

QUANTUM PHYSICS ON A QUANTUM COMPUTER*



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MINI-LECTURE SERIES ON QUANTUM COMPUTING AND
QUANTUM INFORMATION SCIENCE,
Jefferson Laboratory March 11, 2020



U.S. DEPARTMENT OF
ENERGY

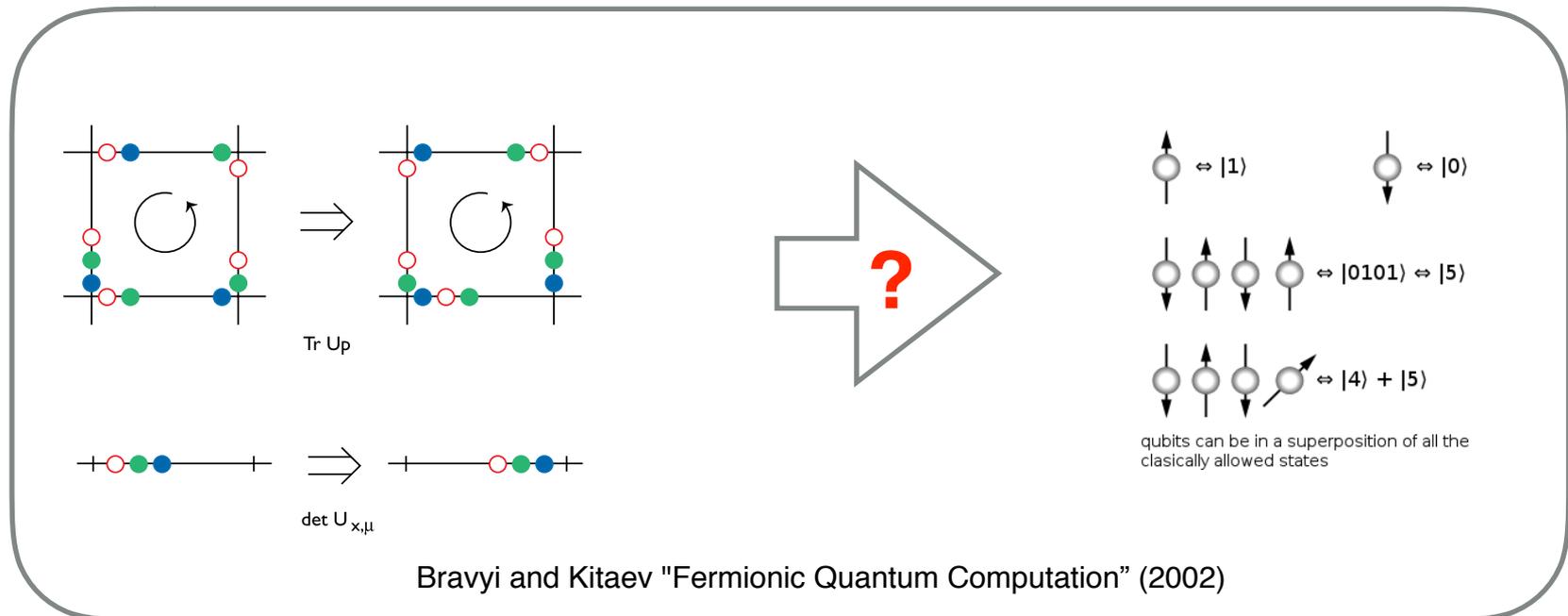
*D-theory: The accidental discovery of a Quantum Algorithm for Lattice Gauge theories: Circa 1998:

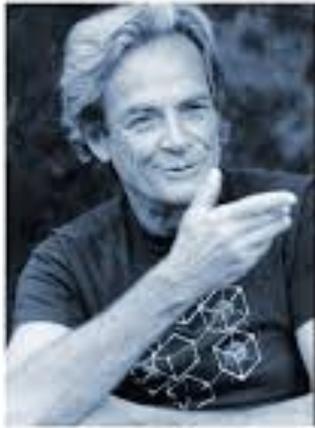
R. C. Brower, S. Chandrasekharan, U-J Wise, *QCD as quantum link model, Phys. Rev D 60 (1999).*

R. C. Brower, S. Chandrasekharan, U-J Wiese, *D-theory: Field quantization ... discrete variable Nucl. Phys. B (2004)*

D-theory :QCD Abacus*

Fermionic Qubit Algorithm ?





“If you think you understand
quantum mechanics,
then you don’t understand
quantum mechanics.”

- Richard Feynman

NEWPLANETCREATIVE.COM

Richard Feynman

On quantum physics and computer simulation

“There is plenty of room to make [computers] smaller. . . . nothing that I can see in the physical laws . . . says the computer elements cannot be made enormously smaller than they are now. In fact, there may be certain advantages. — 1959”

60 years ago!

“trying to find a computer simulation of physics seems to me to be an excellent program to follow out. . . . the real use of it would be with quantum mechanics. . . . Nature isn’t classical . . . and if you want to make a simulation of Nature, you’d better make it quantum mechanical, and by golly it’s a wonderful problem, because it doesn’t look so easy. — 1981”

Feynman's Unfinished Legacy

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

“...The question is, If we wrote a Hamiltonian which *involved only* these [Pauli] operators, locally coupled to corresponding operators on the other space-time points, could we imitate every quantum mechanical system which is discrete and has a finite number of degrees of freedom? *I know, almost certainly, that we could do that for any quantum mechanical system which involves Bose particles. I'm no sure whether Fermi particles could be described by such a system. So, I leave that open...*”

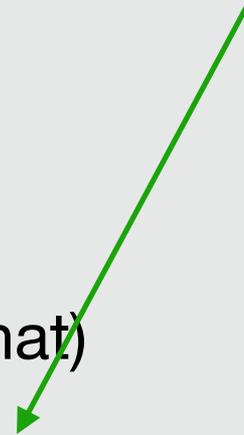
Richard P. Feynman

(*Simulating Physics with Computers (1982)*)

International J. of Theoretical Physics Vol 21, now 6./7, 1982

OUTLINE:

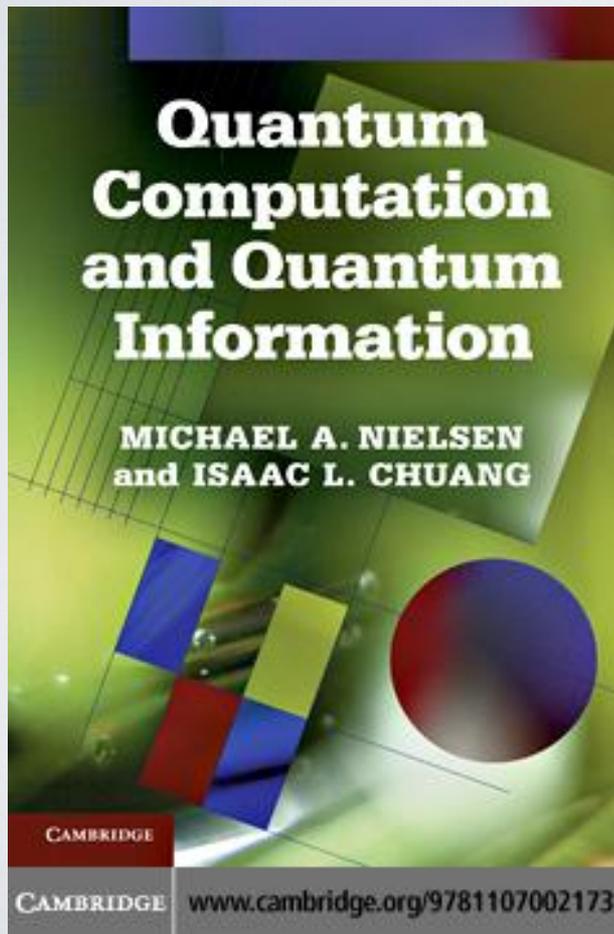
Quantum Computing for Quantum Field Theory



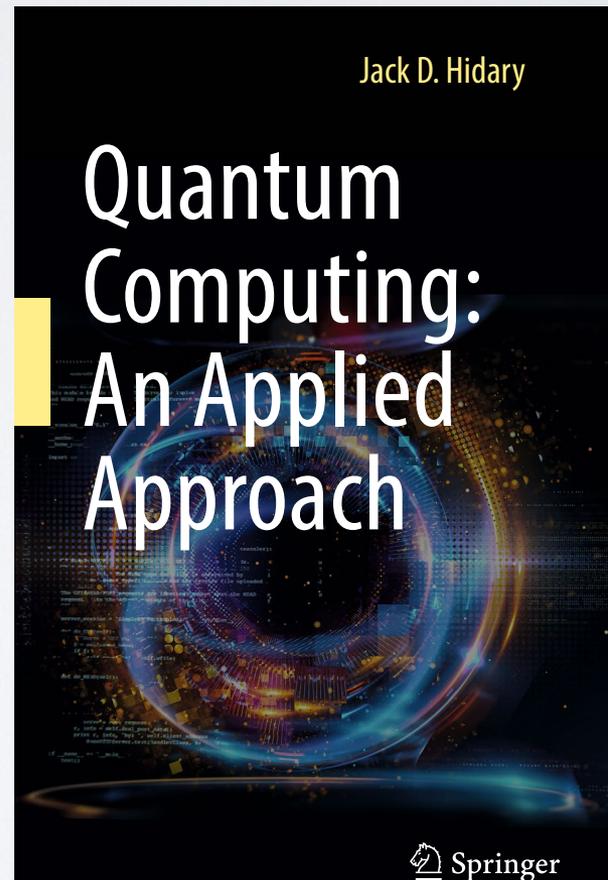
1. What is a Computer?
2. What is Quantum ? (Qubits and all that)
3. Universal vs Efficient/Fault Tol Quantum Computing ? (FFT)
4. What is Quantum Link Lattice Field Theory: Universality?
5. Example for NISQ (Noisy Intermediate Scale Quantum) ?
6. Programming (virtually) the IBM Q ?

GOOD QC REFERENCES

Help you Read this Book!



