Nucleon electric dipole moment on the lattice

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χQCD collaboration

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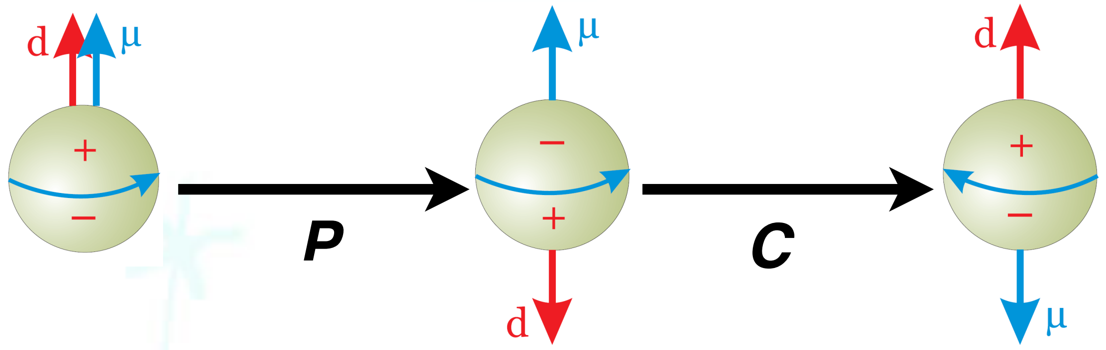
EDM and CP-violation

- The CP violation allowed in the SM (the CKM phase) is **insufficient for Baryogenesis** under Sakharov conditions, BSM interactions?

A. D. Sakharov, JETP Lett. 5 24-27 (1967)

- A non-zero intrinsic electric dipole moment (EDM) of a fundamental particle violates the CP(T) symmetry.



- Nucleon EDM (nEDM) is a sensitive probe of BSM: the contribution to the nEDM from the weak CP-violating (CPV) phase is $\sim 10^{-31}$ e·cm, 10^{-5} of the current experimental limit.

- nEDM is important for theta QCD and the strong CP problem.

- **Lattice QCD: model-independent connection between the CPV interactions (theta term and BSM) and the nEDM.**

Experiments

First experiment: $|d_n| < 5\times10^{-20}$ e·cm

Smith et al., RP108:120-122 (1957)

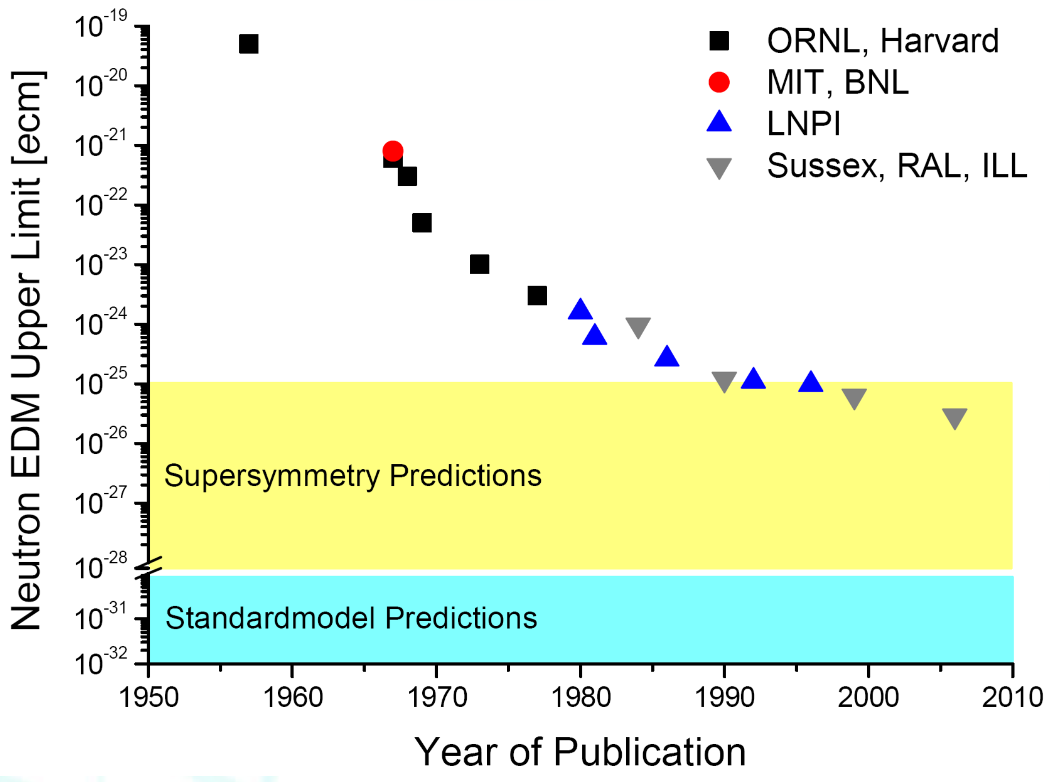
Current neutron EDM limits: $\sim10^{-26}$ e·cm

Baker et al, PRL97:131801 (2006)

Graner et al, PRL116:161601 (2016)

Most recent result: $0.0(1.1)(0.2)\times10^{-26}$ e·cm

C. Abel et al., PRL124:081803 (2020)



During the past ~~50~~ 60 years of experiments, **six orders of magnitude** have been covered.

Several experiments are aiming at improving the limit down to 10^{-28} e·cm in the next ~10 years. It is still a long way to trek to detect a non-zero nEDM but leaves plenty of room for BSM.

CP-violating operators and lattice methodology

Operators at the energy scale of hadronic matter

- theta term $iG_{\mu\nu}\tilde{G}_{\mu\nu}$ dim-4
- quark EDM $i\bar{\psi}[\tilde{F}_{\mu\nu}\sigma^{\mu\nu}]\psi$ dim-5
- quark Chromo-EDM $i\bar{\psi}[\tilde{G}_{\mu\nu}\sigma^{\mu\nu}]\psi$ dim-5
- glue Chromo-EDM (Weinberg term) $f^{abc}\tilde{G}^{a\ \nu}_{\mu}G^{a\ \rho}_{\nu}G^{a\ \mu}_{\rho}$ dim-6
- 4-quark operators? $\bar{\psi}\gamma_5\psi\bar{\psi}\psi$ dim-6

Problematic due to renormalization and mixing

R. Gupta, arXiv:1904.00323

Introducing CPV interactions

- MC simulation with an imaginary θ term
- Taylor expansion in terms of small couplings (theta term and Weinberg term)
- Modifying Dirac operator for inversions and reweighting (quark bilinear terms)

Lattice observables

- CPV EM from factor (FF) from nucleon matrix element
- Nucleon energy shift in the present of a background electric field

