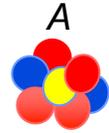
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## NCSM calculations for light *hypernuclei*

- Flexible approach capable performing exact calculations for few-nucleon systems and accurate calculations for nuclei & **hypernuclei** with  $A \leq 25$

PHYSICAL REVIEW LETTERS

*Ab Initio* Description of *p*-Shell Hypernuclei

Roland Wirth,<sup>1,\*</sup> Daniel Gazda,<sup>2,3</sup> Petr Navrátil,<sup>4</sup> Angelo Calci,<sup>1</sup> Joachim Langhammer,<sup>1</sup> and Robert Roth<sup>1,†</sup>

PHYSICAL REVIEW C **97**, 064315 (2018)

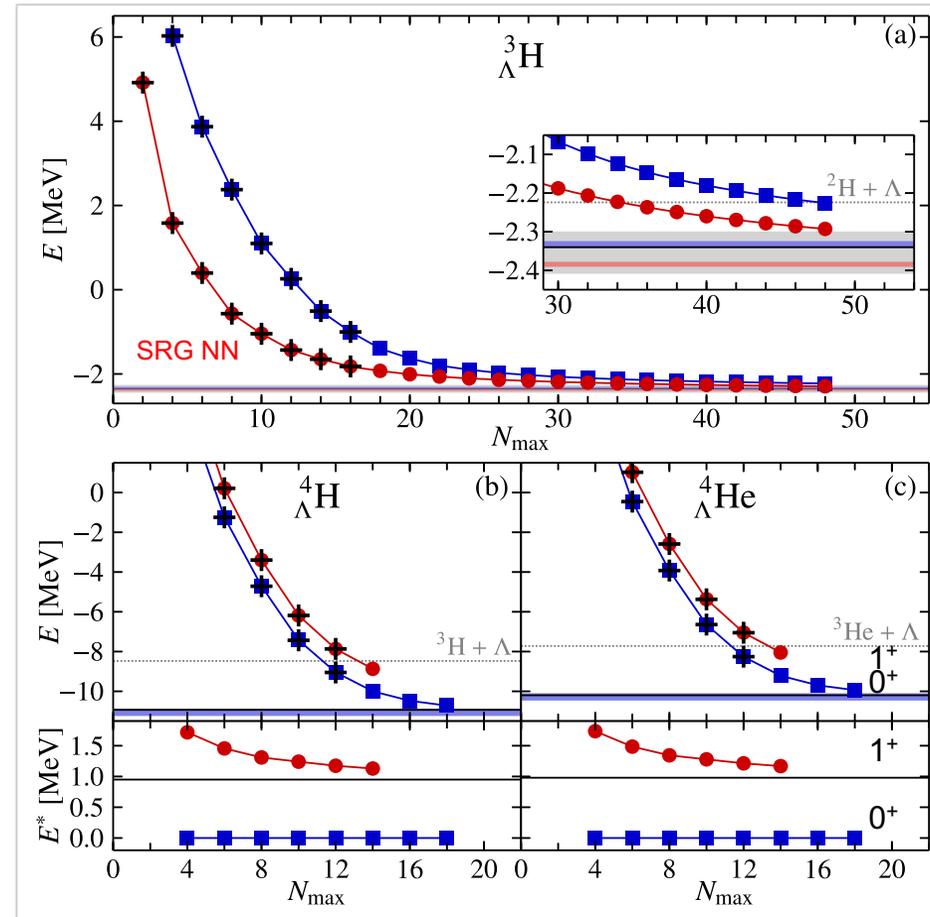
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Testing chiral LO NY potentials with  $\Lambda$ - $\Sigma$  mixing included

...outperform the Julich '04 YN potential

Chiral  $N^3LO$  NN+  $N^2LO$  3N



Jacobi-NCSM vs. Slater-Determinant basis NCSM

$p$  (uud):  $m_p=938.3$  MeV;  $n$  (udd):  $m_n=939.6$  MeV,  $\Lambda$  (uds):  $m_\Lambda=1115.7$  MeV;  $\Sigma$  (uus,uds,dds):  $m_\Sigma=1189.4$  MeV



