$K \to \pi\pi$

in RBC/UKQCD

Masaaki Tomii (UConn)

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G-parity BC result released


• $\text{Re}(\varepsilon'/\varepsilon)_{\text{SM}} = 21.7(2.6)(6.2)(5.0) \times 10^{-4}$ consistent $\text{Re}(\varepsilon'/\varepsilon)_{\text{exp}} = 16.6(2.3) \times 10^{-4}$

• Various improvements to our previous result in 2015
  • 3+ times more configurations
  • multiple $\pi\pi$ operators $\rightarrow$ more accurate $\pi\pi$ phase shift
  • Renormalization scale increased by step scaling
  • ...

• G-parity BC ensures: final ground $\pi\pi$ state is on-shell; $E_{\pi\pi} = m_K$
What’s next?

• Proposal 1: Calculation w/ Periodic BC
  • Important check of G-parity calculation with a different setup
  • Configuration generation & light-quark inversions already done
  • Flavor-unmixed Dirac operator in PBC is 2x cheaper than G-parity’s
  • Challenging because needed on-shell final state is excited (solved by using multiple operators)

• Proposal 2: Improving Wilson Coefficients
  • Perturbative matching $w^4f \rightarrow w^3f$ causes one of the biggest errors
  • Nonperturbative matching
  • Can be applied to both of G-parity and periodic BC calculations

Future works
• Continuum limit
• Isospin breaking / E & M correction
1. $K \rightarrow \pi\pi$ w/ periodic boundaries

- Co-investigators
  - T. Blum (PI, UConn/RBRC), D. Hoying (RBRC), T. Izubuchi (BNL/RBRC), L. Jin (UConn/RBRC), C. Jung (BNL), C. Kelly (BNL), A. Soni (BNL), MT (UConn)

- RBC & UKQCD Collaborations

- Requests
  - 35 M KNL core-hours at JLab or BNL
  - 340 TB new tape storage + current allocation of 386 TB tape & 180 TB disk
Ensembles

• RBC/UKQCD’s 2+1-flavor ensembles with Möbius domain-wall fermions at physical pion & kaon masses

  • $24^3 \times 64$ lattice at $a^{-1} = 1.0$ GeV, 200 confs

  • $32^3 \times 64$ lattice at $a^{-1} = 1.4$ GeV, 200 confs
What to calculate

\[
\text{Re} \left( \frac{e'}{e} \right) = \text{Re} \left\{ \frac{i \omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2e}} \left[ \frac{\text{Im} A_2}{\text{Re} A_2} - \frac{\text{Im} A_0}{\text{Re} A_0} \right] \right\}
\]

- $\delta_1$: $\pi\pi$ phase shifts (measurements finished)

- 2pt functions of 2-pion operators & GEVP $\rightarrow$ $\pi\pi$-state energies
  
  - Lüscher's formalism $\rightarrow$ $\pi\pi$ phase shifts

- $A_I = \langle (\pi\pi)_I \mid H_W \mid K \rangle$ (plan for 20-21 allocation year)

- 4pt functions
Achievements in 19-20

• ππ energies from 2pt functions

• GEVP w/ multiple operators operators

1. K → ππ w/ periodic boundaries
Achievements in 19-20

- $\pi\pi$ phase shifts
  
  - Lüscher's formula: $\tan \delta = -\frac{\pi^{3/2}k}{Z_{00}(1; k)}$

1. $K \rightarrow \pi\pi$ w/ periodic boundaries

Preliminary

\begin{align*}
\delta_0 \\
\delta_2
\end{align*}

\begin{align*}
\sqrt{s} (\text{MeV}) & \quad \sqrt{s} (\text{MeV}) \\
300 & \quad 400 \\
400 & \quad 600 \\
500 & \quad 700 \\
600 & \quad 800 \\
700 & \quad 900 \\
800 & \quad 1000
\end{align*}

\begin{align*}
\text{Preliminary}
\end{align*}
Plan for 20-21

- 4pt functions

- A2A propagators already calculated in ππ scattering work and saved

- Contractions of 4 types of diagrams
A2A propagator w/ V & W vectors

\[
D_{\text{A2A}}^{-1} = \sum_{l=1}^{N_l} |\phi_l\rangle \frac{1}{\lambda} \langle \phi_l | + \frac{1}{N_h} \sum_{h=1}^{N_h} \left( D_{\text{defl}}^{-1} - \sum_{l=1}^{N_l} |\phi_l\rangle \frac{1}{\lambda} \langle \phi_l | \right) |\eta_h\rangle \langle \eta_h |
\]

\[
= \sum_{i=1}^{N_l+N_h} |V_i\rangle \langle W_i|
\]

• V & W vectors

\[
1 \leq i \leq N_l \Rightarrow |V_i\rangle = \frac{1}{\lambda} |\phi_i\rangle, \quad |W_i\rangle = |\phi_i\rangle
\]

\[
N_l + 1 \leq i(=N_l+h) \leq N_l+N_h \Rightarrow |V_i\rangle = \frac{1}{N_h} D_{\text{defl}}^{-1} |\eta_h\rangle, \quad |W_i\rangle = |\eta_h\rangle
\]