Correction of the CPV FF

The CPV terms alter the Dirac equation and spinors

$$(i\not{p} + m' e^{-2i\alpha(\theta)\gamma_5}) u'(p,s) = 0 \qquad u' = e^{i\alpha^1 \theta \gamma_5} u \qquad \bar{u}' = \bar{u} e^{i\alpha^1 \theta \gamma_5}$$

New spinors affect the FF decomposition of nucleon matrix elements

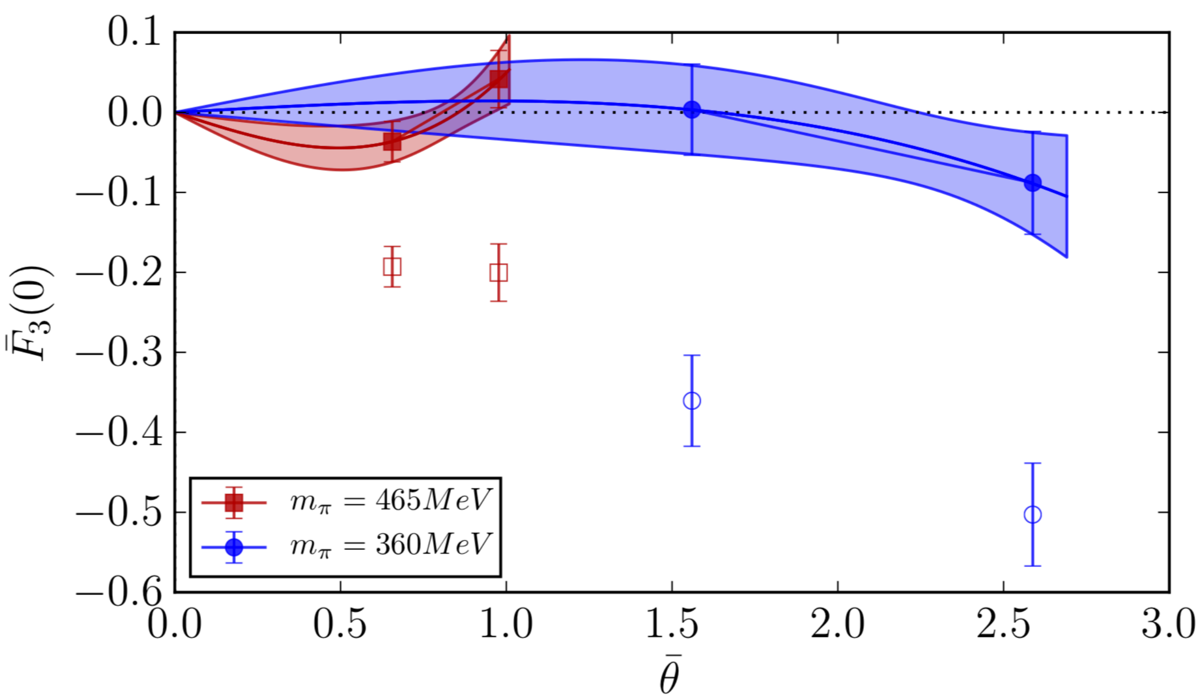
$$\langle N(p') | \bar{\psi}\gamma_\mu \psi | N(p) \rangle_{\not{CP}} = \underline{\bar{u}'(p')} \left[F_1(q^2)\gamma_\mu - \left[F_2(q^2) + i\theta F_3(q^2)\gamma_5 \right] \frac{i\sigma_{\mu\nu} q_\nu}{2m_N} \right] \underline{u'(p)}$$

$$u(p) \to u(\tilde{p}) = \gamma_4 u(p)$$

$$u'(p) \to u'(\tilde{p}) = e^{i\alpha^1 \theta \gamma_5} \gamma_4 u(p)$$

$$F_3^{\mathrm{new}} = F_3 + 2\alpha^1 F_2$$

Abramczyk et al., PRD96:014501 (2017)

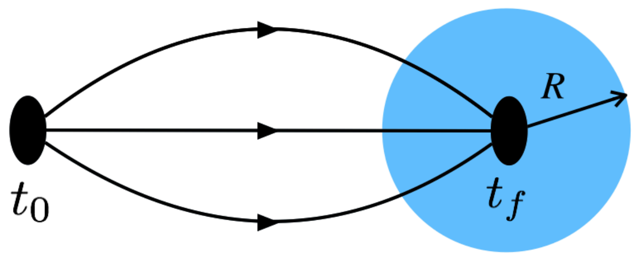


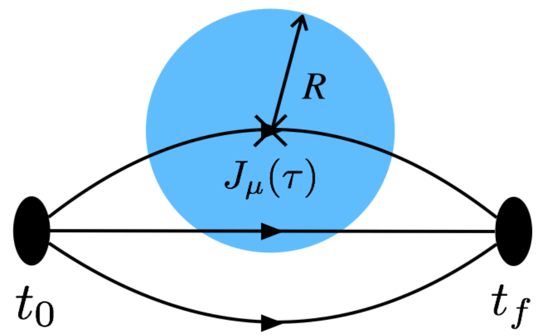
More crucial to reduce error!

Error reduction

The cluster decomposition error reduction (CDER):

*Liu, **Liang** and Yang, PRD97:034507 (2018)*

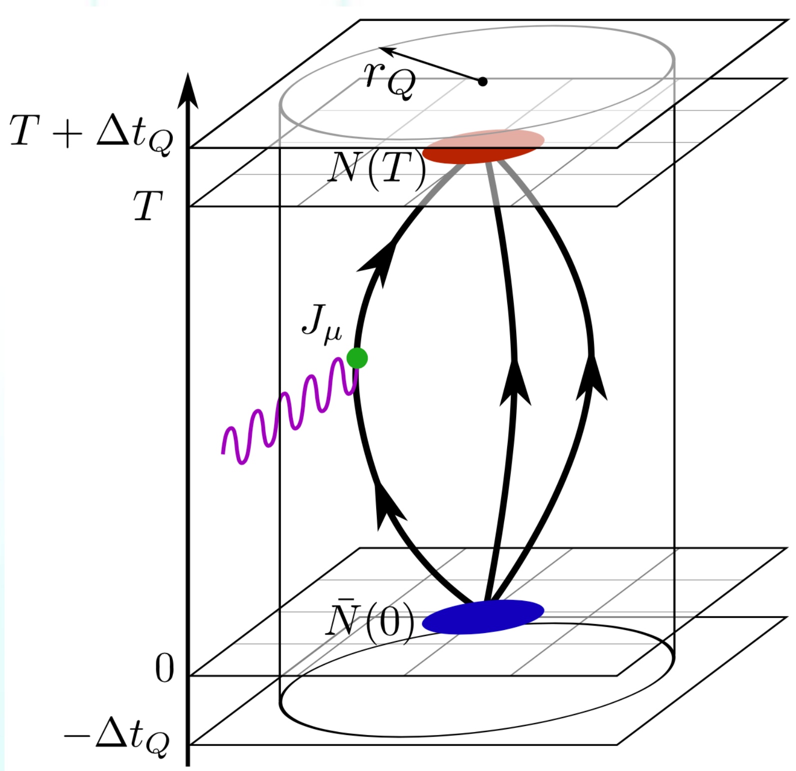




$$C_3^Q(t_f, R) = \sum_{\vec{x}} \left\langle \sum_{r}^{|r|\leq R} q(x+r)\chi(x)\bar{\chi}(0) \right\rangle$$

$$C_4^Q(t_f, \tau, R) = \sum_{\vec{x}\vec{y}} e^{-i\vec{p}\cdot\vec{x}} e^{i\vec{q}\cdot\vec{y}} \left\langle \chi(x) \sum_{r}^{|r|\leq R} q(y+r) J_\mu(y) \bar{\chi}(0) \right\rangle$$

