Quantum Simulation

Uwe-Jens Wiese

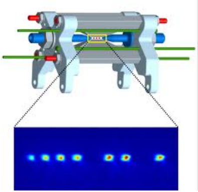
Albert Einstein Center for Fundamental Physics
Institute for Theoretical Physics, Bern University

Mini-Lecture Series on
Quantum Computing and
Quantum Information Science
for Nuclear Physics
JLab, March 18-19, 2020

FNSNF
Swiss National Science Foundation

u^b
b UNIVERSITÄT BERN
AEC ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS

erc European Research Council

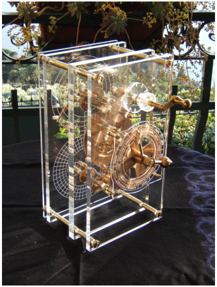


Outline

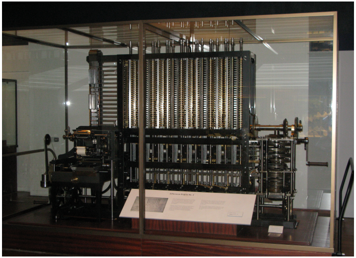
Outline

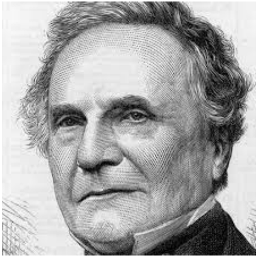
The first “digital computer” in Babylonia about 2400 b.c.

The first “analog computer”: Antikythera for determining the position of celestial bodies, Crete, about 100 b.c.



The first programmable computer: Charles Babbagge's (1791-1871) "difference engine" was realized by his son.





The first software developer: Ada Lovelace (1815-1852).

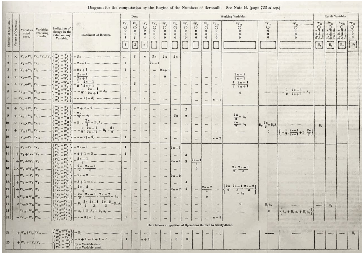
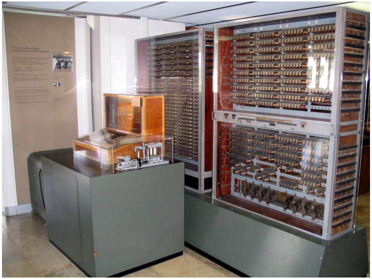
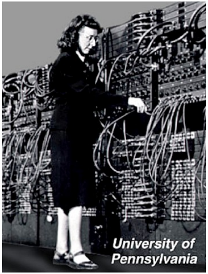


Diagram for the computation by the Engine of the Numbers of Bernoulli. See Note G. (page 722 et seq.)

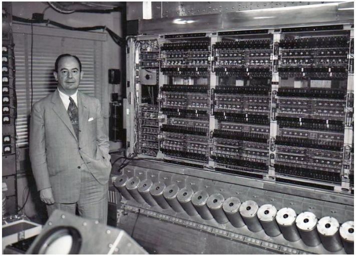
Konrad Zuse's (1910-1992) relay-driven computer Z3

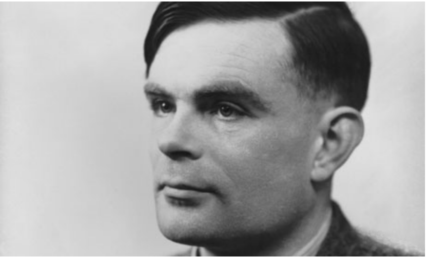


From the vacuum-tube ENIAC to the IBM Blue Gene

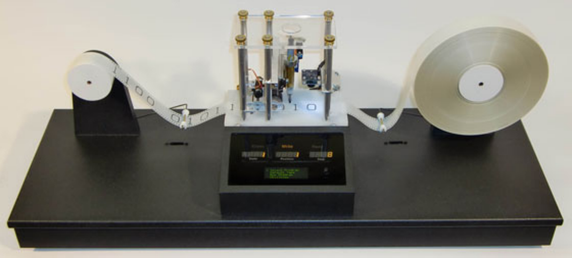


Pioneers of theoretical computer science: John von Neumann (1903-1992) and Alan Turing (1912-1954)





Model of a universal Turing machine



RSA encryption: multiplication is easy, factorization is hard.

RSA decryption challenge in 1991:
factorize the following 174-digit number with 576 bits

$$
\begin{aligned}
RSA576 = \ & 188198812920607963838697239461650439807163563\\
& 379417382700763356422988859715234665485319060\\
& 606504743045317388011303396716199692321205\\
& 734031879550656996221305168759307650257059
\end{aligned}
$$

RSA encryption: multiplication is easy, factorization is hard.

