Pseudoscalar meson production in GK-model

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September 6, 2019

Chiral-even GPDs

- Chiral-even GPDs parametrize the following off-forward matrix elements of quark operators at a light-like separation

D.Müller, D.Robaschik, B.Geyer, F.-M.Dittes, J.Hořejši, Fortsch. Phys. **42**, 101 (1994)
X.D.Ji, PRL **78**, 610 (1997); PRD **55**, 7114 (1997).

A.V.Radyushkin, PLB **380**, 417 & **385**, 333 (1996).

$$P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{z}{2}) \gamma^+ \psi(\frac{z}{2}) | p, \lambda \rangle \Big|_{z^+=0, \vec{z}_T=0} = \bar{u}(p', \lambda') \left[H^q \gamma^+ + E^q \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} \right] u(p, \lambda)$$

$$P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{z}{2}) \gamma^+ \gamma_5 \psi(\frac{z}{2}) | p, \lambda \rangle \Big|_{z^+=0, \vec{z}_T=0} = \bar{u}(p', \lambda') \left[\tilde{H}^q \gamma^+ \gamma_5 + \tilde{E}^q \frac{\gamma_5 \Delta^+}{2m} \right] u(p, \lambda)$$

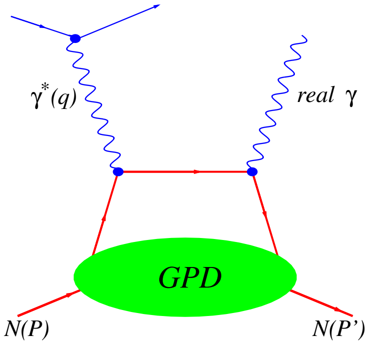
- GPDs depend on three parameters

$$x, \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+}, \qquad t = \Delta^2$$

where $\Delta = p' - p$ *and* $P^+ = (p' + p)^+/2$.

Chiral-even GPDs

- Chiral-even GPDs are accessible through Deeply Virtual Compton Scattering



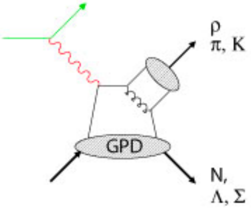
Chiral-odd GPDs

- There are four chiral-odd GPDs $H_T, \tilde{H}_T, E_T, \tilde{E}_T$ at leading twist

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\, i\sigma^{+i}\,\psi(\tfrac{1}{2}z)\;|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[H_T^q\, i\sigma^{+i} + \tilde{H}_T^q\,\frac{P^+\Delta^i - \Delta^+P^i}{m^2} + E_T^q\,\frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^q\,\frac{\gamma^+P^i - P^+\gamma^i}{m}\right] u(p,\lambda).$$

where $i = 1, 2$ is the transversity index [Diehl '03]

- Accessible through exclusive meson production processes



Properties of GPDs

- In the forward limit $\Delta \to 0$, certain GPDs are related to PDFs

$$H^q(x,0,0) = f_1^q(x)$$
$$\tilde{H}^q(x,0,0) = g_1^q(x)$$
$$H_T^q(x,0,0) = h_1^q(x).$$

- It follows from the time reversal invariance that under $\xi \to -\xi$

$$F^q(x,\xi,t) = F^q(x,-\xi,t) \text{ for } F^q = H^q, E^q, \tilde{H}^q, \tilde{E}^q, H_T^q, \tilde{H}_T^q, E_T^q$$
$$F^q(x,\xi,t) = -F^q(x,-\xi,t) \text{ for } F^q = \tilde{E}_T^q$$

- Related to form factors via

$$\int_{-1}^{1} \left\{H, E, \tilde{H}, \tilde{E}\right\}(x,\xi,t)dx = F_1(t), F_2(t), G_A(t), G_P(t)$$
$$\int_{-1}^{1} \left\{H_T, \tilde{H}_T, E_T\right\}(x,\xi,t)dx = H_T(t), \tilde{H}_T(t), E_T(t)$$
$$\int_{-1}^{1} \left\{\tilde{E}_T\right\}(x,\xi,t)dx = 0$$

