K → ππ in RBC/UKQCD

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USQCD All Hands Meeting
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G-parity BC result released

RBC/UKQCD: R. Abbott, T. Blum, P.A. Boyle, M. Bruno, N.H. Christ, D. Hoying, C. Jung, C. Kelly, C. Lehner, R.D. Mawhinney, D.J. Murphy, C.T. Sachrajda, A. Soni, M. Tomii and T. Wang (arXiv:2004.09440)

- $\mathrm{Re}(\varepsilon'/\varepsilon)_{\mathrm{SM}} = 21.7\underbrace{(2.6)}_{\text{stat}}\underbrace{(6.2)}_{\text{sys}}\underbrace{(5.0)}_{\text{isospin breaking}} \times 10^{-4} \overset{\text{consistent}}{\equiv} \mathrm{Re}(\varepsilon'/\varepsilon)_{\mathrm{exp}} = 16.6(2.3) \times 10^{-4}$
- Various improvements to our previous result in 2015
  - 3+ times more configurations
  - multiple ππ operators → more accurate ππ phase shift
  - Renormalization scale increased by step scaling
  - …
- G-parity BC ensures: final ground ππ state is on-shell; $E_{\pi\pi} = m_K$

What's next?

Future works

- Continuum limit
- Isospin breaking / E & M correction

- Proposal 1: Calculation w/ Periodic BC
  - Important check of G-parity calculation with a different setup
  - Configuration generation & light-quark inversions already done
  - Flavor-unmixed Dirac operator in PBC is 2x cheaper than G-parity's
  - Challenging because needed on-shell final state is excited (solved by using multiple operators)

- Proposal 2: Improving Wilson Coefficients
  - Perturbative matching $w^{4f} \rightarrow w^{3f}$ causes one of the biggest errors
  - Nonperturbative matching
  - Can be applied to both of G-parity and periodic BC calculations

1. K → ππ w/ periodic boundaries

- Co-investigators

  T. Blum (PI, UConn/RBRC), D. Hoying (RBRC), T. Izubuchi (BNL/RBRC),
  L. Jin (UConn/RBRC), C. Jung (BNL), C. Kelly (BNL), A. Soni (BNL),
  MT (UConn)

- RBC & UKQCD Collaborations

- Requests
  - 35 M KNL core-hours at JLab or BNL
  - 340 TB new tape storage + current allocation of 386 TB tape & 180 TB disk

Ensembles

- RBC/UKQCD's 2+1-flavor ensembles with Möbius domain-wall fermions at physical pion & kaon masses
  - 24^3 x 64 lattice at a^{-1} = 1.0 GeV, 200 confs
  - 32^3 x 64 lattice at a^{-1} = 1.4 GeV, 200 confs

What to calculate

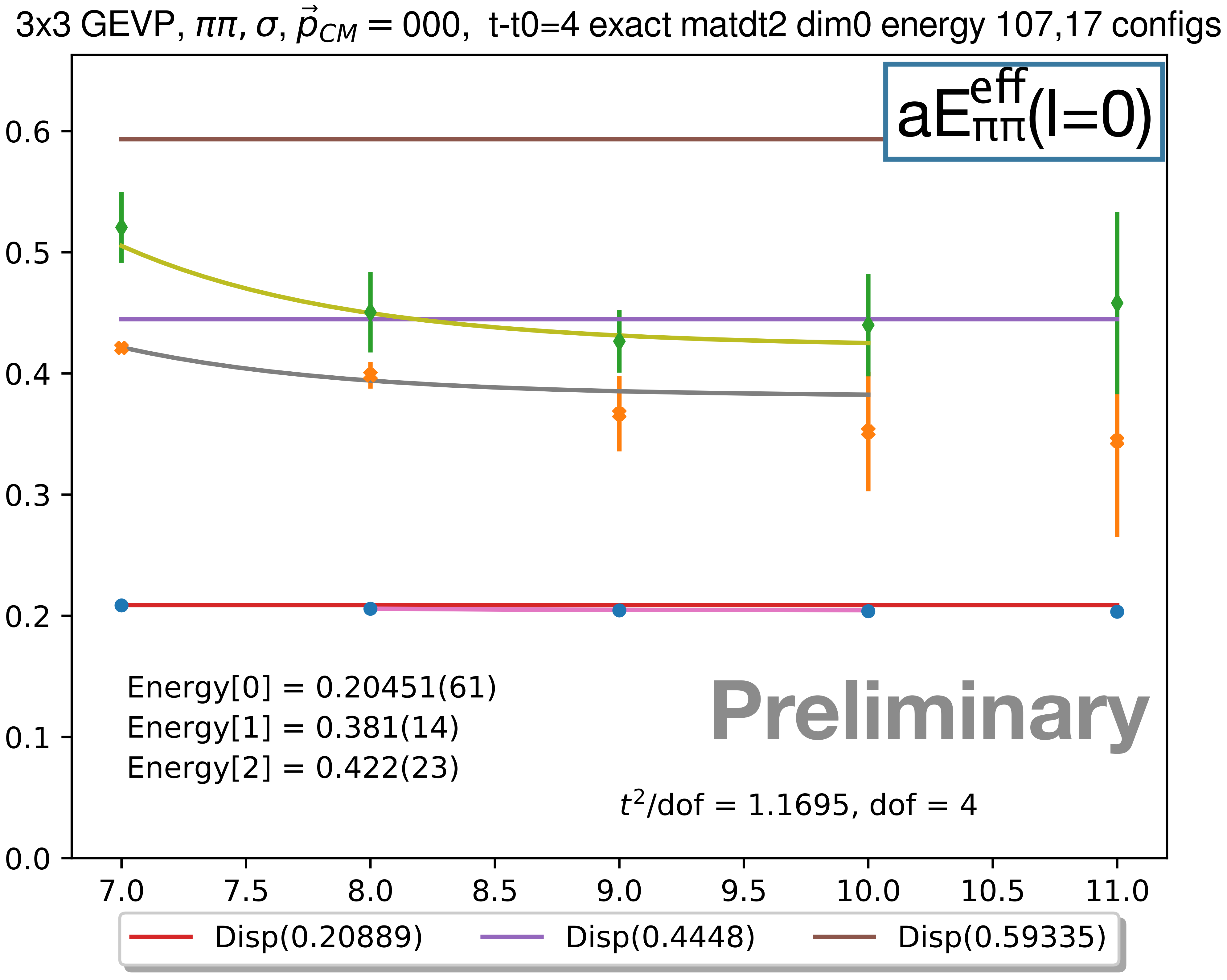
$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \mathrm{Re}\left\{\frac{\mathrm{i}\omega e^{\mathrm{i}(\delta_2-\delta_0)}}{\sqrt{2}\epsilon}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]\right\}$$

