Pions PDFs and Challenges in Implementing Threshold Resummation

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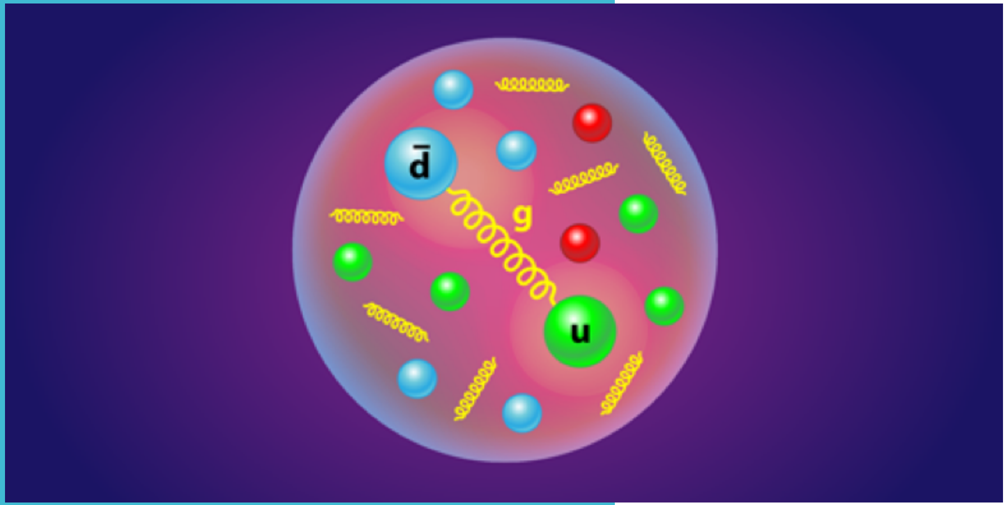
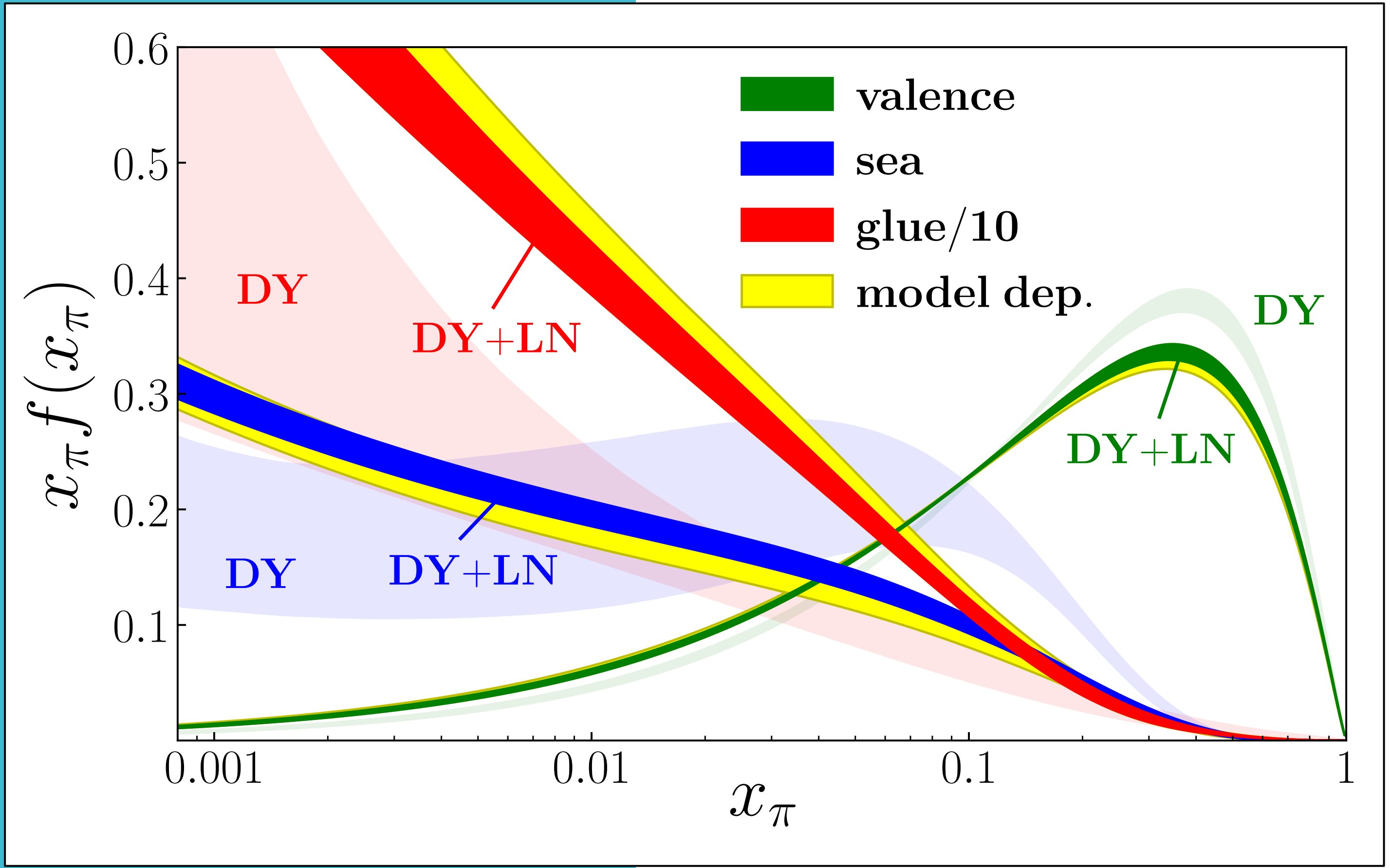