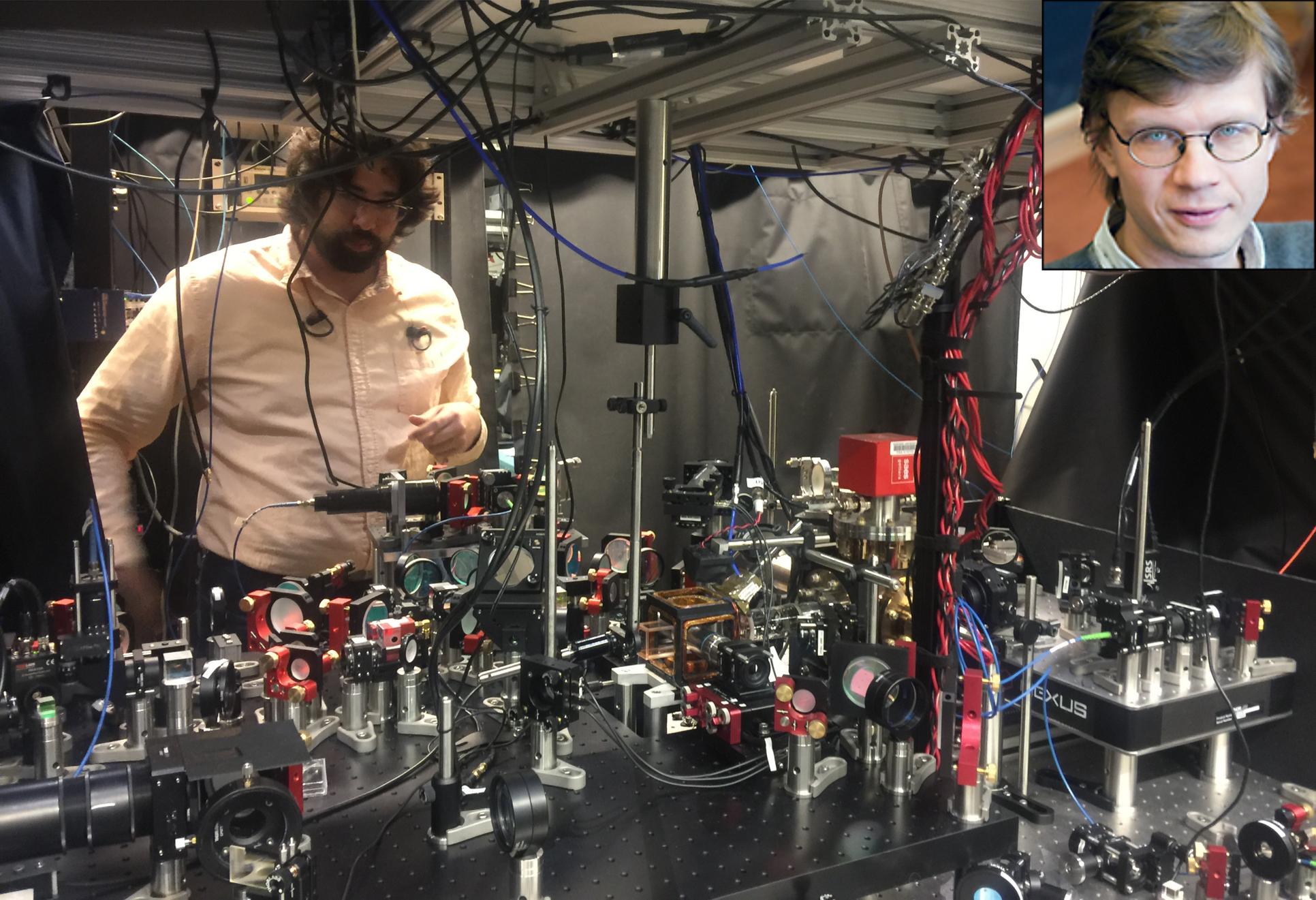
Simple Pedagogical Overview



IBM's new 53-qubit quantum computer is its biggest yet



Mikhail Lukin's Lab Harvard Trapped Ion



TODAY

Oak Ridge National Laboratory's 200 petaflop supercomputer



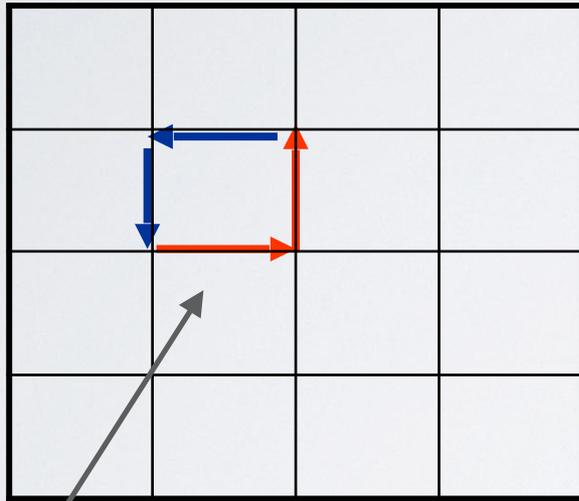
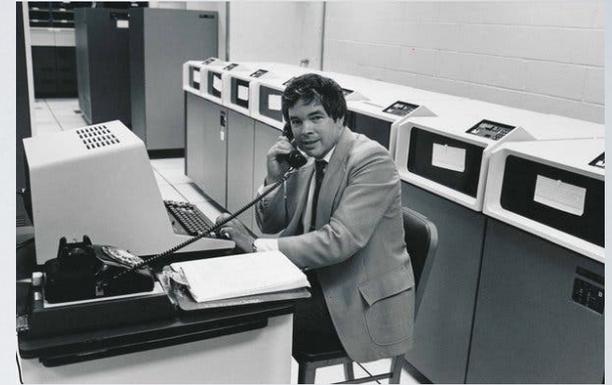
THE COMPETITION!

“Lattice Gauge Theory Machine” 200,000,000,000,000 Floats/sec
9,216 IBM POWER9 CPUs and 27648 NVIDIA GPUs
Each GPU has 5120 Cores and total of 580,608,000,000,000 transistors

WILSON'S LATTICE QCD*

$$Z_{wilson} = \int_{Haar} dU \mathbf{e}^{\frac{6}{g^2} \sum_{\square} Tr[U_{\square} + U_{\square}^{\dagger}]}$$

45 Years Ago



$$U^{ij}(x, x + \mu) = e^{iagA_{\mu}^{ij}(x)}$$

$$i, j = 1, 2, 3$$

SU(3) Gauge Transport on each link.
Exact per site gauge invariance

$$U_{\square_{\mu\nu}}(x) = [U(x, x + \mu)U(x + \mu, x + \mu + \nu)][U(x, x + \nu)U(x + \nu, x + \nu + \mu)]^{\dagger}$$

$$\simeq 1 + a^2 iF_{\mu\nu} - (a^4/2)F_{\mu\nu}^2 + \dots$$

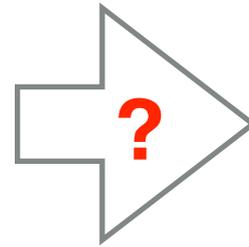
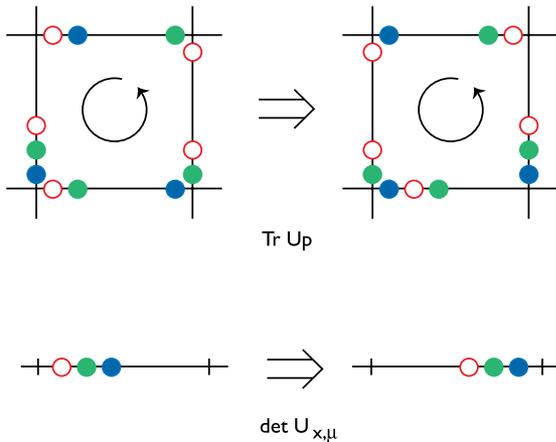
* K.G. Wilson, Phys. Rev. D 10 (1974), 2445.



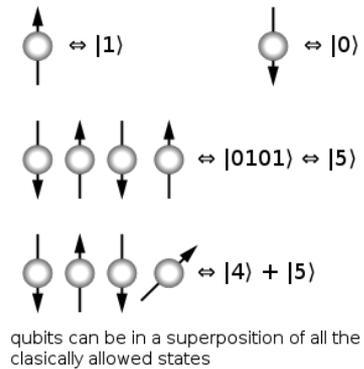
From Bits to Qubits ?



QCD

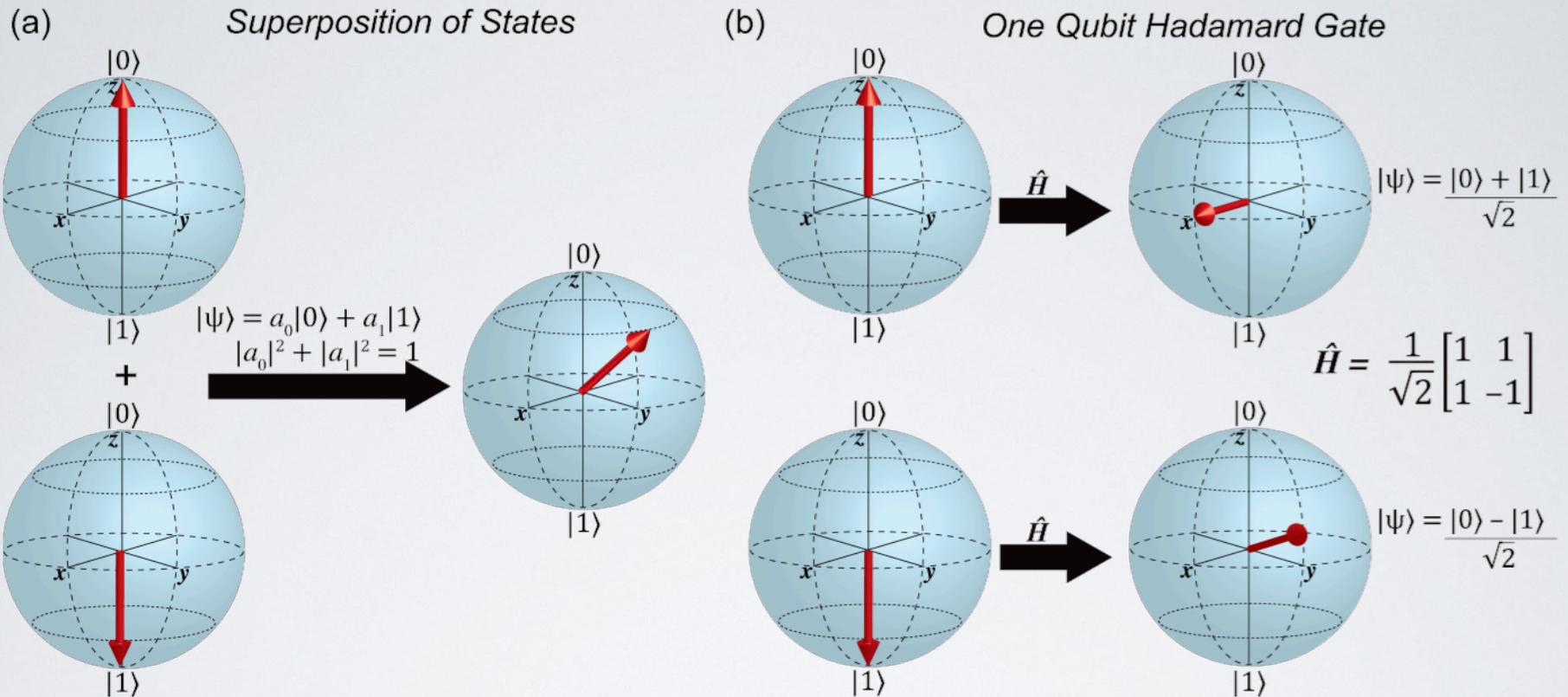


Fermionic Qubit



* See THE QCD ABACUS: A New Formulation for Lattice Gauge Theories R. C. Brower <https://arxiv.org/abs/hep-lat/9711027>
Lecture at "APCTP-ICTP Joint International Conference '97 on Recent Developments in Non-perturbative Method" May, 1997, Seoul, Korea. MIT Preprint CTP 2693.