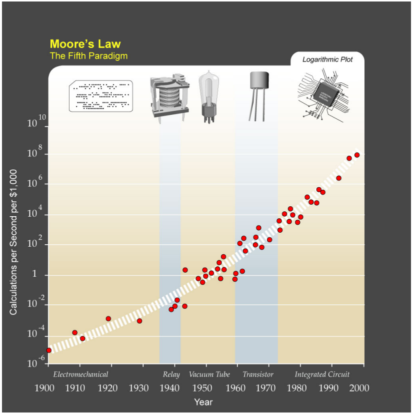
RSA decryption challenge in 1991:
factorize the following 174-digit number with 576 bits

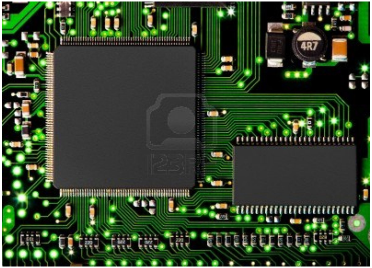
$$
\begin{aligned}
RSA576 &= 18819881292060796383869723946165043980716356\\
&\quad 33794173827007633564229888597152346654853190\\
&\quad 60606504743045317388011303396716199692321205\\
&\quad 734031879550656996221305168759307650257059\\
&= 39807508642406493739712550055038649119906436\\
&\quad 2342526708406385189575946388957261768583317\\
&* \ 47277214610743530253622307197304822463291469\\
&\quad 5302097116459852171130520711256363590397527
\end{aligned}
$$

This problem was solved only in 2003 by two mathematicians in Bonn using very large computer resources.

Only in 2009, when the challenge was no longer active, the 232-digit number RSA768 with 768 bits has finally been factorized.

Moore's law: "Every two years the number of transistors per area increases by a factor of 2."





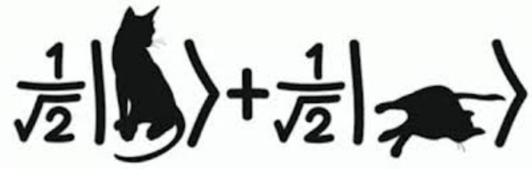
Modern micro chips consist of several billions of transistors, each about 10^{-8} m in size. This is already close to the quantum mechanical limit set by the size of individual atoms.

From bits to qubits

$$|\Psi\rangle = a|1\rangle + b|0\rangle, \quad |a|^2 + |b|^2 = 1$$

Entangled state of two qubits

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right)$$



Outline

Richard Feynman's vision of 1982

"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

A universal quantum computer (David Deutsch's quantum analog of a classical Turing machine) could use Peter Shor's algorithm to solve the factorization problem.

David Deutsch

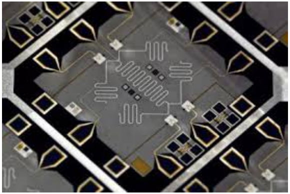
Peter Shor

A universal quantum computer (David Deutsch's quantum analog of a classical Turing machine) could use Peter Shor's algorithm to solve the factorization problem.

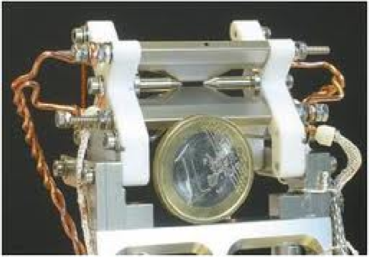
David Deutsch

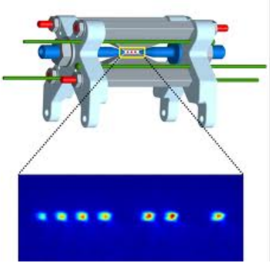
Peter Shor

Until today, only $15 = 3 \cdot 5$ has been correctly factorized by a quantum computer, at least in about 50 % of all trials.



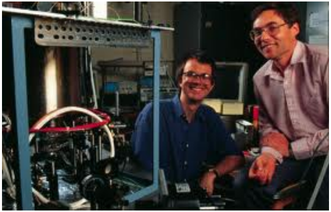
Ion traps as a digital quantum computer?



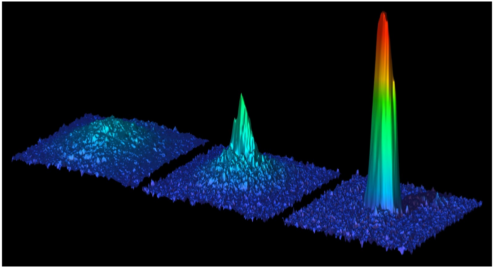


Franklin Medal 2010: I. Cirac, D. Wineland, P. Zoller

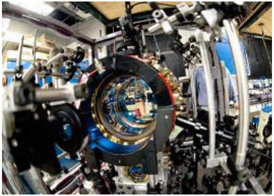
Bose-Einstein condensation in ultra-cold atomic gases

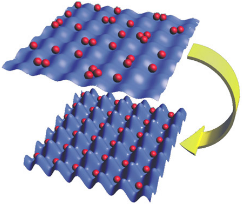
 

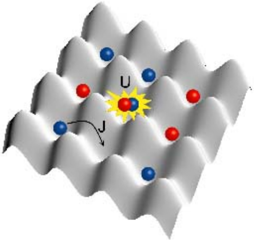
Eric Cornell, Carl Wieman, Wolfgang Ketterle, 1995