- The pion is the lightest bound state of partons
- Also the Goldstone Boson associated with Chiral symmetry breaking
- Makeup can be described in terms of parton distribution functions (PDFs)

Featured in Physics

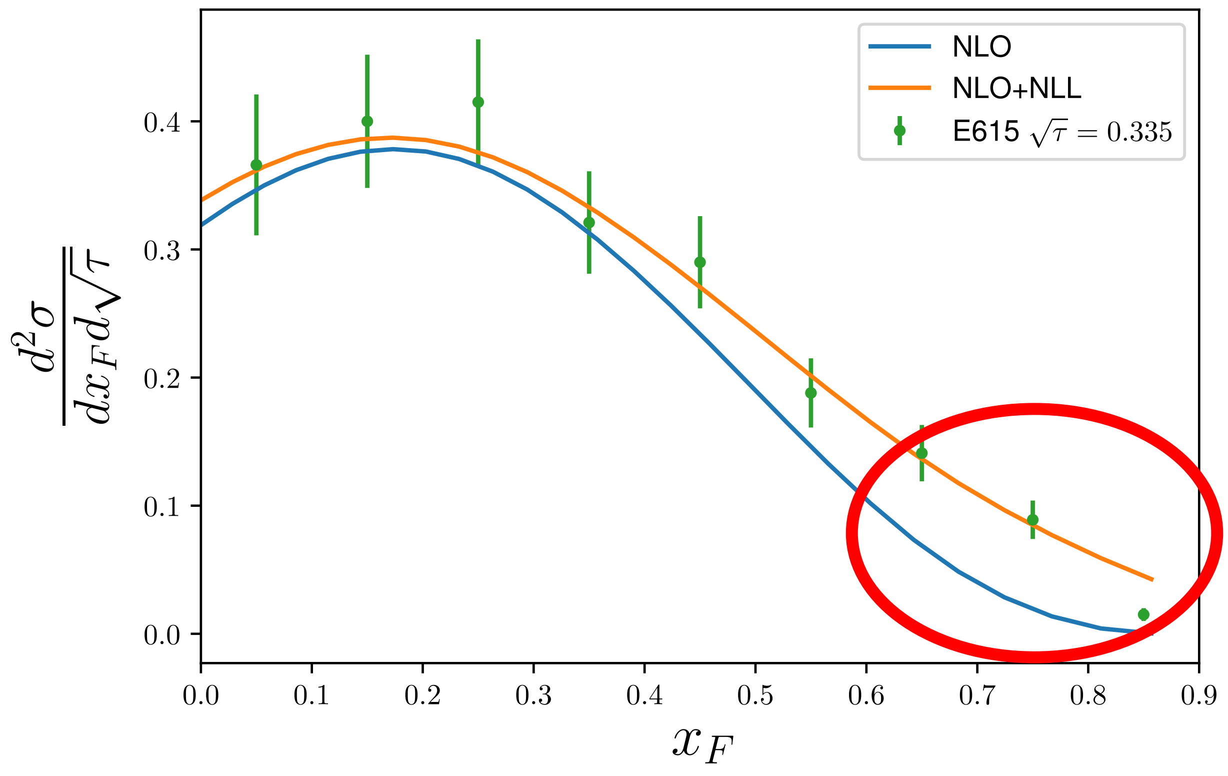


- Pion PDFs have been determined using Drell-Yan and Leading Neutron experimental observables at Next-to-Leading Order
- Want to better determine the large-x_π behavior, dominated by the valence PDF and Drell-Yan kinematics

For example in Drell-Yan:

$$\sigma \propto \sum_{i,j} f_i^\pi(x_\pi,\mu) \otimes f_j^A(x_A,\mu) \otimes \hat{\sigma}_{i,j}(x_\pi, x_A, Q/\mu)$$

Threshold Resummation in Drell-Yan



Noticeably larger contribution to cross-section at **high-*x***

How to calculate these cross-sections in practice?