V & W for light quark generated using CPS/Grid as a part of πππ scattering work
1. $K \rightarrow \pi\pi$ w/ periodic boundaries

Meson fields

- Spin & color contractions leaving mode indices $i, j$
- Multiplied with any other meson fields to construct correlation functions
- Easily summed over time slice $\rightarrow$ savable data size
- Smeared pion fields generated using Grid/CPS by $\pi\pi$ scattering work

<table>
<thead>
<tr>
<th>Meson field</th>
<th>cf. propagator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle W_i</td>
<td>\Gamma</td>
</tr>
</tbody>
</table>
Next steps in 20-21

1. $K \rightarrow \pi\pi$ w/ periodic boundaries

2. 4-quark operator fields
   - too big data due to 4 mode indices
     - generate partially contracted ones $\Pi_A$ & $\Pi_B$

3. Contractions of mode indices

Example:

$$ = \sum_{i,j,k,l} \Pi^i_j(t_K)\Pi^{jk}_\pi(t_{\pi 1})\Pi^{kl}_\pi(t_{\pi 2})\Pi^l_i(t_{H_W})$$
Code preparation

• Strange A2A propagator & kaon field calculation ready (code based on Grid & CPS)

• We create a contraction code based on Grid & Hadrons
  • 4-quark operator fields
    \[ \Pi_A \]
    : almost ready (need a little improvement & test)

    \[ \Pi_B \]
    : need construction (expected to be ready in June)

• Contractions of mode indices: ready — Hadrons is already able to do this part
2. Wilson coefficients

• Co-investigators

  R. Abbott (Columbia), N. Christ (Columbia), C. Jung (BNL), C. Kelly (BNL), MT (PI, UConn)

• RBC & UKQCD Collaborations

• Requests

  • 8.35 M KNL core-hours on KNL at BNL or JLab, or 5.26 M Sky-core-hours on Skylake
2. Wilson coefficients

Motivation

\[ <f|H_W|i> = \sum_i w_i^{3f}(\mu) \frac{<f|O_i(\mu)|i>}{pQCD} \]

How to calculate \( w_i^{3f}(\mu) \) in pQCD

Uncertainty does not decrease with increasing \( \mu \)

Matching needs to be done below charm threshold

\( w_i^{3f}(\mu) \): large uncertainty

12\% uncertainty on \( \text{Re} A_0, \text{Im} A_0 \)

\( w_i^{4f}(\mu) \): precise at HEs
2. Wilson coefficients

Motivation

\[ \langle f | H_W | i \rangle = \sum_i w_i^{3f}(\mu) \frac{\langle f | O_i(\mu) | i \rangle}{LQCD} \]

\[
\begin{align*}
w_i^{4f}(\mu) \text{ from pQCD} \\
+ \text{NP matching: } w_i^{4f}(\mu) \to w_i^{3f}(\mu)
\end{align*}
\]

Matching needs to be done below charm threshold

\[ w_i^{3f}(\mu): \text{large uncertainty} \]

12% uncertainty on Re \( A_0 \), Im \( A_0 \)

\[ w_i^{4f}(\mu): \text{precise at HEs} \]
Methodology

• Neglect sea charm effects — use 2+1-flavor ensemble

• Matching condition
  • Charm decoupling: \( O^{4f}_i \rightarrow \sum_j M_{ij} O^{3f}_j \), i.e. \( \langle f|O^{4f}_i|i\rangle \rightarrow \sum_j M_{ij} \langle f|O^{3f}_j|i\rangle \)
  
  • Weak Hamiltonian with \( M_{ij} \): \( H_W = \sum_{i,j} w^{4f}_i M_{ij} O^{3f}_j \)

• NP Matching condition using long-distance 2pt functions in position space

\[
\langle O^{4f}_i(x) O^{3f}_j(y) \rangle = \sum_k M_{ik} \langle O^{3f}_k(x) O^{3f}_j(y) \rangle
\]
Achievements in 19-20

Exploratory calculation on $16^3$ lattice

- Measurement code (based on Grid & CPS) well tested
- 88 configs, 64 point & 20 noise srcs
  - could result in 10% accuracy of Wilson coefficients
- Plateau seen at LDs as expected
Plan for 20-21

- Calculation on a fine ensemble

- RBC/UKQCD’s 2+1-flavor ensembles with Möbius domain-wall fermions

- $32^3 \times 64$ lattice at $a^{-1} = 3.1$ GeV & $m_\pi = 370$ MeV, 400 confs

- 3 valence charm quark masses

Prospects

- Error on Re $A_0$ & Im $A_0$ from Wilson coefficients

  12% (from PT) $\rightarrow$ 5% (by NP matching) with planned statistics
Summary

- Proposal 1: Calculation w/ Periodic BC
  - Measurements for $\pi\pi$ phase shifts: done
  - Measurements for matrix elements: almost ready (will start in 19-20)
  - Trying to see if PBC calculation works as well as GPBC

- Proposal 2: Improving Wilson Coefficients
  - Exploratory calculation on $16^3$ lattice gave promising results
  - Main calculation on $32^3$ lattice about to begin
  - Aimed at 5% precision in $w_i^3f(\mu)$