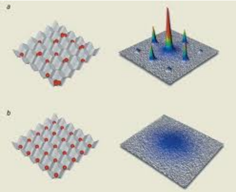
Ultra-cold atoms in optical lattices as analog quantum simulators

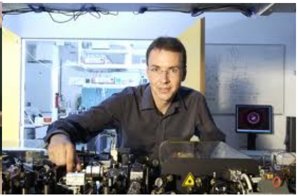






Transition from a superfluid to a Mott insulator





Theodor Hänsch Immanuel Bloch

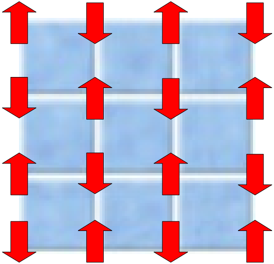
Can one understand high-T_c superconductivity in this way?

Outline

Richard Feynman, Int. J. Theor. Phys. 21 (1982) 467

"Can quantum systems be probabilistically simulated by a classical computer? This is the hidden variable problem: it is impossible to represent the results of quantum mechanics with a classical universal device."

The spin $\frac{1}{2}$ quantum Heisenberg model



Quantum spins $[S_x^a, S_y^b] = i\delta_{xy}\varepsilon_{abc}S_x^c$ and their Hamiltonian

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

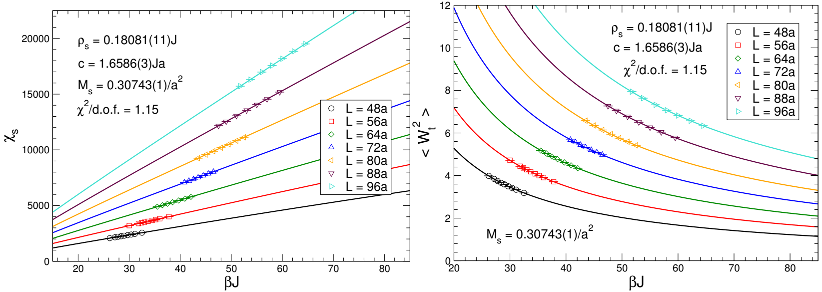
Partition function at inverse temperature $\beta = 1/T$

$$Z = \mathrm{Tr} \exp(-\beta H)$$

Low-energy effective action for antiferromagnetic magnons

$$S[\vec{e}] = \int_0^\beta dt \int d^2x \; \frac{\rho_s}{2} \left(\partial_i \vec{e} \cdot \partial_i \vec{e} + \frac{1}{c^2} \partial_t \vec{e} \cdot \partial_t \vec{e} \right)$$

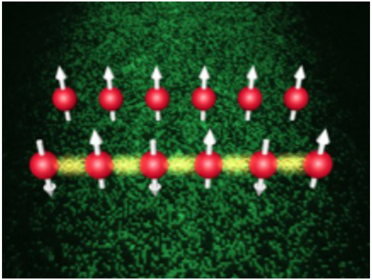
Fit to analytic predictions of effective theory

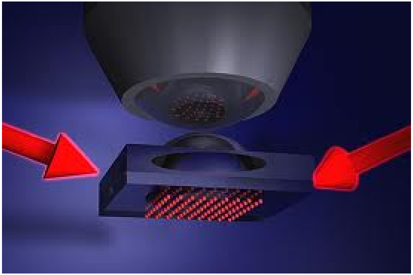


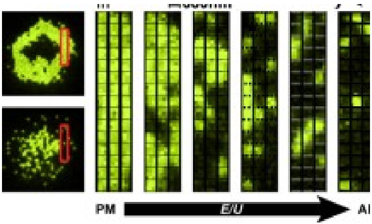
$$\mathcal{M}_s = 0.30743(1), \quad \rho_s = 0.18081(11)J, \quad c = 1.6586(3)Ja$$

UJW, H.-P. Ying (1994); F.-J. Jiang, UJW (2010)

Optical lattice quantum simulation of quantum spin systems







J. Simon, W. S. Bakir, R. Ma, M. E. Tal, P. M. Preis, M. Greiner, Nature 472 (2011) 307.

Homework 1:

Show that the Heisenberg Hamiltonian H commutes with the total spin $\vec{S}$

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y, \quad \vec{S} = \sum_x \vec{S}_x.$$

Show that ferromagnetic spin waves $|p_1 p_2\rangle$ are eigenstates of H and determine their energy-momentum dispersion relation,

