Quantum Simulation

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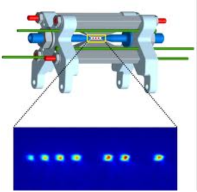

Outline

LECTURE 1:

A Brief History of Computing

Pioneers of Quantum Computing and Quantum Simulation

Classical and Quantum Simulations of Quantum Spin Systems

LECTURE 2:

High-Temperature Superconductors versus QCD

The Nature of the Sign Problem

From Wilson's Lattice Gauge Theory to Quantum Link Models

LECTURE 3:

Quantum Simulators for Abelian Lattice Gauge Theories

Non-Abelian Quantum Link Models

Quantum Simulators for non-Abelian Gauge Theories

LECTURE 4:

Quantum Simulators for $\mathbb{C}P(N-1)$ Models

Continuum Limit of Quantum Link QCD

References and Conclusions

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LECTURE 1:

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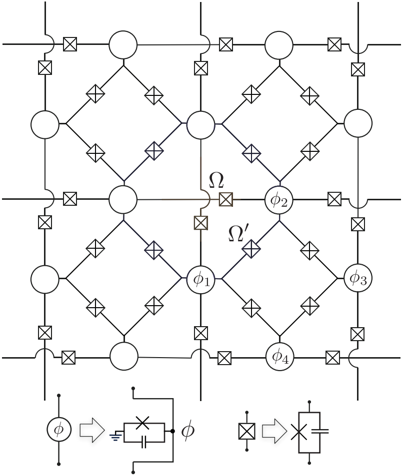
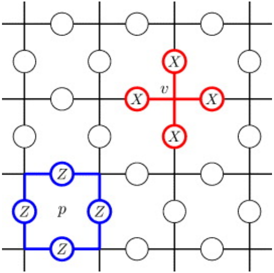
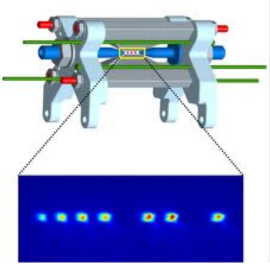
"String theory on a chip" with superconducting circuits



D. Marcos, P. Rabl, E. Rico, P. Zoller,
Phys. Rev. Lett. 111 (2013) 110504 (2013).
D. Marcos, P. Widmer, E. Rico, M. Hafezi, P. Rabl, UJW, P. Zoller,
arXiv:1407.6066.

Digital quantum simulation of Kitaev's toric code (a $\mathbb{Z}(2)$ quantum link model) with trapped ions





• Precisely controllable many-body quantum device, which can execute a prescribed sequence of quantum gate operations.

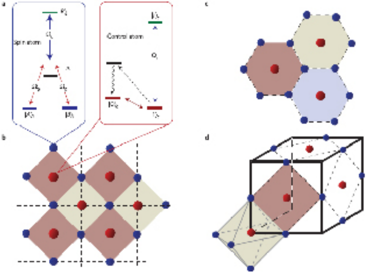
• State of simulated system encoded as quantum information.

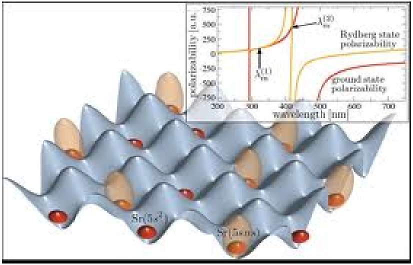
• Dynamics is represented by a sequence of quantum gates, following a stroboscopic Trotter decomposition.

A. Y. Kitaev, Ann. Phys. 303 (2003) 2.

B. P. Lanyon, C. Hempel, D. Nigg, M. Müller, R. Gerritsma, F. Zähringer, P. Schindler, J. T. Barreiro, M. Rambach, G. Kirchmair, M. Hennrich, P. Zoller, R. Blatt, C. F. Roos, Science 334 (2011) 6052.

$U(1)$ quantum link models can also be simulated with Rydberg atoms in an optical lattice





- Lasers can excite atoms to high-lying Rydberg states.
- Rydberg atoms are large and have collective interactions.
- Ensemble Rydberg atoms represent qubits at link centers.
- Control atoms at lattice sites ensure the Gauss' law.