Cylinder shape



T. Izubuchi, H. Ohki and S. Syritsyn, arXiv:2004.10449

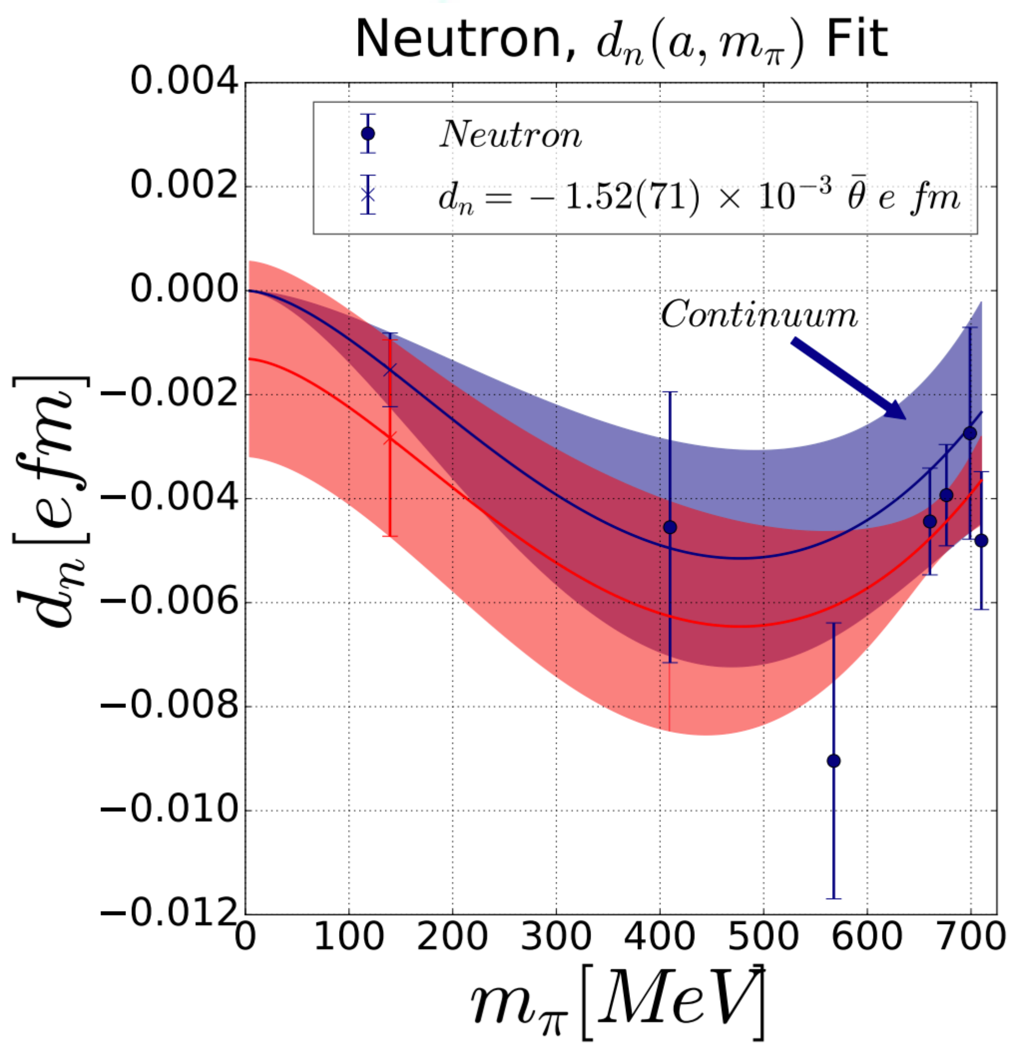
Truncation in *t*-direction

$$\overline{G}_3^{(\bar{Q})}(\boldsymbol{p}', t, \boldsymbol{q}, \tau, \Pi, \gamma_\mu, t_f, t_s)$$

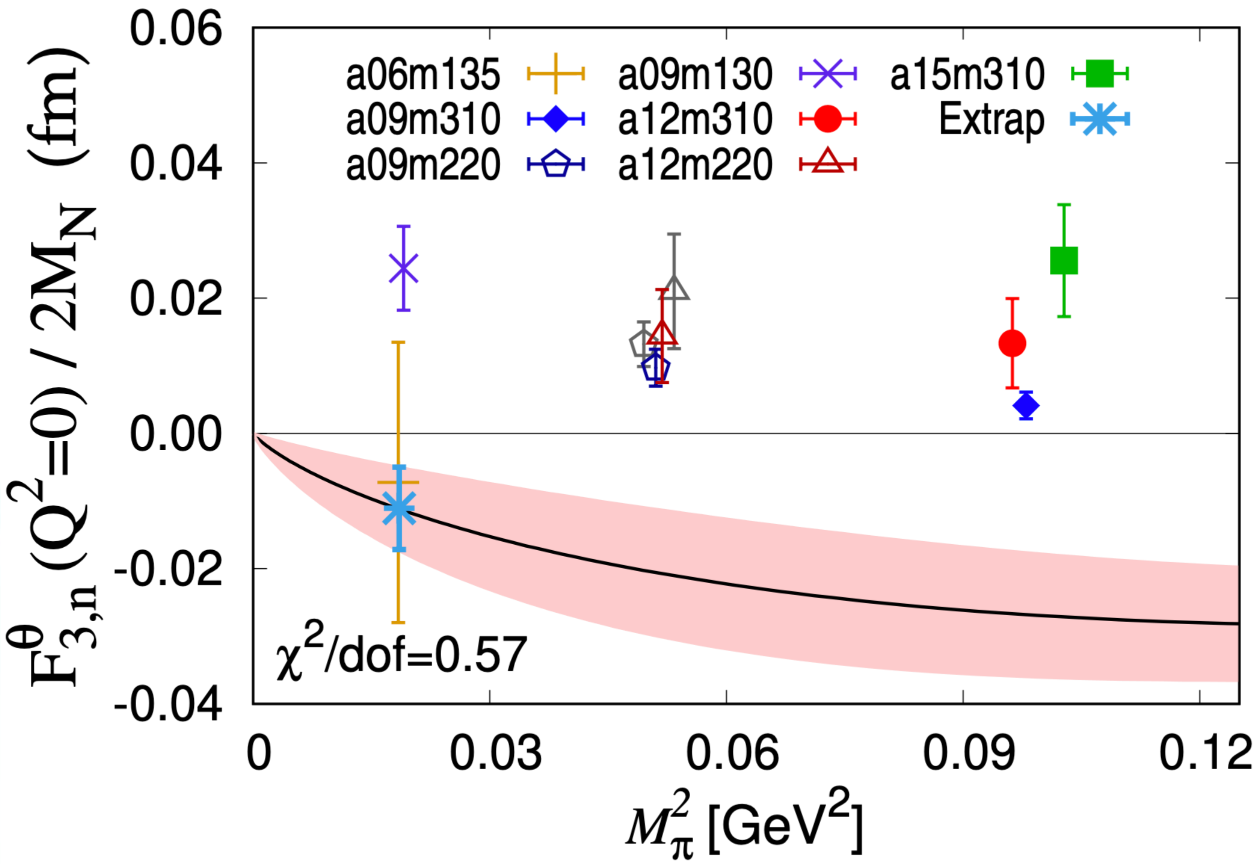
$$= a \sum_{\frac{\tau_Q}{a}=0}^{t_s/a} \Big[\Delta_3^{(\bar{Q})}(\boldsymbol{p}', t, \boldsymbol{q}, \tau, \tau_Q, \Pi, \gamma_\mu, t_f) + \Delta_3^{(\bar{Q})}(\boldsymbol{p}', t, \boldsymbol{q}, \tau, T - \tau_Q, \Pi, \gamma_\mu, t_f) \Big]$$