Goloskokov-Kroll Model

- Goloskokov-Kroll(GK) model for pseudoscalar meson production considers the region of small ξ and small $-t$, but large Q^2 and W.
  [Goloskokov-Kroll '11]
- For longitudinally polarized photons and longitudinally polarized mesons, the differential cross section is given by

$$\frac{d\sigma_L}{dt} = \kappa[\mathcal{M}^2_{0+,0+} + \mathcal{M}^2_{0-,0+}]\,,$$

for π^0 production

$$\mathcal{M}_{0+,0+} = \sqrt{1-\xi^2}\frac{e}{Q}[\langle\tilde{H}\rangle - \frac{\xi^2}{1-\xi^2}\langle\tilde{E}\rangle]$$

$$\mathcal{M}_{0-,0+} = \frac{e}{Q}\frac{-t'}{2m}[\xi\langle\tilde{E}\rangle]]$$

Goloskokov-Kroll Model

- Generically, $\langle F \rangle$ represents a convolution of a GPD F with an appropriate subprocess amplitude

$$\langle F \rangle = \sum_{\lambda} \int_{-1}^{1} dx \, \mathcal{H}_{0\lambda,0\lambda}(x, \xi, Q^2, t=0) \, F(x, \xi, t)$$

where λ denotes unobserved helicities of the partons.

- Subprocesses are calculated in the so-called modified perturbative approach: Transverse momenta of the quark and the anti-quark in the meson are kept and gluon radiations are taken into account through Sudakov factor
- In impact space

$$\mathcal{H}_{\pi} = \int d\tau d^2\vec{b} \, \hat{\Psi}_{\pi}(\tau, -\vec{b}) \hat{\mathcal{F}}^{i}_{\pi}(\bar{x}, \xi, \tau, Q^2, \vec{b}) \alpha_s(\mu_R) \, exp\big(- S(\tau, \vec{b}, Q^2)\big)$$

Goloskokov-Kroll Model

- Hard scattering kernels, for example $\mathcal{F}_V^q$, has the following form in momentum space

$$\mathcal{F}_\pi^q = \frac{N_c^2-1}{2N_c}\sqrt{\frac{2}{N_c}}\frac{Q}{\xi}\left[\frac{1}{k_\perp^2+\bar{\tau}(\bar{x}+\xi)Q^2/(2\xi)-i\epsilon} - \frac{1}{k_\perp^2-\tau(\bar{x}-\xi)Q^2/(2\xi)-i\epsilon}\right]$$

- A Gaussian meson wave function is used

$$\Psi_\pi(\tau,\vec{b}) \sim \tau(1-\tau)exp\left[-\frac{\tau(\tau-1)}{4}\frac{\vec{b}^2}{a_\pi^2}\right]$$

- Sudakov factor has the form

$$S(\tau,b,Q) = s(\tau,b,Q) + s(\bar{\tau},b,Q) - \frac{4}{\beta_0}\; ln\,\frac{ln(\mu_R/\Lambda_{QCD})}{\hat{b}}$$

where

$$s(\tau,b,Q) = \frac{8}{3\beta_0}\left(\hat{q}\; ln\left(\frac{\hat{q}}{\hat{b}}\right) - \hat{q} + \hat{b}\right) + NLL$$

$$\hat{b} = -ln(b\,\Lambda_{QCD})$$

$$\hat{q} = ln\left(\tau Q/(\sqrt{2}\Lambda_{QCD})\right)$$

Goloskokov-Kroll Model

- Contributions from transversely polarized photons: Twist-3 effects
- Twist-3 contribution consists of transversity GPDs

$$\mathcal{M}_{0-,++} = \sqrt{1-\xi^2}e\,\langle H_T \rangle$$
$$\mathcal{M}_{0+,\mu+} = -\frac{e}{4m}\sqrt{-t'}\langle \bar{E}_T \rangle$$

with twist-3 meson wave function

$$\Psi_\pi(\tau,\vec{b}) \sim exp\Big[-\frac{\vec{b}^2}{8a_\pi^2}\Big]\, I_0(\frac{\vec{b}^2}{8a_\pi^2})$$

Goloskokov-Kroll Model

- Partial cross sections

$$\frac{d\sigma_T}{dt} = \frac{\kappa}{2}[\mathcal{M}^2_{0-,-+} + \mathcal{M}^2_{0-,++} + \mathcal{M}^2_{0+,-+} + \mathcal{M}^2_{0+,++}]\,,$$

$$\frac{d\sigma_{TT}}{dt} = -\frac{\kappa}{2}Re[\mathcal{M}^*_{0-,++} \cdot \mathcal{M}_{0-,-+} + \mathcal{M}^*_{0+,++} \cdot \mathcal{M}_{0+,-+}]\,,$$

$$\frac{d\sigma_{LT}}{dt} = \frac{\kappa}{\sqrt{2}}Re[\mathcal{M}^*_{0-,0+} \cdot (\mathcal{M}_{0-,++} - \mathcal{M}_{0-,-+}) + \mathcal{M}^*_{0+,0+} \cdot (\mathcal{M}_{0+,++} - \mathcal{M}_{0+,-+})]\,.$$

