OLIVIER PFISTER - U. Virginia. 3/9/20. #1

www.scottaaronson.com/blog/?p=3943 → lecture notes by him, John Preskill, et al.

4 lectures : Qubits and qumodes

"quantum algorithm zoo" online

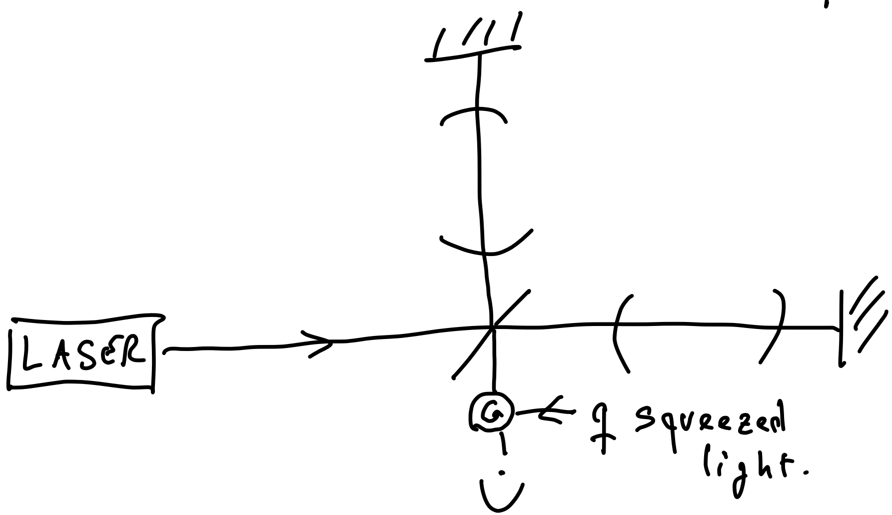
Nielsen & Chuang's book

David Mermin's book

Qubits	Qumodes
2-state systems	continuous-variable; really: fields
$\{\|0\rangle, \|1\rangle\}$ computational basis	$\{\|x\rangle\}_{x \in \mathbb{R}}$ e.g. position eigenbasis
e.g. $\{\|\uparrow\rangle, \|\downarrow\rangle\}$ for a spin-$\frac{1}{2}$	typically for a harmonic oscillator

Quantum Information Science: a spectrum of applications

* QUANTUM SENSING: lowering the noise floor in physical measurements: squeezed states, LIGO.



* Q COMMUNICATION : - q key distribution (Micius Chinese satellite)

  ↑ "quantum"

  - q teleportation
  - multipartite protocols:
    - ↪ q anonymous broadcasting
    - ↪ q voting, etc.

* Q SIMULATION (Feynman 1982) : N qubits = 2^N-dim Hilbert space

  INCREASE N: "SCALABILITY"

* Q COMPUTATION : SOME Q ALGORITHMS GIVE AN EXPONENTIAL SPEEDUP OVER CLASSICAL COMPUTING:
  - SHOR ALGORITHM FOR INTEGER FACTORING

QUANTUM INFORMATION: encoding in probability amplitudes

$\{|0\rangle, |1\rangle\}$

$$|\psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle \qquad |\psi_0|^2 + |\psi_1|^2 = 1$$

not accessible via a single measurement.

Repeated measurements of say, S_z (diagonal in $\{|0\rangle, |1\rangle\}$)
(on many identical copies of $|\psi\rangle$)

will yield $\rightarrow$ $|\psi_0|^2$ & $|\psi_1|^2$

Then, more measurement of, say, S_x, will yield the phase difference b/w ψ_0 & ψ_1

"QUANTUM STATE TOMOGRAPHY, or ESTIMATION"

Note: QUDITS : $\{|0\rangle, |1\rangle, \dots |d-1\rangle\}$

D-dim Hilbert space

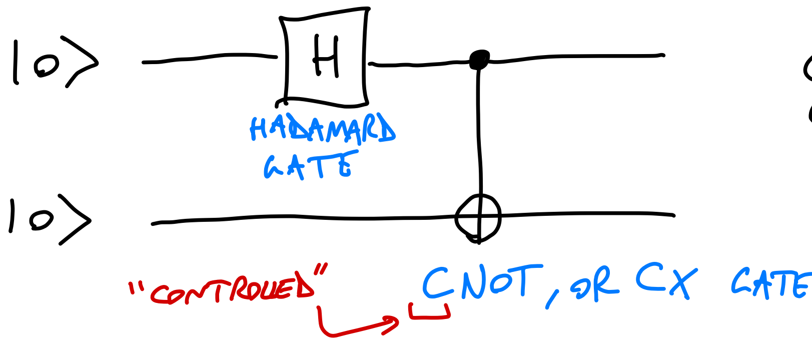
$$|\psi\rangle = \sum_{j=0}^{d-1} \psi_j |j\rangle$$

QUMODES: $\{|x\rangle\}_{x\in\mathbb{R}}$

$$|\psi\rangle = \int \underbrace{\psi(x)}_{\text{wave function}} |x\rangle \, dx$$

qumode-based quantum computing can be made fault tolerant.

QUANTUM CIRCUITS : evolution from left to right



?

Universal gate set:

Any N-qubit can be made w/ just any one-qubit gate and one well-chosen two-qubit gate.