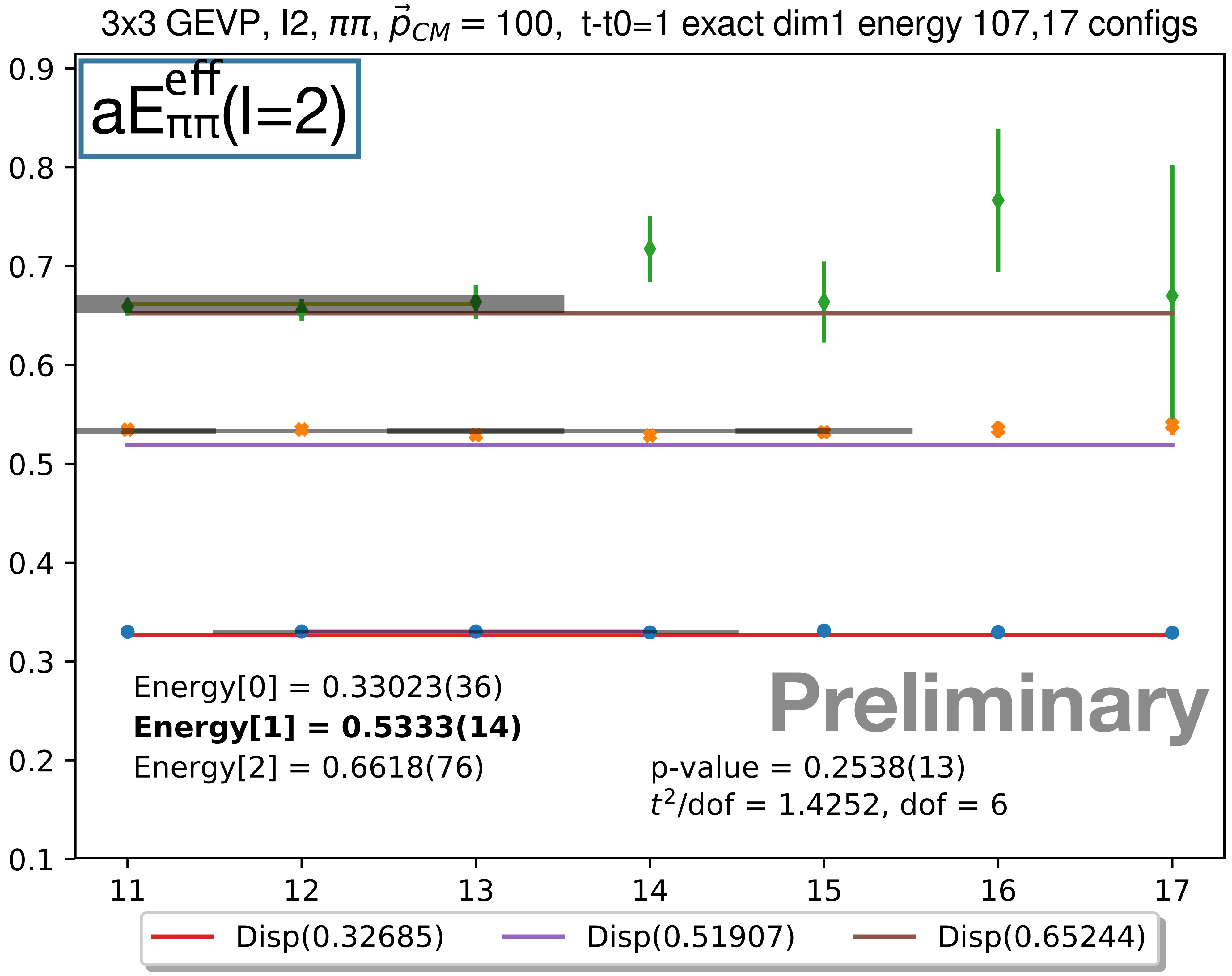
- δ_I : ππ phase shifts (measurements finished)
  - 2pt functions of 2-pion operators & GEVP → ππ-state energies
  - ‣ Lüscher's formalism → ππ phase shifts
- $A_I = \langle (\pi\pi)_I \,|\, H_W \,|\, K \rangle$ (plan for 20-21 allocation year)
  - 4pt functions

