Error correction schemes for fully correlated quantum channels

Chi-Kwong Li
Department of Mathematics, The College of William and Mary.
Institute for Quantum Computing, University of Waterloo.

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- A quantum channel (operation) $\mathcal{E} : M_n \to M_n$ is a trace preserving completely positive map with the following operator sum representation

$$\mathcal{E}(\rho) = E_1 \rho E_1^\dagger + \cdots + E_r \rho E_r^\dagger,$$

for some $E_1, \ldots, E_r \in M_n$ such that $E_1^\dagger E_1 + \cdots + E_r^\dagger E_r = I_n$.

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- The matrices $E_1, \ldots, E_r$ are the error operators of the channel.

Quantum error correction code

- A subspace $\mathbf{V} \subseteq \mathbb{C}^n$ is a quantum error correction code for $\mathcal{E}$ if there is a recovery channel $\mathcal{R} : M_n \to M_n$ such that

$$\mathcal{R} \circ \mathcal{E}(\rho) = \rho \qquad \text{whenever} \quad \mathrm{range}(\rho) \subseteq \mathbf{V}.$$

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- So, we want:

$\rho \to$ [Encoding ρ as $\hat{\rho}$] $\to$ [Noisy quantum channel $\mathcal{E}$] $\to$ [Decoding $\mathcal{E}(\hat{\rho})$] $\to \rho$.

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Knill-Laflamme condition

There is an error correction code **V** of dimension k for $\mathcal{E}$ if and only if there is a unitary $U \in M_n$ such that

$$U^\dagger E_i^\dagger E_j U = \begin{bmatrix} \gamma_{ij} I_k & * \\ * & * \end{bmatrix} \quad \text{with } \gamma_{ij} \in \mathbb{C} \ \text{ for all } i,j \in \{1,\dots,r\},$$

and **V** will be spanned by the first k columns of U.

Fully correlated channels

Denote the Pauli's matrices by

$$\sigma_0 = I_2, \ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

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A fully correlated channel $\mathcal{E}$ on n-qubit states are defined by

$$\mathcal{E}(\rho) = p_0(\sigma_0)^{\otimes n}\rho(\sigma_0^\dagger)^{\otimes n} + \cdots + p_3(\sigma_3)^{\otimes n}\rho(\sigma_3^\dagger)^{\otimes n} \quad \text{for any } \rho \in M_{2^n},$$

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Note that every $\rho \in D_{2^n}$ is a linear combination of the product state $\rho_1 \otimes \cdots \otimes \rho_n$ with $\rho_1, \ldots, \rho_n \in D_2$, and

$$\mathcal{E}(\rho_1 \otimes \cdots \otimes \rho_n) = \sum_{j=0}^3 p_j(\sigma_j\rho_1\sigma_j^\dagger) \otimes \cdots \otimes (\sigma_j\rho_n\sigma_j^\dagger).$$

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- If n is even, there is a quantum error correction code of dimension 2^{n-2}.

Implementation

Denote by $X_n = \sigma_1^{\otimes n}, Y_n = \sigma_2^{\otimes n}, Z_n = \sigma_3^{\otimes n}$.

When n is odd, encode $\rho \in D_{2^{n-1}}$ as $\hat{\rho} = U^\dagger(\sigma \otimes \rho)U$ with $\sigma = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

When n is even, encode $\rho \in D_{2^{n-2}}$ as $\hat{\rho} = U^\dagger(\sigma \otimes \rho)U$ with $\sigma = E_{11} \in D_4$.

In both cases, we have $$U\mathcal{E}(\hat{\rho})U^\dagger = \hat{\sigma} \otimes \rho.$$

The circuit diagram:

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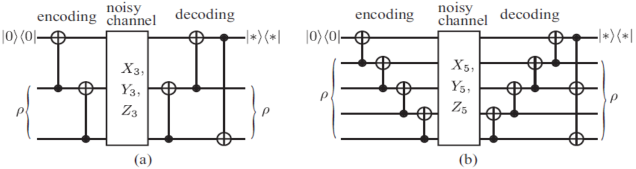

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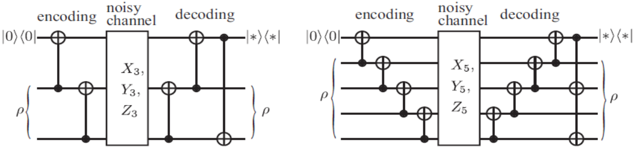

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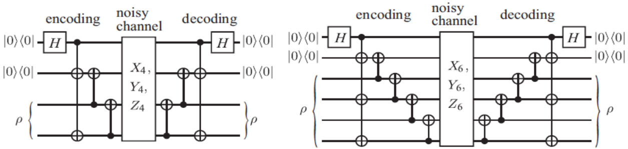

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1. If $n = 2k + r$ with $r \in \{1, 2\}$, one can use r arbitary qubit to protect $2k$ qubits of data.
2. If $n = 2k + 2$, one can use two pure states to transmit two classical bit of information in $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ and to protect $2k$ qubits of quantum information.