Building blocks : $D^{(\frac{1}{2})}$ of $SU(2)$

Pauli group of Pauli matrices, generated by Z & X

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad Z|j\rangle = e^{ij\pi}|j\rangle \qquad j = 0, 1$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad X|j\rangle = |j \oplus 1\rangle \qquad \begin{matrix} 0 \oplus 1 = 1 \\ 1 \oplus 1 = 0 \end{matrix}$$

$$\mathcal{P} = e^{i\theta}\{\mathbb{1}, X, Y, Z\} \quad \theta = 0, \tfrac{\pi}{2}, \pi, \tfrac{3\pi}{2}$$ PAULI GROUP

HADAMARD :

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = R_y\left(\frac{\pi}{2}\right) R_z(\pi)$$

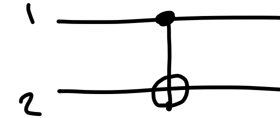
$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = |+\rangle$$

$$\underbrace{H|1\rangle}_{\text{EIGENSTATES of } Z} = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = \underbrace{|-\rangle}_{\text{EIGENSTATES of } X}$$

CONTROLLED - X GATE :

$C_{X_{12}} \, |j\rangle_1 \, |k\rangle_2$

$\uparrow$ CONTROL $\quad \uparrow$ TARGET



- if $|0\rangle_1$

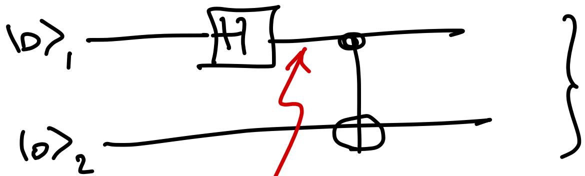
  $\hookrightarrow$ do nothing on 2 $\Leftrightarrow \mathbb{I}_2$

- if $|1\rangle_1$

  $\hookrightarrow$ apply X_2

matrix representation :

$$C_X = \begin{array}{c|cccc} & |00\rangle & |01\rangle & |10\rangle & |11\rangle \\ \hline \langle 00| & 1 & 0 & 0 & 0 \\ \langle 01| & 0 & 1 & 0 & 0 \\ \langle 10| & 0 & 0 & 0 & 1 \\ \langle 11| & 0 & 0 & 1 & 0 \end{array}$$



$$\frac{1}{\sqrt{2}}\left(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2\right) \neq |\varphi\rangle_1|\chi\rangle_2$$

"product state"

$|+\rangle_1|0\rangle_2$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle_1 + |1\rangle_1\right)|0\rangle_2$$

Not a product state

"verschränkung" = entanglement

Schrödinger (1935),

in response to

Einstein, Podolski, & Rosen (1935)

& Bohr (1935)

EPR state

$$\int |x\rangle_1 |x\rangle_2 \, dx = \int |p\rangle_1 |-p\rangle_2 \, dp$$

position

momentum

Randomness prevents non-causality!

Neither partner can tell whether they measured their spin first or just got the collapsed state from an instantaneous measurement projection of the other lab. (IF they do not communicate).

This is because the results look random in the same way in both cases!

However, if both partners share their measurement results, they'll find these are perfectly correlated.

WHAT'S THE PROBLEM ?

↳ WE MEASURED S_z AND FOUND CORRELATIONS

↳ THESE CORRELATIONS ARE STILL HERE IF WE MEASURE S_x !

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \pm |1\rangle\right) \iff |{0 \atop 1}\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle \pm |-\rangle\right) \rightarrow \left\{ \begin{array}{l} \text{perfectly} \\ \text{correlated} \\ \text{measurements} \end{array} \right.$$

"Bell State" $\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = (\ldots) = \frac{1}{\sqrt{2}}\left(|++\rangle + |--\rangle\right)$

still entangled !

EPR paradox: • Quantum is valid.

• Assume measurement results explained by a local "element of reality"

• EoR cannot be given by Q mech b/c of Heisenberg inequality.

• Hence, Q mech would be an incomplete theory if EoR exist.

