Non perturbative studies of Parton Distribution Functions

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Introduction

- Goal: Compute hadron structure properties from QCD
  - Parton distribution functions (PDFs)
- Operator product: Mellin moments are local matrix elements that can be computed in Lattice QCD
  - Power divergent mixing limits us to few moments
- Few years ago X. Ji suggested an approach for obtaining PDFs from Lattice QCD
- First calculations already available

X. Ji, Phys.Rev.Lett. 110, (2013)

Y.-Q. Ma J.-W. Qiu (2014) 1404.6860

H.-W. Lin, J.-W. Chen, S. D. Cohen, and X. Ji, Phys.Rev. D91, 054510 (2015)

C. Alexandrou, et al, Phys. Rev. D92, 014502 (2015)

- A new approach for obtaining PDFs from LQCD introduced by A. Radyushkin

A. Radyushkin Phys.Lett. B767 (2017)

- Hadronic tensor methods

K-F Liu et al Phys. Rev. Lett. 72 (1994) , Phys. Rev. D62 (2000) 074501
Detmold and Lin 2005
M. T. Hansen et al arXiv:1704.08993.
UKQCD-QCDSF-CSSM Phys. Lett. B714 (2012), arXiv:1703.01153

Ma and Qiu : arXiv:1709.03018

PDFs: Definition

Light-cone PDFs:

$$f^{(0)}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+\omega^-} \left\langle P \left| T\,\overline{\psi}(0,\omega^-,\mathbf{0}_\mathrm{T}) W(\omega^-,0)\gamma^+ \frac{\lambda^a}{2}\psi(0) \right| P \right\rangle_\mathrm{C} .$$

$$W(\omega^-,0) = \mathcal{P}\exp\left[-ig_0\int_0^{\omega^-}\mathrm{d}y^- A_\alpha^+(0,y^-,\mathbf{0}_\mathrm{T})T_\alpha\right]$$

$$\langle P'|P\rangle = (2\pi)^3 2P^+\delta\left(P^+ - P'^+\right)\delta^{(2)}\left(\mathbf{P}_\mathrm{T} - \mathbf{P}'_\mathrm{T}\right)$$

Moments:

$$a_0^{(n)} = \int_0^1 \mathrm{d}\xi\,\xi^{n-1}\left[f^{(0)}(\xi) + (-1)^n\overline{f}^{(0)}(\xi)\right] = \int_{-1}^1 \mathrm{d}\xi\,\xi^{n-1} f(\xi)$$

Local matrix elements:

$$\left\langle P|\mathcal{O}_0^{\{\mu_1\ldots\mu_n\}}|P\right\rangle = 2a_0^{(n)}\left(P^{\mu_1}\cdots P^{\mu_n} - \mathrm{traces}\right)$$