J. Dragos et al., arXiv:1902.03254

Recent results (theta term)



J. Dragos et al., arXiv:1902.03254

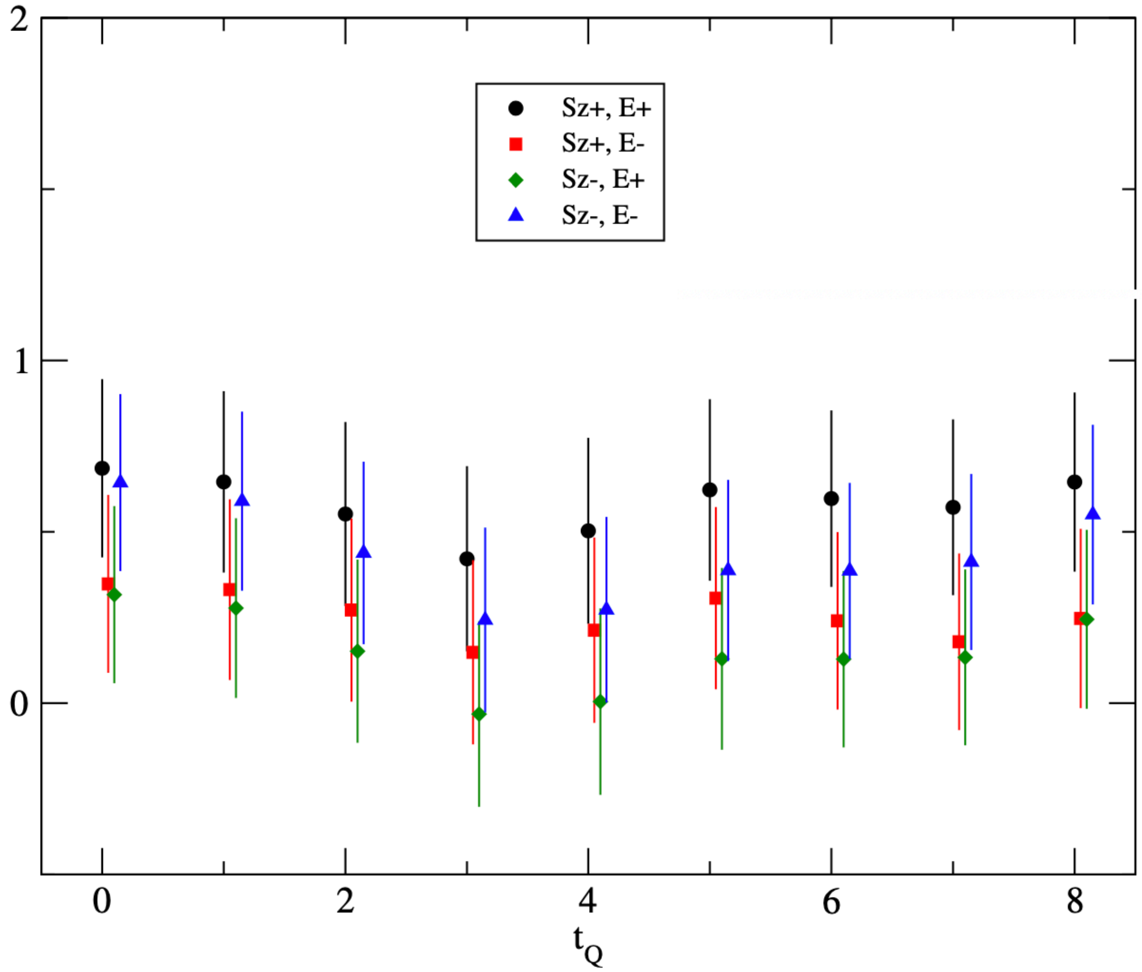


B. Yoon et. al., arXiv:2003.05390

Chiral extrapolation with heavy pion masses and with non-chiral fermion

Non-zero signal at the physical point, but large order *a* extrapolation