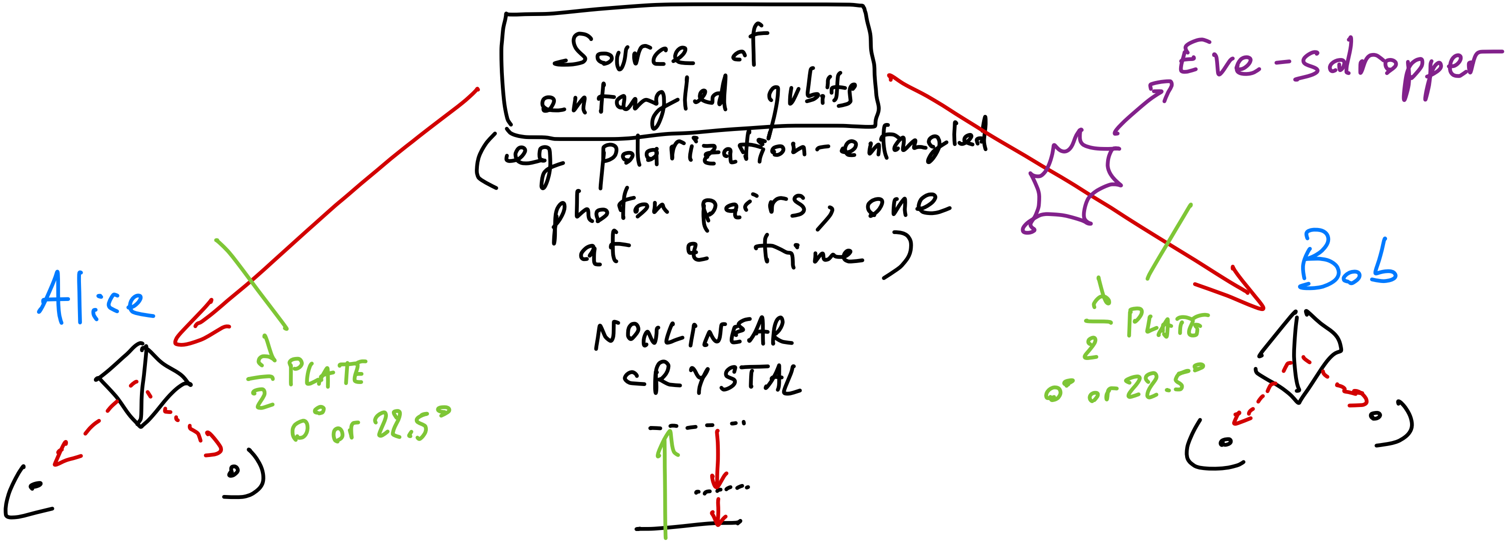
#2

<u>Bell</u> (1964):
- assume EoR exist
- Show this entail an inequality, testable in lab
- All tests violated inequality $\Rightarrow$ $\nexists$ EoR.

Entanglement is weird, but also useful!

$\rightarrow$ Quantum Key Distribution : sharing securely a random bit string.

Cryptography : encrypt some message:
"Hello, World".
with a (very long) random key : 0111001010000



$$\frac{1}{\sqrt{2}}\left(|\updownarrow\rangle_1 |\leftrightarrow\rangle_2 + |\leftrightarrow\rangle_1 |\updownarrow\rangle_2\right)$$

$$= \frac{1}{\sqrt{2}}\left(|\nearrow\rangle_1 |\searrow\rangle_2 + |\searrow\rangle_1 |\nearrow\rangle_2\right)$$

Alice & Bob talk over open channel:

- discard all meas'ts made in ≠ bases; keep rest
- "sacrifice" some same-basis results to make sure they <u>are</u> correlated
  - ↳ If not, an eavesdropper <u>must</u> be present.

Q teleportation : Sending $|\psi\rangle$ w/o state measurement (full = tomography)

Bennett et al. PRL (1993)

Bell-state Measurement

$\frac{1}{\sqrt{2}}\left(|00\rangle_{23}+|11\rangle_{23}\right)$

Bell basis

$$\frac{1}{\sqrt{2}}\left(|00\rangle_{12} \pm |11\rangle_{12}\right) = |\phi^{\pm}\rangle_{12}$$

$$\frac{1}{\sqrt{2}}\left(|01\rangle_{12} \pm |10\rangle_{12}\right) = |\chi^{\pm}\rangle_{12}$$

Rewrite in

Initial state :

$$|\psi\rangle_1 \otimes \frac{1}{\sqrt{2}}\left(|0\rangle_2 \otimes |0\rangle_3 + |1\rangle_2 \otimes |1\rangle_3\right)$$

$$= \frac{1}{\sqrt{2}}\Big(\psi_0 \underbrace{|0\rangle_1 |0\rangle_2}_{\frac{1}{\sqrt{2}}(|\phi^+\rangle_{12}+|\phi^-\rangle_{12})} |0\rangle_3 + \psi_0 \underbrace{|0\rangle_1 |1\rangle_2}_{\frac{1}{\sqrt{2}}(|\chi^+\rangle_{12}+|\chi^-\rangle_{12})} |1\rangle_3 + \psi_1 \underbrace{|1\rangle_1 |0\rangle_2}_{\frac{1}{\sqrt{2}}(|\chi^+\rangle_{12}-|\chi^-\rangle_{12})} |0\rangle_3 + \psi_1 \underbrace{|1\rangle_1 |1\rangle_2}_{\frac{1}{\sqrt{2}}(|\phi^+\rangle_{12}-|\phi^-\rangle_{12})} |1\rangle_3 \Big)$$

$$= \frac{1}{2}\Big[|\phi^+\rangle_{12}\left(\psi_0|0\rangle_3 + \psi_1|1\rangle_3\right) + |\phi^-\rangle_{12}\left(\psi_0|0\rangle_3 - \psi_1|1\rangle_3\right) + |\chi^+\rangle_{12}\left(\psi_0|1\rangle_3 + \psi_1|0\rangle_3\right) + |\chi^-\rangle\left(\psi_0|1\rangle_3 - \psi_1|0\rangle_3\right) \Big]$$

$$= \frac{1}{2}\Big[|\phi^+\rangle_{12} \underbrace{\left(\psi_0|0\rangle_3 + \psi_1|1\rangle_3\right)}_{|\psi\rangle_3} + |\phi^-\rangle_{12} \underbrace{\left(\psi_0|0\rangle_3 - \psi_1|1\rangle_3\right)}_{Z_3|\psi\rangle_3} + |\chi^+\rangle_{12} \underbrace{\left(\psi_0|1\rangle_3 + \psi_1|0\rangle_3\right)}_{X_3|\psi\rangle_3} + |\chi^-\rangle \underbrace{\left(\psi_0|1\rangle_3 - \psi_1|0\rangle_3\right)}_{Y_3|\psi\rangle_3} \Big]$$