Achievements in 19-20

Led by D. Hoying

- ππ energies from 2pt functions
  - GEVP w/ multiple operators operators

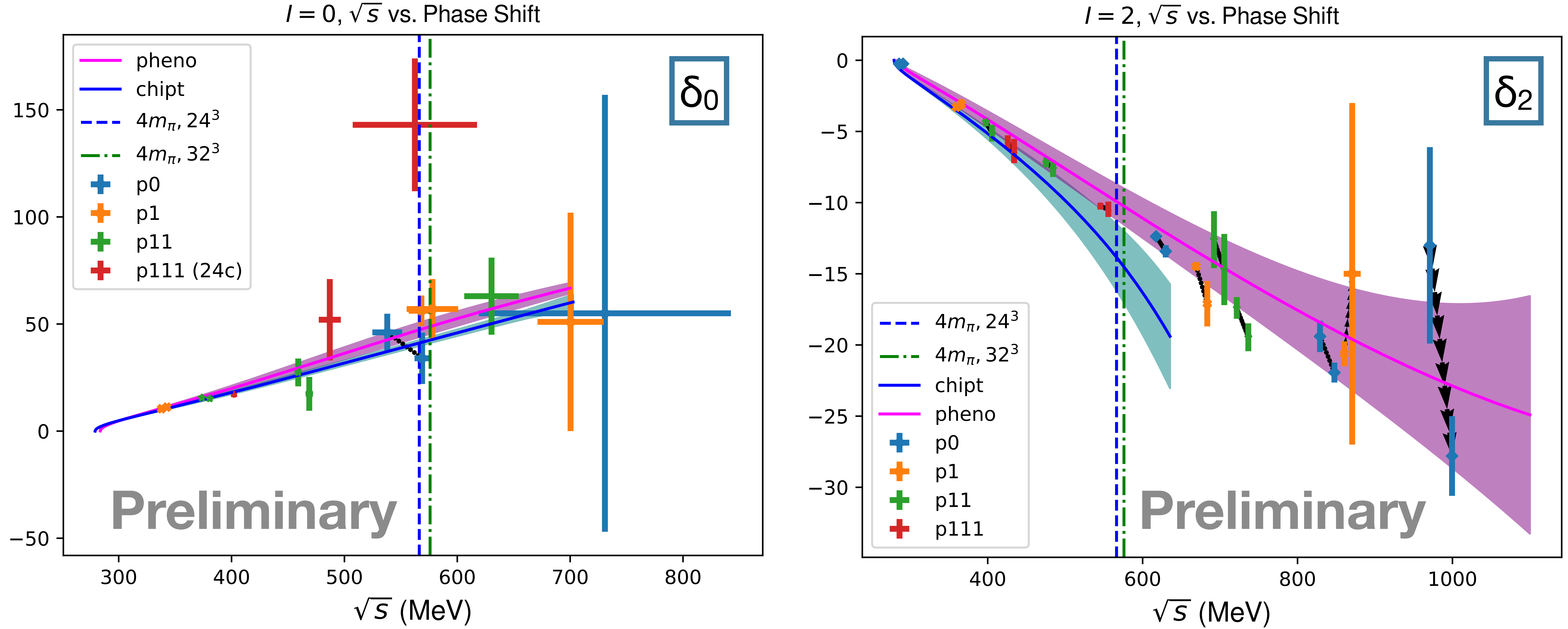




Achievements in 19-20

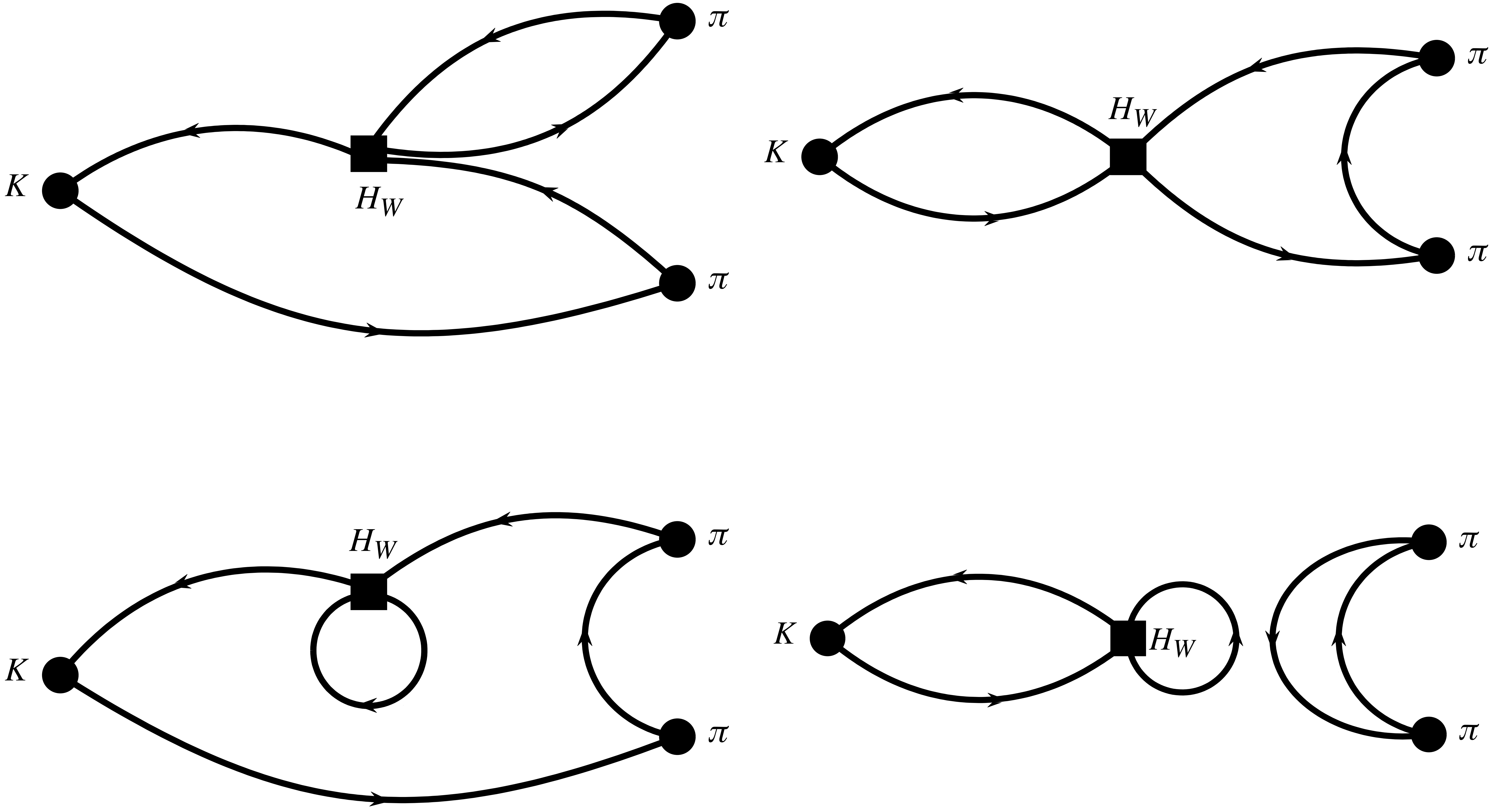
Led by D. Hoying

- ππ phase shifts
  - Lüscher's formula: $\tan\delta = -\dfrac{\pi^{3/2}\mathrm{k}}{\mathrm{Z}_{00}(1;\mathrm{k})}$



Plan for 20-21

- 4pt functions
  - A2A propagators already calculated in ππ scattering work and saved
  - Contractions of 4 types of diagrams



A2A propagator w/ V & W vectors