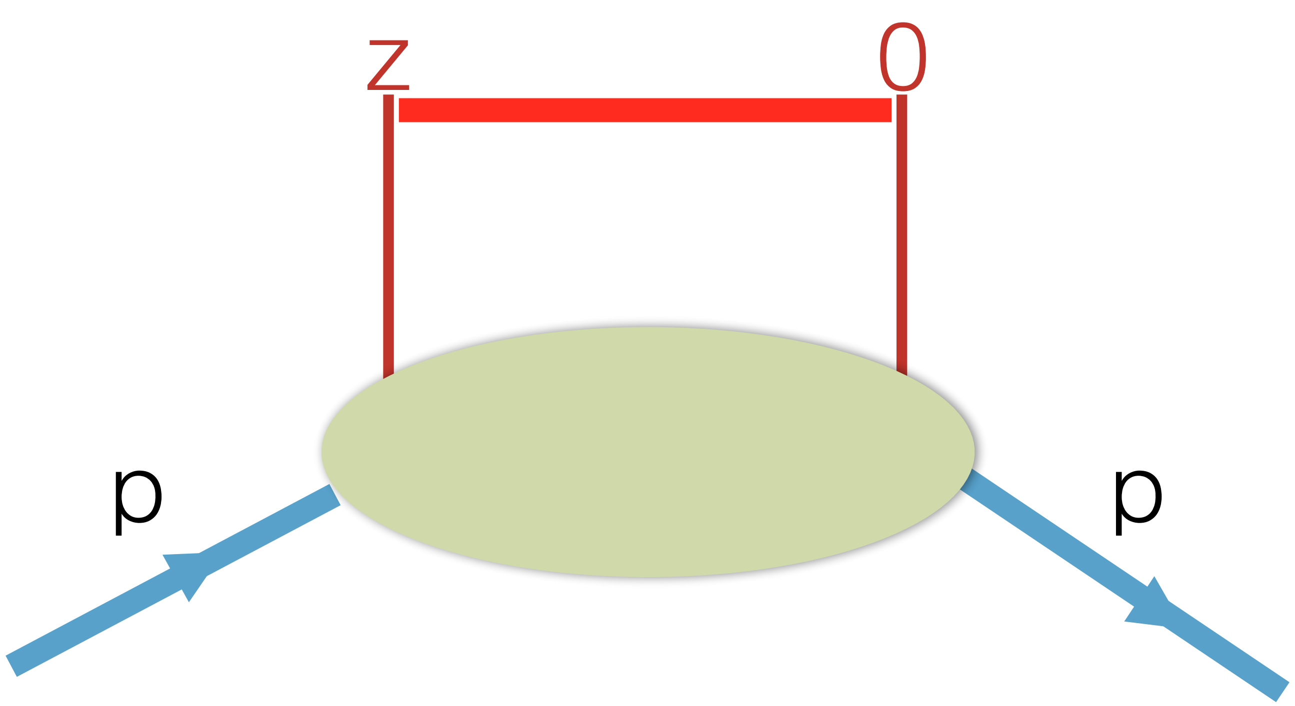
$$\mathcal{O}_0^{\{\mu_1\cdots\mu_n\}} = i^{n-1}\overline{\psi}(0)\gamma^{\{\mu_1}D^{\mu_2}\cdots D^{\mu_n\}}\frac{\lambda^a}{2}\psi(0) - \mathrm{traces}$$

Pseudo-PDFs

Unpolarized PDFs proton:

$$\mathcal{M}^{\alpha}(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^{\alpha}\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

$$\hat{E}(0,z;A) = \mathcal{P}\exp\left[-ig\int_0^z \mathrm{d}z'_{\mu}\,A^{\mu}_{\alpha}(z')T_{\alpha}\right]$$



Lorentz decomposition:

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(-(zp),-z^2) + z^\alpha \mathcal{M}_z(-(zp),-z^2)$$

Collinear PDFs: Choose

$$z = (0, z_-, 0)$$

$$p = (p_+, 0, 0)$$

$$\gamma^+$$

$$\mathcal{M}^+(z,p) = 2p^+ \mathcal{M}_p(-p_+ z_-, 0)$$

Definition of PDF:

$$\mathcal{M}_p(-p_+ z_-, 0) = \int_{-1}^{1} dx\, f(x)\, e^{-ixp_+ z_-}$$

$$\mathcal{M}_p(\nu, z^2) = \int_0^1 d\alpha\, \mathcal{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\, \mathcal{Q}(\alpha\nu, \mu) \;\; + \mathcal{O}(z^2\Lambda_{qcd}^2)$$

$\mathcal{Q}(\nu, \mu)$ is called the Ioffe time PDF

V. Braun, et. al Phys. Rev. D 51, 6036 (1995)

$$\mathcal{Q}(\nu, \mu) = \int_{-1}^1 dx\, e^{-ix\nu} f(x, \mu)$$

Matching to $\overline{MS}$

Radyushkin Phys.Rev. D98 (2018) no.1, 014019
Izubuchi et al. Phys.Rev. D98 (2018) no.5, 056004
Zhang et al. Phys.Rev. D97 (2018) no.7, 074508

A. Radyushkin Phys.Lett. B767 (2017)

Lattice QCD calculation:

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

Choose
$$p = (p_0, 0, 0, p_3)$$
$$z = (0, 0, 0, z_3)$$
γ^0

On shell equal time matrix element computable in Euclidean space

Briceno *et al* arXiv:1703.06072

Obtaining only the relevant

$$\mathcal{M}_p(\nu, z_3^2) = \frac{1}{2p_0}\mathcal{M}^0(z_3, p_3)$$

Chosing γ^0 was also suggested also by M. Constantinou at GHP2017 based on an operator mixing argument for the renormalized matrix element

Alexandrou *et al* arXiv:1706.00265

Consider the ratio $\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}_p(\nu, z_3^2)}{\mathcal{M}_p(0, z_3^2)}$

UV divergences will cancel in this ratio resulting a renormalization group invariant (RGI) function

The lattice regulator can now be removed

$\mathfrak{M}^{cont}(\nu, z_3^2)$ Universal independent of the lattice

$\mathcal{M}_p(0,0) = 1$ Isovector matrix element

Matching to $\overline{MS}$ computed at 1-loop

Radyushkin Phys.Rev. D98 (2018) no.1, 014019
Zhang et al. Phys.Rev. D97 (2018) no.7, 074508

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, q_v(x,\mu)\mathcal{K}(x\nu, z^2\mu^2) \; + \sum_{k=1}^{\infty} \mathcal{B}_k(\nu)(z^2)^k \,.$$

$$\mathcal{K}(x\nu, z^2\mu^2) = \cos(x\nu) - \frac{\alpha_s}{2\pi} C_F \left[\ln(e^{2\gamma_E+1} z^2\mu^2/4)\tilde{B}(x\nu) + \tilde{D}(x\nu) \right]$$

$$\tilde{B}(x) = \frac{1-\cos(x)}{x^2} + 2\sin(x)\frac{x\mathrm{Si}(x)-1}{x} + \frac{3-4\gamma_E}{2}\cos(x) + 2\cos(x)\left[\mathrm{Ci}(x) - \ln(x)\right]$$

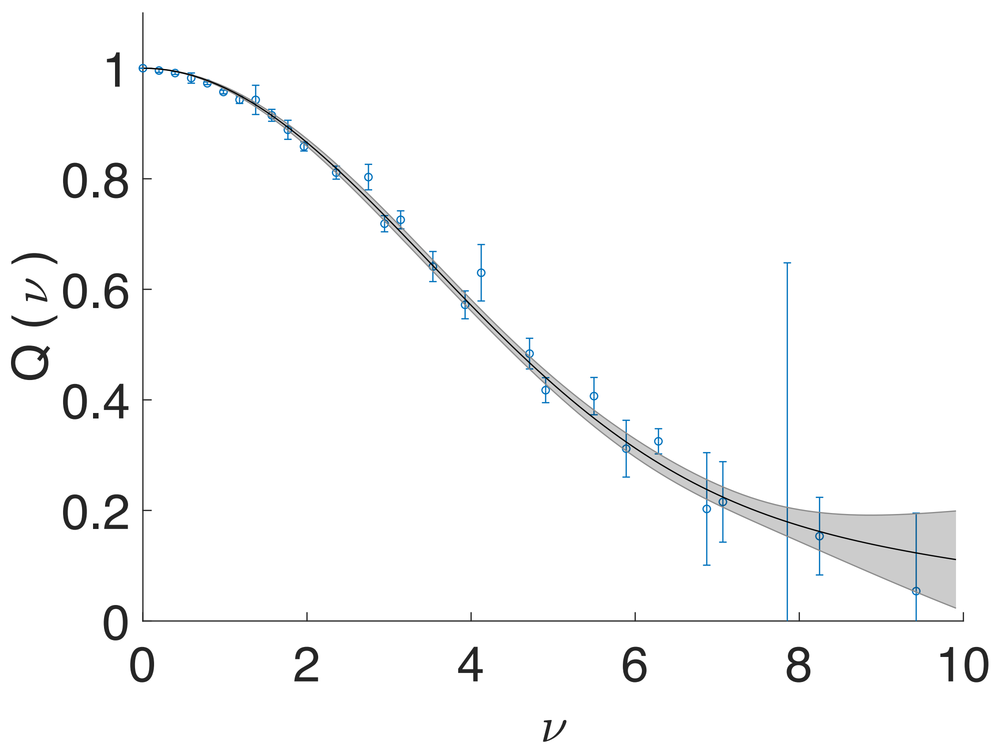
$$\tilde{D}(x) = x\mathrm{Im}\left[e^{ix}{}_3F_3(111;222;-ix)\right] - \frac{2-(2+x^2)\cos(x)}{x^2}$$