* Alice gets either one of 4 Bell state, at random.
  ↳ From meas^t result, she knows what Pauli operator ("by product") Bob got on his side

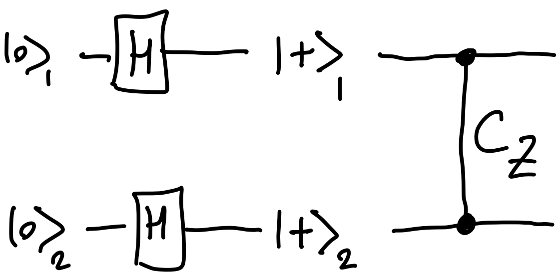
* Alice sends her meas^t results to Bob (classical comm only)
  ↳ Bob can "fix" the state, eg $Z(Z|\psi\rangle) = |\psi\rangle$

<u>HW</u>: Does it work for an N-qbit state?

Quantum gate teleportation : a primitive for quantum computing (besides the circuit model)

Gottesman & Chuang, Nature (98 or 99)

First, a more compact teleportation protocol that uses CLUSTER STATES



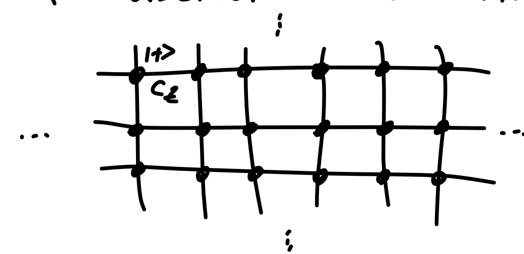
$C_Z|00\rangle = |00\rangle$

$C_Z|01\rangle = |01\rangle$

$C_Z|10\rangle = |10\rangle$

$C_Z|11\rangle = -|11\rangle$

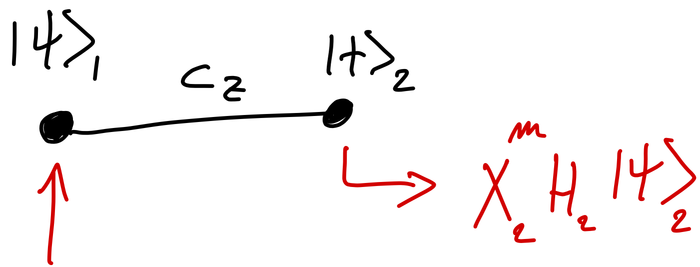
Universal Q computing can be performed on a 2D cluster state and measts + classical communication



$$C_Z|+\rangle_1|+\rangle_2 = \frac{1}{2}C_Z\left(|00\rangle+|01\rangle+|10\rangle+|11\rangle\right)$$

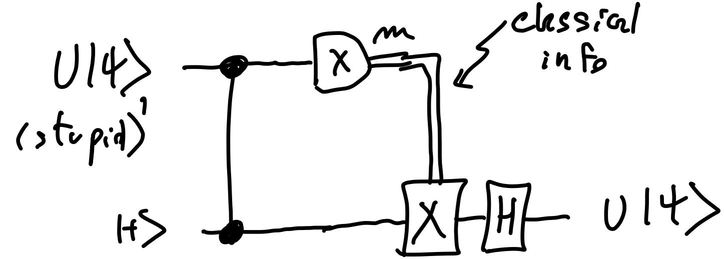
$$= \frac{1}{2}\left(|00\rangle+|01\rangle+|10\rangle-|11\rangle\right) = \frac{1}{\sqrt{2}}\left(|0+\rangle+|1-\rangle\right) = \frac{1}{\sqrt{2}}\left(|+0\rangle+|-1\rangle\right)$$

Compact teleportation: 2 qubits

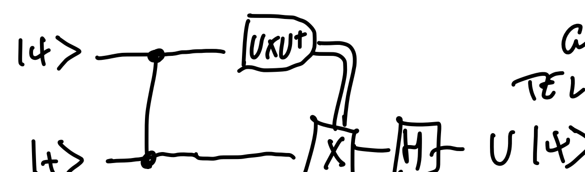


measure
X
ie in $\{|+\rangle, |-\rangle\}$
↳ $m = 0, 1$

Gate teleportation idea



Assume U only depends on Z (more clever)
then we have



GATE TELEPORTATION

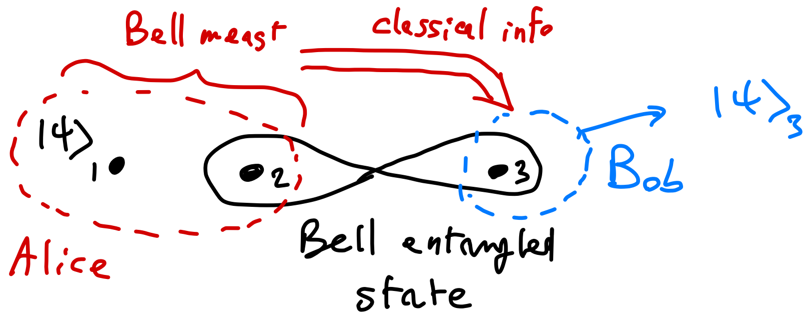
3/10/20
\# 3

Quantum dense coding & q sensing

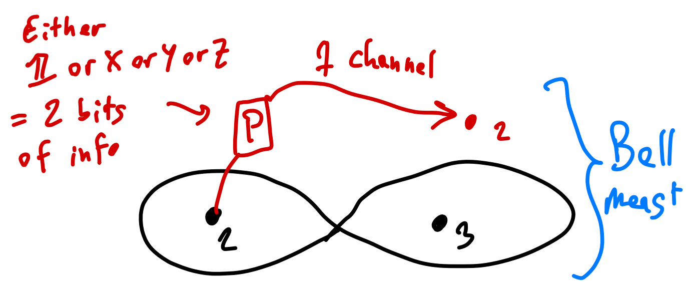
(a good reason to use continuous variables...)