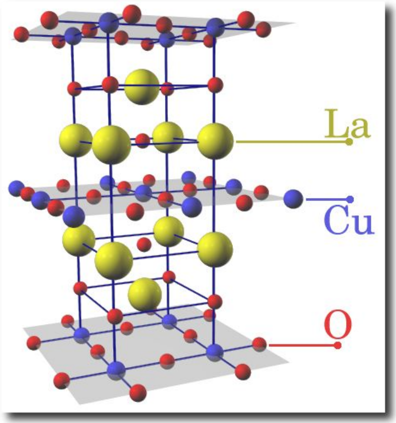
$$|p_1 p_2\rangle = \sum_x \exp(i(p_1 x_1 + p_2 x_2)) S_x^+ |\uparrow\uparrow \ldots \uparrow\rangle.$$

Some important lessons from lecture 1:

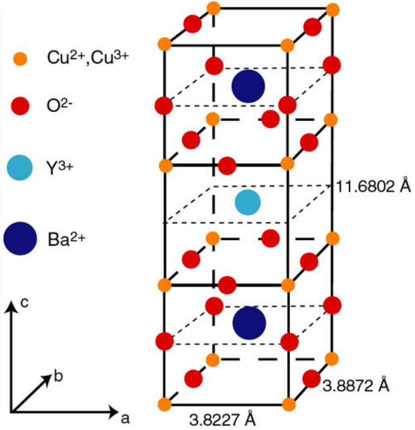
- Quantum computers or quantum simulators are potentially much more powerful than classical computers.
- The Heisenberg quantum spin model in thermal equilibrium can be simulated very efficiently using classical computers.
- The collective dynamics of discrete quantum spin degrees of freedom can give rise to an emergent quantum field theory for the low-energy spin wave Goldstone boson excitations.

