Extracting Hypernuclear Properties from the $(e, e'K^+)$ Cross Section

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Outline

- ⋆ Motivation
- ⋆ The $(e, e'K^+)$ cross-section
  - ▷ Factorisation of the nuclear cross section
  - ▷ Nuclear and hypernuclear dynamics
- ▷ Kinematics: combining $(e, e'p)$ and $(e, e'K^+)$
- ⋆ From nuclei to nuclear matter: the case for ^{208}Pb
- ⋆ Outlook

Motivation

- Understanding hypernuclear dynamics is a fundamental problem, with important implications for the understanding of neutron star properties

- Experimental studies of the $(e, e'K^+)$ reaction have the potential to provide information needed to improve the available models of interactions involving nucleons and hyperons, and shed light on issues such as isospin dependence and the role of thee-body forces

- The dynamical information extracted from experimental data must be as model independent as posible

- Besides being a good proxy for uniform nuclear matter, ^{208}Pb has been extensively studied by high resolution $(e, e'p)$ experiments. The data obtained from these analyses provide the baseline for the determination of hypernuclear properties

The $A(e, e'K^+)_\Lambda A$ Cross Section

- ⋆ Consider the process

$$e(k) + A(p_A) \to e'(k') + K^+(p_K) + {}_\Lambda A(p_{{}_\Lambda A})$$

- ⋆ Cross section $(i, j = 1,\ 2,\ 3)$

$$d\sigma \propto L_{ij} W^{ij}$$

  - ▷ The lepton tensor L_{ij} is fully specified by the *measured* electron kinematical variables
  - ▷ The tensor W^{ij}, describing the nuclear response, contains all the information on both nuclear and hypernuclear dynamics

⋆ Lepton tensor

$$L = \begin{pmatrix} \eta_+ & 0 & -\sqrt{\epsilon_L \eta_+} \\ 0 & \eta_- & 0 \\ -\sqrt{\epsilon_L \eta_+} & 0 & \epsilon_L \end{pmatrix} ,$$

$$\eta_\pm = \frac{1}{2}(1 \pm \epsilon) \ , \ \epsilon = \left(1 + 2\frac{|\mathbf{q}|^2}{Q^2} \tan^2\frac{\theta_e}{2}\right)^{-1} \ , \ \epsilon_L = \frac{Q^2}{\omega^2}\,\epsilon$$

⋆ Target response tensor

$$W^{ij} = \langle 0|{J_A^i}^\dagger(q)|F\rangle\langle F|J_A^j(q)|0\rangle\, \delta^{(4)}(q + p_0 - p_F)$$

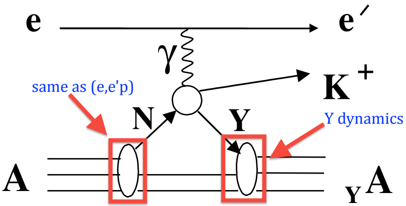
⋆ Building blocks

$$|0\rangle = |A\rangle \ , \ J_A^i = \sum_{n=1}^{A} j^i(n) \ , \ |F\rangle = |K^+\rangle \otimes |_\Lambda \mathrm{A}\rangle$$

▷ According to the paradigm of nuclear many-body theory, nuclear and hypernuclear states should be obtained from dynamical models based on phenomenological microscopic Hamiltonians

Impulse Approximation and Factorization

- ⋆ At momentum transfer $|\mathbf{q}|^{-1} \ll d$, $d \sim 1.5$ fm being the average nucleon-nucleon separation distance in the target nucleus, the beam particles interact with individual (bound, moving) nucleons



- ⋆ Within this scheme, the nuclear transition amplitude factorizes into the amplitude of the elementary process, a purely nuclear amplitude and a hypernuclear amplitude
- ⋆ The effects of Final State Interactions (FSI) between the outgoing K^+ and the recoiling system can be included using an optical potential