"Dense" coding = teleportation flipped on its head

teleportation



"dense" coding
$\downarrow$
2 bits of classial info sent w/ "just" one qubit



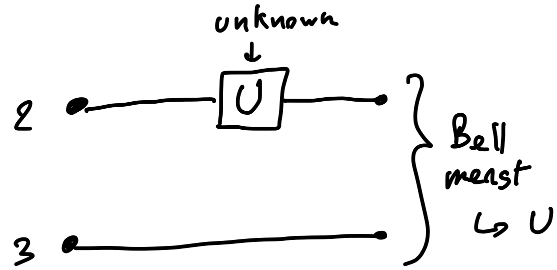
* Alice applies one of 4 Pauli ops. to qubit 2

* Alice sends qubit 2 to Bob

* Bob does BSM to find out what Pauli op. Alice did.

Using qumodes instead of qubits: dense coding as Q sensing

= e.m. Fields



qubit, or qumode 2 probes, say, the environment's action U.

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \to X \to \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$$

$$Y \to \frac{1}{\sqrt{2}}(-|10\rangle + |01\rangle)$$

$$Z \to \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

Pb: qubit Hilbert space only 4-dimensional

↳ cannot detect an arbitrary qubit by an angle $\neq \frac{\pi}{2}, 0, \pi$ or around axis $\neq \hat{x}, \hat{y}, \hat{z}$

Enter qumodes!

comp. basis $\{|0\rangle, |1\rangle\}$ eg eigenstates of S_z

$\updownarrow$ Hadamard op.

conjugate basis $\{|+\rangle, |-\rangle\}$ of S_x

Pauli group $P = \langle Z, X \rangle$

$\{|q\rangle\}_{q\in\mathbb{R}}$ eigenstates of position Q esp. harm. osc.

$\updownarrow$ F.T.

$Q = \frac{1}{\sqrt{2}}(a + a^\dagger)$

annihilation creation

$\{|p\rangle\}_{p\in\mathbb{R}}$ $|p\rangle = \frac{1}{\sqrt{2\pi}}\int e^{ipq}\,|q\rangle\, dq$

Fourier trasform

$P = \frac{i}{\sqrt{2}}(a^\dagger - a)$

Weyl-Heisenberg group of displacements in phase space

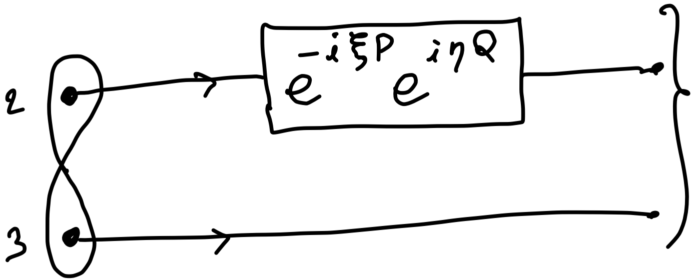
$\langle Z(\eta), X(\xi) \rangle$

translation operators

$X = e^{-i\xi P}$

$Z = e^{i\eta Q}$

Back to dense coding



$$\int e^{i\eta q} |q+\xi\rangle_2 |q\rangle_3 \, dq = \int e^{-i\xi p} |p+\eta\rangle_2 |-p\rangle_3 \, dp$$

still an EPR state.

$\downarrow$

Can <u>sense</u> any value of ξ and η by measuring $Q_1 - Q_2$ and $P_1 + P_2$

How small can they be?

$$\int_{-\infty}^{+\infty} |q\rangle_2 |q\rangle_3 \, dq = \int |p\rangle_2 |-p\rangle_3 \, dp$$

EPR state (1935)
(unphysical)

$\uparrow$

Bell: $\sum_{j=0}^{1} |j\rangle_2 |j\rangle_3$

Concretely.

The quantized E.M. field (in our case, optical) is

$$\vec{E}(\vec{r}) = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\,\hat{e}\left(a\,u(\vec{r})\,e^{i\vec{k}\cdot\vec{r}} + a^\dagger u^*(\vec{r})\,e^{-i\vec{k}\cdot\vec{r}}\right)$$

quantization volume

$$[a, a^\dagger] = \mathbb{1}$$

1 field mode defined by $\vec{k}\,(\omega)$, $\hat{e}$

For a harmonic oscillator = free field

$$H = \hbar\omega\left(a^\dagger a + \frac{\mathbb{1}}{2}\right) \longrightarrow a(t) = a(0)\,e^{-i\omega t}$$