WHAT IS A QUBIT



$$|\psi_{block}\rangle = e^{i\gamma} \left(\cos \frac{\theta}{2} |0\rangle + ie^{i\phi} \sin \frac{\theta}{2} |1\rangle \right)$$

$$|\psi\rangle = e^{i(\theta/2)\hat{n} \cdot \vec{\sigma}} |0\rangle = (\cos \theta/2 + i\hat{n} \cdot \vec{\sigma} \sin \theta/2) |0\rangle$$

Math Stuff: $U(2) = U(1) \times SU(2)^*$

$$U = e^{i\phi} e^{i(\theta/2)\hat{n} \cdot \vec{\sigma}} = e^{i\phi} [\cos(\theta/2) + i \hat{n} \cdot \vec{\sigma} \sin(\theta/2)]$$

$$U = e^{i\phi} \begin{bmatrix} \cos \theta/2 + i n_z \sin \theta/2 & i(n_x - i n_y) \sin \theta/2 \\ i(n_x + i n_y) \sin \theta/2 & \cos \theta/2 - i n_z \sin \theta/2 \end{bmatrix}$$

THEREFORE:

$$U|0\rangle = e^{i\phi} [\cos(\theta/2) + i n_z \sin(\theta/2)]|0\rangle + (i n_x + n_y) \sin(\theta/2)|1\rangle$$

$$\alpha\alpha^* + \beta\beta^* = 1 \implies \cos^2(\theta) + \hat{n} \cdot \hat{n} \sin^2(\theta) = 1$$

* OR $U = e^{i\phi} [k_0 \sigma_0 + i \vec{k} \cdot \vec{\sigma}] \implies k_\mu k_\mu = 1 \implies S^1 \otimes S^3$

UNIVERSAL GATE SET

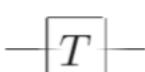
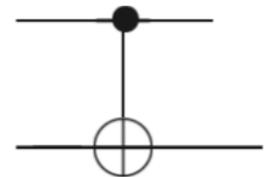
*	Hadamard		$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	
	Pauli-X		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	NOT
	Pauli-Y		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	
	Pauli-Z		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	
	Phase		$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	
*	$\pi/8$		$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$	

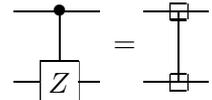
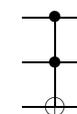
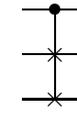
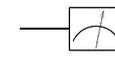
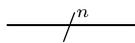
Figure 4.2. Names, symbols, and unitary matrices for the common single qubit gates.

CNOT

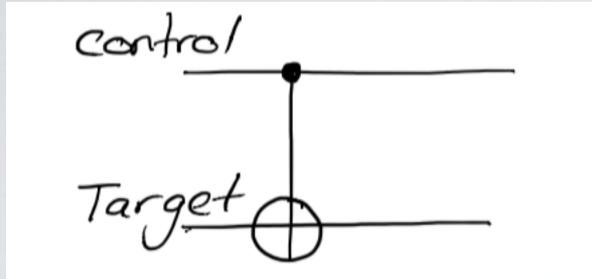
*

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} .$$



controlled-NOT		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
swap		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
controlled-Z		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
controlled-phase		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{bmatrix}$
Toffoli		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$
Fredkin (controlled-swap)		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
measurement		Projection onto $ 0\rangle$ and $ 1\rangle$
qubit		wire carrying a single qubit (time goes left to right)
classical bit		wire carrying a single classical bit
n qubits		wire carrying n qubits

WHAT IS CNOT ? UNITARY (GENERALIZED) XOR



c = 1 (true) activates Not t *

$$|c\rangle \otimes |t\rangle \equiv |ct\rangle$$

XOR / Mod 2 add

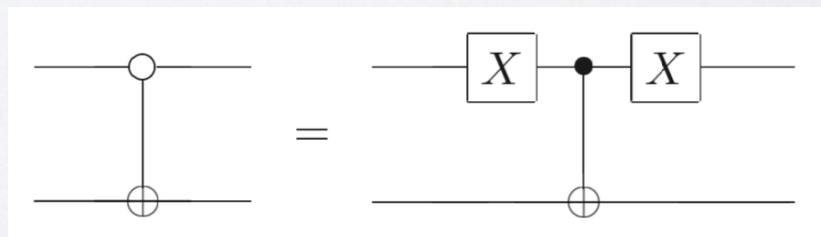


Cnot is $|c\rangle|t\rangle \rightarrow |c\rangle|c \oplus t\rangle$

$$CNOT = \begin{matrix} & 00 & 01 & 10 & 11 \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{matrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = Cnot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

WARNING
NC has a NotCnot with 0



UNIVERSAL QUANTUM COMPUTER

involving only those gates. We now describe three universality constructions for quantum computation. These constructions build upon each other, and culminate in a proof that any unitary operation can be approximated to arbitrary accuracy using Hadamard, phase, CNOT, and $\pi/8$ gates. You may wonder why the phase gate appears in this list, since it can be constructed from two $\pi/8$ gates; it is included because of its natural role in the fault-tolerant constructions described in Chapter 10.

The first construction shows that an arbitrary unitary operator may be expressed *exactly* as a product of unitary operators that each acts non-trivially only on a subspace spanned by two computational basis states. The second construction combines the first construction with the results of the previous section to show that an arbitrary unitary operator may be expressed *exactly* using single qubit and CNOT gates. The third construction combines the second construction with a proof that single qubit operation may be approximated to arbitrary accuracy using the Hadamard, phase, and $\pi/8$ gates. This in turn implies that any unitary operation can be approximated to arbitrary accuracy using Hadamard, phase, CNOT, and $\pi/8$ gates.

Our constructions say little about efficiency – how many (polynomially or exponentially many) gates must be composed in order to create a given unitary transform. In Section 4.5.4 we show that there *exist* unitary transforms which require exponentially many gates to approximate. Of course, the goal of quantum computation is to find interesting families of unitary transformations that *can* be performed efficiently.

Universal Quantum Computing

$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle = e^{-itH} |\Psi(0)\rangle$$

- I. Any $d \times d$ unitary $U(d)$ is product of 2×2 $U(2)$ unitary by GAUSSIAN ELIMINATION!

$$2^n(2^n - 1)/2 = 2^{n-1}(2^n - 1) \quad \text{Qubit Rotations}$$

- II. General $U(2)$ rotation: Gray coding with *cnot's and one Qubit rotation*

$$n^2 \quad \text{cnot gates} + \text{one rotation}$$

- III. But Solovay-Kitaev theorem approx $U(2)$ on single Qubit can be approximated by *Hadamard and $\pi/8$ gates*

$$\text{No of gates} \in O(\log^c(1/\epsilon)) \quad c \simeq 2$$

Gaussian Elimination: Rotate Qubit at a time!

Label: $|s_n\rangle \otimes \cdots \otimes |s_2\rangle \otimes |s_1\rangle$

$$i = 1, \cdots, 2^n \rightarrow 000, 001, 010, 011, 100, 101, 111, \cdots, 2^n - 1$$

$$UX = b \implies u_{ij}x_j = b_i$$

$$u_{11}x_1 + u_{12}x_2 + u_{13}x_3 + u_{14}x_4 + \cdots = b_1$$

$$u_{21}x_1 + u_{22}x_2 + u_{23}x_3 + u_{24}x_4 + \cdots = b_2$$

$$u_{31}x_1 + u_{32}x_2 + u_{33}x_3 + u_{34}x_4 + \cdots = b_3$$

...

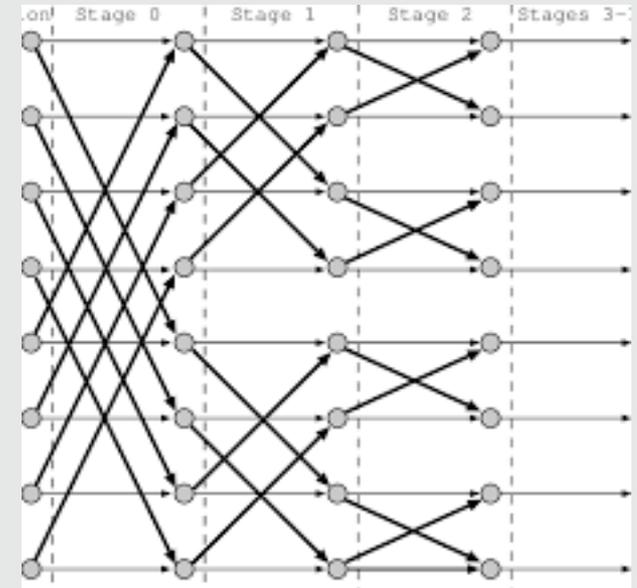
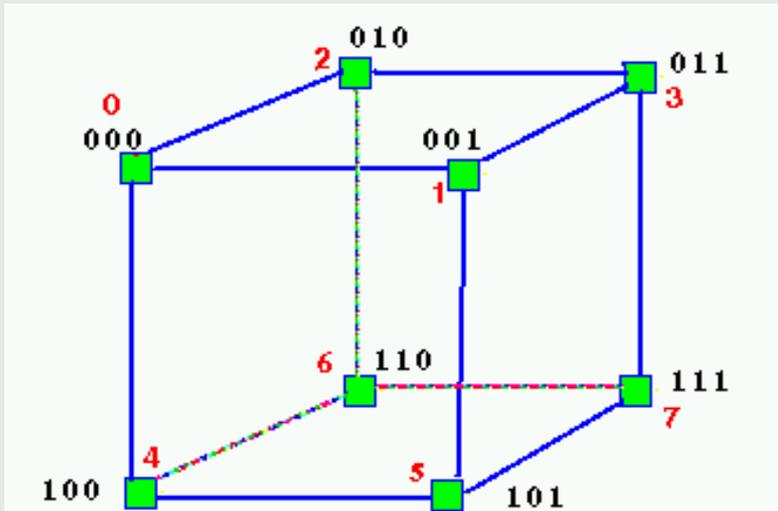
Now multiply $U_1UX = U_1b$ where

$$U_1 = \begin{bmatrix} \alpha & \beta & 0 & 0 & 0 & 0 & \cdots \\ -\beta^* & \alpha^* & 0 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 1 & \cdots \\ \cdots & & & & & \end{bmatrix}$$

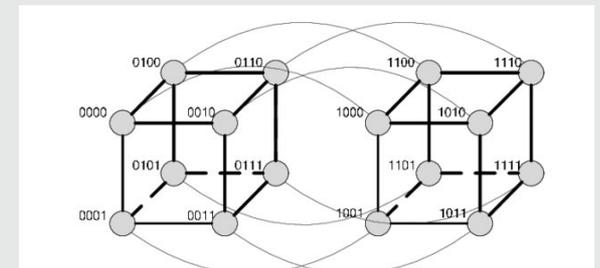
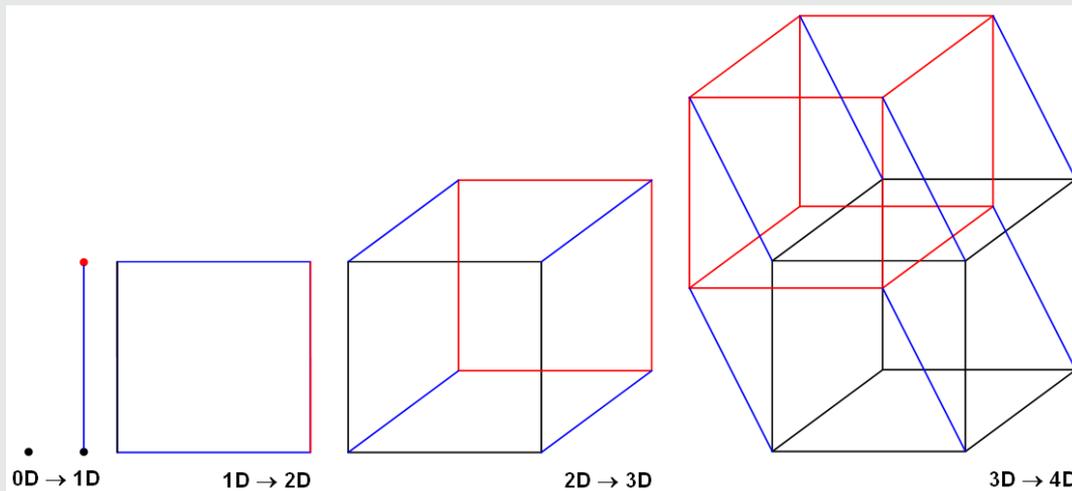
$$\text{with } -\beta^* u_{11} + \alpha^* u_{12} = 0 \implies u_{21} = 0$$

Hypercube - Gray Codes - FFT and all that

- Gray Coding: Adjacent Vertices Differ by one bit



Texi Cab distance = Hamming Distance.