$$D_{A2A}^{-1} = \sum_{l=1}^{N_l} |\phi_l\rangle \frac{1}{\lambda} \langle \phi_l| + \frac{1}{N_h} \sum_{h=1}^{N_h} \underbrace{\left(D^{-1} - \sum_{l=1}^{N_l} |\phi_l\rangle \frac{1}{\lambda} \langle \phi_l| \right)}_{D_{defl}^{-1}} |\eta_h\rangle\langle\eta_h|$$

$$= \sum_{i=1}^{N_l+N_h} |V_i\rangle\langle W_i|$$

- V & W vectors

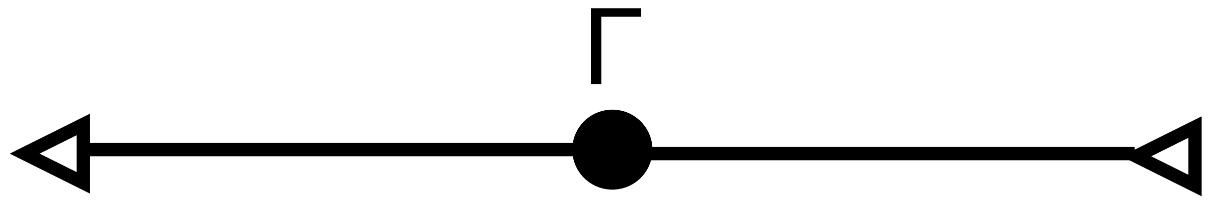
$$1 \le i \le N_l \ \Rightarrow \ \underline{|V_i\rangle = \frac{1}{\lambda}|\phi_i\rangle, \quad |W_i\rangle = |\phi_i\rangle}$$

$$N_l + 1 \le i (= N_l + h) \le N_l + N_h \ \Rightarrow \ \underline{|V_i\rangle = \frac{1}{N_h} D_{defl}^{-1} |\eta_h\rangle, \quad |W_i\rangle = |\eta_h\rangle}$$