Heisenberg-picture evolution

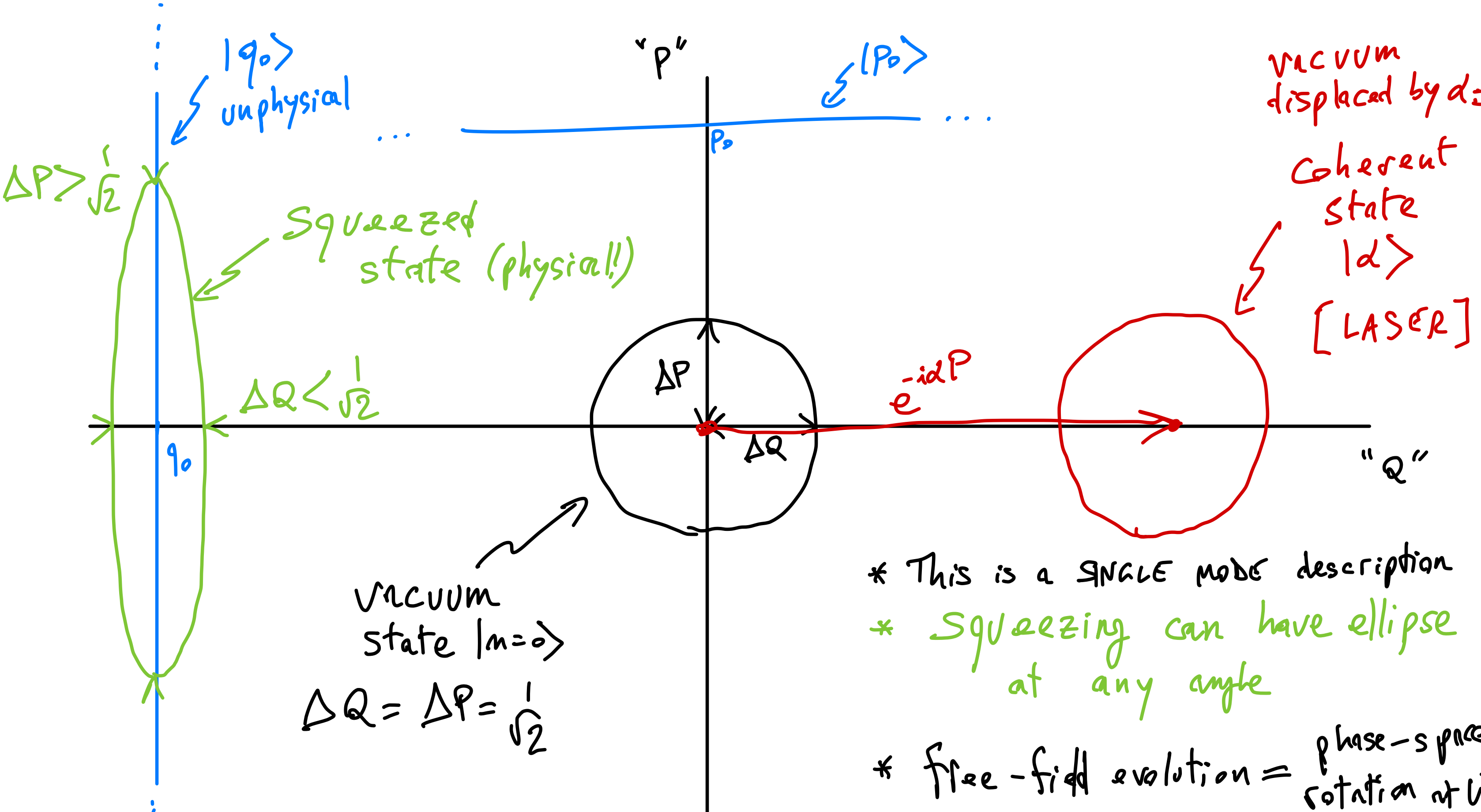
Plane-wave field:

$$\vec{E}(\vec{r},t) = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\,\hat{e}\left(Q(0)\cos(\vec{k}\cdot\vec{r}-\omega t) + P(0)\sin(\vec{k}\cdot\vec{r}-\omega t)\right)$$

General boson mode commutator

$$\left[a_{\hat{k},\omega,\hat{e}} , a^{\dagger}_{\hat{k}',\omega',\hat{e}'} \right] = \delta(\hat{k}-\hat{k}')\,\delta(\omega-\omega')\left(1-\delta_{\hat{e}',\hat{e}_{\perp}}\right)$$

#4 Quantum noise : $\Delta Q \, \Delta P \geq \frac{1}{2}$ $\quad [Q, P] = i \mathbb{1}$



* This is a SINGLE MODE description
* Squeezing can have ellipse at any angle
* free-field evolution = phase-space rotation at ω

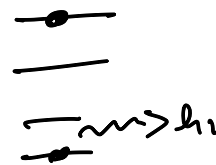
Quantum state generation

* Coherent states : $e^{\alpha a^\dagger - \alpha^* a}|0\rangle$ — $n=0$ vacuum — same $e^{-i\xi P} e^{i\eta Q} \times e^{\frac{i\eta\xi}{2}}$

↳ Hamiltonian eg electric dipole $-\vec{d}\cdot\vec{\mathcal{E}}$, $\vec{p}\cdot\vec{A}$

α — $a, a^\dagger$

Single-photon emission (at a time)



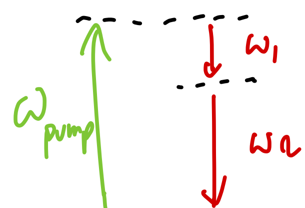
* Squeezed states : $e^{\frac{r}{2}(a^{\dagger 2} - a^2)}|0\rangle$