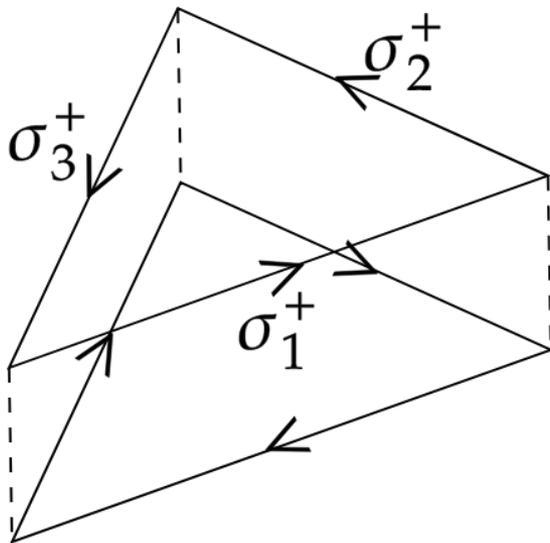


Plaquette term for U(1) is an example

For single triangle:

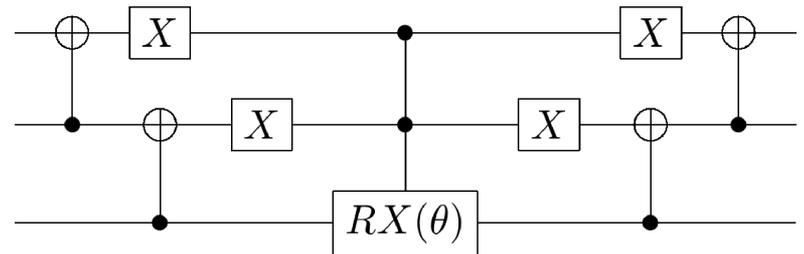
$$\hat{H} = \sum_s \left[\frac{g^2}{2} \sum_{j=1}^3 (\sigma_{j,s}^z + \sigma_{j,s+1}^z)^2 + \frac{\alpha}{2g^2} \sum_{j=1}^3 (\sigma_{j,s}^+ \sigma_{j,s+1}^- + \sigma_{j,s}^- \sigma_{j,s+1}^+) \right]$$

$$- \frac{1}{2g^2} (\sigma_{1,s}^+ \sigma_{2,s}^+ \sigma_{3,s}^+ + \sigma_{1,s}^- \sigma_{2,s}^- \sigma_{3,s}^-)$$



+h.c.

Circuit



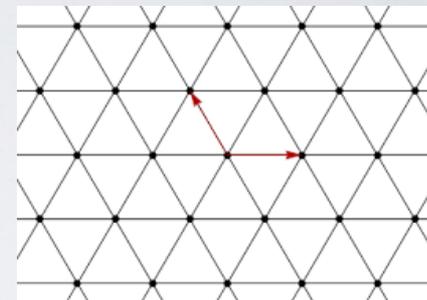
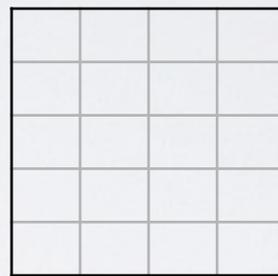
\mathbb{R}

$$e^{-i\hat{H}_B\theta}$$

Universality == Many equivalent LFT

- Different space-time + field discretizations **define exactly** the same continuum quantum field theory

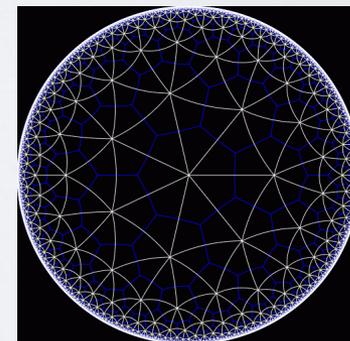
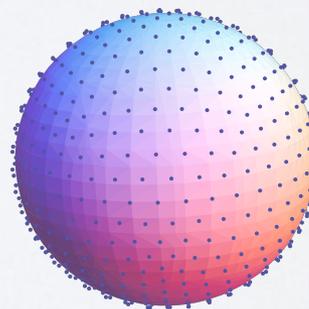
- eg. Lattice different lattice give identical $c = 1/2$ CFT — square, triangle or spherical lattice!



- Fields: Continuum ϕ 4th field and single bit Ising fields are equivalent

$$s \in \pm 1 \iff \phi_x \in \mathbb{R}$$

- Bosonic Sine Gordon Theory = Fermionic Thirring Theory



- $N = 4$ SUSY is dual AdS Gravity

WARM UP: O(3) HEISENBERG SPIN MODEL

Classical O(3) MODEL

Quantum antiferromagnet O(3) MODEL

$$Z = \int dS \exp\left[\frac{1}{g^2} \sum_{\langle x,y \rangle} \vec{S}_x \cdot \vec{S}_y\right] \quad \rightarrow \quad \text{Tr} \left[e^{-\frac{\beta}{g^2} \sum_{x,\mu} \vec{\sigma}_x \cdot \vec{\sigma}_{x+\mu}} \right]$$

Warm up with Fermionic Quantum Operator

$$\vec{S}_x \rightarrow a_i^\dagger(x) \sigma^{ij} a_j(x) \quad \text{or} \quad \hat{S}^{ij}(x) = a_i^\dagger(x) a_j(x)$$

$$\{a_i(x), a_j^\dagger(x)\} = \delta_{ij} \delta_{xy} \quad , \quad \{a_i^\dagger(x) a_j^\dagger(x)\} = 0 \quad , \quad \{a_i(x), a_j(x)\} = 0$$

$$Z = \text{Tr} \exp(-\beta \hat{H}). \quad \hat{H} = \sum_{\langle x,y \rangle} \text{Tr} [\hat{S}_x \hat{S}_y] \quad , \quad \text{Tr} [\hat{S}_x] = 0$$

$$\text{Global Rotation: } \vec{J} = \sum_x \text{Tr} [\vec{\sigma} \hat{S}_x] \implies [\vec{J}, \hat{H}] = 0$$

$$\text{Local Fermion No: } \hat{F}_x = \text{Tr} [\hat{S}_x] \implies [\hat{F}_x, \hat{H}] = 0$$

B.B. Beard and U-J Wiese

<https://arxiv.org/abs/cond-mat/9602164>

FERMIONIC QUBIT GATES

Each Fermion

$$a^\dagger a + a a^\dagger = 1$$

$$a a = a^\dagger a^\dagger = 0$$

On Each qubit

$$a^\dagger(\alpha|1\rangle + \beta|0\rangle) = \beta|1\rangle$$

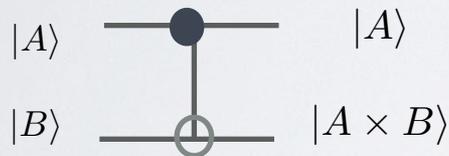
$$a(\alpha|1\rangle + \beta|0\rangle) = \alpha|0\rangle$$

$$a^\dagger + a \implies X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

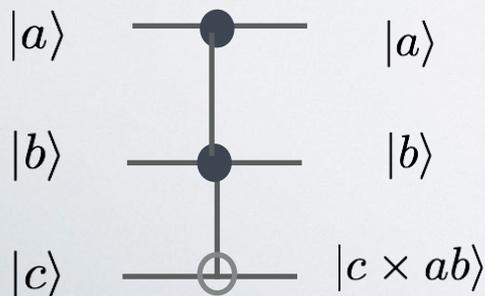
$$(a + a^\dagger)^2 - 1 \implies H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$(a^\dagger + a)/i \implies Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$(2a^\dagger a - 1) \implies Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



$$(1 - a^\dagger a) + a^\dagger a(b + b^\dagger) \implies \text{ContrNOT}$$



$$(1 - a^\dagger a b^\dagger b) + a^\dagger a b^\dagger b(c^\dagger + c) \implies \text{Toffoli}$$

SYMPLECTIC/GAUGE ALGEBRA

$$H = \frac{g^2}{2} \sum_{\langle x, x+\mu \rangle} (Tr[E_L^2(x, \mu)] + Tr[E_R^2(x, \mu)]) - \frac{1}{2g^2} \sum_{\square} Tr[U_{\square} + U_{\square}^{\dagger}]$$

Example: U(1) Abelian P/Q symplectic operators in Q-basis are

$$E = i \frac{d}{d\theta} \quad , \quad U = \exp[i\theta]$$

U(N) generalization of Gauge Algebra is

$$E^{ij} \equiv \lambda_{\alpha}^{ij} E^{\alpha} \quad \Longrightarrow \quad [E^{\alpha}, E^{\beta}] = 2if^{\alpha\beta\gamma} E^{\gamma}$$

$$[E_L, U] = -E_L U \quad , \quad [E_R, U] = -U E_R$$

$$E_R = U^{\dagger} E_L U \quad , \quad [U, U^{\dagger}] = 0 \quad \quad U U^{\dagger} = 1$$

DISCRETE QUANTUM LINK

On each link $(x, x + \mu)$ introduce $2N_c$ complex fermion a_i, a_i^\dagger right(+) moving and b_j, b_j^\dagger left(-) moving fluxon

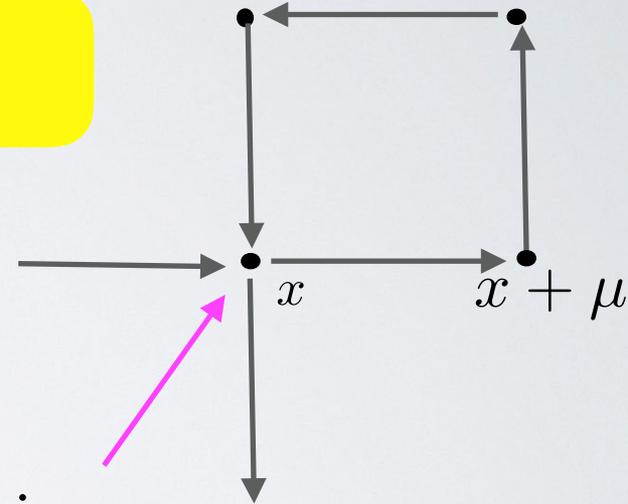
LINK:



$$U_{ij}(x, x + \mu) \rightarrow \hat{U}_{ij} = a_i(x) b_j^\dagger(x + \mu)$$

$$\{a_i, a_j^\dagger\} = \delta_{ij} \quad \{b_i, b_j^\dagger\} = \delta_{ij}$$

Local Gauge Operators $\Omega_{ij}(x) = a_i^\dagger(x) a_j(x) + \dots$



For SU(3) QCD have SU(6) per Link Lie Algebra

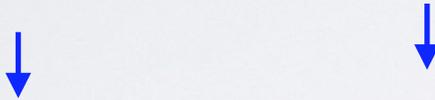
$$\begin{bmatrix} a_i a_j^\dagger & a_i b_j^\dagger \\ b_i a_j^\dagger & b_i b_j^\dagger \end{bmatrix}$$

WHAT ABOUT ANTI-SYMMETRIC FERMIONIC FOCK SPACE?

