AI for Jets in JAM

Yiyu Zhou

College of William & Mary and Jefferson Lab

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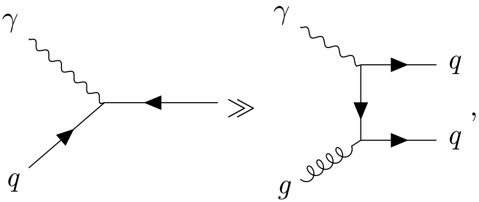
Outline

There are three parts in my talk today:

- motivation,
- challenges,
- results.

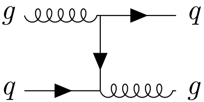
DIS and Jet

In DIS, sensitivity to gluon only appears in NLO



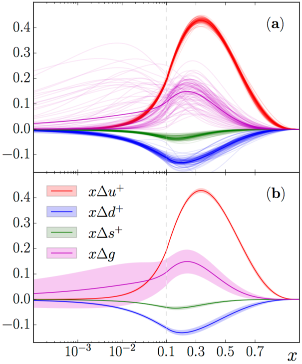
(1)

while for Jet, gluon diagrams appears at LO



Δg

JAM 2015 fits [1] on Δg has larger error bands than other flavors



with the RHIC Jet A_{LL} data, we can narrow down the uncertainty significantly.

Simultaneous Extraction of PDFs and polarized PDFs

The RHIC [2] measured Jet A_{LL} is

$$A_{LL} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}, \tag{2}$$

where the denominator is the spin averaged cross section. Therefore we can fit spin averaged and spin dependent PDFs at the same time.

Why Mellin

DGLAP equations are convoluted in x space

$$\frac{\partial \boldsymbol{q}(x,t)}{\partial t} = \frac{\alpha_s(t)}{2\pi} \int_x^1 \boldsymbol{P}\left(\frac{x}{z}, \alpha_s(t)\right) \boldsymbol{q}(z,t) \frac{1}{z} \mathrm{d}z, \tag{3}$$

apply Mellin transformation $\int_0^1 x^{N-1} \mathrm{d}x$ and the right side becomes nothing but a Mellin convolution

$$\frac{\partial \widetilde{\boldsymbol{q}}(N,t)}{\partial t} = \frac{\alpha_s(t)}{2\pi} \widetilde{\boldsymbol{P}}(N, \alpha_s(t)) \widetilde{\boldsymbol{q}}(N,t), \tag{4}$$

and is no longer convoluted.

Jet calculation

The single jet inclusive cross section is given by [3]

$$\begin{aligned} \frac{E_J \mathrm{d}^3\sigma}{\mathrm{d}^3 p_J} = & \frac{1}{16\pi^2 s} \sum_{i,j,k,l} \int_0^1 \frac{\mathrm{d}x}{x} \frac{\mathrm{d}y}{y} f_i\left(x, \mu^2\right) f_j\left(y, \mu^2\right) \cdot \\ & \overline{\sum} \left|\mathcal{M}_{ij\to kl}\right|^2 \frac{1}{1+\delta_{kl}} \delta\left(\hat{s}+\hat{t}+\hat{u}\right), \end{aligned} \tag{5}$$

where f_i and f_j are the parton distribution functions.

Double Mellin Transformation

Writing equation (5) in a Mellin inverted form, we can single out the x dependence of f_i and f_j from their shapes, the cross section can be written proportional to $\frac{E_J \mathrm{d}^3\sigma}{\mathrm{d}^3 p_J} \propto \sigma_1 + \sigma_2$

$$\sigma_1 = \frac{-1}{2\pi^2} \mathrm{Re}\left(\int_0^\infty \mathrm{d}r_N \mathrm{d}r_M \, \widetilde{f}_i(M) \, e^{2\imath\phi} \, \widetilde{f}_j(N) \int \mathrm{d}x \mathrm{d}y x^{-M} y^{-N} \mathcal{H} \right),$$

$$\sigma_2 = \frac{1}{2\pi^2} \mathrm{Re}\left(\int_0^\infty \mathrm{d}r_N \mathrm{d}r_M \, \widetilde{f}_i(M) \, \widetilde{f}_j(N)^* \int \mathrm{d}x \mathrm{d}y x^{-M} y^{-N^*} \mathcal{H} \right),$$

where $\mathcal{H}$ represents the perturbative part and we define Mellin tables as

$$T_{M,N} = \int \mathrm{d}x \mathrm{d}y x^{-M} y^{-N} \mathcal{H}, \quad T^*_{M,N} = \int \mathrm{d}x \mathrm{d}y x^{-M} y^{-N^*} \mathcal{H}.$$

Mellin Moments of PDFs

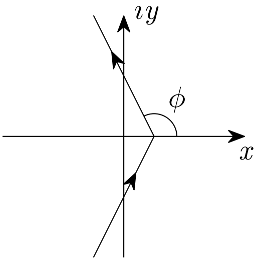
Generally when fitting PDFs, they were taken the form $f(x) = x^{\alpha}(1-x)^{\beta}$, the Mellin transformation turns out to be simple

$$\widetilde{f}(N) = \int_0^1 x^{N-1} f(x)\,\mathrm{d}x = B\left(N+\alpha, \beta+1\right), \tag{6}$$

the shape of the PDFs is now contained in the Beta function.