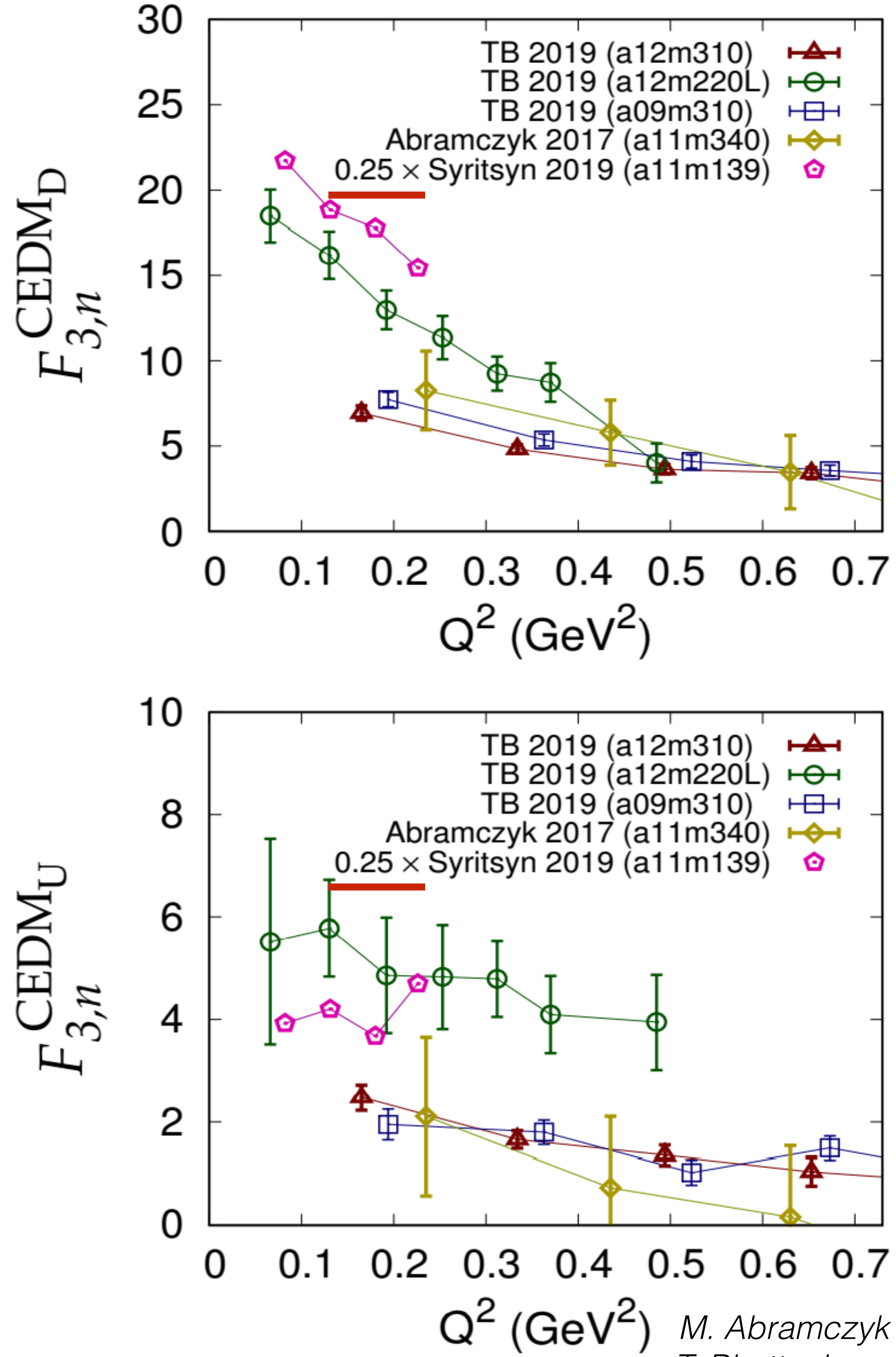
Recent results (theta term)

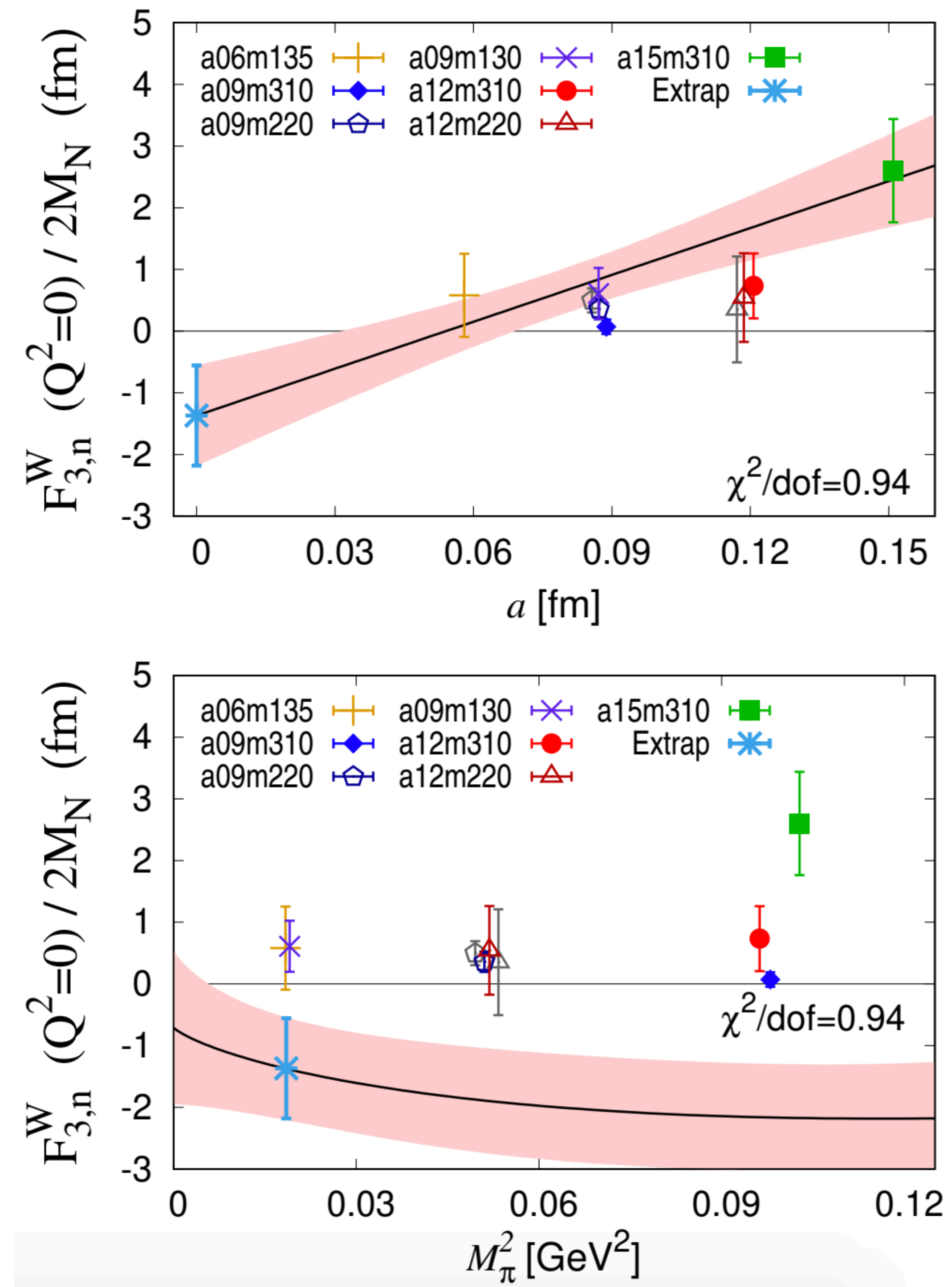


T. Izubuchi, H. Ohki and S. Syritsyn, arXiv:2004.10449

Energy shift method in the presence of a background electric field with local topological charge (no *momentum transfer* extrapolation is required), F3(0) at pion mass ~330 MeV