Nuclear Transition Amplitude

- Isolate the building blocks

$$\begin{aligned}\mathcal{M}_{0\to F} &= \langle K^+, {}_\Lambda A | J_A^i | 0\rangle \\ &= \sum_n \sum_{k_p,k_\Lambda} \Big\{ \langle {}_\Lambda A | (A-1)_n \rangle | Y \rangle \Big\} \langle K^+ Y | j^i | p \rangle \Big\{ \langle p | \langle (A-1)_n | 0 \rangle \Big\}\end{aligned}$$

- Relation to the spectral function formalism of $(e, e'p)$

$$P_N(k_p, E_p) = \sum_n |\langle p | \langle (A-1)_n | 0 \rangle|^2 \delta(E_p - E_n + E_0)$$

  - probability of removing a proton of momentum k_p from the nuclear target, leaving the residual nucleus with energy E_p

$$P_\Lambda(k_\Lambda, E_\Lambda) = \sum_n |\langle \Lambda | \langle (A-1)_n | {}_\Lambda A \rangle|^2 \delta(E_\Lambda - E_n + E_0)$$

  - probability of removing the Λ, carrying momentum k_Λ from the final state hypernucleus, leaving the residual nucleus with energy E_Λ

KINEMATICS

- Conservation of energy requires

$$\omega + M_A = E_{K+} + E_{\Lambda A} \quad , \quad \omega = E_e - E_{e'}$$

- From the nuclear and hypernuclear amplitudes

$$M_A = E_p + E_n \quad , \quad E_{\Lambda A} = E_\Lambda + E_n$$

- Missing energy, determined from measured kinematical quantities

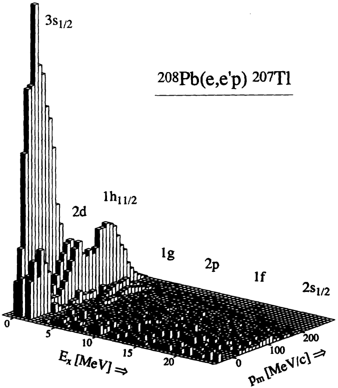
$$E_{\text{miss}} = \omega - E_{K+}$$
$$\omega = E_{K+} + E_\Lambda - E_p \Longrightarrow E_{\text{miss}} = E_\Lambda - E_p$$

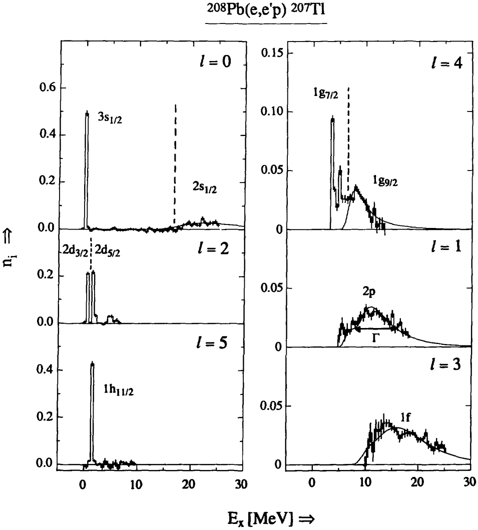
- Compare to $(e, e'p)$

$$E_{\text{miss}}^{(e,e'p)} = -E_p \Longrightarrow E_\Lambda = E_{\text{miss}} + E_{\text{miss}}^{(e,e'p)}$$

- $(e, e'p)$ data provide the baseline for the model independent determination of the Λ binding energy $B_\Lambda = -E_\Lambda$

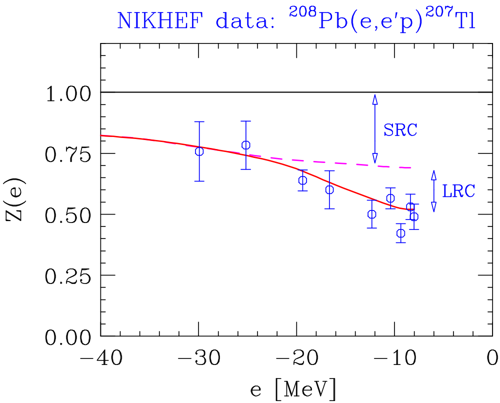
$^{208}\mathrm{Pb}(e, e'p)^{207}\mathrm{Tl}$ DATA