Goloskokov-Kroll Model

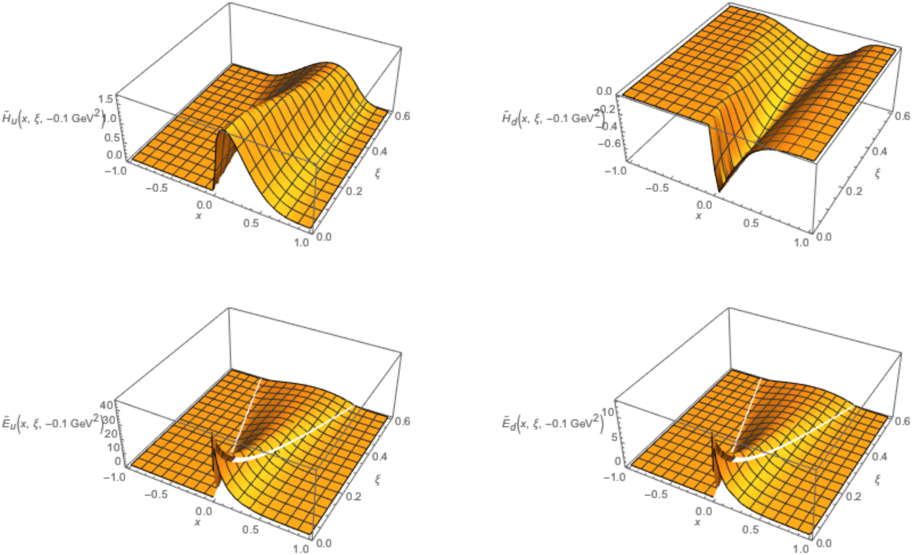
- Lastly, GPDs are constructed from double distribution ansatz

$$H_i(\bar{x}, \xi, t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \; \delta(\beta + \xi\alpha - \bar{x}) f_i(\beta, \alpha, t)$$

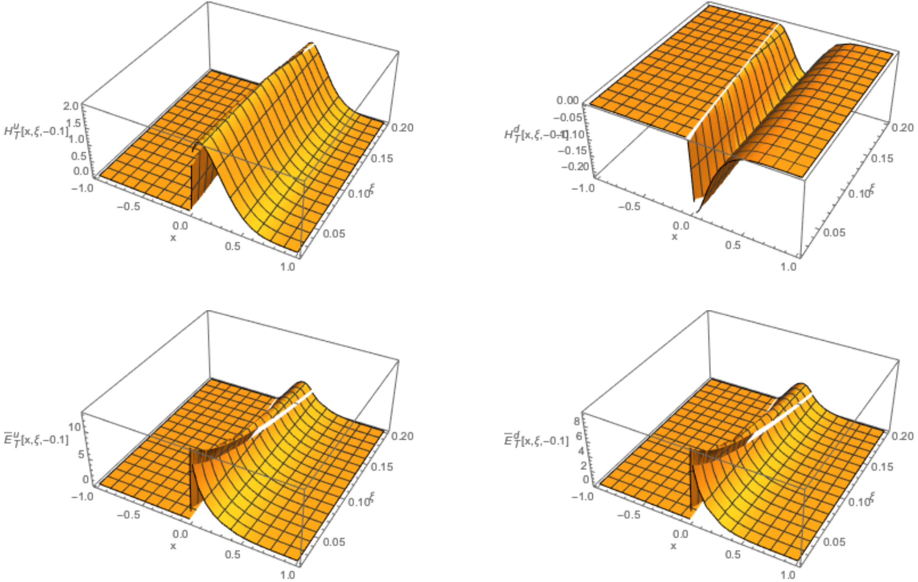
with

$$f_i(\beta, \alpha, t) = exp[(b_i - \alpha'_i \, ln\, \beta)t] \, F_i(\beta, \xi = t = 0) \, \frac{3}{4} \frac{(1-\beta)^2 - \alpha^2}{(1-\beta)^3} \Theta(\beta)$$