Recent results (BSM)





M. Abramczyk et. al., PRD96: 014501 (2017)
T. Bhattacharya, R. Gupta and B. Yoon, arXiv:2003.08490
S. Syritsyn, T. Izubuchi and H. Ohki, ArXiv:1901.05455

B. Yoon et. al., arXiv:2003.05390

Strong quark mass dependence (or other unknown systematic uncertainties) of cEDM

Large statistical error in the Weinberg term case

Theta QCD with chiral fermions

Overlap operator D_{ov} satisfies the Ginsparg-Wilson relation, and the effective quark propagators we use are $1/(D_c + m)$ where $D_c = D_{ov}/(1 - 1/2D_{ov})$ which anti-commutes with γ_5, the same as in the continuum.

For overlap fermions, the anomalous Ward identity (AMI) has been proven (with chiral axial vector current) and numerically checked (with local axial current plus a normalization constant the same as the iso-vector case) at finite lattice spacings.

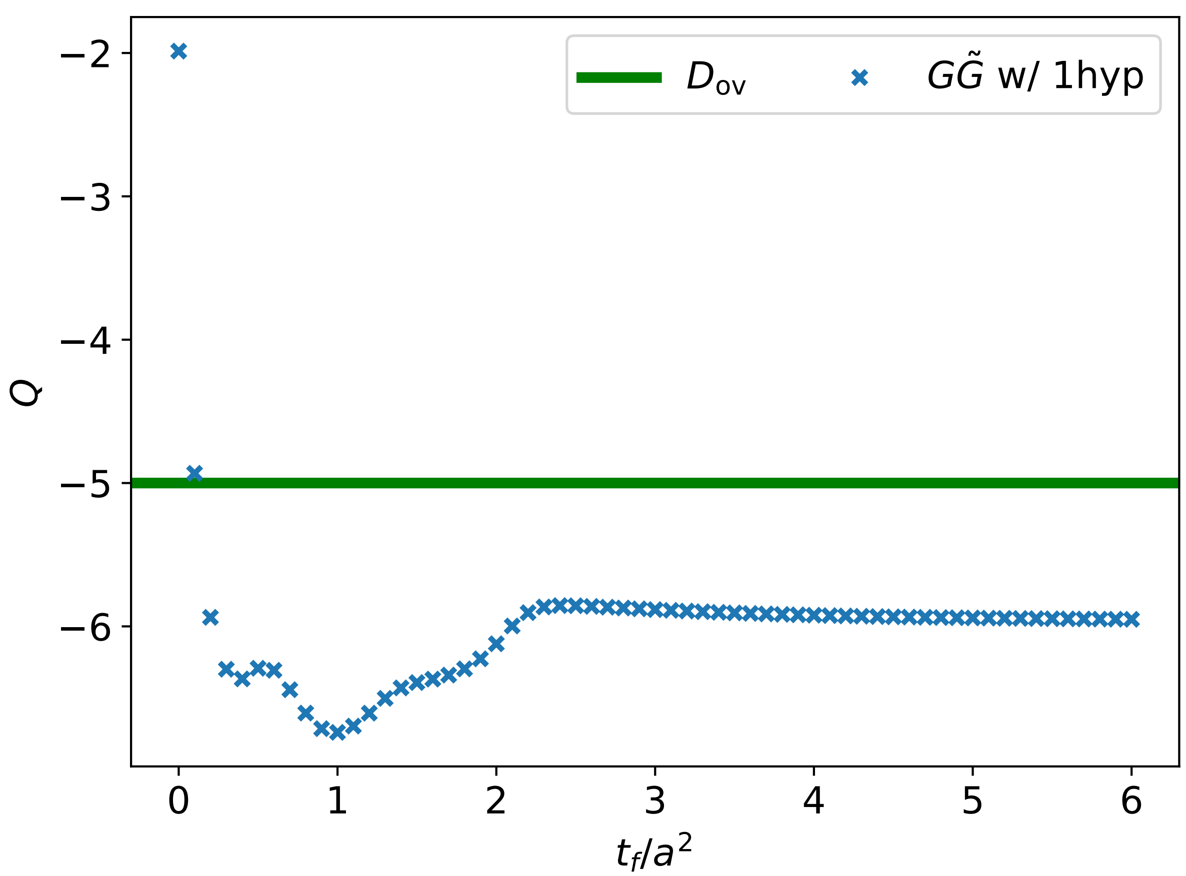
P. Hasenfratz, et. al., NPB643:280 (2002)
***J. Liang** et. al., PRD98:074505 (2018)*

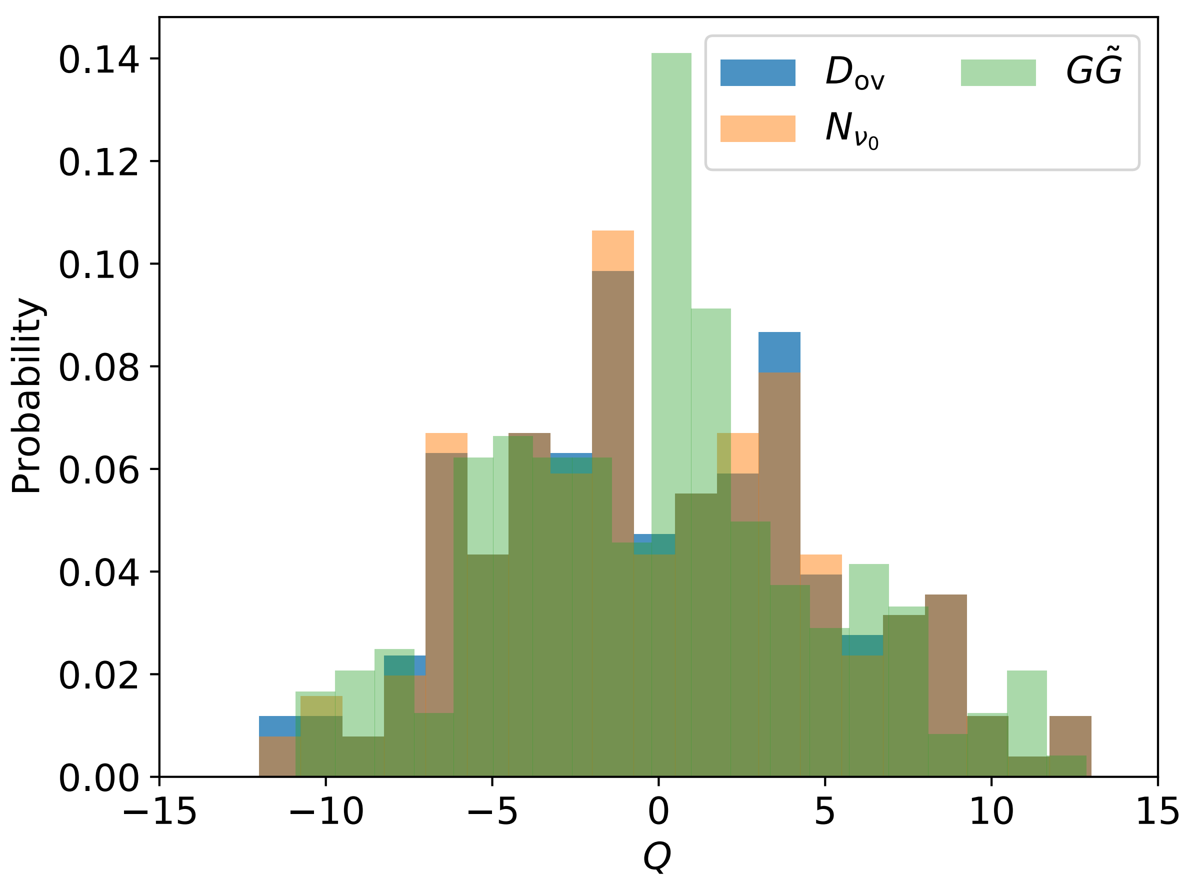
With the AWI, it can be shown that the topological charge term can be replaced with the *2mP* term, which grantees that **EDM→0 when m→0 even at finite lattice spacings**.