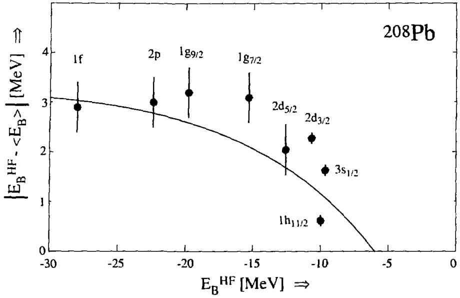
Spectroscopic Factors of ^{208}Pb

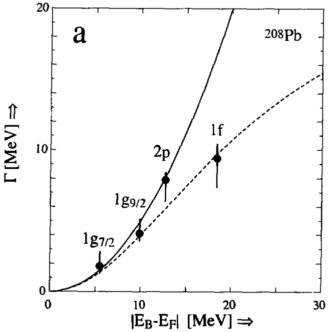
- Data: Quint, PhD Thesis (1988), Lapikas, NPA **553**, 297 (1993).
  Theory: OB *et al* PRC **41**, R24 (1990)



- Deeply bound hole states largely unaffected by finite size and shell effects

More $^{208}\mathrm{Pb}(e,e'p)^{207}\mathrm{Tl}$ Data





⋆ Deviation of the observed binding energies from mean field predictions

⋆ Widths of hole states

MODELING THE $(e, e'K^+)$ CROSS SECTION

- ⋆ The Λ binding energy in hypernuclei can be obtained in a largely model independent fashion combining $(e, e'K^+)$ and $(e, e'p)$ data

- ⋆ Exploiting the experimental information to improve the models of hyperon-nucleon dynamics will require the extension of the theoretical framework developed to describe $(e, e'p)$ data

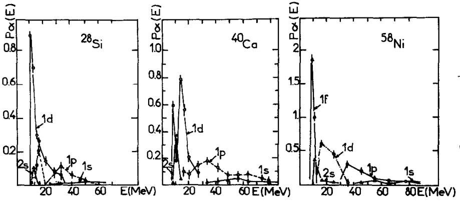
- ⋆ The main elements entering the calculation of the nuclear $(e, e'K^+)$ cross section are
  - ▷ The cross section of the elementary process $e + p \to e' + \Lambda + K^+$ on a bound, moving nucleon (see talk of P. Bydžovský)
  - ▷ A *fully realistic*—that is, beyond the mean-field approximation—description of the hypernuclear amplitudes determining the Λ spectral function

Comparison Between $(e, e'p)$ and $(e, e'K^+)$

- ⋆ Within the independent particle model

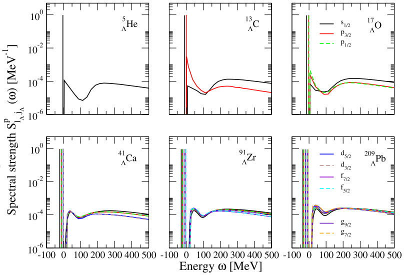
$$P_N(k_p, E_p) \sim \sum_\alpha \delta(E_p - \epsilon_p^\alpha) \quad , \quad P_Y(k_Y, E_Y) \sim \sum_\alpha \delta(E_Y - \epsilon_Y^\alpha)$$

- ▷ $P_N(k_p, E_p)$ from $(e, e'p)$



- ▷ Accurate theoretical calculations available for light nuclei and infinite matter

- ▷ $P_\Lambda(k_\Lambda, E_\Lambda)$, I. Vidaña, NPA **958**, 48 (2017)



Summary & Outlook

- ⋆ The available $(e, e'p)$ data provide the baseline for a model independent determination of the Λ binding energy with the $(e, e'K^+)$ in hypernuclei

- ⋆ The information obtained from these studies has the potential to greatly help the development of accurate models of hyperon-nucleon interactions