Outline

Antiferromagnetic precursors of high-T_c superconductors

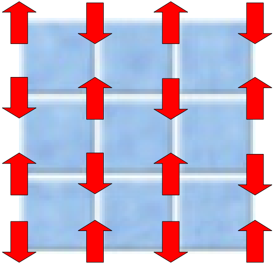


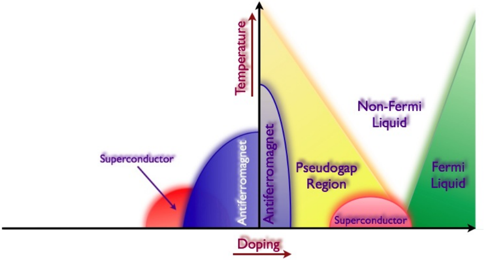
LaCuO

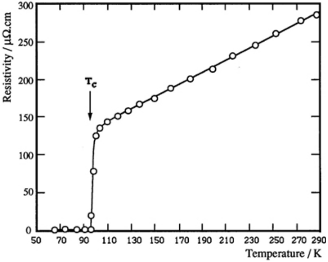


YBaCuO

Properties of cuprates

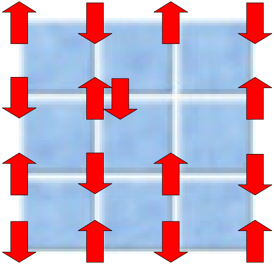


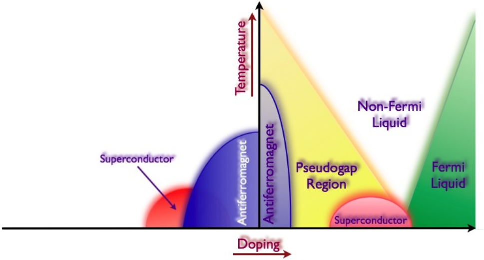


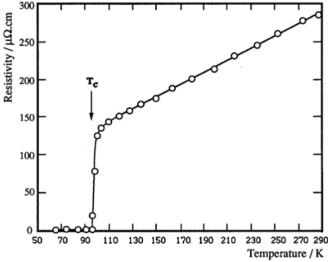


Temperature-dependence of resistivity

Properties of cuprates

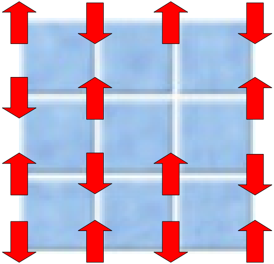


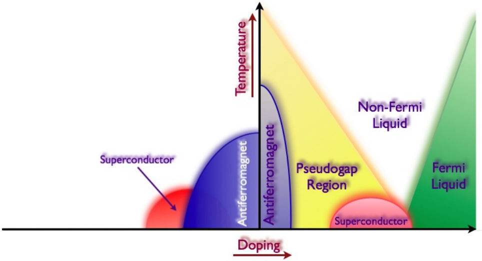


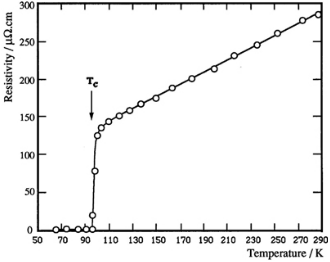


Temperature-dependence of resistivity

Properties of cuprates

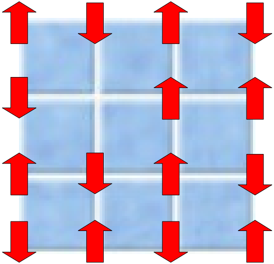


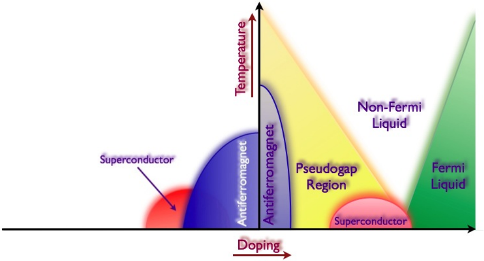


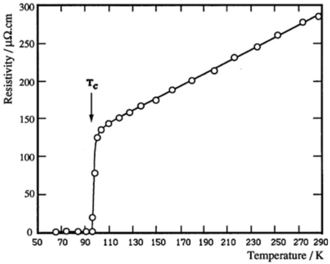


Temperature-dependence of resistivity

Properties of cuprates

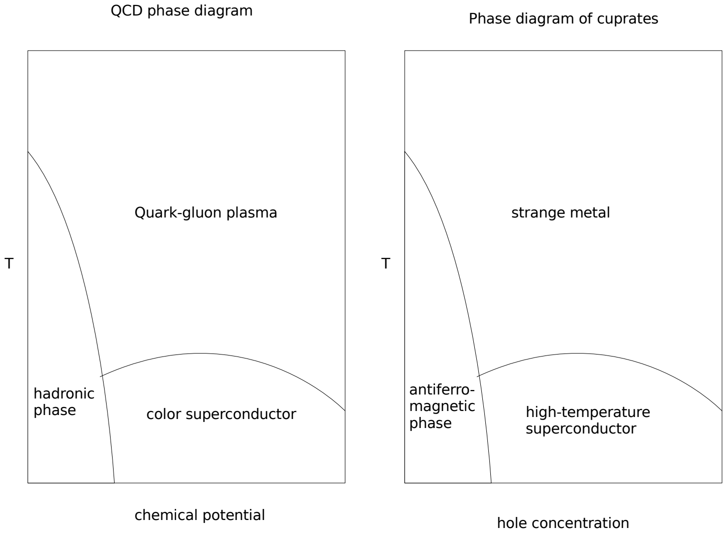






Temperature-dependence of resistivity

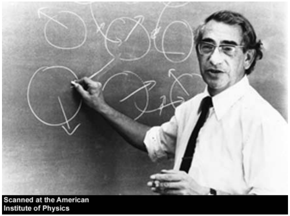
Phase diagrams of QCD and of doped antiferromagnets

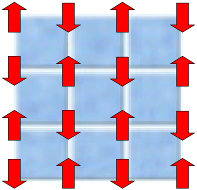


Correspondences between QCD and Antiferromagnetism

	QCD	Antiferromagnetism
broken phase	hadronic vacuum	antiferromagnetic phase
global symmetry	chiral symmetry	spin rotations
symmetry group G	$SU(2)_L \otimes SU(2)_R$	$SU(2)_s$
unbroken subgroup H	$SU(2)_{L=R}$	$U(1)_s$
Goldstone boson	pion	magnon
Goldstone field in G/H	$U(x) \in SU(2)$	$\vec{e}(x) \in S^2$
order parameter	chiral condensate	staggered magnetization
coupling strength	pion decay constant F_π	spin stiffness ρ_s
propagation speed	velocity of light	spin-wave velocity c
conserved charge	baryon number $U(1)_B$	electric charge $U(1)_Q$
charged particle	nucleon or antinucleon	electron or hole
long-range force	pion exchange	magnon exchange
dense phase	nuclear or quark matter	high-T_c superconductor
microscopic description	lattice QCD	Hubbard or t-J model
effective description of Goldstone bosons	chiral perturbation theory	magnon effective theory
effective description of charged fields	baryon chiral perturbation theory	magnon-hole effective theory

The Hubbard Model for doped antiferromagnets





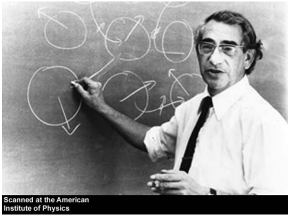
$$H = -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + U \sum_x (c_x^\dagger c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

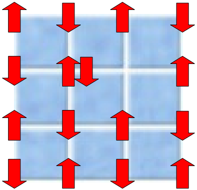
reduces to the Heisenberg model at half-filling for $U \gg t$

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

Important open question:
Does the Hubbard model explain high-T_c superconductivity?

The Hubbard Model for doped antiferromagnets





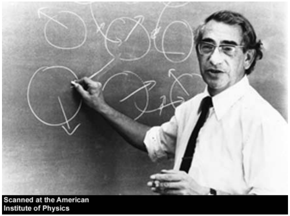
$$H = -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + U \sum_x (c_x^\dagger c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

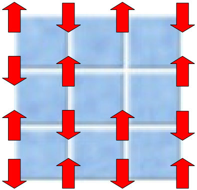
reduces to the Heisenberg model at half-filling for $U \gg t$

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

Important open question:
Does the Hubbard model explain high-T_c superconductivity?