Goloskokov-Kroll Model



Goloskokov-Kroll Model

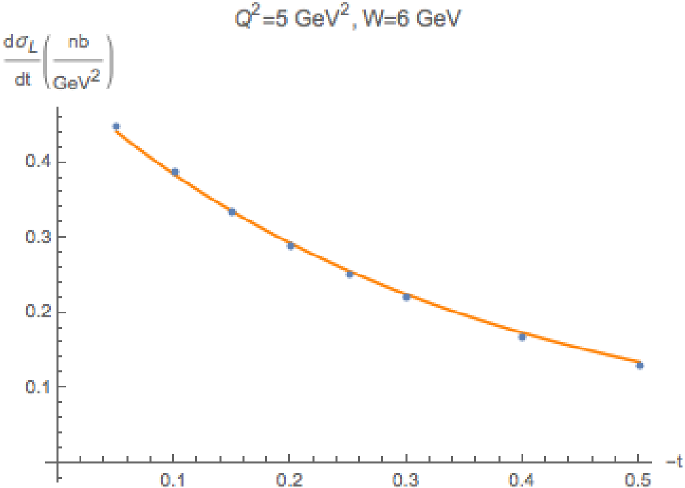


Goloskokov-Kroll Model

- So altogether, to compute the longitudinal cross section, we need: Twist-2 meson wave function, kernel, running coupling, Sudakov factor and GPDs $\tilde{H}$ and $\tilde{E}$.
- To compute the transverse cross section, we need: Twist-3 meson wave function, kernel, running coupling, Sudakov factor and GPDs H_T and $\bar{E}_T$.
- 3 dimensional integrals, over $\bar{x}, \tau$ and b, are performed in impact space
- In many different processes, like the vector meson or pseudoscalar meson production, meson wavefunction has the same structure.
- Sudakov factor has also the same structure

Goloskokov-Kroll Model

- Comparision between hepgen and our Mathmematica code



Goloskokov-Kroll Model

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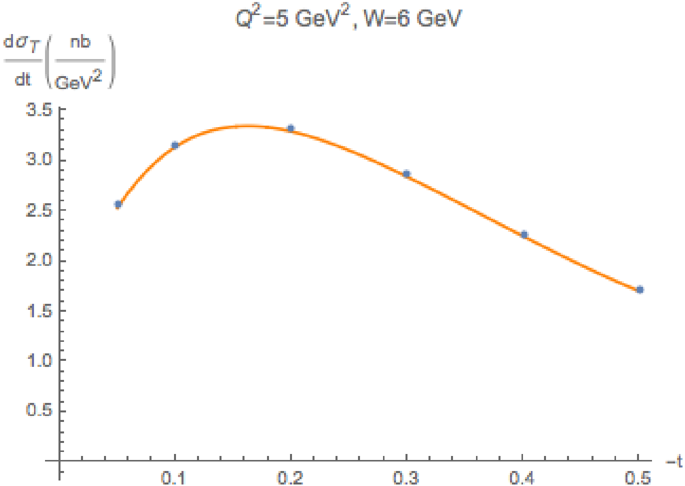

Goloskokov-Kroll Model

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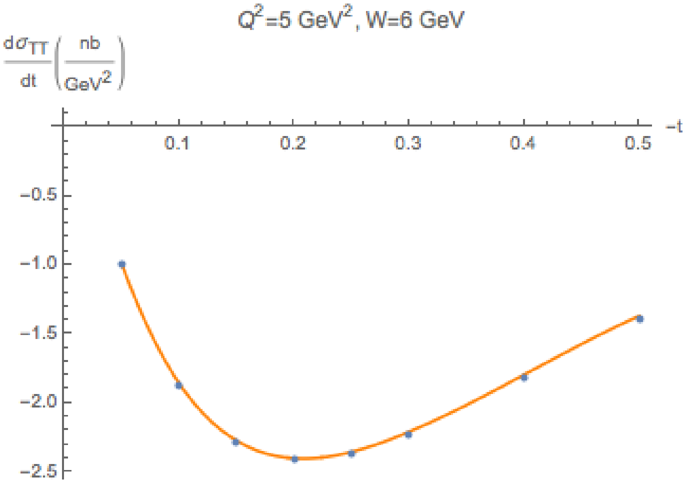

PARTONS

Eur.Phys.J. C78 (2018) 6, 478

- PARTONS is a computing framework for the phenomenology of GPDs
- Written in C++: Inheritance and polymorphism
- Layered structure:
  - GPD Layer
  - CFF Layer
  - Process Layer
  - Observable Layer
- Has already been implemented for DVCS

Goloskokov-Kroll Model in PARTONS (Conclusions)

- Inclusion of partial cross sections and total cross section into PARTONS for pseudoscalar meson (π^0) production
- Electroproduction of π^+ has a similar scenario; GPDs appear in a different flavor combination. However, there is also a pion pole contribution
- General structure:
    - GPDs
    - Sudakov factor
    - Meson wave functions
    - Running coupling
    - Subprocess amplitudes
    - Convolutions
    - Amplitudes
    - Cross section