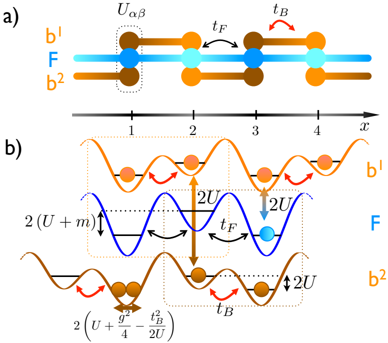
M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502.
H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, H. P. Büchler, Nat. Phys. 6 (2010) 382.
L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Nature Communications 4 (2013) 2615.
L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein, Ann. Phys. 330 (2013) 160.

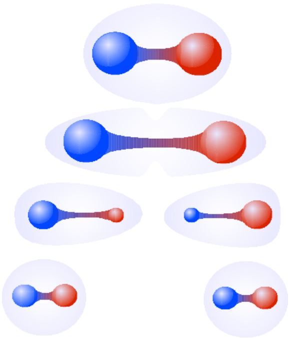
Hamiltonian for staggered fermions and $U(1)$ quantum links

$$H = -t \sum_x \left[\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \text{h.c.} \right] + m \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x E_{x,x+1}^2$$

$$U_{x,x+1} = b_x b_{x+1}^\dagger, \quad E_{x,x+1} = \frac{1}{2} \left(b_{x+1}^\dagger b_{x+1} - b_x^\dagger b_x \right)$$

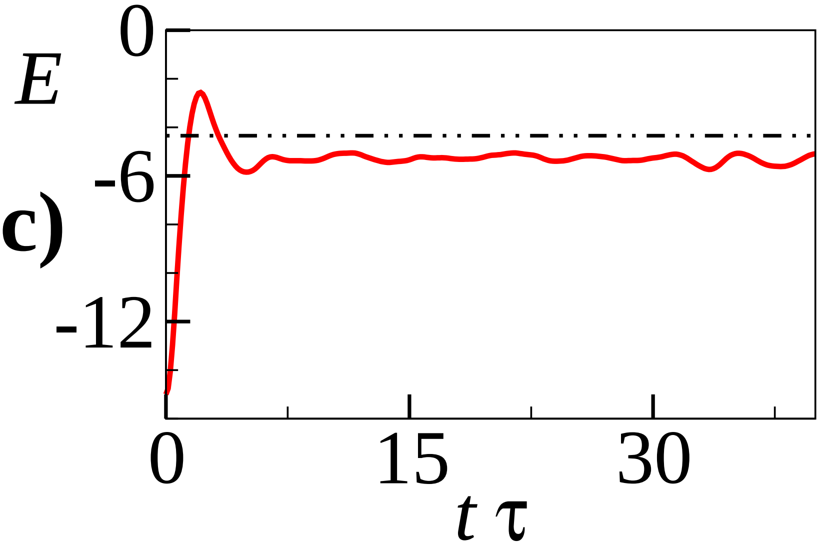
Optical lattice with Bose-Fermi mixture of ultra-cold atoms



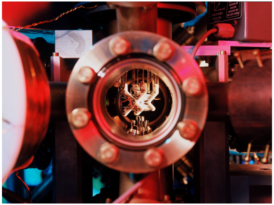


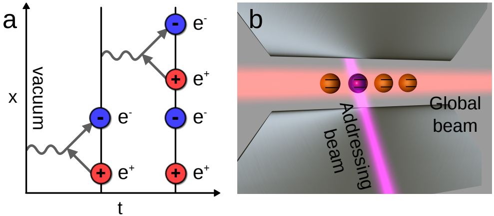
D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.

Quantum simulation of the real-time evolution of string breaking



Digital 4-qubit ion-trap quantum computation of pair creation





• Quantum computer consisting of four trapped Ca ions that act as four qubits, which are manipulated by external laser beams.
• Precisely controllable many-body quantum device, executing a prescribed sequence of quantum gate operations.
• State of simulated system is encoded as quantum information.
• Dynamics is represented by a sequence of quantum gates, following a stroboscopic Trotter decomposition.