V & W for light quark generated using CPS/Grid as a part of ππ scattering work

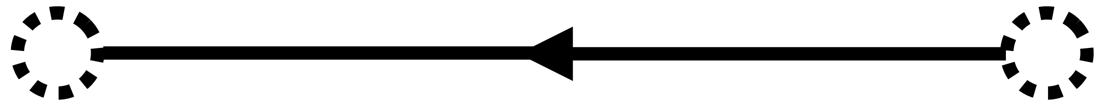
Meson fields

meson field



$$\langle W_i | \Gamma | V_j \rangle$$

cf. propagator

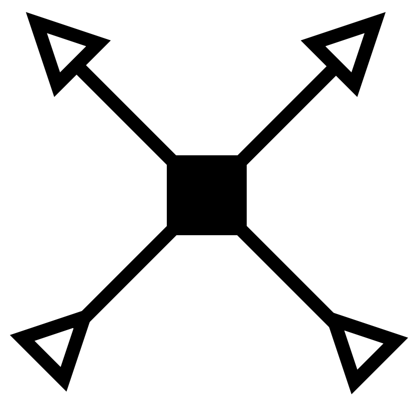


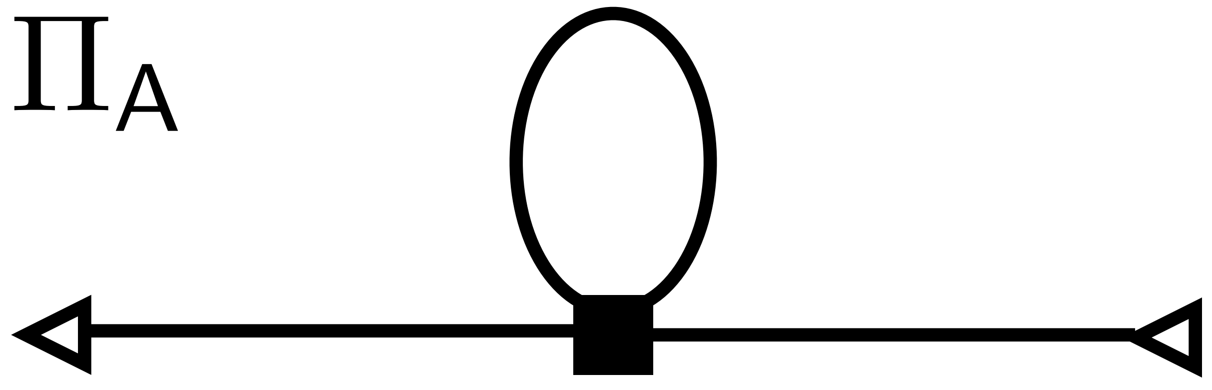
$$\sum_i |V_i\rangle\langle W_i|$$

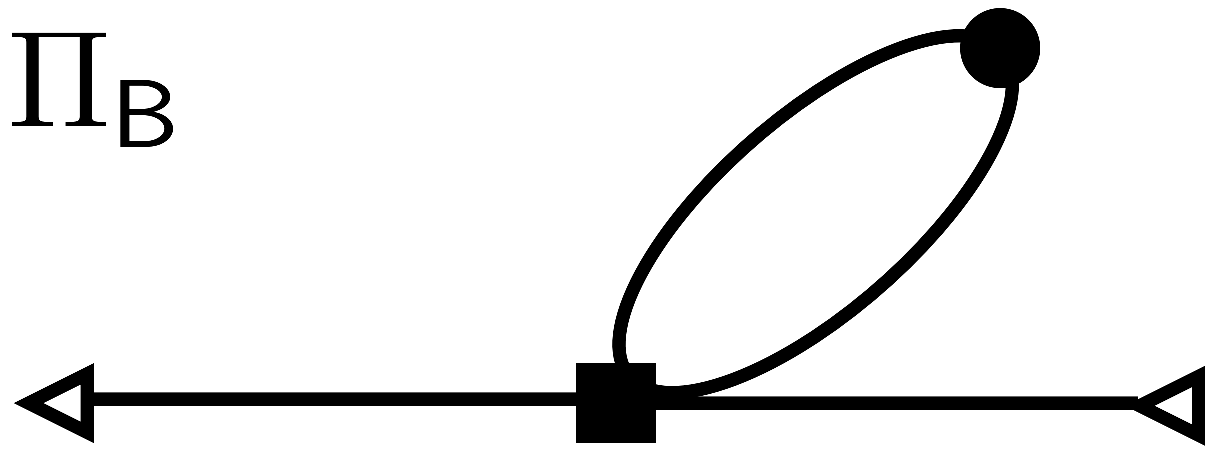
- Spin & color contractions leaving mode indices i, j
- Multiplied with any other meson fields to construct correlation functions
- Easily summed over time slice → savable data size
- Smeared pion fields generated using Grid/CPS by ππ scattering work

Next steps in 20-21

1. A2A propagator for strange & kaon fields
2. 4-quark operator fields
   - too big data due to 4 mode indices
     - generate partially contracted ones Π_A & Π_B