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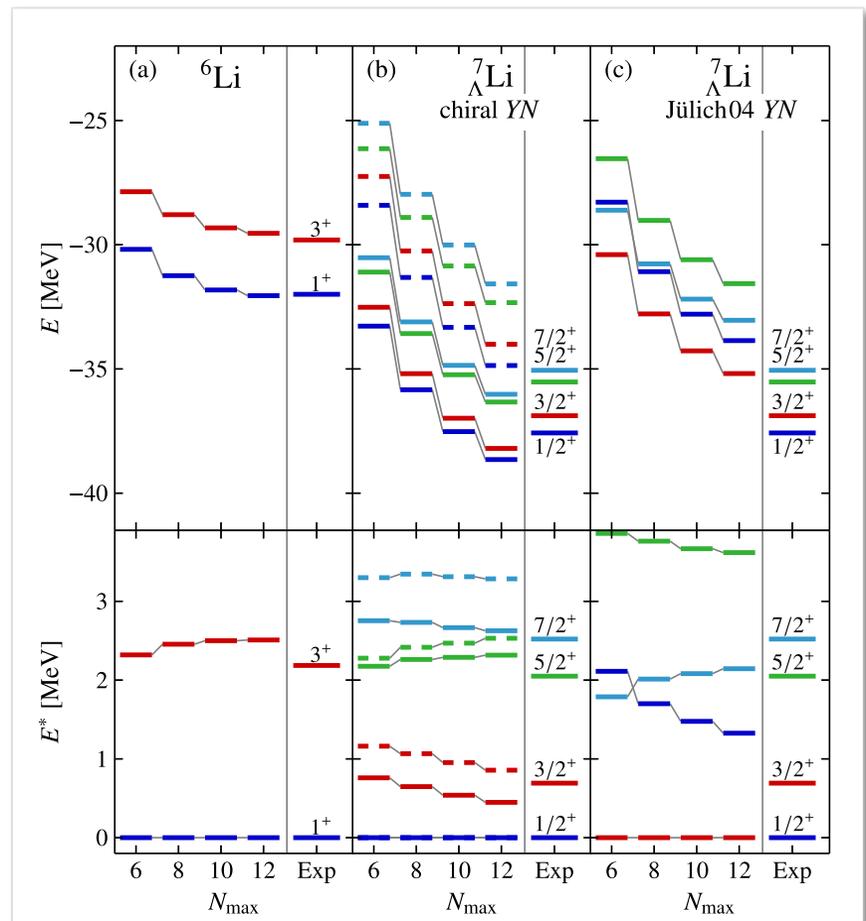
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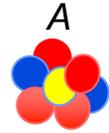
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Slater-Determinant basis NCSM