E. A. Martinez, C. A. Muschik, P. Schindler, D. Nigg, A. Erhard, M. Heyl, P. Hauke, M. Dalmonte, T. Monz, P. Zoller, R. Blatt, Nature 534 (2016) 516.

Outline

$U(N)$ quantum link operators

$$U^{ij} = S_1^{ij} + iS_2^{ij}, \ U^{ij\dagger} = S_1^{ij} - iS_2^{ij}, \ i,j \in \{1,2,\dots,N\}, \ [U^{ij},(U^\dagger)^{kl}] \neq 0$$

$SU(N)_L \times SU(N)_R$ gauge transformations of a quantum link

$$[L^a, L^b] = if_{abc}L^c, \ [R^a, R^b] = if_{abc}R^c, \ a,b,c \in \{1,2,\dots,N^2-1\}$$

$$[L^a, R^b] = [L^a, E] = [R^a, E] = 0$$

Infinitesimal gauge transformations of a quantum link

$$[L^a, U] = -\lambda^a U, \ [R^a, U] = U\lambda^a, \ [E, U] = U$$

Algebraic structures of $U(N)$ quantum link models

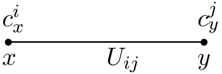
$U^{ij}, \ L^a, \ R^a, E, \ 2N^2 + 2(N^2-1) + 1 = 4N^2 - 1$ $SU(2N)$ generators

R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502

Fermionic rishons at the two ends of a link

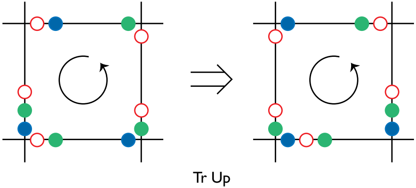
$$\{c_x^i, c_y^{j\dagger}\} = \delta_{xy}\delta_{ij}, \ \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0$$

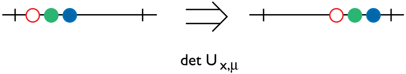
Rishon representation of link algebra



$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \ L_{xy}^a = c_x^{i\dagger}\lambda_{ij}^a c_x^j, \ R_{xy}^a = c_y^{i\dagger}\lambda_{ij}^a c_y^j, \ E_{xy} = \frac{1}{2}(c_y^{i\dagger}c_y^i - c_x^{i\dagger}c_x^i)$$

Can a “rishon abacus” implemented with ultra-cold atoms be used as a quantum simulator?





Homework 5:

Show that the $SU(2)$ embedding algebra of the $U(1)$ quantum link model can be realized with bosonic rishons

$$U_{xy} = b_x b_y^\dagger,\ E_{xy} = \frac{1}{2}(b_y^\dagger b_y - b_x^\dagger b_x)$$

Homework 6:

Show that the $SU(2N)$ embedding algebra of the $U(N)$ quantum link model can be realized with fermionic rishons

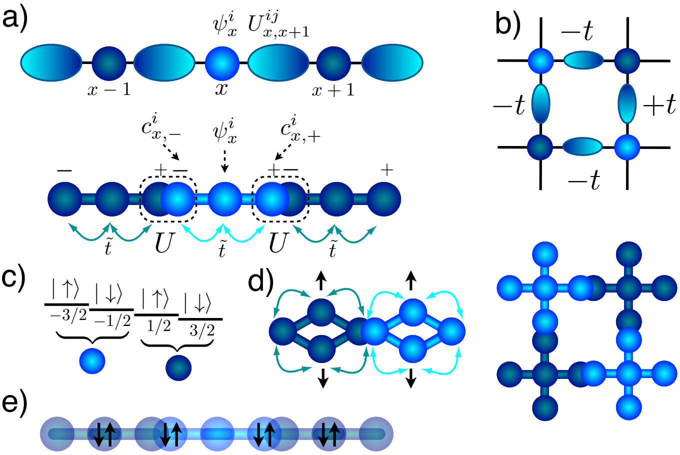
$$U_{xy}^{ij} = c_x^i c_y^{j\dagger},\ L_{xy}^a = c_x^{i\dagger}\lambda_{ij}^a c_x^j,\ R_{xy}^a = c_y^{i\dagger}\lambda_{ij}^a c_y^j,\ E_{xy} = \frac{1}{2}(c_y^{i\dagger} c_y^i - c_x^{i\dagger} c_x^i)$$