Step #1: PARA-STATISTICS:

D-Theory only require anti-commutator within each a's and b's set

$$[a_i^\dagger a_j, a_p^\dagger b_q] = [a_i^\dagger a_j, a_p^\dagger] b_q + a_p^\dagger [a_i^\dagger a_j, b_q]$$



$$\Rightarrow [E_L, U] = -E_L U \quad \text{same for } a \rightarrow b$$

Step #2; Jordan-Wigner:

Apply to Locally to each set of 3 a's and b's.

$$a_1^\dagger = \sigma_1^+ \quad , \quad a_2^\dagger = -\sigma_1^z \sigma_2^+ \quad , \quad a_3^\dagger = \sigma_1^z \sigma_2^z \sigma_3^+ \quad \text{same for } a \rightarrow b$$

PARA-STATISTICS & JORDAN-WIGNER TO THE RESCUE

Math Stuff: “Gamma Matrices are Jordan-Wigner”

$$d = 2 : \quad \vec{\gamma}_i^{(2)} \rightarrow (\sigma_1, \sigma_2)$$

$$\gamma_{d+1}^{(2)} \rightarrow \sigma_3 = -i\sigma_1\sigma_2$$

$$d = 4 : \quad \vec{\gamma}_i^{(4)} \rightarrow (\sigma_3 \otimes \vec{\gamma}^{(2)}, \sigma_1 \otimes I_2, \sigma_2 \otimes I_2)$$

$$\gamma_{d+1}^{(4)} \rightarrow \gamma_1\gamma_2\gamma_3\gamma_4 = \gamma_5$$

$$d \leftarrow d + 2 : \quad \vec{\gamma}_i^{(d)} \rightarrow (\sigma_3 \otimes \vec{\gamma}^{(d-2)}, \sigma_1 \otimes I_{2^d}, \sigma_2 \otimes I_{2^d})$$

$$\gamma_{d+1}^{(d)} \rightarrow \gamma_1\gamma_2\gamma_3 \cdots \gamma_d = \gamma_{d+1}$$

KOGUT SUSKIND HAMILTONIAN

$$H = \frac{g^2}{2} \sum_{\langle x, x+\mu \rangle} (Tr[E_L^2(x, \mu)] + Tr[E_R^2(x, \mu)]) - \frac{1}{2g^2} \sum_{\square} Tr[U_{\square} + U_{\square}^{\dagger}]$$

Example: U(1) Abelian P/Q symplectic operators in Q-basis are

$$E = i \frac{d}{d\theta} \quad , \quad U = \exp[i\theta]$$

U(N) generalization of Gauge Algebra is

$$E^{ij} \equiv \lambda_{\alpha}^{ij} E^{\alpha} \quad \Longrightarrow \quad [E^{\alpha}, E^{\beta}] = 2if^{\alpha\beta\gamma} E^{\gamma}$$

$$[E_L, U] = -E_L U \quad , \quad [E_R, U] = -U E_R$$

$$E_R = U^{\dagger} E_L U \quad , \quad [U, U^{\dagger}] = 0 \quad \quad U U^{\dagger} = 1$$

STARTING TO TEST REAL TIME QUBIT ALGORITHM FOR U(1) QUANTUM LINK GAUGE THEORY

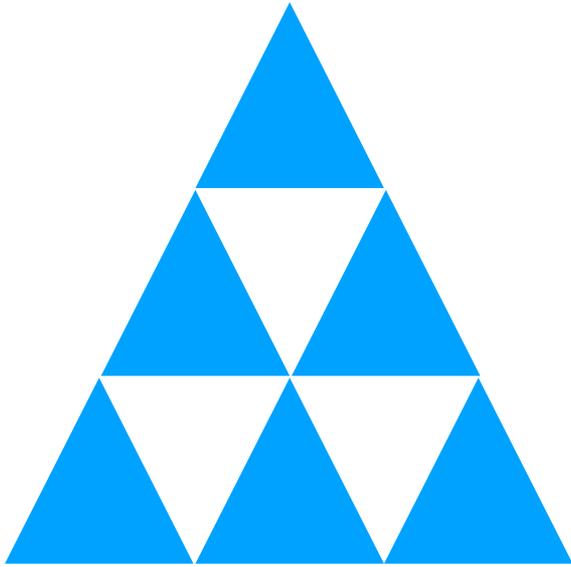
See Gauge Theory for a Quantum Computer

R. C. Brower, D. Berenstein & H. Kawai (Lattice 2020)

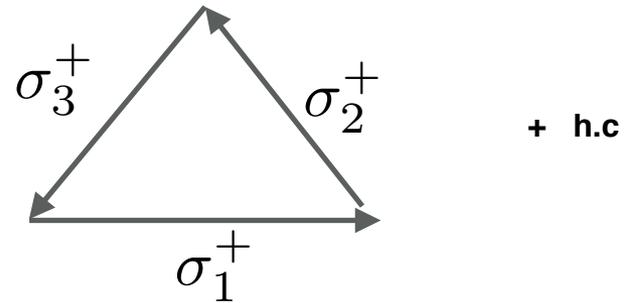
$$\hat{H} = \frac{e^2}{2} \sum_{links, s} (\sigma_s^z + \sigma_{s+1}^z)^2 + \alpha \sum_{links, s} [\sigma_s^+ \otimes \sigma_{s+1}^- + \sigma_s^- \otimes \sigma_{s+1}^+] \\ - \frac{1}{2e^2} \sum_{\Delta, s} [\sigma_s^+ \otimes \sigma_s^+ \otimes \sigma_s^+ + \sigma_s^- \otimes \sigma_s^- \otimes \sigma_s^-]$$

Few very simple kernels in Trotter factorization into
Gauge invariant Unitary operators with very few Qubit width

Choose 2 + 1 on U(1) Hamiltonian on a Triangular spacial lattice



To evaluate lattice: alternate between two coloring of triangles.



Total: about 15-20 consecutive gate operations (coherence time) per qubit per Trotter step

Estimate of current machines: 3 Trotter steps on Lattice

Extra dimension builds local field rep. from XYZ ferromagnetic chain

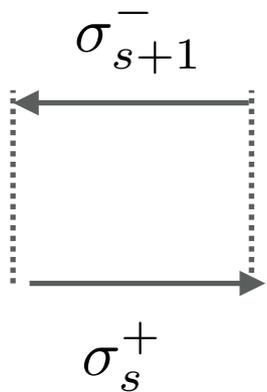
Using

$$\sigma^+ \otimes \sigma^- + \sigma^- \otimes \sigma^+ = \frac{1}{2}(\sigma^x \otimes \sigma^x + \sigma^y \otimes \sigma^y)$$

$$(\sigma^z \otimes 1 + 1 \otimes \sigma^z)^2 = 2(1 + \sigma^z \otimes \sigma^z)$$

The each links between two triangle are coupled by 2 Qubit ferromagnetic interaction operator to align them.

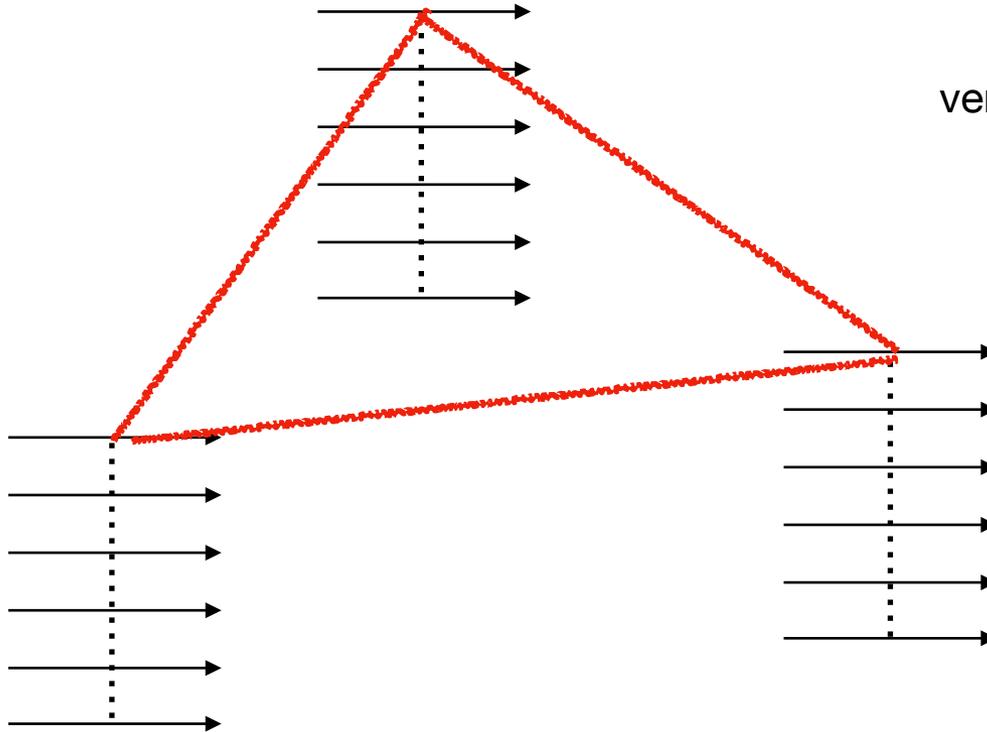
$$H_{align} \simeq -\alpha_{align} \sum (\sigma^x \otimes \sigma^x + \sigma^y \otimes \sigma^y) - \beta_{align} \sum \sigma^z \otimes \sigma^z$$



XY coupling of gauge fixed extra dimension squares!