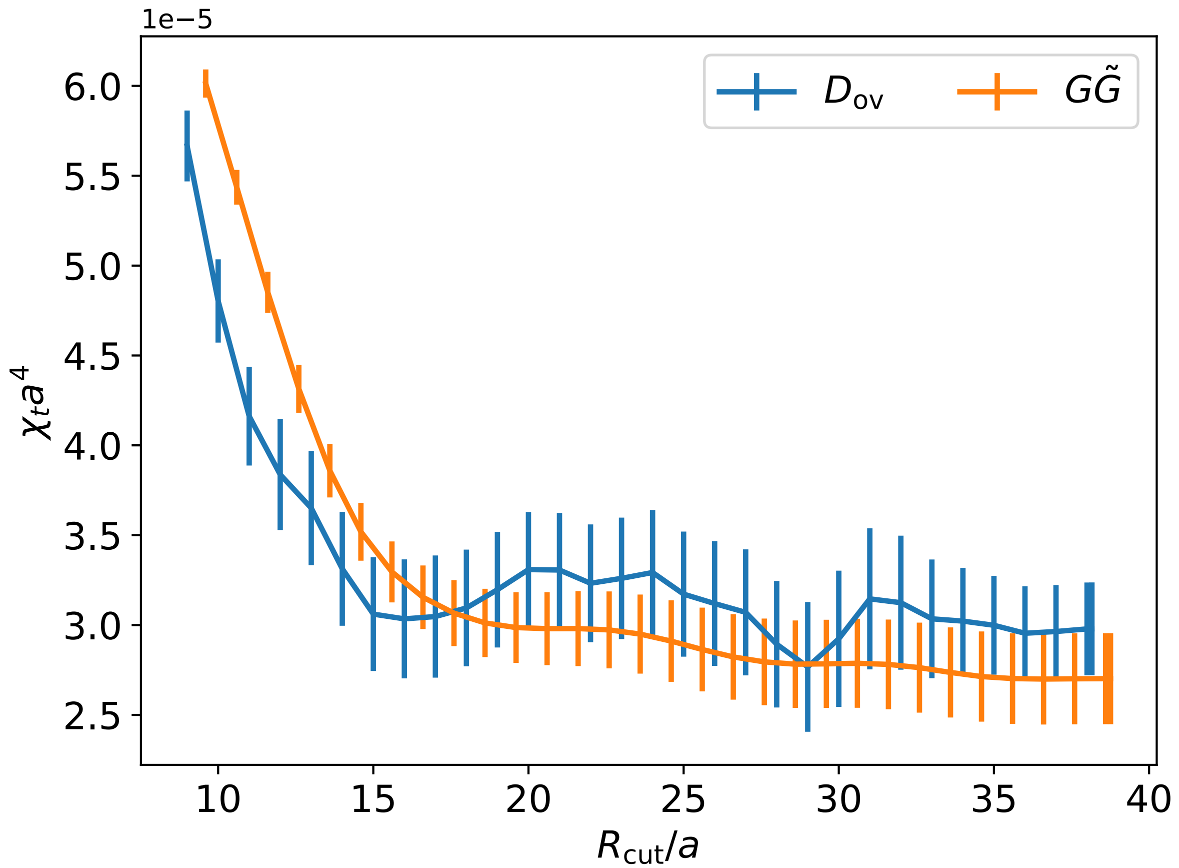
D. Guadagnoli, et. al., JHEP 0304, 019 (2003)

Topological charge can be defined from the overlap operator: $\frac{1}{2}\mathrm{Tr}[\gamma_5 D_{\mathrm{ov}}]$