Li, Lyles, Poon, Error correction schemes for fully correlated quantum channels protecting both quantum and classical information, 18 pages, Quantum Information Processing. https://arxiv.org/pdf/1905.10228.pdf

Implementation

The scheme was implemented using Matlab, Mathematica, Python, and the IBM's quantum computing framework `qiskit`.

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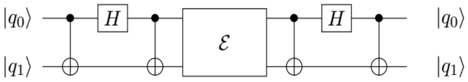
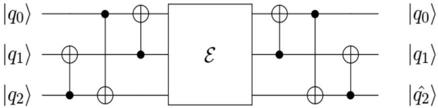
For $n = 2$, if $|q_1 q_0\rangle \in \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, then circuit diagram will be:



For $n = 3$,



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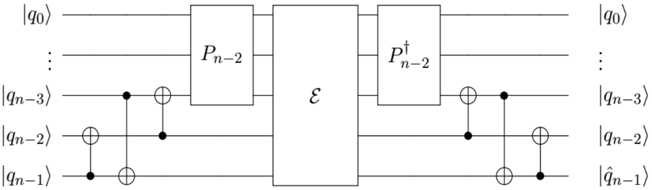
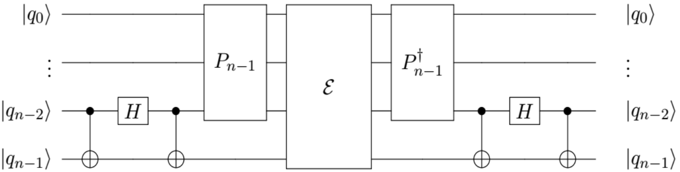


For even n, if $|q_{n-1}q_{n-2}\rangle \in \{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}$, then the circuit diagram will look like:



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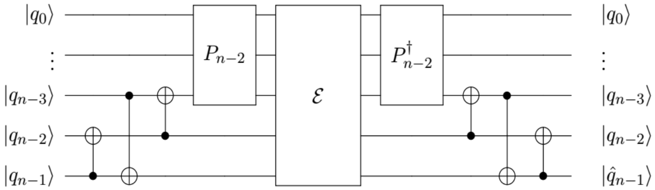

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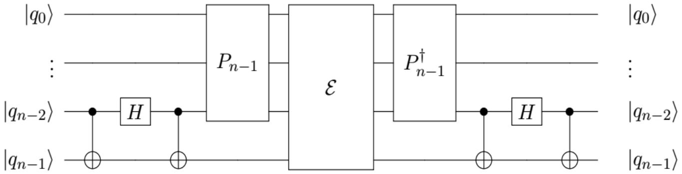


Note that our scheme is good for multiple times of quantum error correction without syndrome measurement.

Experimental results

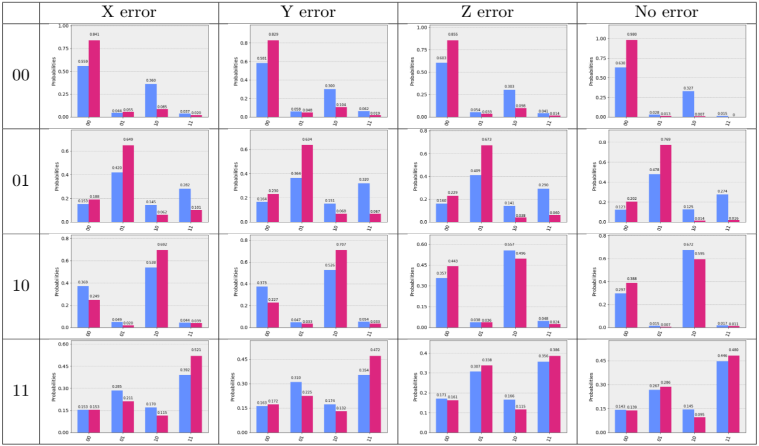


Table 1: Inputs and Errors on sigma = 0, Legend: Tenerife (pink) and Yorktown (blue)