The Hubbard Model for doped antiferromagnets





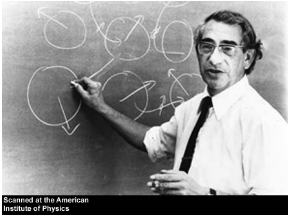
$$H = -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + U \sum_x (c_x^\dagger c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

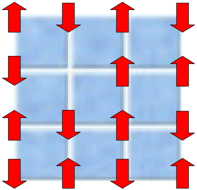
reduces to the Heisenberg model at half-filling for $U \gg t$

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

Important open question:
Does the Hubbard model explain high-T_c superconductivity?

The Hubbard Model for doped antiferromagnets





$$H = -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + U \sum_x (c_x^\dagger c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

reduces to the Heisenberg model at half-filling for $U \gg t$

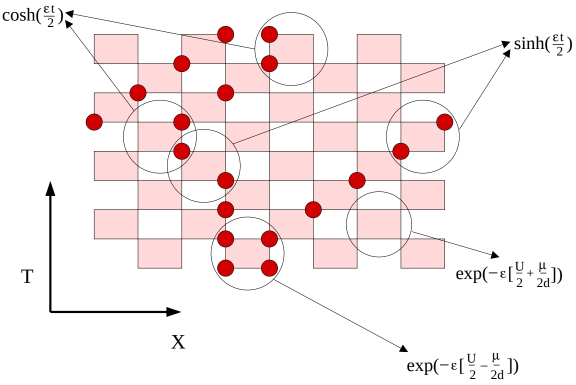
$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

Important open question:
Does the Hubbard model explain high-T_c superconductivity?

Outline

Path integral

$$Z_f = \mathrm{Tr}[\exp(-\varepsilon H_1)\exp(-\varepsilon H_2)\ldots\exp(-\varepsilon H_M)]^N$$
$$= \sum_{[n]} \mathrm{Sign}[n]\exp(-S[n])$$



Sign problem of fermionic path integrals

$$Z_f = \mathrm{Tr}\exp(-\beta H) = \sum_{[n]} \mathrm{Sign}[n]\exp(-S[n]) \;, \quad \mathrm{Sign}[n] = \pm 1$$

Average sign is exponentially small

$$\langle \mathrm{Sign} \rangle = \frac{\sum_{[n]} \mathrm{Sign}[n]\exp(-S[n])}{\sum_{[n]} \exp(-S[n])} = \frac{Z_f}{Z_b} = \exp(-\beta V \Delta f)$$

The statistical error is exponentially large

$$\frac{\sigma_{\mathrm{Sign}}}{\langle \mathrm{Sign} \rangle} = \frac{\sqrt{\langle \mathrm{Sign}^2 \rangle - \langle \mathrm{Sign} \rangle^2}}{\sqrt{N}\langle \mathrm{Sign} \rangle} = \frac{\exp(\beta V \Delta f)}{\sqrt{N}} \;.$$

Some very hard sign problems are NP complete

M. Troyer, UJW, Phys. Rev. Lett. 94 (2005) 170201.

Homework 2:

Show that the anti-commutation relations $\{c^\dagger_{x,s}, c_{y,s'}\} = \delta_{xy}\delta_{ss'}$ of fermionic creation and annihilation operators imply angular momentum commutation relations

$$[S^a_x, S^b_y] = i\delta_{xy}\varepsilon_{abc}S^c_x, \quad \vec{S}_x = \sum_x c^\dagger_x \frac{\vec{\sigma}}{2} c_x, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}.$$

Homework 3:

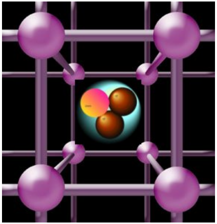
Show that the Hubbard Hamiltonian H commutes with the total spin $\vec{S}$ and with the particle number N

$$H = -t \sum_{\langle xy \rangle} (c^\dagger_x c_y + c^\dagger_y c_x) + U \sum_x (c^\dagger_x c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

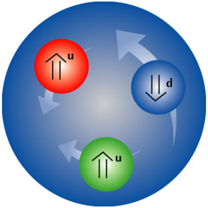
$$\vec{S} = \sum_x \vec{S}_x = \sum_x c^\dagger_x \frac{\vec{\sigma}}{2} c_x, \quad N = \sum_x n_x = \sum_x c^\dagger_x c_x.$$

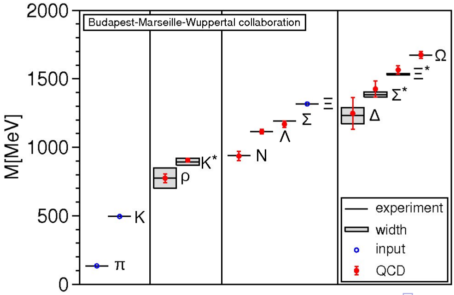
Outline

Kenneth Wilson's lattice QCD describes confinement of quarks and gluons inside protons und neutrons