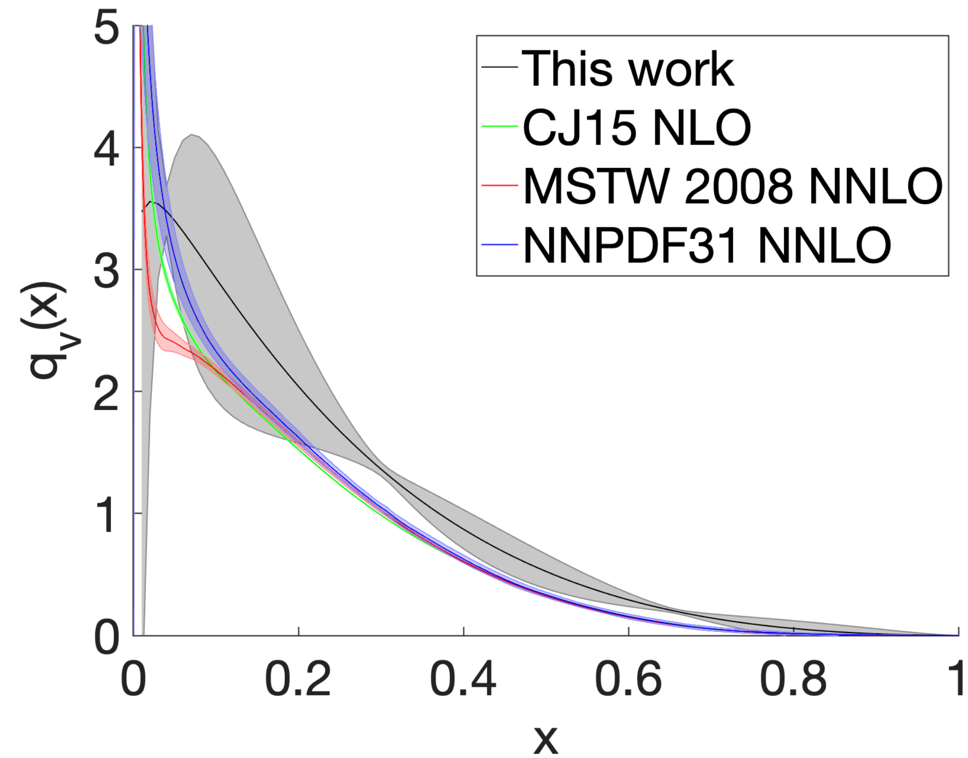
Polynomial corrections to the Ioffe time PDF may be suppressed

B. U. Musch, *et al* Phys. Rev. D 83, 094507 (2011)
M. Anselmino et al. 10.1007/JHEP04(2014)005
A. Radyushkin Phys.Lett. B767 (2017)

Ioffe time PDF

PDF





Fit to:

$N(a,b)x^a(1-x)^b$

a=0.00(22) b=2.46(68)

β=6.1 a=0.125fm dynamical fermions Nf=2+1

$$\mathfrak{M}(\nu, z^2) = \int_0^1 d\alpha\, \mathfrak{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\, \mathcal{Q}(\alpha\nu, \mu)$$

PDF reconstruction

PDFs cannot be directly computed

PDFs can only be reconstructed from matrix elements

Just they are obtained from cross-sections

Reason: Fourier transform cannot be done with limited data

work done with: *Joe Karpie, Alexander Rothgopf, Savvas Zafeiropoulos*

The inverse problem:

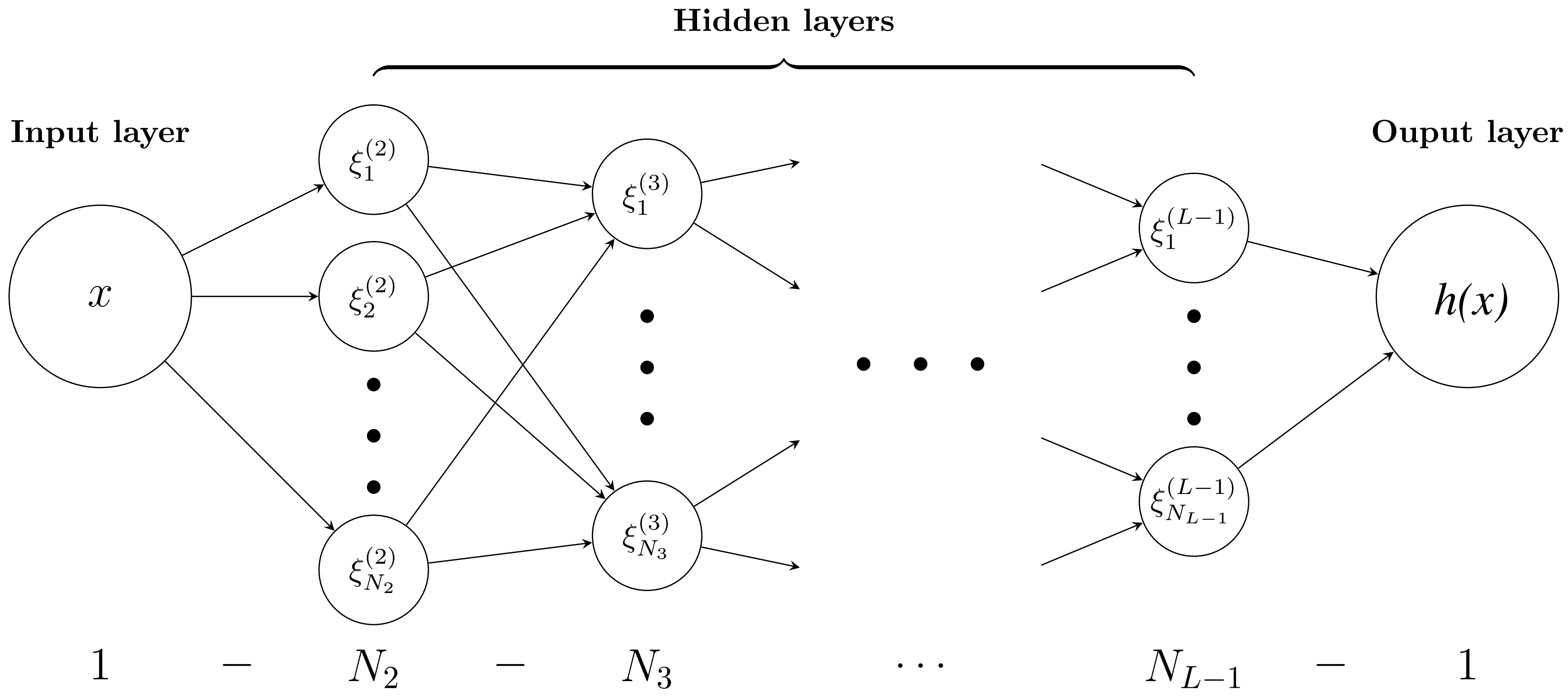
$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, q_v(x,\mu)\mathcal{K}(x\nu, z^2\mu^2) \; + \sum_{k=1}^{\infty} \mathcal{B}_k(\nu)(z^2)^k \,.$$

$$\mathcal{K}(x\nu, z^2\mu^2) = \cos(x\nu) - \frac{\alpha_s}{2\pi} C_F \left[\ln(e^{2\gamma_E+1} z^2\mu^2/4)\tilde{B}(x\nu) + \tilde{D}(x\nu)\right]$$

Leading order in a_s it is just a cosine transform.