SRG NN+3N



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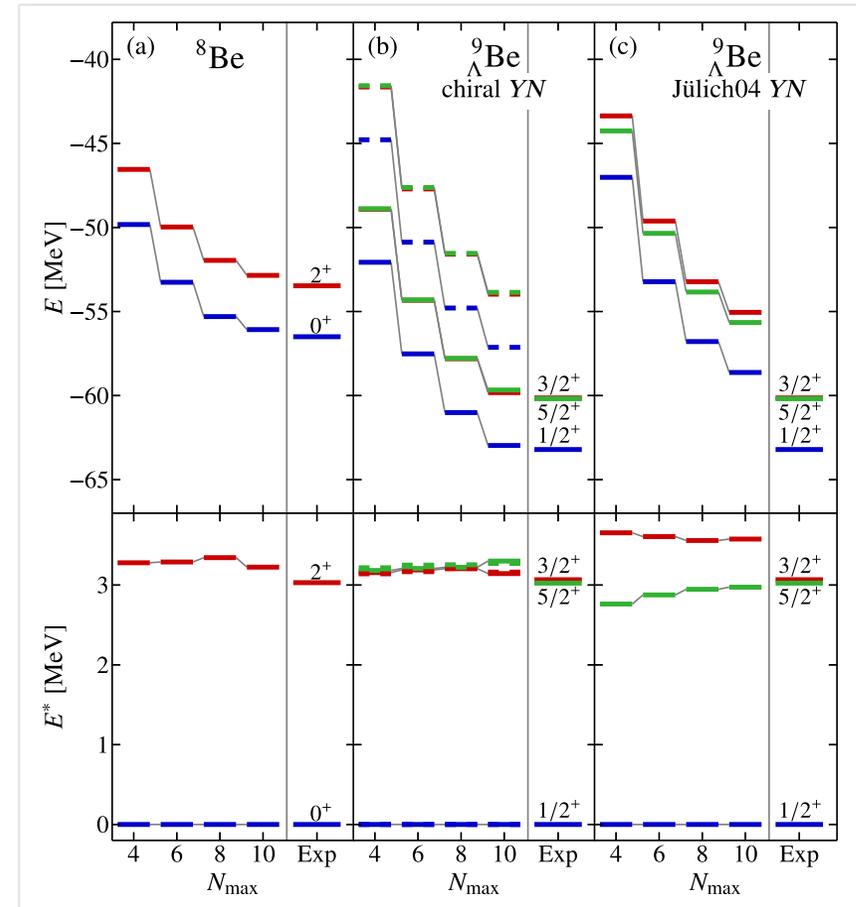
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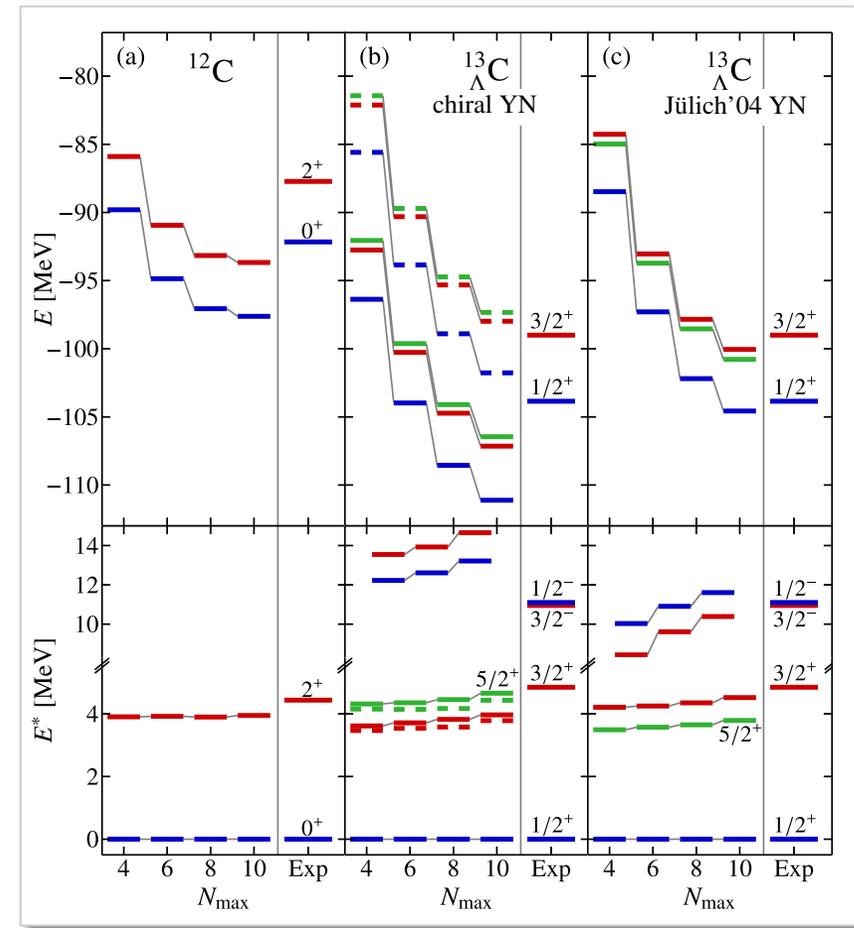
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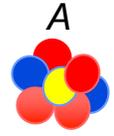