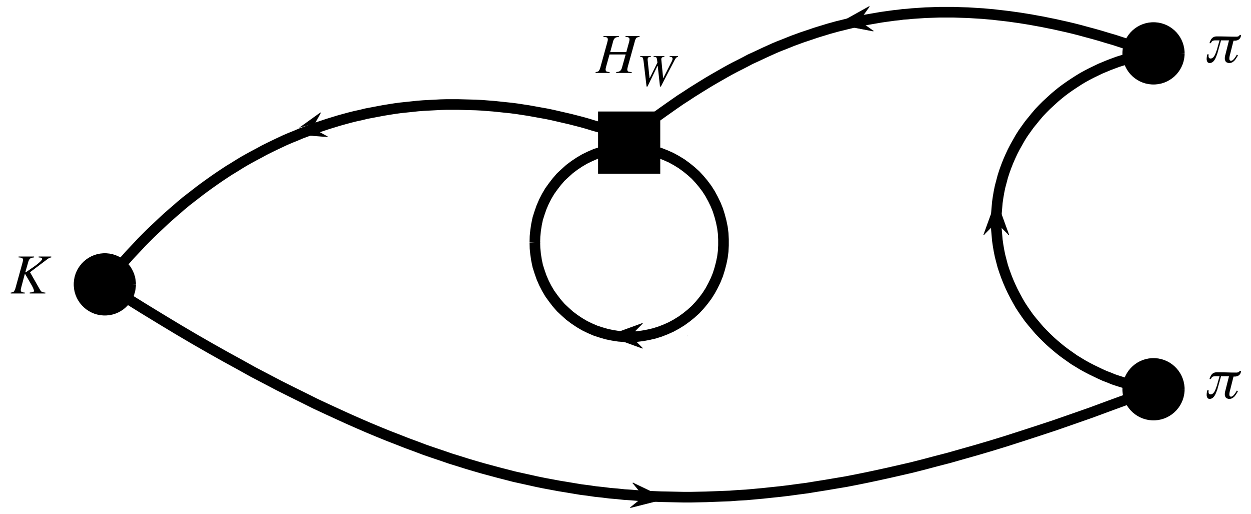






3. Contractions of mode indices

Example

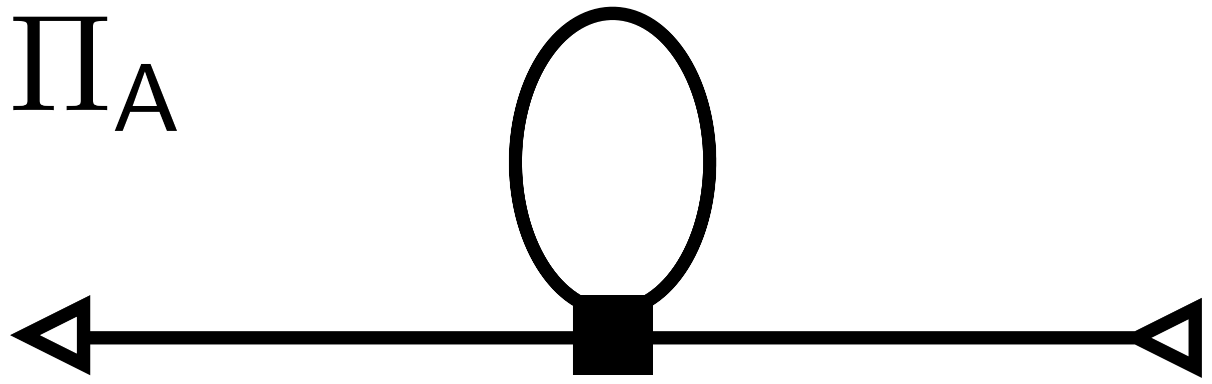


$$= \sum_{i,j,k,l} \Pi_K^{ij}(t_K)\Pi_\pi^{jk}(t_{\pi 1})\Pi_\pi^{kl}(t_{\pi 2})\Pi_A^{li}(t_{H_W})$$

Code preparation

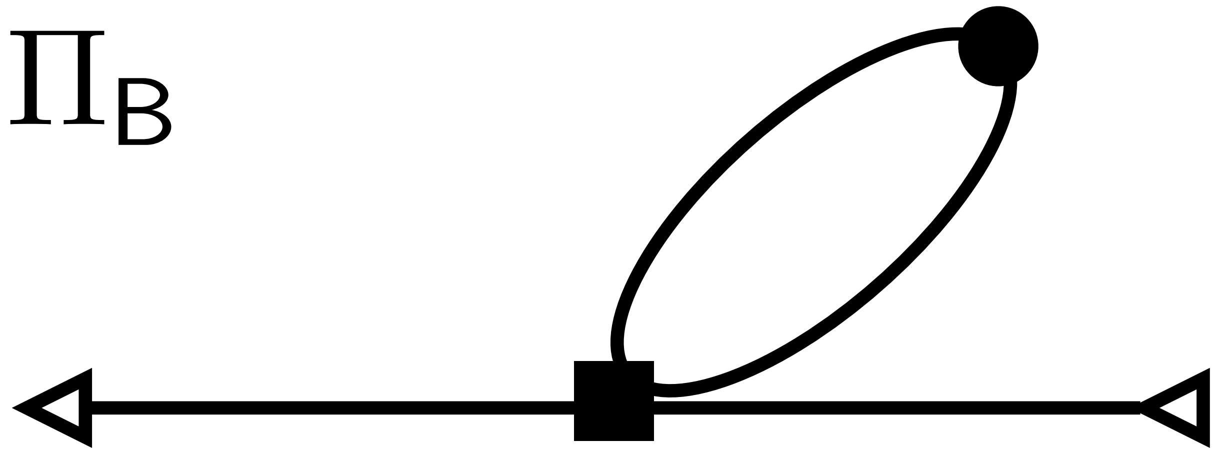
- Strange A2A propagator & kaon field calculation ready (code based on Grid & CPS)
- We create a contraction code based on Grid & Hadrons
  - 4-quark operator fields