Fixed order Drell-Yan and the Mellin Transform

- We use the Mellin transform to perform DGLAP evolution on the pion PDF in order to compare with data at many Q^2 values

$$\frac{d\sigma^{\text{NLO}}}{dx_F dQ^2} = \sigma_0 \sum_{i,j} \frac{1}{2\pi i} \int_{C_N} dN \, f_i^{\pi}(N) \boxed{\int dx_\pi x_\pi^{-N} \int dx_A f_j^A(x_A) C_{i,j}(x_\pi, x_A)}$$

- Requires the use of saved Mellin tables for *each datapoint* given an external Tungsten PDF
- Need to include 168 saved points for the Mellin integrand
- Total data points: 149 -> 25,032 entries to grab from storage for 1 single iteration of fit
  - Manageable, but could improve

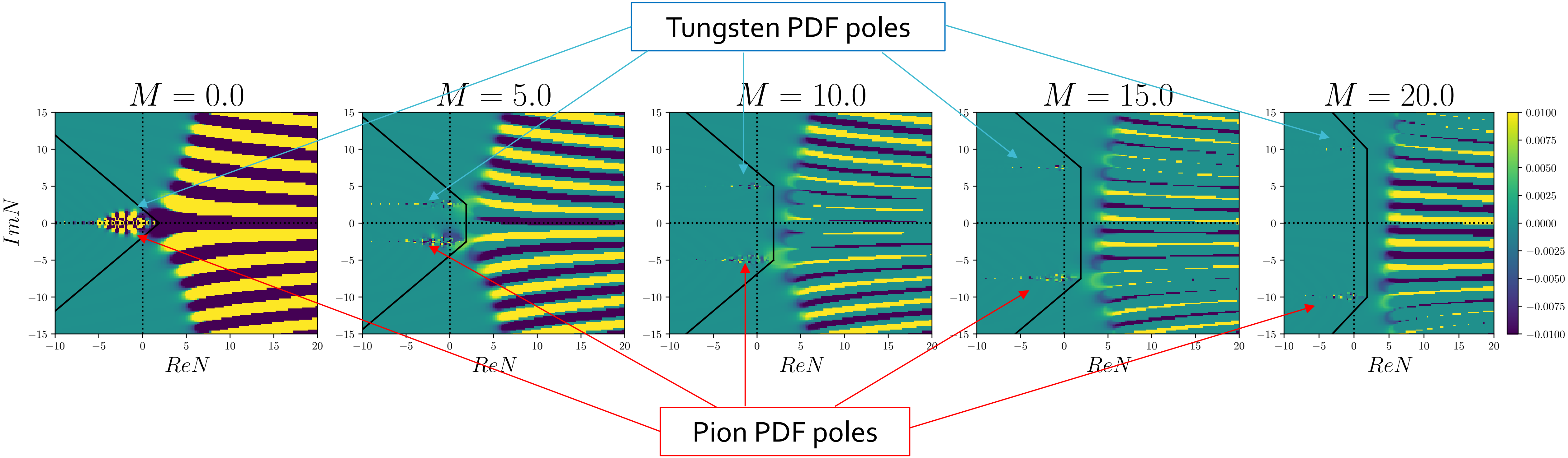
Resummation and the Mellin Transform

- The Mellin transform is a necessity to resum higher order effects, in addition to DGLAP evolution