Topological charges



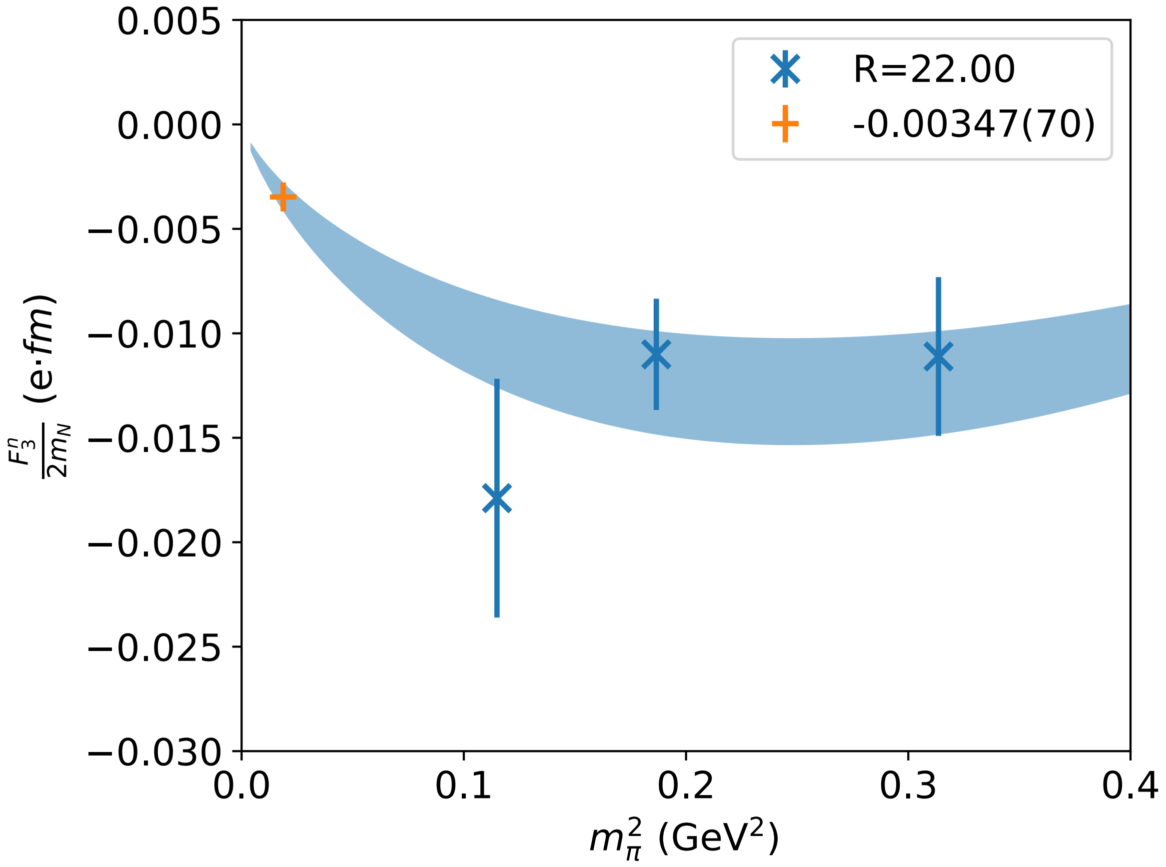


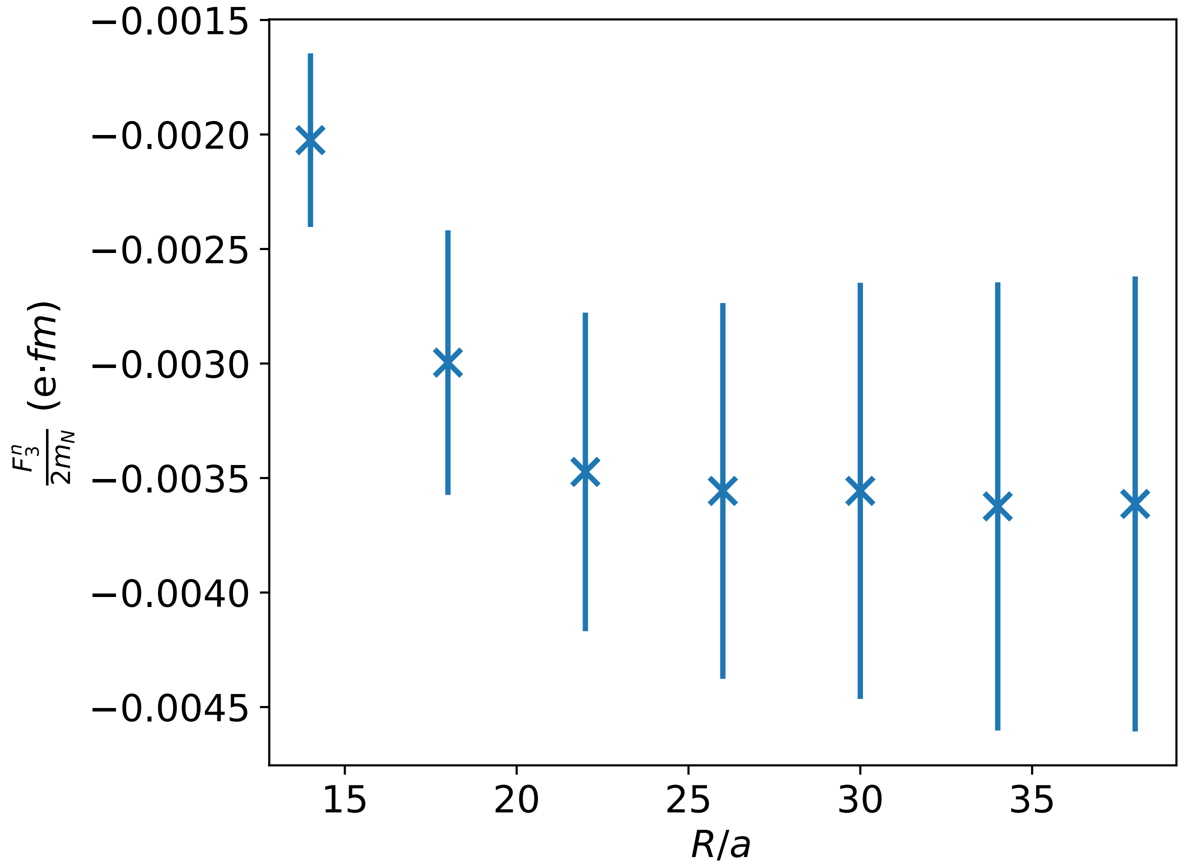


The topological charges of individual configurations with different definitions are different, which is natural as they involve different regulations. Distributions are similar.

For physical quantities such as the topological susceptibility, **different definitions agree within statistical errors**.

Preliminary results





$$d_{p/n}(m_\pi) = C_1\ m_\pi^2 + C_2\ m_\pi^2 \log\left(\frac{m_\pi^2}{m_{N,phys}^2}\right)$$

W. H. Hockings and U. van Kolck, PLB605, 273 (2005)

We can have more **partially-quenched pion mass points** thanks to the multi-mass algorithm.

$$d_n^{(PQ)} = \frac{e\ \overline{\theta}\ m_{\rm sea}}{4\pi^2 f^2}\left[F_\pi\ \log\left(\frac{m_\pi^2}{\mu^2}\right) + F_J\ \log\left(\frac{m_J^2}{\mu^2}\right)\right] + \overline{\theta}\ \frac{e}{\Lambda_\chi^2}\left[\frac{m_{\rm sea}}{2}\ c(\mu) + d\,(m_{\rm sea} - m_{\rm val}) + f q_{jl}\ (m_{\rm sea} - m_{\rm val})\right]$$

D.O'Connell and M. J. Savage, PLB633:319 (2006)

Summary and outlook

- ✦ Many efforts in the community are made to study the nucleon EDM using lattice QCD.
- ✦ However, direct calculation at the physical point is quite challenging.
- ✦ *U(1)* chiral symmetry is of special importance in the calculation of nEDMs.
- ✦ Using overlap fermions ensures a correct chiral limit of nEDM even at finite lattice spacings.
- ✦ More statistics and more (partially-quenched) pion mass points will be added to have a more reliable chiral extrapolation.
- ✦ Systematic uncertainties should be carefully estimated.

Thank you for your attention!