• a_1 & a_2 can be $\neq$ modes
two-mode squeezing

• we can also have single-mode squeezing where $a_1 = a_2 = a$

$$H = i\hbar\chi\, a_{pump}\, a_1^\dagger a_2^\dagger + H.c.$$

$\omega_{pump} = \omega_1 + \omega_2$



PARAMETRIC DOWNCONVERSION

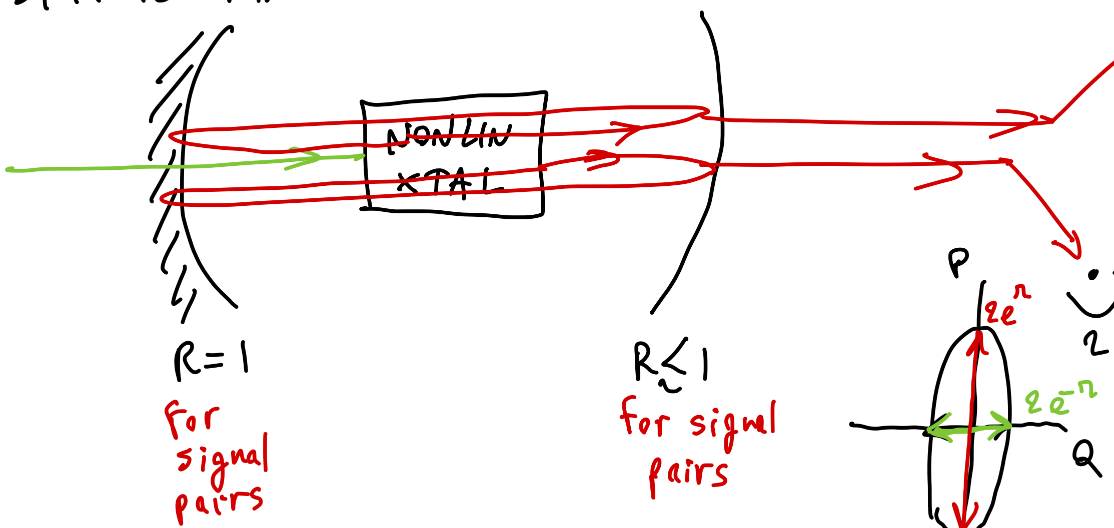
↳ typically in a nonlinear crystal.

Two-mode squeezed state → best possible approximation to the unphysical EPR state

$$|EPR\rangle = \int |q\rangle|q\rangle\, dq = \int |p\rangle|-p\rangle\, dp = \sum_{n=0}^{\infty} |n\rangle|n\rangle$$

Schmitt decomposition

OPTICAL PARAMETRIC OSCILLATOR:



$$\sum_{n=0}^{\infty} \frac{\tanh^n r}{\cosh r} |n\rangle_1 |n\rangle_2$$

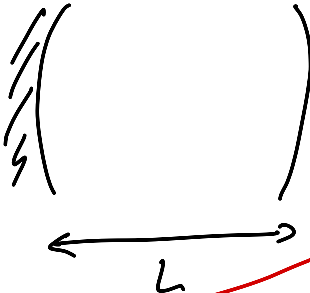
$$r = \chi\, \alpha_{pump}\, t$$

boosted by optical resonator

Squeezing parameter

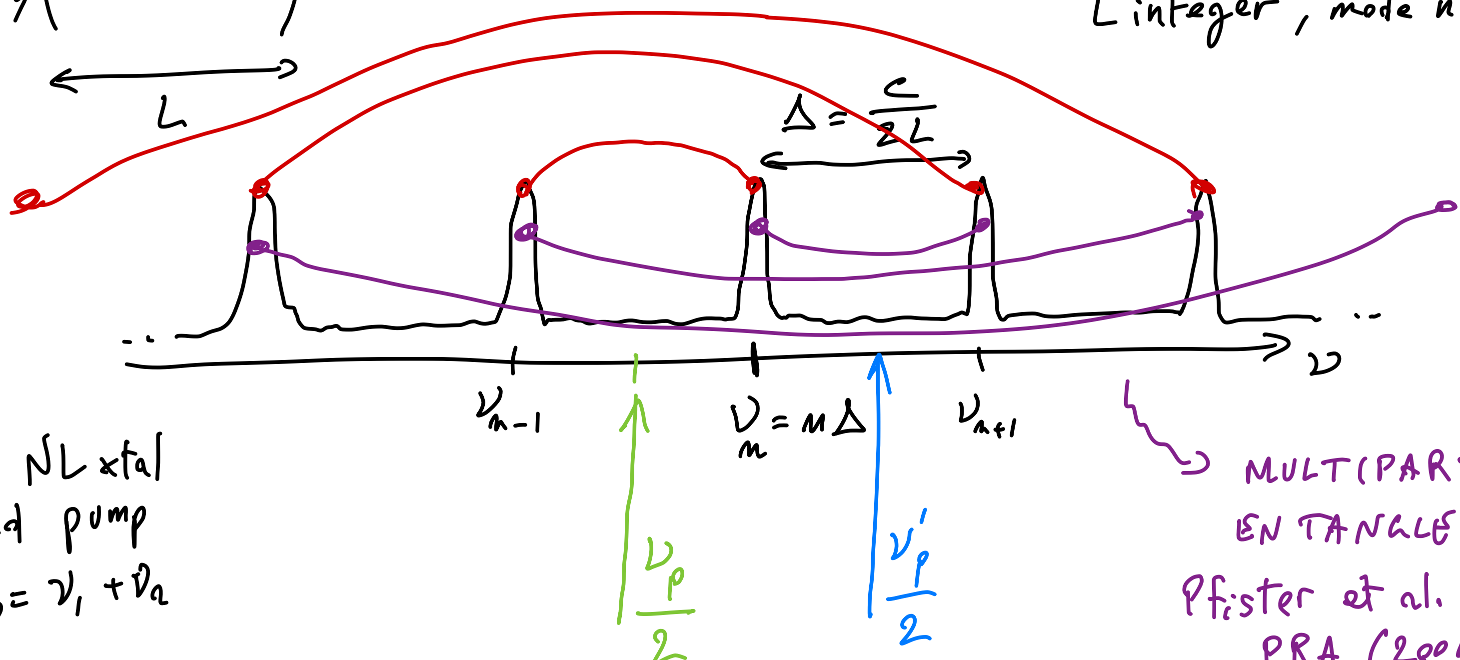
The optical resonator has a "frequency comb" spectrum