Some important lessons from lecture 3:

- Thanks to their finite-dimensional Hilbert space per link, quantum links can be embodied by ultra-cold fermionic or bosonic atoms in an optical lattice or by superconducting flux cubits.
- Non-Abelian quantum link models are formulated in terms of discrete quantum links, but have an exact $SU(N)$ gauge symmetry.
- Quantum links have fermionic "rishon" constituents.

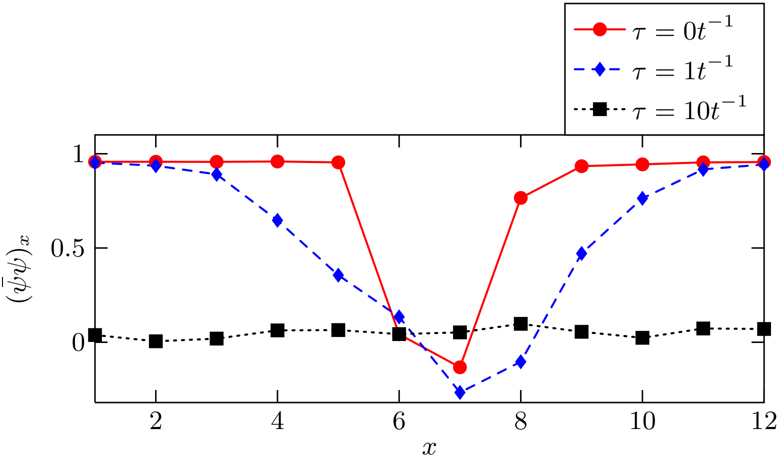
Outline

Optical lattice with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) with color encoded in nuclear spin

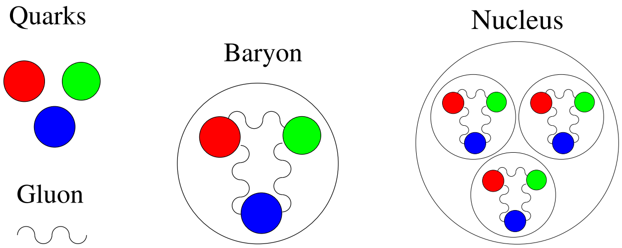


D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303

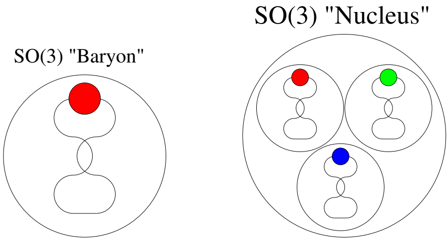
Expansion of a "fireball" mimicking a hot quark-gluon plasma



Nuclear Physics from $SU(3)$ QCD



“Nuclear Physics” in an $SO(3)$ lattice gauge theory?

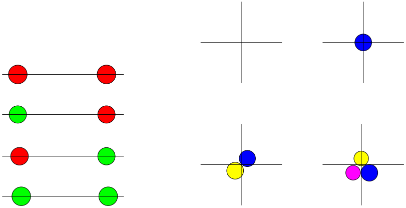


1-d $SO(3)$ quantum link model with adjoint triplet-fermions

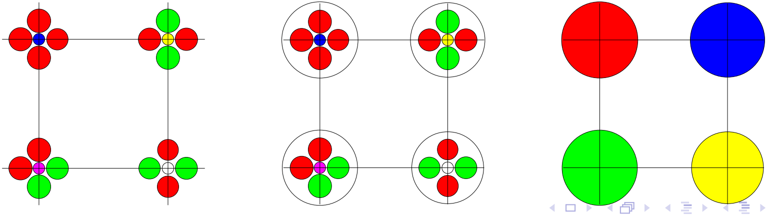
$$H = -t \sum_x \left[\psi_x^{i\dagger} O_{x,x+1}^{ij} \psi_{x+1}^j + \text{h.c.} \right] + m \sum_x (-1)^x \psi_x^{i\dagger} \psi_x^i$$