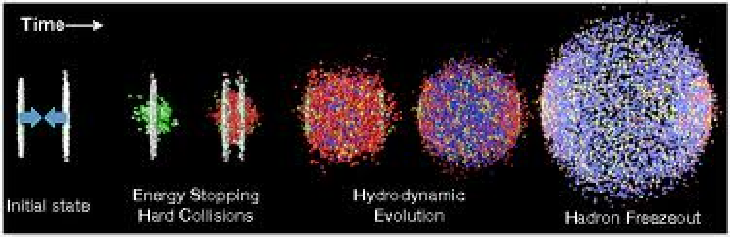


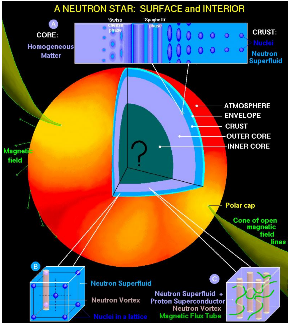
and confirms the experimentally measured mass spectrum

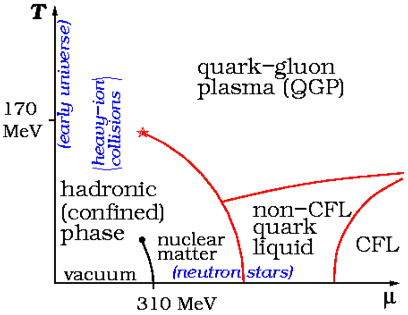




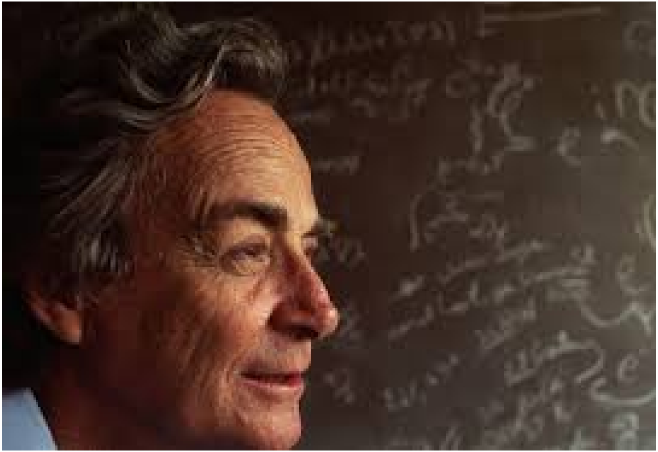
Can heavy-ion collision physics or nuclear astrophysics benefit from quantum simulations in the long run?







Richard Feynman's vision of 1982



"It does seem to be true that all the various field theories have the same kind of behavior, and can be simulated in every way, apparently, with little latticeworks of spins and other things."

Different descriptions of dynamical Abelian gauge fields:

Maxwell's classical electromagnetic gauge fields

$$\vec{\nabla} \cdot \vec{E}(\vec{x}, t) = \rho(\vec{x}, t), \quad \vec{\nabla} \cdot \vec{B}(\vec{x}, t) = 0, \quad \vec{B}(\vec{x}, t) = \vec{\nabla} \times \vec{A}(\vec{x}, t)$$

Quantum Electrodynamics (QED) for perturbative treatment

$$E_i = -i \frac{\partial}{\partial A_i}, \quad [E_i(\vec{x}), A_j(\vec{x}\,')] = i\delta_{ij}\delta(\vec{x} - \vec{x}\,'), \quad \left[\vec{\nabla} \cdot \vec{E} - \rho\right] |\Psi[A]\rangle = 0$$

Wilson's $U(1)$ lattice gauge theory for classical simulation

$$U_{xy} = \exp\left(ie \int_x^y d\vec{l} \cdot \vec{A}\right) = \exp(i\varphi_{xy}) \in U(1), \quad E_{xy} = -i \frac{\partial}{\partial \varphi_{xy}},$$

$$[E_{xy}, U_{xy}] = U_{xy}, \quad \left[\sum_i (E_{x,x+\hat{i}} - E_{x-\hat{i},x}) - \rho\right] |\Psi[U]\rangle = 0$$

$U(1)$ quantum link models for quantum simulation

$$U_{xy} = S^+_{xy}, \quad U^\dagger_{xy} = S^-_{xy}, \quad E_{xy} = S^3_{xy},$$

$$[E_{xy}, U_{xy}] = U_{xy}, \quad [E_{xy}, U^\dagger_{xy}] = -U^\dagger_{xy}, \quad [U_{xy}, U^\dagger_{xy}] = 2E^\dagger_{xy}$$

Hamiltonian formulation of Wilson's $U(1)$ lattice gauge theory

$$U = \exp(i\varphi), \ U^\dagger = \exp(-i\varphi) \in U(1)$$

Electric field operator E

$$E = -i\partial_\varphi, \ [E, U] = U, \ [E, U^\dagger] = -U^\dagger, \ [U, U^\dagger] = 0$$