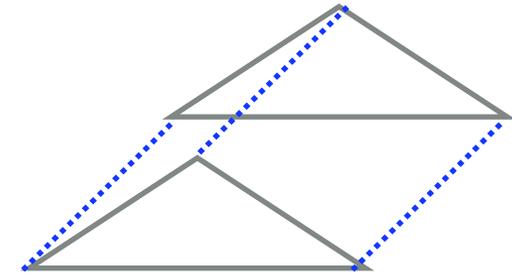
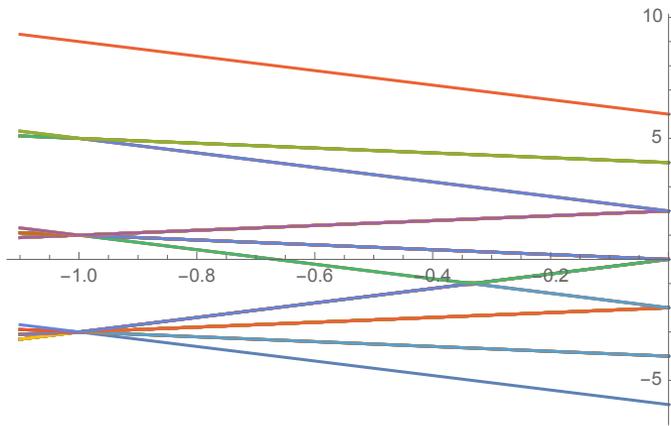
E^2 coupling term

Plaquette

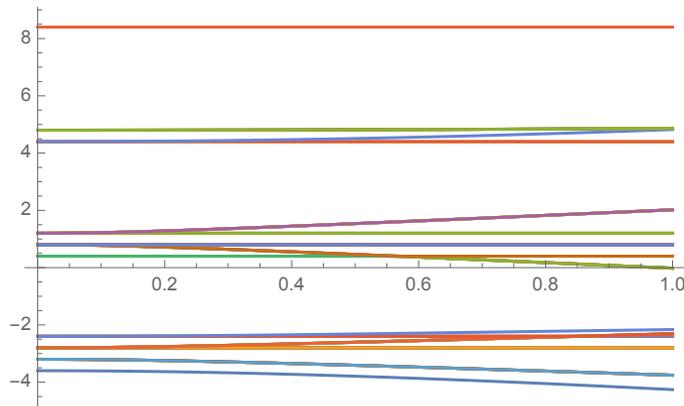


Still gauge invariant if broken vertically: few operations per qubit.
(Not all to all)

Parameter fitting Two Triangle couple Hamiltonian needing 6 Qubits & eigenvalues of 64x64 matrices



Just XXZ piece: need to
avoid level crossings,
close to XXX is better



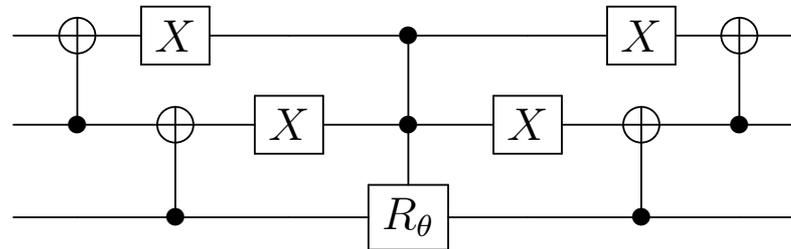
Together with plaquette operator:
Gap persists: suggests simulation will
not be too polluted by UV

$$\mathcal{H}_\Delta \propto \sigma^+ \otimes \sigma^+ \otimes \sigma^+ + \sigma^- \otimes \sigma^- \otimes \sigma^-$$

$$U_\Delta(t) = \exp(-itH_\Delta)$$

This is a rotation on a 2-plane of 8 dimensional Hilbert space (+++) rotating into (- - -).

Can be written in terms of a double control gate after some bit flips which need to be undone



Depending on details of architecture: it can take anywhere between 5 computing cycles and 20 (depth).

For experts: May be done efficiently with ancillas if Toffoli gates available.

Now we have the spin model

For single triangle:

$$\hat{H} = \sum_s \left[\overset{\hat{H}_E}{\frac{g^2}{2} \sum_{j=1}^3 (\sigma_{j,s}^z + \sigma_{j,s+1}^z)^2} + \overset{\hat{H}_B}{\frac{\alpha}{2g^2} \sum_{j=1}^3 (\sigma_{j,s}^+ \sigma_{j,s+1}^- + \sigma_{j,s}^- \sigma_{j,s+1}^+)} - \frac{1}{2g^2} (\sigma_{1,s}^+ \sigma_{2,s}^+ \sigma_{3,s}^+ + \sigma_{1,s}^- \sigma_{2,s}^- \sigma_{3,s}^-) \right] \overset{\hat{H}_{XY}}{}$$

Now we have the spin model

For single triangle:

$$\hat{H} = \sum_s \left[\frac{g^2}{2} \sum_{j=1}^3 (\sigma_{j,s}^z + \sigma_{j,s+1}^z)^2 + \frac{\alpha}{2g^2} \sum_{j=1}^3 (\sigma_{j,s}^+ \sigma_{j,s+1}^- + \sigma_{j,s}^- \sigma_{j,s+1}^+) \right. \\ \left. - \frac{1}{2g^2} (\sigma_{1,s}^+ \sigma_{2,s}^+ \sigma_{3,s}^+ + \sigma_{1,s}^- \sigma_{2,s}^- \sigma_{3,s}^-) \right]$$

We are interested in time evolution:

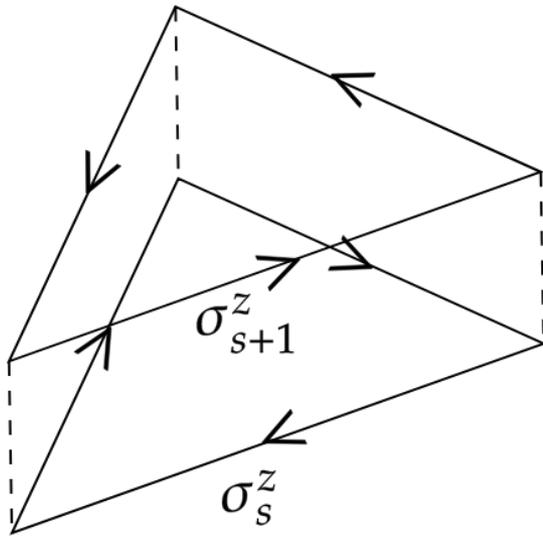
$$|\psi(t)\rangle = e^{-i\hat{H}t} |\psi(0)\rangle \\ \approx \left(e^{-i\hat{H}_E \frac{t}{n}} e^{-i\hat{H}_{XY} \frac{t}{n}} e^{-i\hat{H}_B \frac{t}{n}} \right)^n |\psi(0)\rangle$$

with t/n small enough.

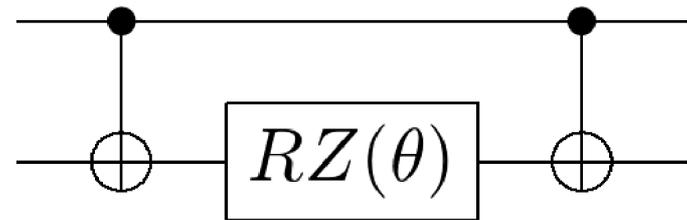
Electric term:

For single triangle:

$$\hat{H} = \sum_s \left[\frac{g^2}{2} \sum_{j=1}^3 (\sigma_{j,s}^z + \sigma_{j,s+1}^z)^2 + \frac{\alpha}{2g^2} \sum_{j=1}^3 (\sigma_{j,s}^+ \sigma_{j,s+1}^- + \sigma_{j,s}^- \sigma_{j,s+1}^+) - \frac{1}{2g^2} (\sigma_{1,s}^+ \sigma_{2,s}^+ \sigma_{3,s}^+ + \sigma_{1,s}^- \sigma_{2,s}^- \sigma_{3,s}^-) \right]$$



Circuit



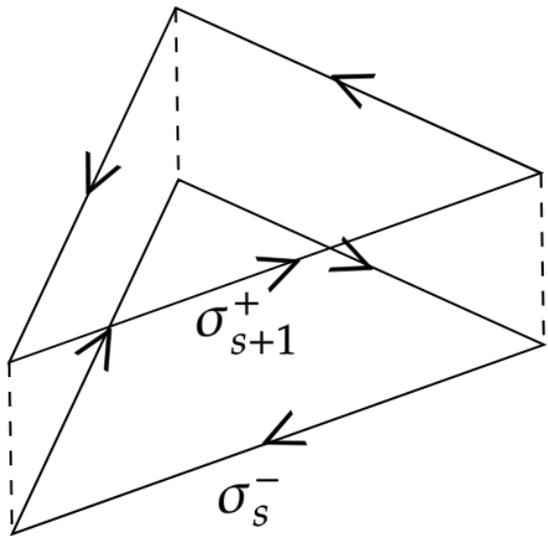
\mathbb{R}

$$e^{-i\hat{H}_E\theta}$$

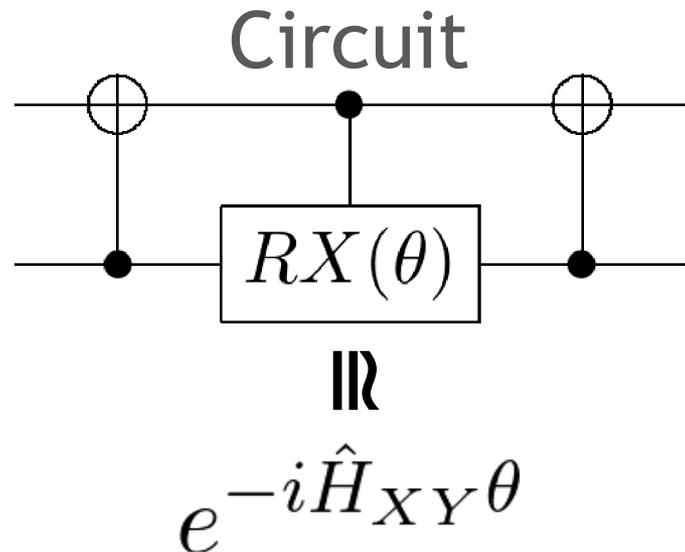
Coupling term:

For single triangle:

$$\hat{H} = \sum_s \left[\frac{g^2}{2} \sum_{j=1}^3 (\sigma_{j,s}^z + \sigma_{j,s+1}^z)^2 + \frac{\alpha}{2g^2} \sum_{j=1}^3 (\sigma_{j,s}^+ \sigma_{j,s+1}^- + \sigma_{j,s}^- \sigma_{j,s+1}^+) \right. \\ \left. - \frac{1}{2g^2} (\sigma_{1,s}^+ \sigma_{2,s}^+ \sigma_{3,s}^+ + \sigma_{1,s}^- \sigma_{2,s}^- \sigma_{3,s}^-) \right]$$



+h.c.