$SO(3)$ quantum links

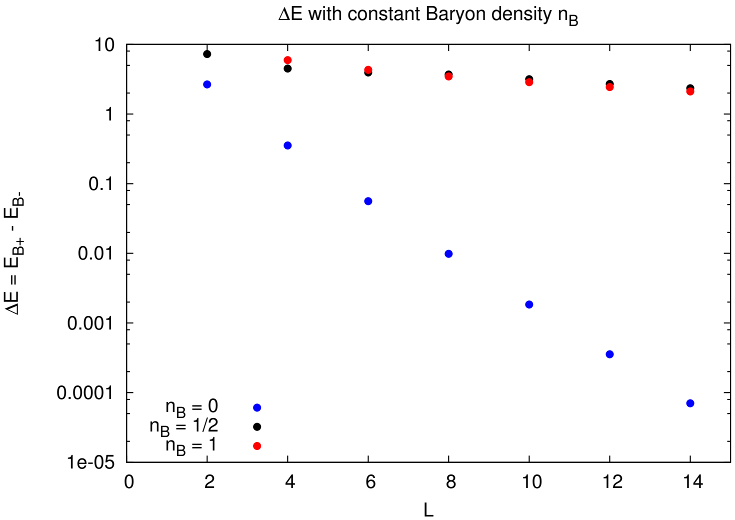
$$O_{x,x+1}^{ij} = \sigma_{x,L}^i \sigma_{x+1,R}^j$$



Encoding manifestly gauge invariant states obeying Gauss' law



Restoration of chiral symmetry at baryon density $n_B \geq \frac{1}{2}$

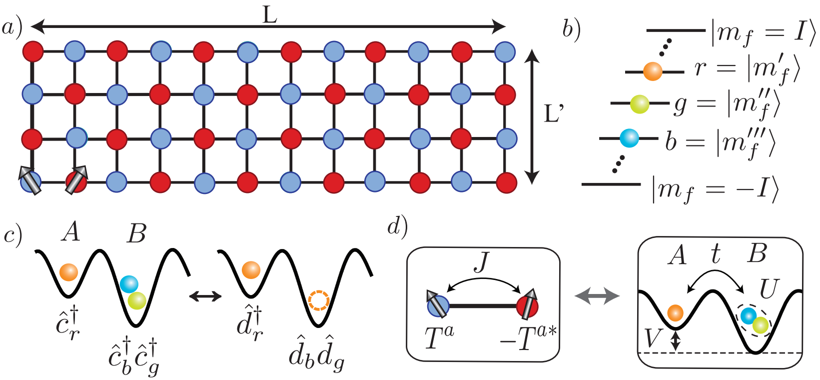


E. Rico, M. Dalmonte, P. Zoller, D. Banerjee, M. Bögli, P. Stebler, UJW, Annals Phys. 393 (2018).

Outline

Ladder of $SU(N)$ quantum spins embodied with alkaline-earth atoms

$$H = -J \sum_{\langle xy \rangle} T_x^a T_y^{a*}, \quad [T_x^a, T_y^b] = i\delta_{xy} f_{abc} T_x^c$$



C. Laflamme, W. Evans, M. Dalmonte, U. Gerber, H. Mejia-Diaz, W. Bietenholz, UJW, P. Zoller, Annals Phys. 360 (2016) 117.

Goldstone boson fields in $\mathbb{C}P(N-1) = SU(N)/U(N-1)$

$$P(x)^\dagger = P(x), \quad \text{Tr}P(x) = 1, \quad P(x)^2 = P(x)$$

Low-energy effective action

$$\begin{aligned} S[P] &= \int_0^\beta dt \int_0^L dx \int_0^{L'} dy \ \text{Tr} \left\{ \rho_s' \partial_y P \partial_y P \right. \\ &+ \left. \rho_s \left[\partial_x P \partial_x P + \frac{1}{c^2} \partial_t P \partial_t P \right] \right\} \end{aligned}$$