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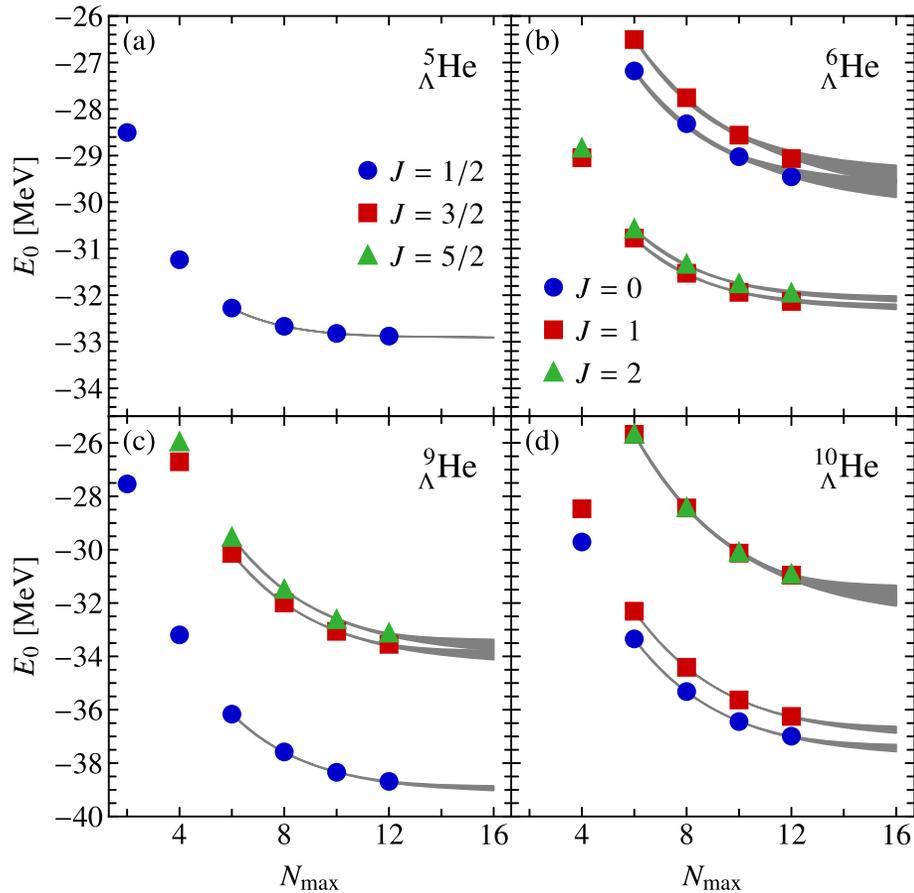
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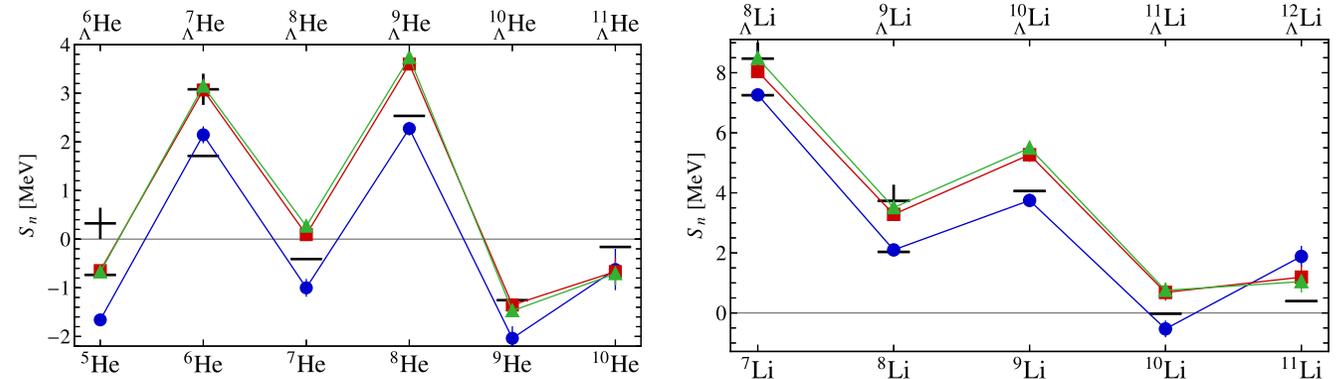
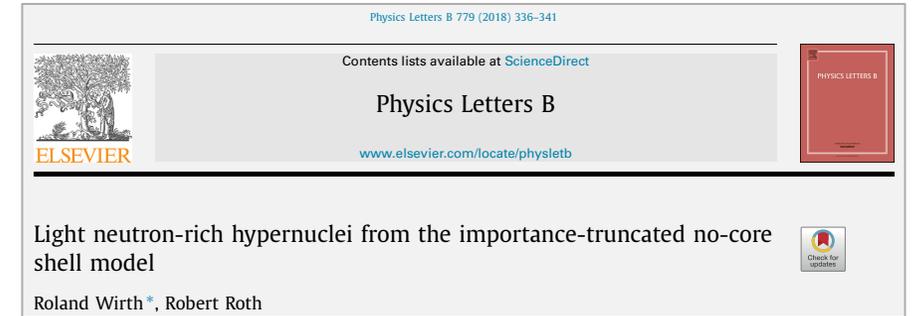
# NCSM calculations for light hypernuclei