Table Construction

We need to compute the integrals over x and y for each M and N along the contour to construct Mellin tables



There are 68×68 points in total for M and N.

For all 6 channels and 4 components in each channel, we compute 68×68 integrals. And this is only one data point.

We have in total 196 data points.

Jet Data

Here we are showing the single inclusive Jet data from CDF 2002 to 2006 Run II 1.96 TeV data from [4]

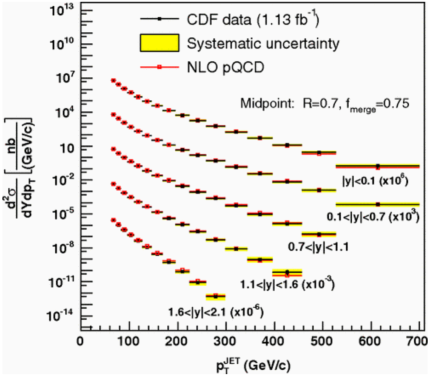
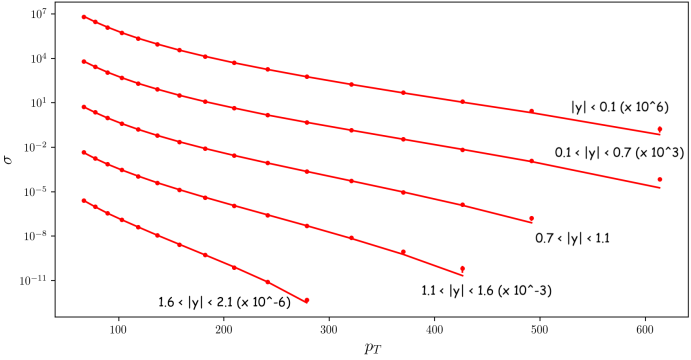


Figure 1: different multipliers are applied to distinguish η bins

Jet Observable

We have carried out a new global analysis using the multi step strategy as explained by Carlota



AI in Jet

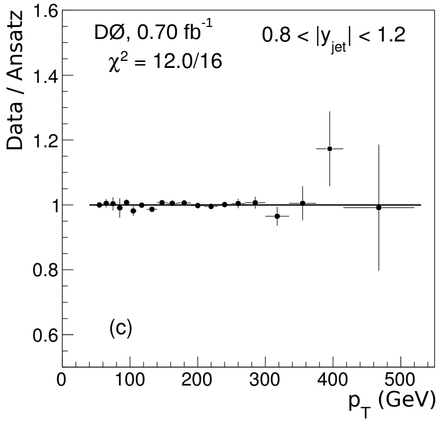
As we have seen in the previous slides, a lot of computation was put into constructing the Mellin tables. It would be great if we can somehow cleverly avoid this hurdle. And there is, it is machine learning.

We can let the machine learn the Mellin tables and once new data points are added, we will be able to find their Mellin tables by interpolation.

Also, by doing machine learning, we save the hard disk space, because we would store only one model for many data points.

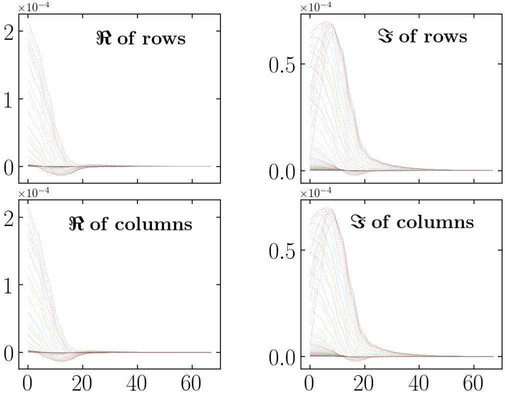
p_T bins

Using machine learning models to predict, we can also narrow down the p_T bins [5], which is very time consuming in previous method



Challenges

Now the challenge for Jet is, using double Mellin table, the values at large M and N are very suppressed, can be as small as 10^{-69}. Learning such small values is a challenge for the neutral network



AI in JAM

The work shown here is in collaboration with Nobuo Sato, Wally Melnitchouk and Carlota Andrés, thanks to Patrick Barry and Christopher Cocuzza for helpful discussions!

Reference

[1] Nobuo Sato, W. Melnitchouk, et al. In: *Physical Review D* 93.074005 (Apr. 2016). DOI: 10.1103/PhysRevD.93.074005 (cit. on p. 4).

[2] J. Adam et al. In: *Physical Review D* 100.052005 (Sept. 2019). DOI: 10.1103/PhysRevD.100.052005 (cit. on p. 5).

[3] R. K. Ellis, W. J. Stirling, and B. R. Webber. *QCD and Collider Physics*. First Edition. Cambridge University Press, 2003. ISBN: 9780511628788 (cit. on p. 7).

[4] T. Aaltonen et al. In: *Physical Review D* 78.052006 (Sept. 2008). DOI: 10.1103/PhysRevD.78.052006. **Erratum**: [**a.aaltonen.erratum_jet_sigma**] (cit. on p. 11).

[5] V. M. Abazov et al. In: *Physical Review D* 85.052006 (Mar. 2012). DOI: 10.1103/PhysRevD.85.052006 (cit. on p. 14).