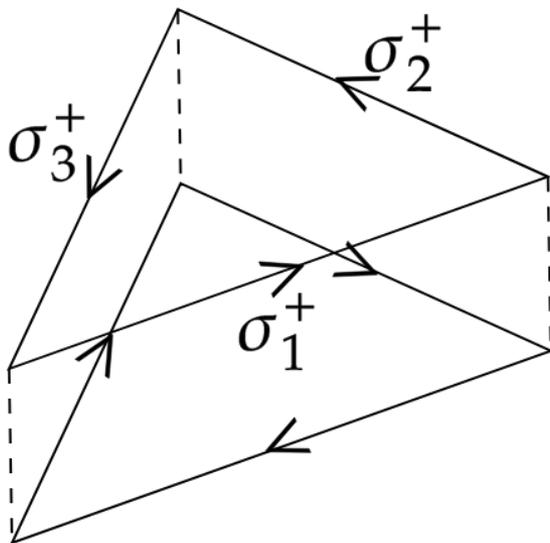


Plaquette term:

For single triangle:

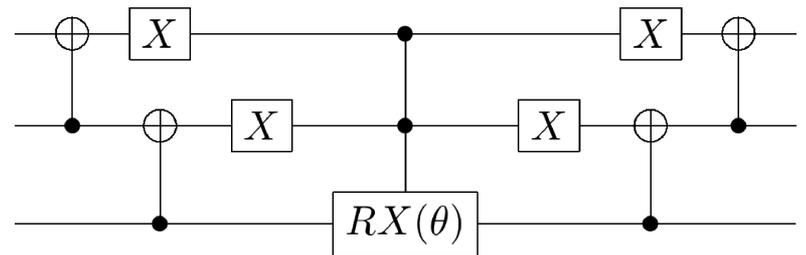
$$\hat{H} = \sum_s \left[\frac{g^2}{2} \sum_{j=1}^3 (\sigma_{j,s}^z + \sigma_{j,s+1}^z)^2 + \frac{\alpha}{2g^2} \sum_{j=1}^3 (\sigma_{j,s}^+ \sigma_{j,s+1}^- + \sigma_{j,s}^- \sigma_{j,s+1}^+) \right]$$

$$- \frac{1}{2g^2} (\sigma_{1,s}^+ \sigma_{2,s}^+ \sigma_{3,s}^+ + \sigma_{1,s}^- \sigma_{2,s}^- \sigma_{3,s}^-)$$



+h.c.

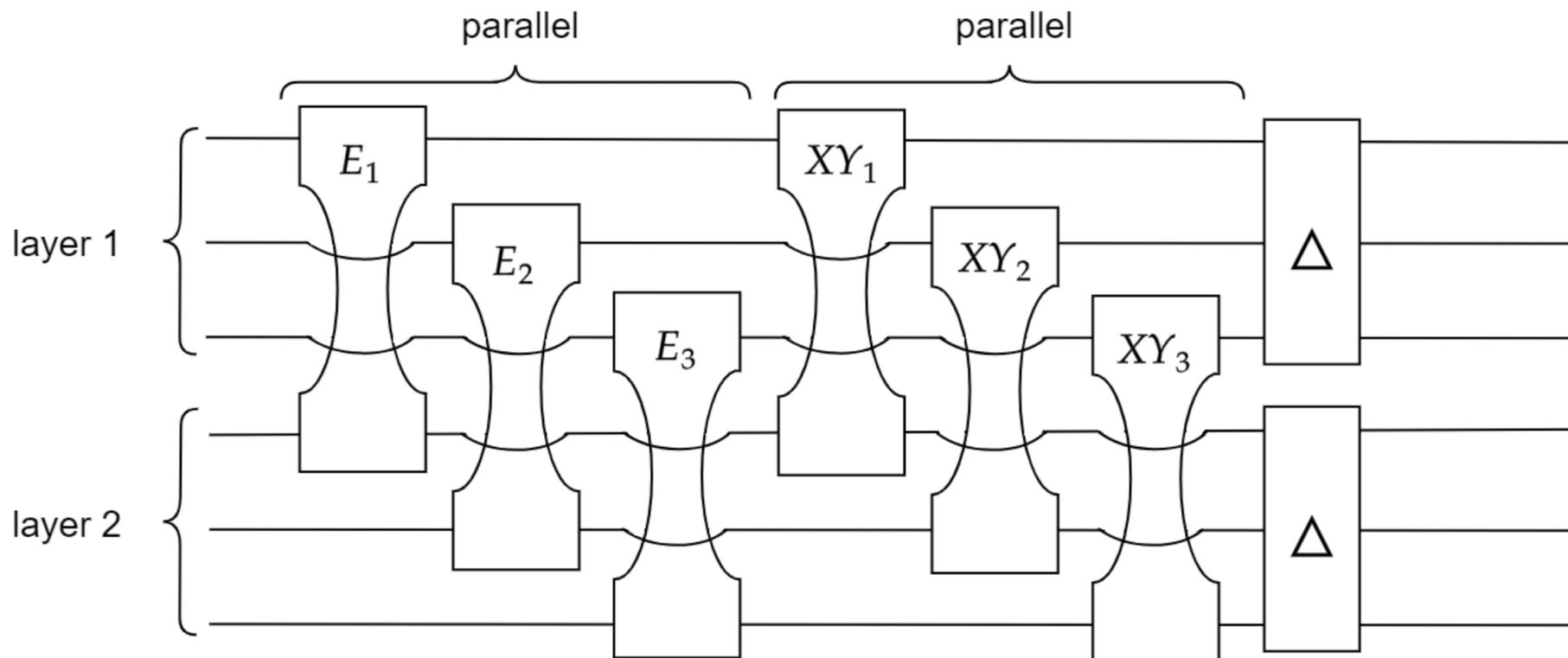
Circuit



\mathbb{R}

$$e^{-i\hat{H}_B\theta}$$

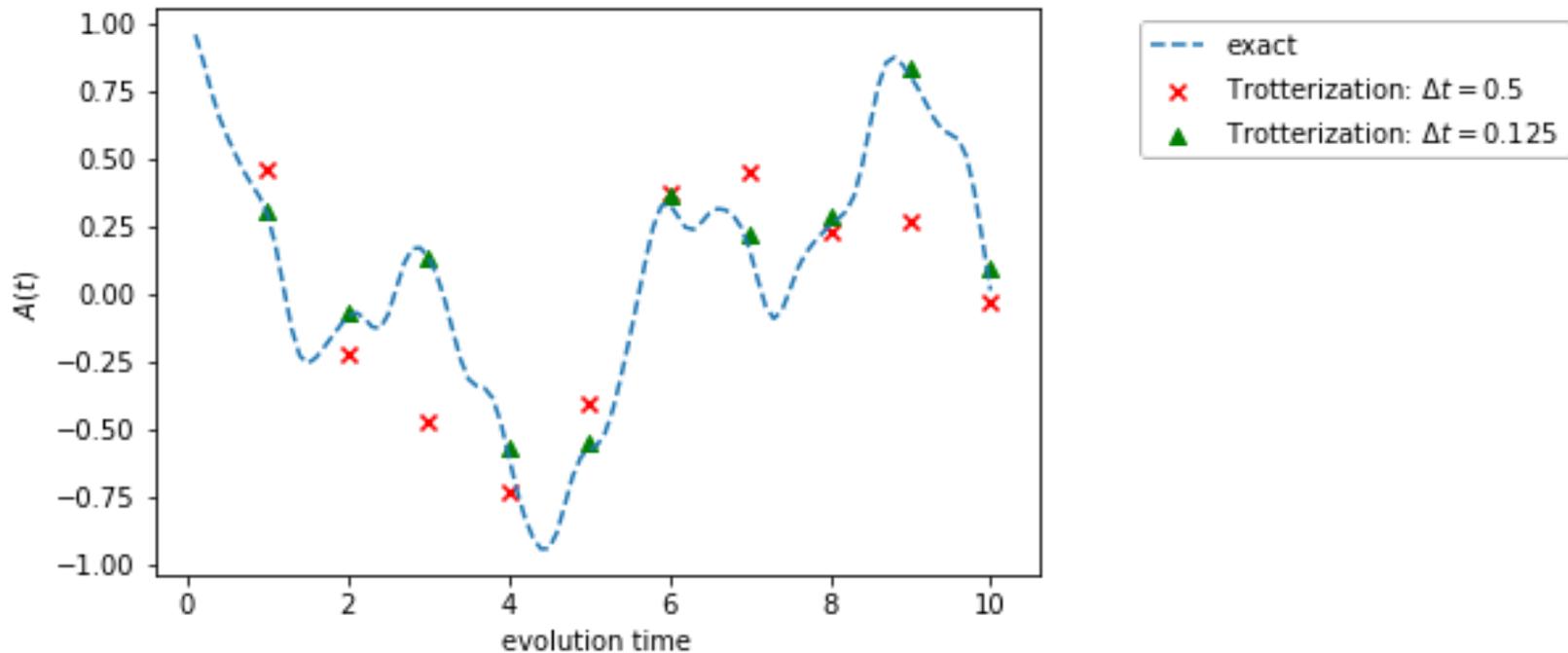
Overall one Trotter step looks like:



50 “physical” qubits per step.

Trotterization

$$U(t) = \left(e^{-i\hat{H}_E \frac{t}{n}} e^{-i\hat{H}_{XY} \frac{t}{n}} e^{-i\hat{H}_B \frac{t}{n}} \right)^n$$

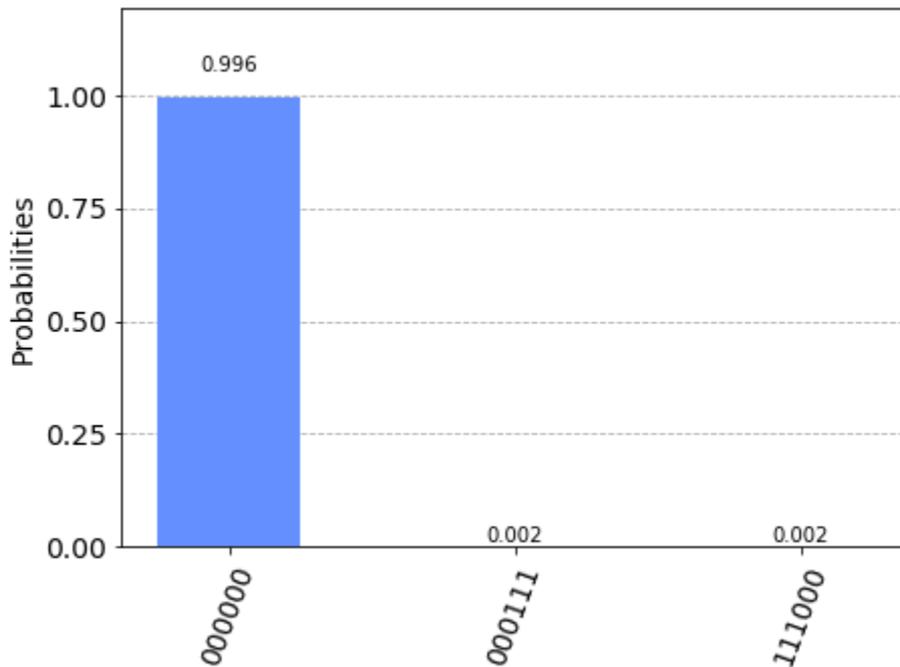


Computation of $\langle 0 | U(t) | 0 \rangle$ (exact).
Around $t/n \approx 0.1$ for useful simulation.