$$\frac{d\sigma^{\text{NLL}}}{dQ^2 dY} = \sigma_0 \sum_{i,j} \int_{-\infty}^{\infty} \frac{dM}{2\pi} e^{-iMY} \int_{C_N} \frac{dN}{2\pi i} \tau^{-N} f_i^{\pi}(N + i\frac{M}{2}) f_j^{A}(N - i\frac{M}{2}) \tilde{C}_{i,j}(N, M)$$

- Poles of PDFs are now in complex N space instead of on the real axis *and* a function of the rapidity conjugate, M

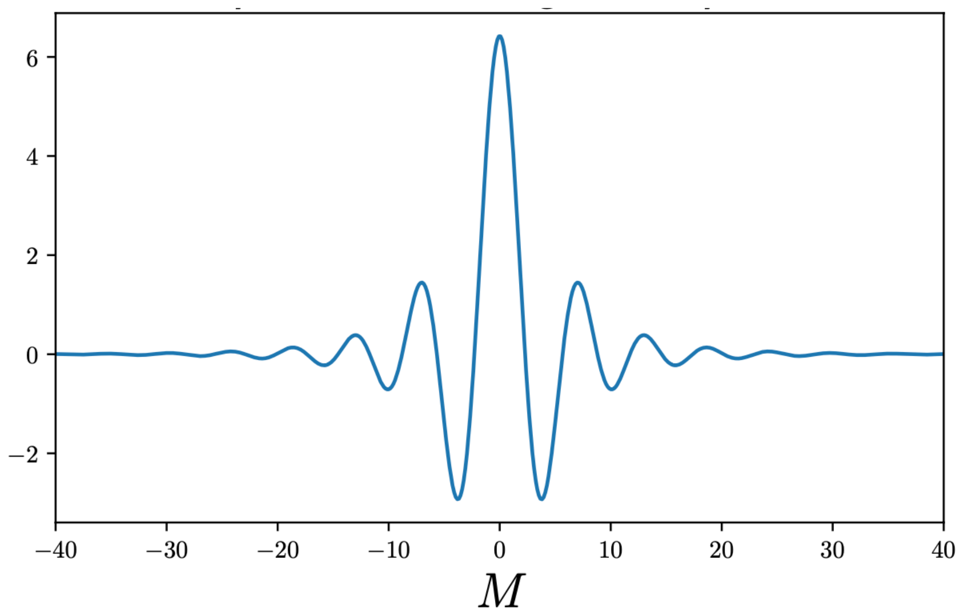
$$\frac{d\sigma^{\text{NLL}}}{dQ^2 dY} = \sigma_0 \sum_{i,j} \int_{-\infty}^{\infty} \frac{dM}{2\pi} e^{-iMY} \int_{C_N} \frac{dN}{2\pi i} \tau^{-N} f_i^{\pi}(N + i\frac{M}{2}) f_j^{A}(N - i\frac{M}{2}) \tilde{C}_{i,j}(N, M)$$



Need to capture the pole structure, and be mold the contour to be close to the poles while not touching them

Complications in Resummation Cacluation

- We need enough points in M space to describe well the integrand



- Going to such large values of M requires 1024 evaluations of the integrand
- Implementing the Fast Fourier Transform (FFT) allows evaluation for each bin of Q^2
- 21 Q^2 bins -> 21504 evaluations of M integrand
- 3 legs of 168 Mellin N points -> $\mathcal{O}(10^8)$ evaluations for each iteration of fit