Π_A



: almost ready (need a little improvement & test)

Π_B



: need construction (expected to be ready in June)

  - Contractions of mode indices: ready — Hadrons is already able to do this part

2. Wilson coefficients

- Co-investigators

  R. Abbott (Columbia), N. Christ (Columbia), C. Jung (BNL), C. Kelly (BNL), MT (PI, UConn)

- RBC & UKQCD Collaborations

- Requests

  - 8.35 M KNL core-hours on KNL at BNL or JLab, or 5.26 M Sky-core-hours on Skylake

Motivation

$$\langle f|H_W|i\rangle = \Sigma_i \underbrace{w_i^{3f}(\mu)}_{\text{pQCD}} \underbrace{\langle f|O_i^{3f}(\mu)|i\rangle}_{\text{LQCD}}$$

How to calculate $w_i^{3f}(\mu)$ in pQCD

Uncertainty does not decrease with increasing μ

$w_i^{3f}(\mu)$: large uncertainty

12% uncertainty on Re A_0, Im A_0

$w_i^{4f}(\mu)$: precise at HEs

matching needs to be done below charm threshold

Motivation

$$\langle f|H_W|i\rangle = \Sigma_i \underbrace{w_i^{3f}(\mu)}_{\cancel{\text{pQCD}}} \underbrace{\langle f|O_i^{3f}(\mu)|i\rangle}_{\text{LQCD}}$$

$w_i^{4f}(\mu)$ from pQCD

+

NP matching: $w_i^{4f}(\mu) \to w_i^{3f}(\mu)$

$w_i^{3f}(\mu)$: large uncertainty

12% uncertainty on Re A_0, Im A_0

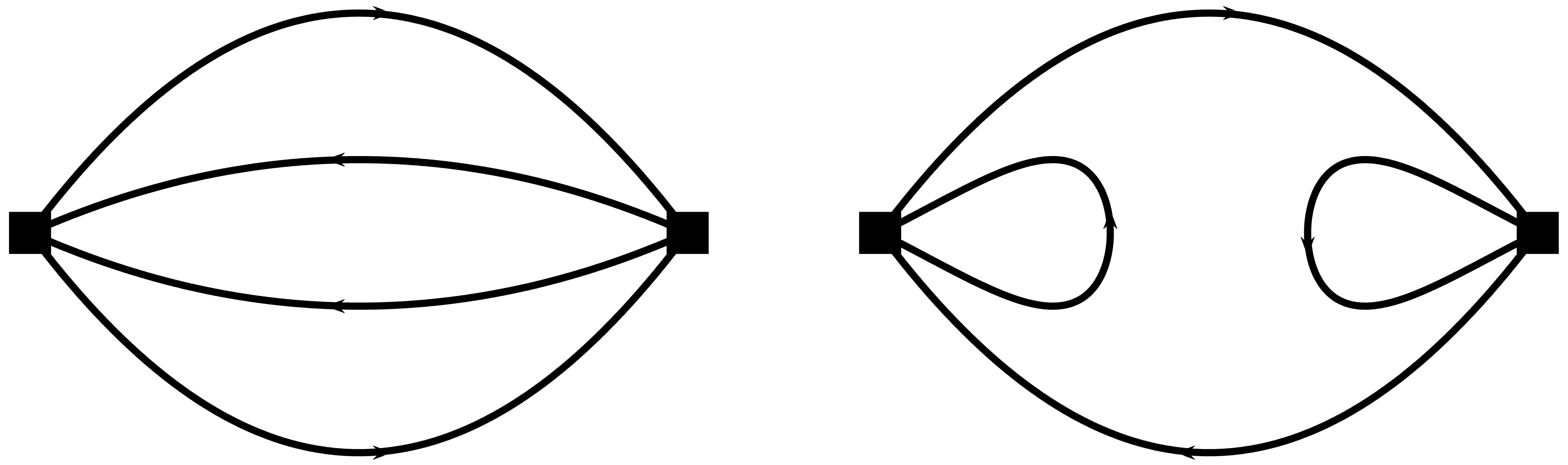
$w_i^{4f}(\mu)$: precise at HEs

matching needs to be done below charm threshold

Methodology

- Neglect sea charm effects — use 2+1-flavor ensemble
- Matching condition
  - Charm decoupling: $O_i^{4f} \to \sum_j M_{ij} O_j^{3f}$, i.e. $\langle f | O_i^{4f} | i \rangle \to \sum_j M_{ij} \langle f | O_j^{3f} | i \rangle$
  - Weak Hamiltonian with Mij: $H_W = \sum_{i,j} \underbrace{w_i^{4f} M_{ij}}_{= w_i^{3f}} O_j^{3f}$
  - NP Matching condition using long-distance 2pt functions in position space