Very large correlation length

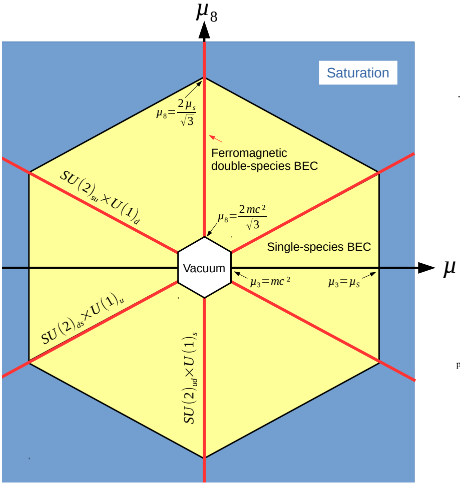
$$\xi \propto \exp(4\pi L' \rho_s / cN) \gg L', \quad \frac{1}{g^2} = L' \rho_s$$

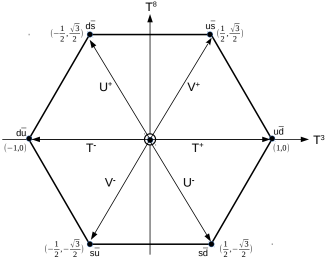
Dimensional reduction to the $(1+1)$-d $\mathbb{C}P(N-1)$ model

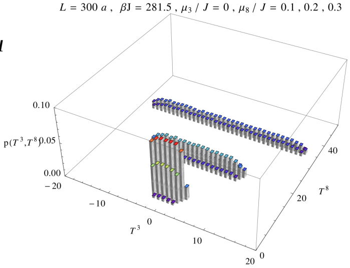
$$S[P] = \int_0^\beta dt \int_0^L dx \ \frac{1}{g^2} \text{Tr} \left[\partial_x P \partial_x P + \frac{1}{c^2} \partial_t P \partial_t P \right]$$

K. Harada, N. Kawashima, M. Troyer, PRL 90 (2003) 117203.
B. B. Beard, M. Pepe, S. Riederer, UJW, PRL 94 (2005) 010603.

Ferromagnetic Double-Species BEC in the $\mathbb{C}P(2)$ Model







W. Evans, U. Gerber, M. Hornung, UJW, Annals Phys. 398 (2018) 92.

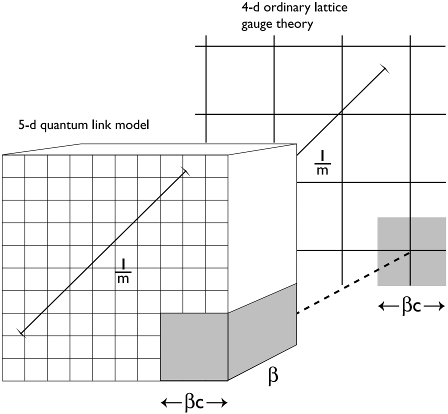
Outline

Low-energy effective action of a quantum link model

$$S[G_\mu] = \int_0^\beta dx_5 \int d^4x \, \frac{1}{2e^2} \left(\text{Tr } G_{\mu\nu} G_{\mu\nu} + \frac{1}{c^2} \text{Tr } \partial_5 G_\mu \partial_5 G_\mu \right), \; G_5 = 0$$

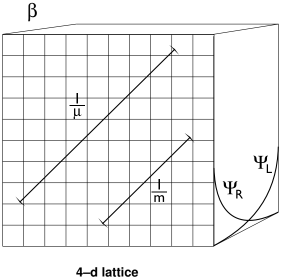
undergoes dimensional reduction from $4+1$ to 4 dimensions

$$S[G_\mu] \to \int d^4x \, \frac{1}{2g^2} \text{Tr } G_{\mu\nu} G_{\mu\nu}, \; \frac{1}{g^2} = \frac{\beta}{e^2}, \; \frac{1}{m} \sim \exp\left(\frac{24\pi^2\beta}{11Ne^2}\right)$$