Similarity renormalization group transformation applied consistently up to the three-baryon level to improve the model-space convergence



Chiral N<sup>3</sup>LO NN+ N<sup>2</sup>LO 3N + LO YN

## Slater-Determinant basis NCSM



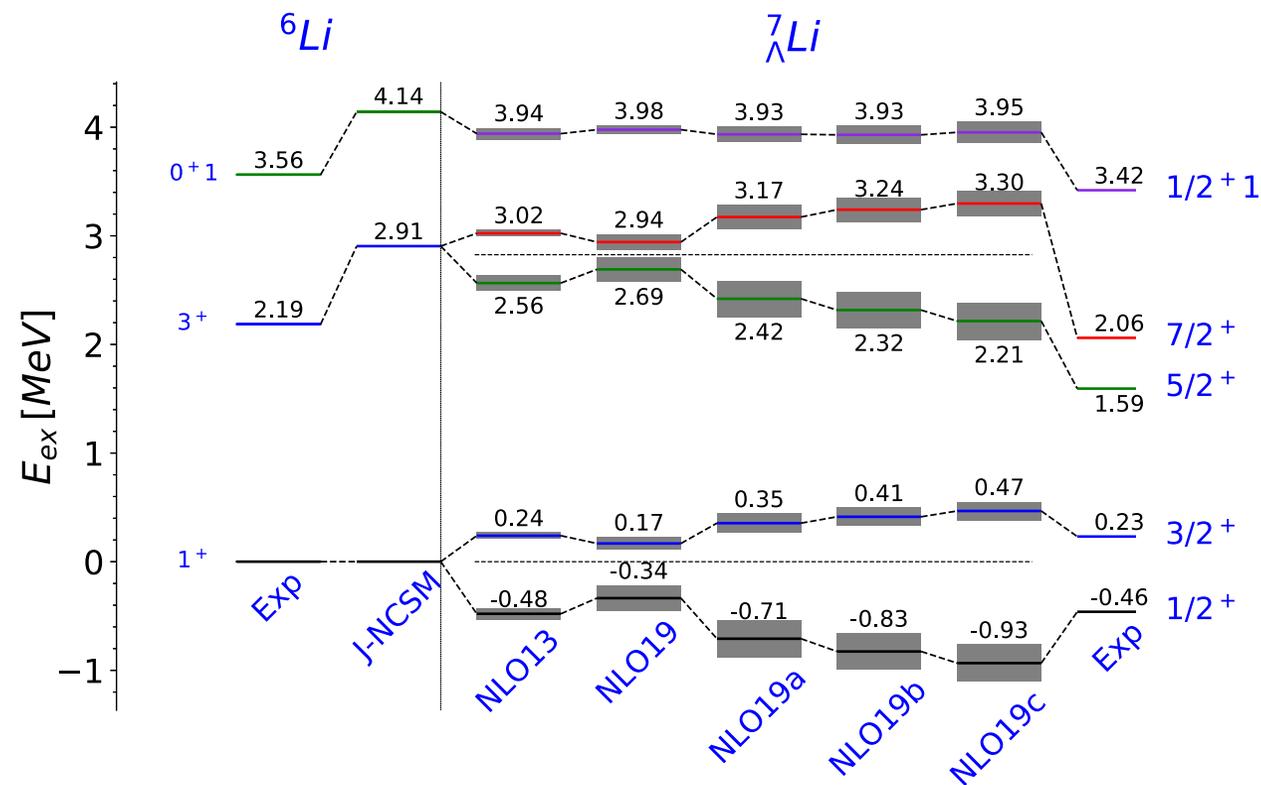
- Neutron separation energies stable and in good agreement with available data for both nucleonic parents and their daughter hypernuclei
- Neutron drip lines in the helium and lithium isotopic chains not changed by the addition of a hyperon



# NCSM calculations for light hypernuclei

Chiral N<sup>4</sup>LO NN + NLO YN  
SRG transformed at two-baryon level

Jacobi-coordinate NCSM



Physics Letters B 801 (2020) 135189

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Implications of an increased  $\Lambda$ -separation energy of the hypertriton

Hoai Le<sup>a</sup>, Johann Haidenbauer<sup>a,\*</sup>, Ulf-G. Meißner<sup>b,a,c</sup>, Andreas Nogga<sup>a</sup>

Check for updates

STAR collaboration -  
larger  ${}^3\text{H}_\Lambda$  binding energy (0.13 -> 0.41 MeV)

YN NLO19a,b,c fitted to obtain the larger value

${}^4\text{He}_\Lambda(0^+)$ ,  ${}^4\text{He}_\Lambda(1^+)$  and  ${}^7\text{Li}_\Lambda$  still well described

## Conclusions

- *Ab initio* calculations of nuclear structure and reactions becoming feasible beyond the lightest nuclei
  - Make connections between the low-energy QCD, many-body systems, and nuclear astrophysics
- *Ab initio* nuclear theory essential for precision applications such as tests of fundamental symmetries
  - Quenching of  $g_A$
  - Double beta decay matrix elements
  - Isospin mixing correction  $\delta_C$
  - ...
- Initial applications of *ab initio* methods to  $p$ -shell hypernuclei
- It is feasible to extend the reach and precision of *ab initio* hypernuclei calculations in a near future

**In synergy with experiments, *ab initio* nuclear theory is the right approach to understand low-energy properties of atomic nuclei**

Thank you!  
Merci!