↳ Hall & Hänsch (2005 Nobel)



constructive interference $\Longleftrightarrow k\,2L = n\,2\pi$

$\uparrow$ integer, mode number



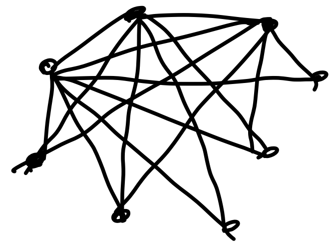
Add NL xtal and pump

$\nu_p = \nu_1 + \nu_2$

MULTIPARTITE ENTANGLEMENT!

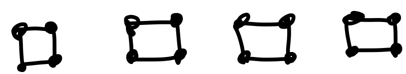
Pfister et al. PRA (2004)

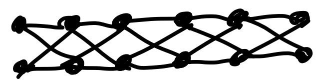
Initially, "too connected" states:



not useful for q computing!

But we figured out how to do cluster states

PRL (2011)  ....

PRL (2014) ...  ...

} 60 qumodes (measured), probably 10^4 total.

Arxiv 19.... new stuff

Review paper: O. Pfister, Continuous-variable quantum computing in the quantum optical frequency comb, Journal of Physics B: Atomic, Molecular, and Optical Physics 53, 012001 (2020); invited topical review.

PHYSICAL REVIEW A **92**, 063825 (2015)

Quantum simulation of quantum field theory using continuous variables

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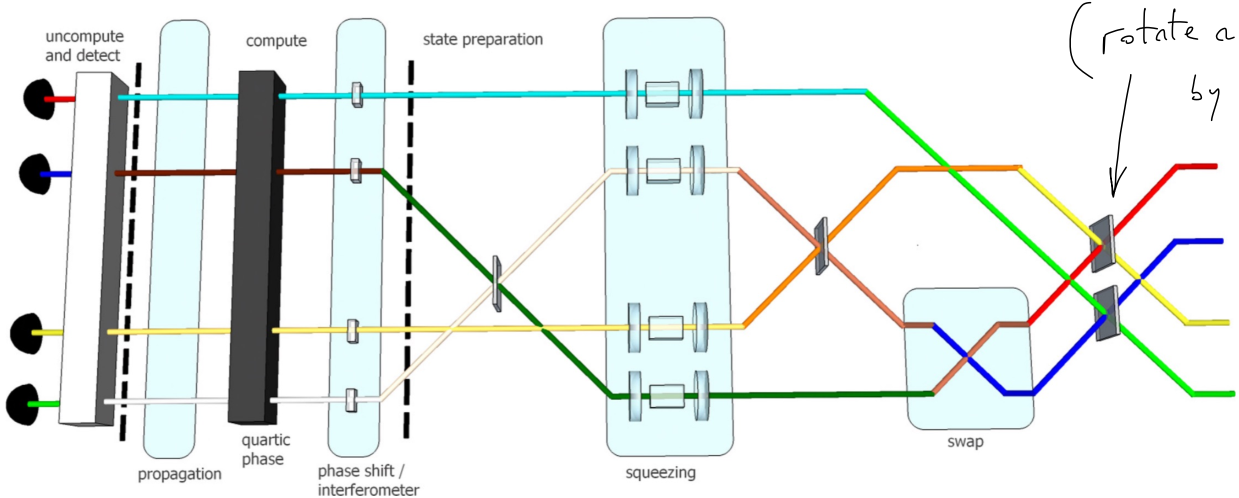
The year 1982 is often credited as the year that theoretical quantum computing was started with a keynote speech by Richard Feynman, who proposed a universal quantum simulator, the idea being that if you had such a machine you could in principle "imitate any quantum system, including the physical world." With that in mind, we present an algorithm for a continuous-variable quantum computing architecture which gives an exponential speedup over the best-known classical methods. Specifically, this relates to efficiently calculating the scattering amplitudes in scalar bosonic quantum field theory, a problem that is believed to be hard using a classical computer. Building on this, we give an experimental implementation based on continuous-variable states that is feasible with today's technology.

$$\mathcal{A} = \langle \text{out}|T \exp\left\{ i \int_{-T}^{T} dt [H_{\text{int}}(t) + H_{\text{c.t.}}(t)] \right\} |\text{in}\rangle$$

ϕ^4 term



(rotate all beamsplitters by 90°...)