Quarks as Domain Wall Fermions

$$H = J \sum_{x,\mu\neq\nu} \mathrm{Tr}[U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu}] + J' \sum_{x,\mu} [\det U_{x,\mu} + \det U^\dagger_{x,\mu}]$$
$$+ \frac{1}{2} \sum_{x,\mu} [\Psi^\dagger_x \gamma_0 \gamma_\mu U_{x,\mu} \Psi_{x+\hat{\mu}} - \Psi^\dagger_{x+\hat{\mu}} \gamma_0 \gamma_\mu U^\dagger_{x,\mu} \Psi_x] + M \sum_x \Psi^\dagger_x \gamma_0 \Psi_x$$
$$+ \frac{r}{2} \sum_{x,\mu} [2\Psi^\dagger_x \gamma_0 \Psi_x - \Psi^\dagger_x \gamma_0 U_{x,\mu} \Psi_{x+\hat{\mu}} - \Psi^\dagger_{x+\hat{\mu}} \gamma_0 U^\dagger_{x,\mu} \Psi_x].$$



$$\mu = 2M \exp(-M\beta), \ \frac{1}{m} \propto \exp(\frac{24\pi^2\beta}{(11N - 2N_f)e^2}), \ M > \frac{24\pi^2}{(11N - 2N_f)e^2}$$

Homework 7:

Show that the $-T^{a*}$ (with $a \in \{1, 2, ..., N^2 - 1\}$), form a representation of $SU(N)$, assuming that the T^a do, i.e. $[T^a, T^b] = if_{abc} T^c$. Show that the conjugate anti-fundamental representation $-T^{a*}$ is unitarily equivalent to the fundamental representation T^a only for $SU(2)$.

Homework 8:

Show that the $SU(N)$ spin ladder Hamiltonian commutes with the total $SU(N)$ spin $T^a = \sum_{x \in A} T_x^a - \sum_{x \in B} T_x^{a*}$.

Some important lessons from lecture 4:

• Quantum links have fermionic "rishon" constituents which can be embodied by alkaline-earth atoms.

• Continuous gluon fields emerge as low-energy collective excitations of the discrete quantum link variables, just as magnetic spin wave fields emerge from discrete quantum spins.

• 4-d QCD emerges by dimensional reduction from a $(4 + 1)$-d $SU(3)$ quantum link model. Quarks arise as domain wall fermions.

Outline

Some analog quantum simulator proposals

H. P. Büchler, M. Hermele, S. D. Huber, M. P. A. Fisher, P. Zoller, Phys. Rev. Lett. 95 (2005) 040402.

E. Zohar, B. Reznik, Phys. Rev. Lett. 107 (2011) 275301.

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 109 (2012) 125302.

D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.

D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303.

V. Kasper, F. Hebenstreit, F. Jendrzejewski, M. K. Oberthaler, J. Berges, New J. Phys. 19 (2017) 023030; A. Mil et al., arXiv:1909.07641.

Some digital quantum simulator proposals

M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502; Nat. Phys. 6 (2010) 382.

L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Nature Communications 4 (2013) 2615.

Reviews on quantum simulators for lattice gauge theories

UJW, Annalen der Physik 525 (2013) 777, arXiv:1305.1602.

M. C. Banuls et al., arXiv:1911.00003.

Conclusions

• Quantum link models provide an alternative formulation of lattice gauge theory with a finite-dimensional Hilbert space per link, which allows implementations with ultra-cold atoms in optical lattices.

• Quantum simulator constructions have already been presented for Wilson's lattice gauge theory as well as for the $U(1)$ quantum link model with fermionic matter using ultra-cold Bose-Fermi mixtures. $\mathbb{C}P(N-1)$ models as well as non-Abelian $U(N)$ and $SU(N)$ quantum link models can be embodied by alkaline-earth atoms.

• This allows the quantum simulation of the real-time evolution of string breaking as well as false vacuum decay. Accessible effects also include chiral symmetry restoration at high baryon density or the expansion of a hot quark-gluon plasma.

• In quantum spin and quantum link models regularizing asymptotically free theories, including $(1+1)$-d $\mathbb{C}P(N-1)$ models and $(3+1)$-d QCD, the continuum limit is taken by dimensional reduction of discrete variables.

• The path towards quantum simulation of QCD will be a long one.

However, with a lot of interesting physics along the way.