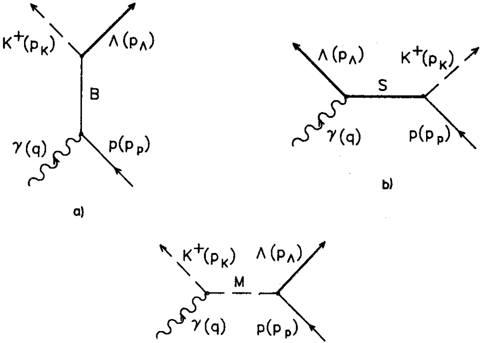
- ⋆ Achieving this goal requires a theoretical framework for the description of the $(e, e'K^+)$ cross section, including effects beyond the mean-field approximation. The development of such a framework within nuclear many-body theory does not involve severe conceptual problems, and the results of early efforts in this direction are quite promising, see talks of D. Lonardoni and I. Vidaña.

- ⋆ The astrophysical implications of hypernuclear dynamics can be best addresses using heavy and neutron rich targets, providing a good proxy for neutron star matter. The nucleus of $^{208}\mathrm{Pb}$, which has been extensively studied by $(e, e'p)$ experiments, appears to be the obvious choice.

Backup slides

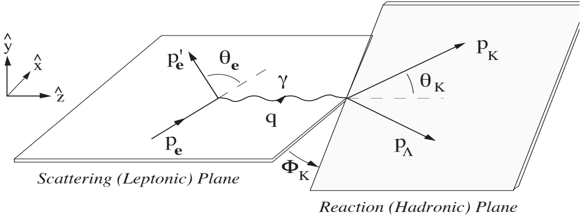
The Elementary $(e, e'k^+)$ Process

$$e + p \to e' + \Lambda + K^+$$



- B, S and M denote a nonstrange baryon, a strange baryon and a strange meson, respectively

⋆ Elementary cross section



$$\frac{d\sigma_N}{dE_{e'}d\Omega_{e'}d\Omega_K} \propto \left[\eta^+ W^{xx} + \eta^- W^{yy} + \epsilon_L W^{zz} + \sqrt{\epsilon_L \eta^+}\left(W^{xz} + W^{zx}\right)\right]$$

$$W^{\mu\nu} \propto \sum_{spins} j^{\mu\dagger} j^{\nu}$$

example: s-channel

$$j^\mu = \bar{u}(p_\Lambda)\Gamma(p_K)S_F(p_p + q)\Gamma^\mu(q)u(p_p)$$

- Hadron tensor [J. Adam *et al*, Czech. J. Phys **42**, 1167 (1992), D. Skoupil and P. Bydžovský, PRC **93**, 025204 (2016), **97**, 025202 (2018)]

$$j^\mu = \sum_i A_i(s,t,u)\bar{u}(p_\Lambda)M_i^\mu u(p_p)$$

$$s = (q+p_p)^2 \quad , \quad s = (q-p_K)^2 \quad , \quad s = (q-p_\Lambda)^2$$

$$M_1^\mu = \frac{1}{2}\gamma_5\left[\not{q}, \gamma^\mu\right]$$

$$M_2^\mu = \gamma_5\left[q^2 p_p^\mu - (q\cdot p_p)q^\mu\right] \quad , \quad M_3^\mu = \gamma_5\left[q^2 p_\Lambda^\mu - (q\cdot p_\Lambda)q^\mu\right]$$

$$M_4^\mu = \gamma_5\left[\gamma^\mu(q\cdot p_p) - \not{q}p_p^\mu\right] \quad , \quad M_5^\mu = \gamma_5\left[\gamma^\mu(q\cdot p_\Lambda) - \not{q}p_\Lambda^\mu\right]$$

$$M_6^\mu = \frac{1}{2}\gamma_5\left[\not{q}q^\mu - \gamma^\mu q^2\right]$$

- The model parameters involved in the definition of the current, e.g. the strong coupling constants, are obtained from a fit to the existing data

FROM NUCLEI TO NUCLEAR MATTER AND NEUTRON STARS