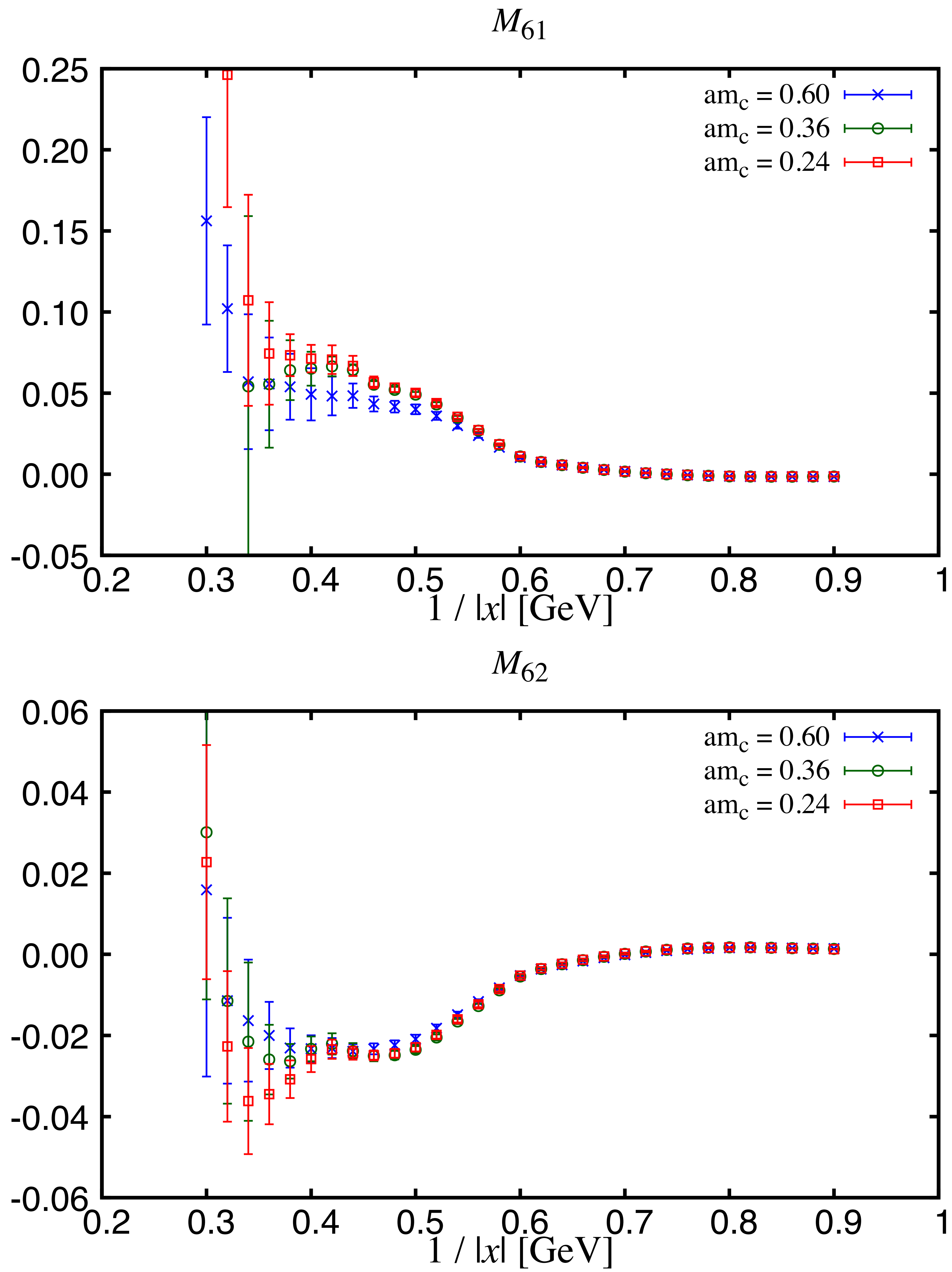
$$\langle O_i^{4f}(x) O_j^{3f}(y)^\dagger \rangle = \sum_k M_{ik} \langle O_k^{3f}(x) O_j^{3f}(y)^\dagger \rangle$$



Achievements in 19-20

Exploratory calculation on 16^3 lattice

- Measurement code (based on Grid & CPS) well tested
- 88 configs, 64 point & 20 noise srcs
  - could result in 10% accuracy of Wilson coefficients
- Plateau seen at LDs as expected



Plan for 20-21

- Calculation on a fine ensemble
  - RBC/UKQCD's 2+1-flavor ensembles with Möbius domain-wall fermions
  - 32^3 x 64 lattice at a^{-1} = 3.1 GeV & m_π = 370 MeV, 400 confs
  - 3 valence charm quark masses
- Prospects
  - Error on Re A_0 & Im A_0 from Wilson coefficients

    12% (from PT) → 5% (by NP matching) with planned statistics

Summary

- Proposal 1: Calculation w/ Periodic BC
  - Measurements for ππ phase shifts: done
  - Measurements for matrix elements: almost ready (will start in 19-20)
  - Trying to see if PBC calculation works as well as GPBC
- Proposal 2: Improving Wilson Coefficients
  - Exploratory calculation on 16^3 lattice gave promising results
  - Main calculation on 32^3 lattice about to begin
  - Aimed at 5% precision in $w_i^{3f}(\mu)$