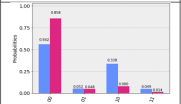
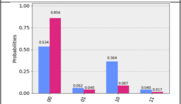
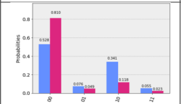
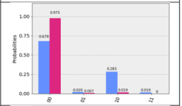
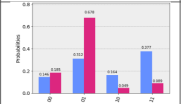
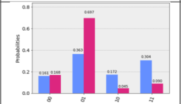
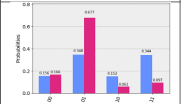
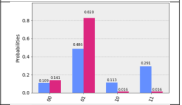
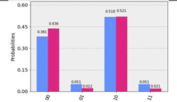
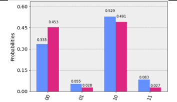
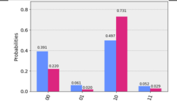
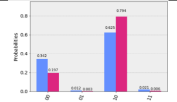
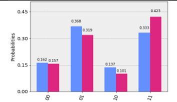
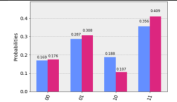
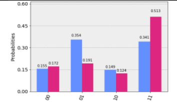
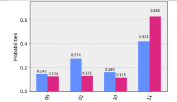
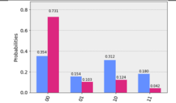
	X error	Y error	Z error	No error
00				
01				
10				
11				

Table 2: Inputs and Errors on sigma = 1, Legend: Tenerife (pink) and Yorktown (blue)

<table>
<tr><th></th><th>X error</th><th>Y error</th><th>Z error</th><th>No error</th></tr>
<tr><td>00</td>
<td>
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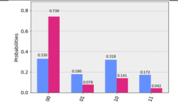
<td>
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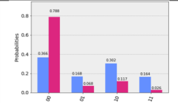
<td>
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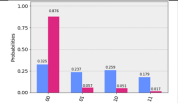
<td>
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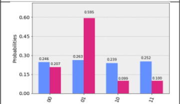
<td>
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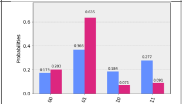
<td>
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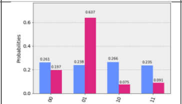
<td>
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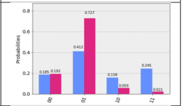
<td>
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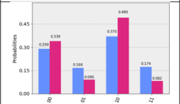
<td>
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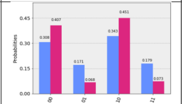
<td>
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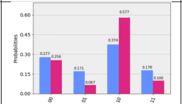
<td>
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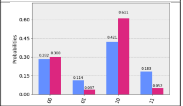
<td>
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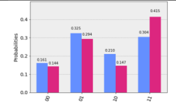
<td>
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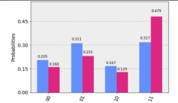
<td>
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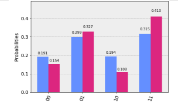
<td>
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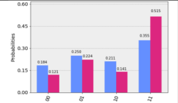
<td>
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Table 3: Inputs and Errors on random sigma, Legend: Tenerife (pink) and Yorktown (blue)

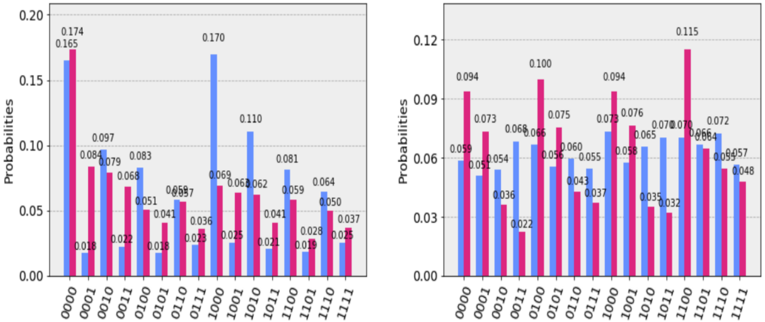

Figure 1: QECC on 4 and 5 qubits

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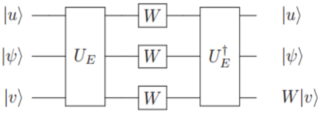

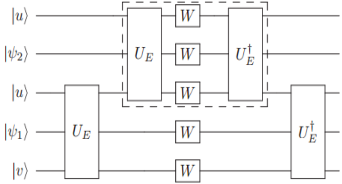

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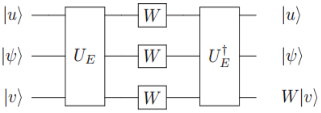

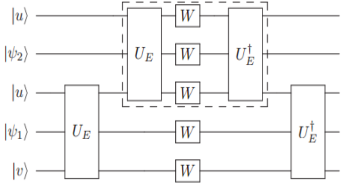

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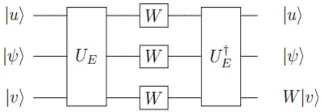

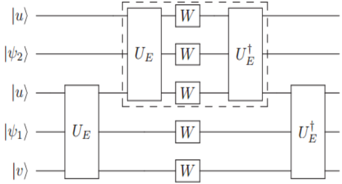

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Thank you for your attention!