Generator of $U(1)$ gauge transformations

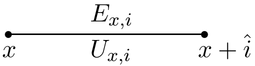
$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

$U(1)$ gauge invariant Hamiltonian

$$H = \frac{g^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2g^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j} + \text{h.c.})$$

operates in an infinite-dimensional Hilbert space per link

$U(1)$ quantum links from spins $\frac{1}{2}$

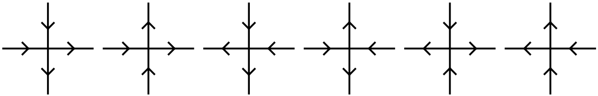


$$U = S_1 + iS_2 = S_+, \; U^\dagger = S_1 - iS_2 = S_-$$

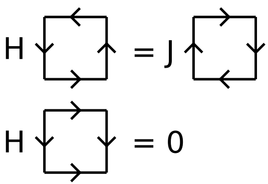
Electric flux operator E

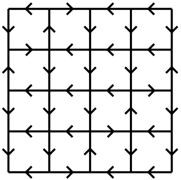
$$E = S_3, \; [E, U] = U, \; [E, U^\dagger] = -U^\dagger, \; [U, U^\dagger] = 2E$$

Gauss law



Ring-exchange plaquette Hamiltonian



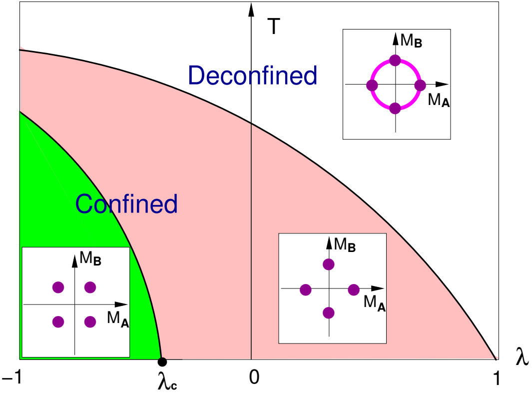


D. Horn, Phys. Lett. B100 (1981) 149
P. Orland, D. Rohrlich, Nucl. Phys. B338 (1990) 647
S. Chandrasekharan, UJW, Nucl. Phys. B492 (1997) 455

Hamiltonian with Rokhsar-Kivelson term

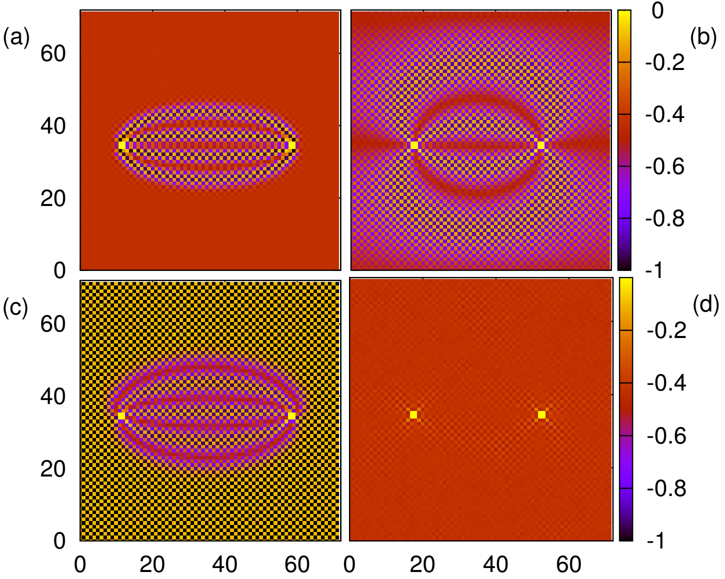
$$H = -J \left[\sum_{\square} (U_{\square} + U_{\square}^{\dagger}) - \lambda \sum_{\square} (U_{\square} + U_{\square}^{\dagger})^2 \right]$$

Phase diagram



D. Banerjee, F.-J. Jiang, P. Widmer, UJW, JSTAT (2013) P12010.

Energy density of charge-anti-charge pair $Q = \pm 2$



D. Banerjee, F.-J. Jiang, P. Widmer, UJW, JSTAT (2013) P12010.

Homework 4:

Show that the Hamiltonian of the 2-d $U(1)$ quantum link model commutes with the local generators of gauge transformations $G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i})$

$$H = -\frac{1}{2g^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j} + \text{h.c.}).$$

Some important lessons from lecture 2:

- QCD shares some qualitative features with high-temperature superconductors.
- Wilson's lattice QCD allows the precise determination of static hadron properties using Monte Carlo simulations. It also allows to simulate QCD at finite temperature. Simulations of dynamical processes or of the physics at non-zero baryon density suffer from very severe sign problems.
- Gauge theories with exact continuous gauge symmetry can be formulated in terms of discrete quantum link degrees of freedom.

Outline

Outline

Outline

Outline

Outline

Outline