Machine Learning

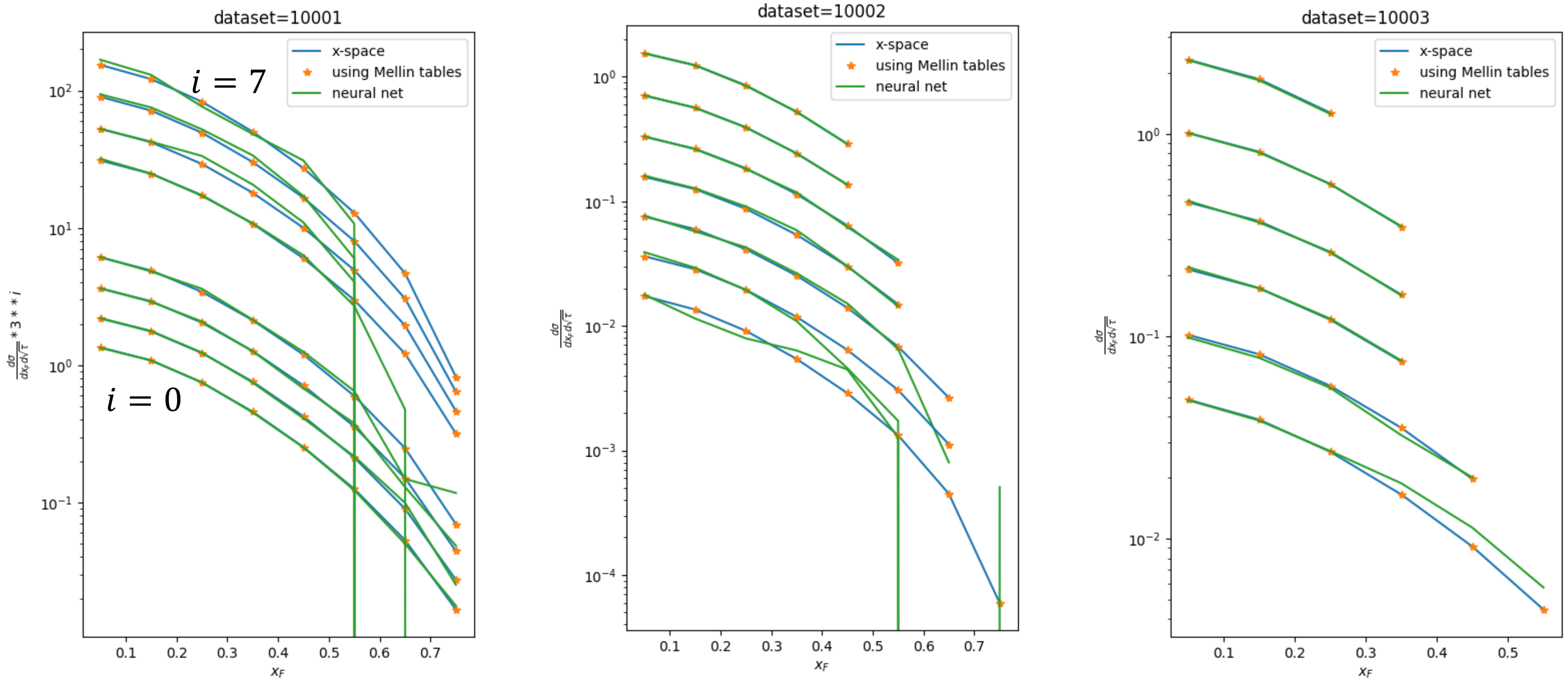
$$\sigma^{\text{NLO+NLL}} = \boxed{\sigma^{\text{NLO}}} + \sigma^{\text{NLL}} - \sigma^{\text{overlap}}$$

Still needs to be calculated

- We have hope
- We are in the early stages of implementing machine learning to this calculation
- Fixed order is conceivable, by having a neural net *learned* on the Mellin tables
- Instead of huge data to search for, a neural net can predict the values directly

Success in Machine Learning

- In an example of the full NLO calculation, we can test the neural net's prediction to the Mellin tables and an x-space calculation for all data points



Remarkable agreement!

More fine tuning to come…