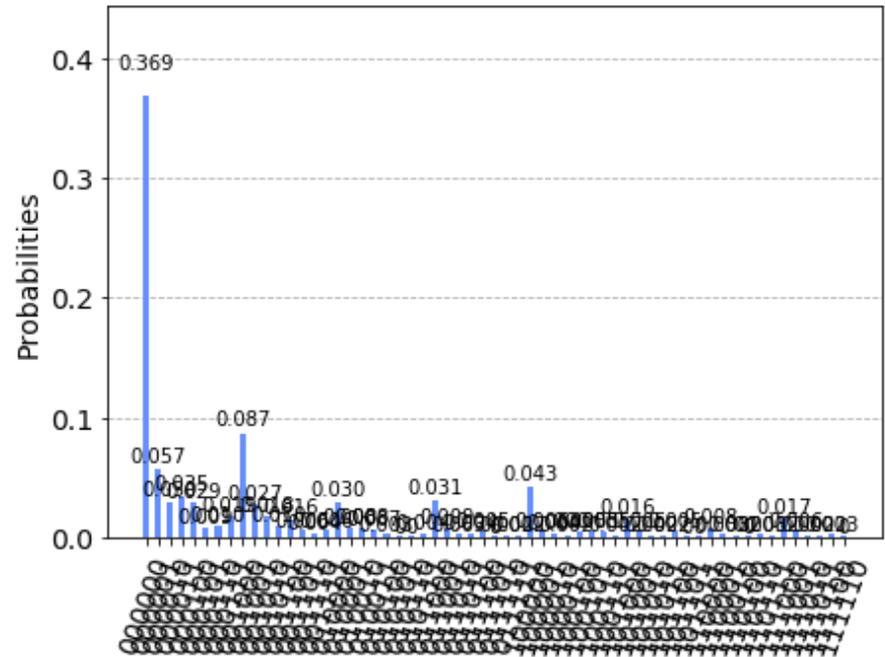
On the actual IBM Q machine

Again, $U(t) = \left(e^{-i\hat{H}_E \frac{t}{n}} e^{-i\hat{H}_{XY} \frac{t}{n}} e^{-i\hat{H}_B \frac{t}{n}} \right)^n$ (t=0.1, n=1)

The distribution $\rho(t) = \text{diag}(U(t) |0\rangle \langle 0| U(t)^\dagger)$



Ideal

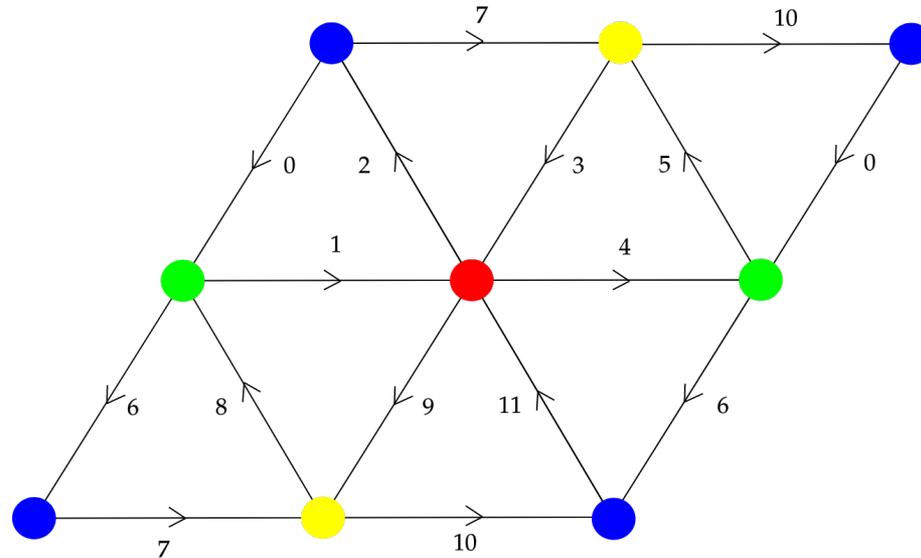


IBM Q

What I have to work on from now

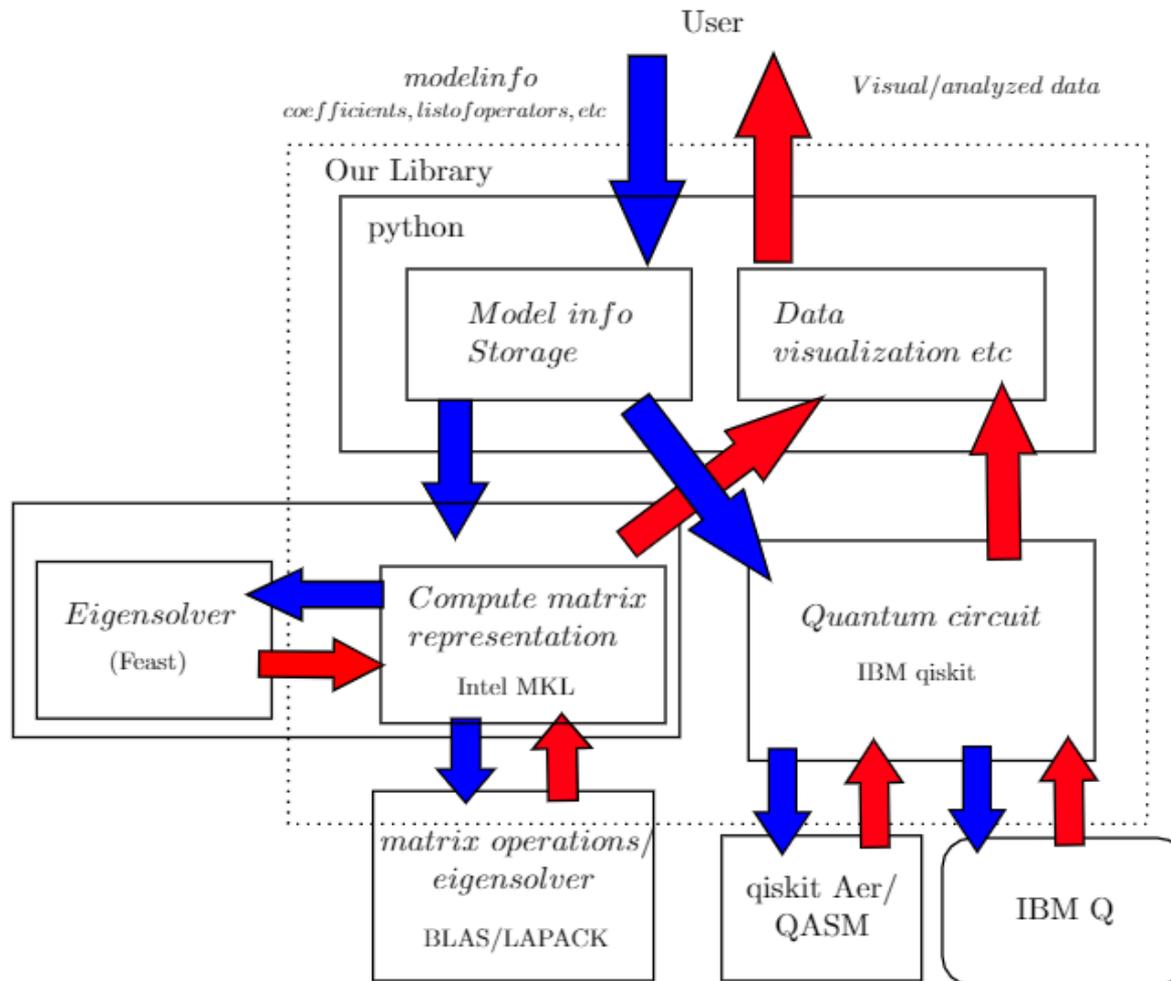
Theory side:

- What if we extend the model to the spacial direction?



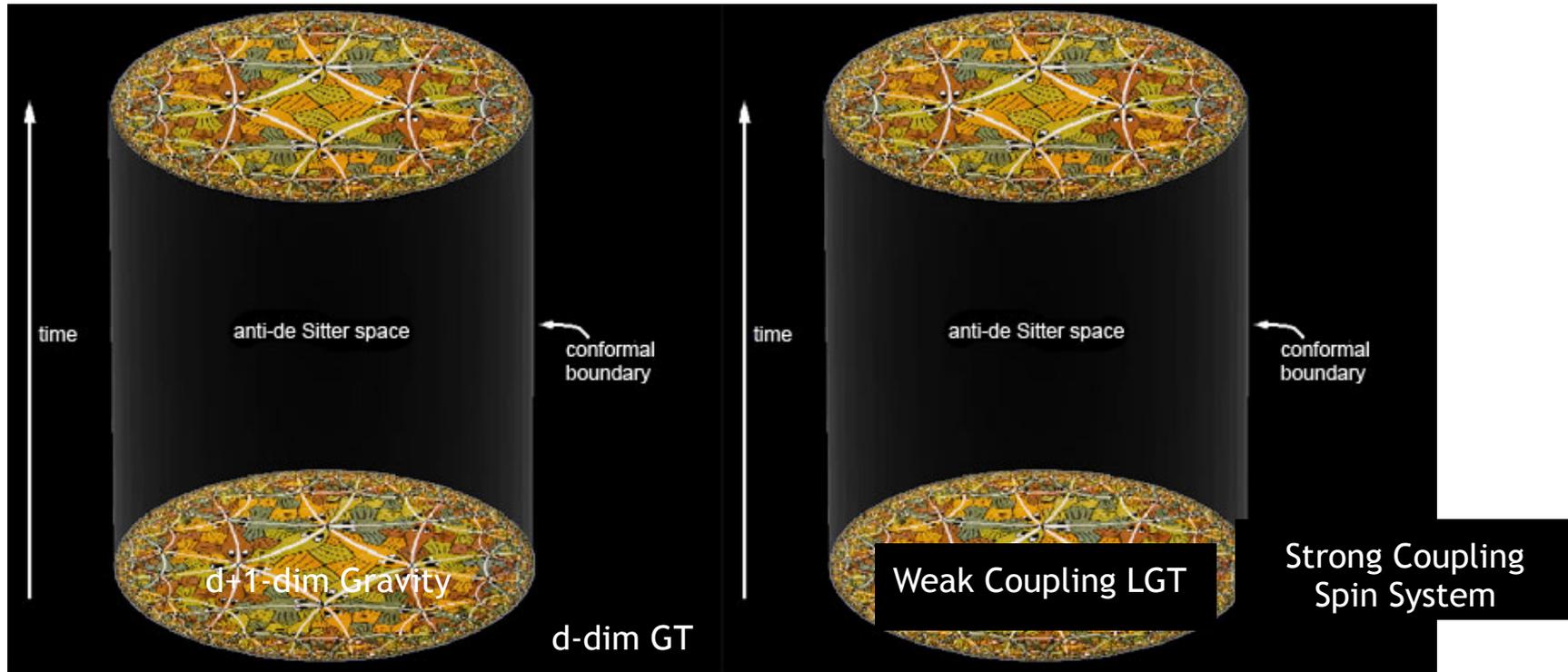
$Z_2 \times Z_2$ lattice equivalent to $C_2 \times C_2$ torus with PBC (i.e. quotient mapping)

The architecture of the library



IBM-Q Links for Excellent Python Tutorials, compiler and simulators

Also Developing Quantum Computing for Hyperbolic Lattice Hamiltonian using triangle group to do strong coupling AdS/CFT or Gravity/Gauge Duality in Minkowski



See **Lattice Setup for Quantum Field Theory in AdS2**
[Richard C. Brower](#), [Cameron V. Coghurn](#), [A. Liam Fitzpatrick](#), [Dean Howarth](#),
[Chung-I Tan](#) <https://arxiv.org/abs/1912.07606>

Questions?