We used a toy model to inversigate inversion algorithms

Neural networks as fitting form



$$q(x) = N(a,b)\, x^a\, (1\text{-}x)^b\, h(x)$$

Loss function

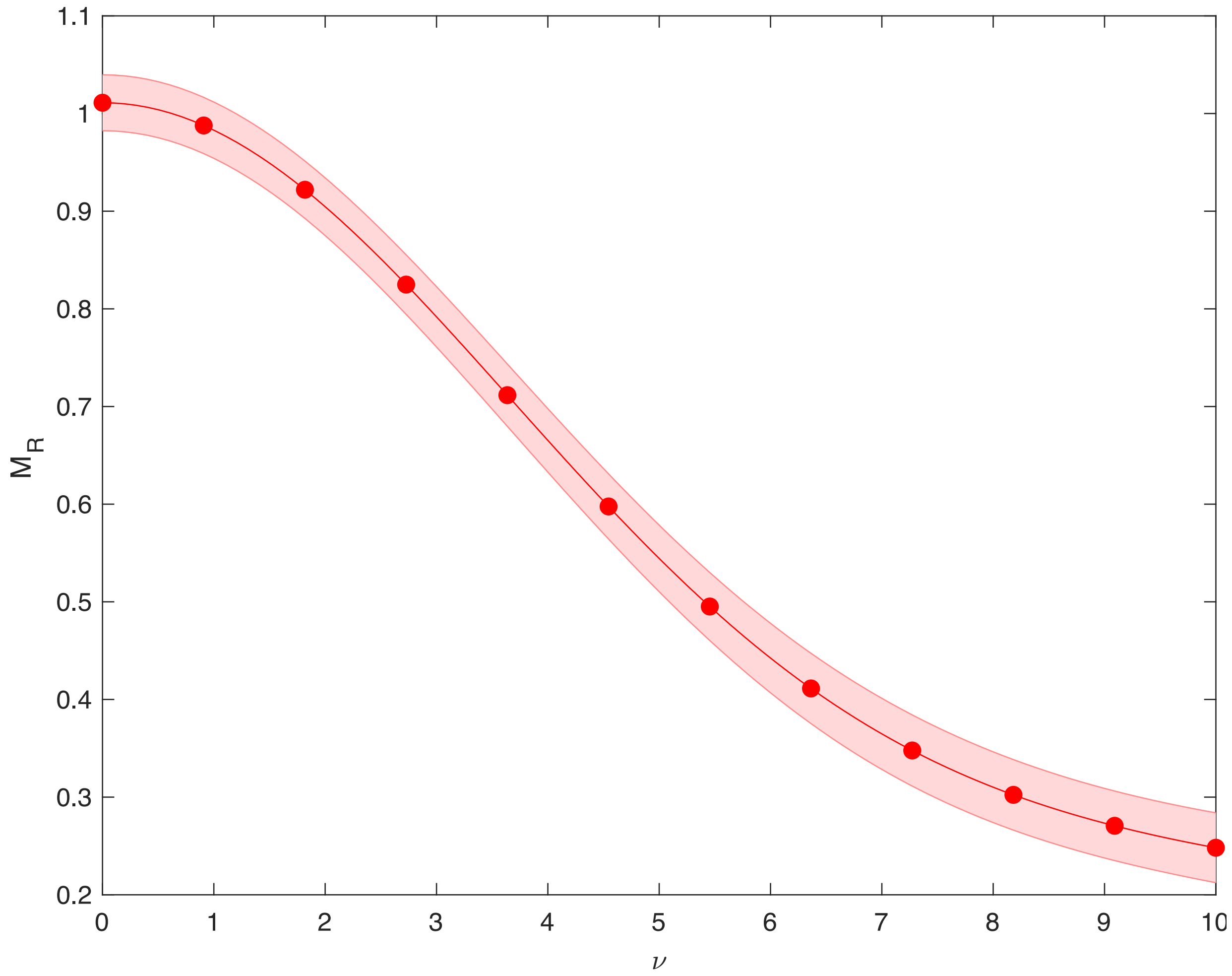
$$\mathcal{L}(\mathcal{M}_{lat}, \mathcal{M}_{model}) = (|\mathcal{M}_{lat} - \mathcal{M}_{model}|)^2$$

Covariance can be added

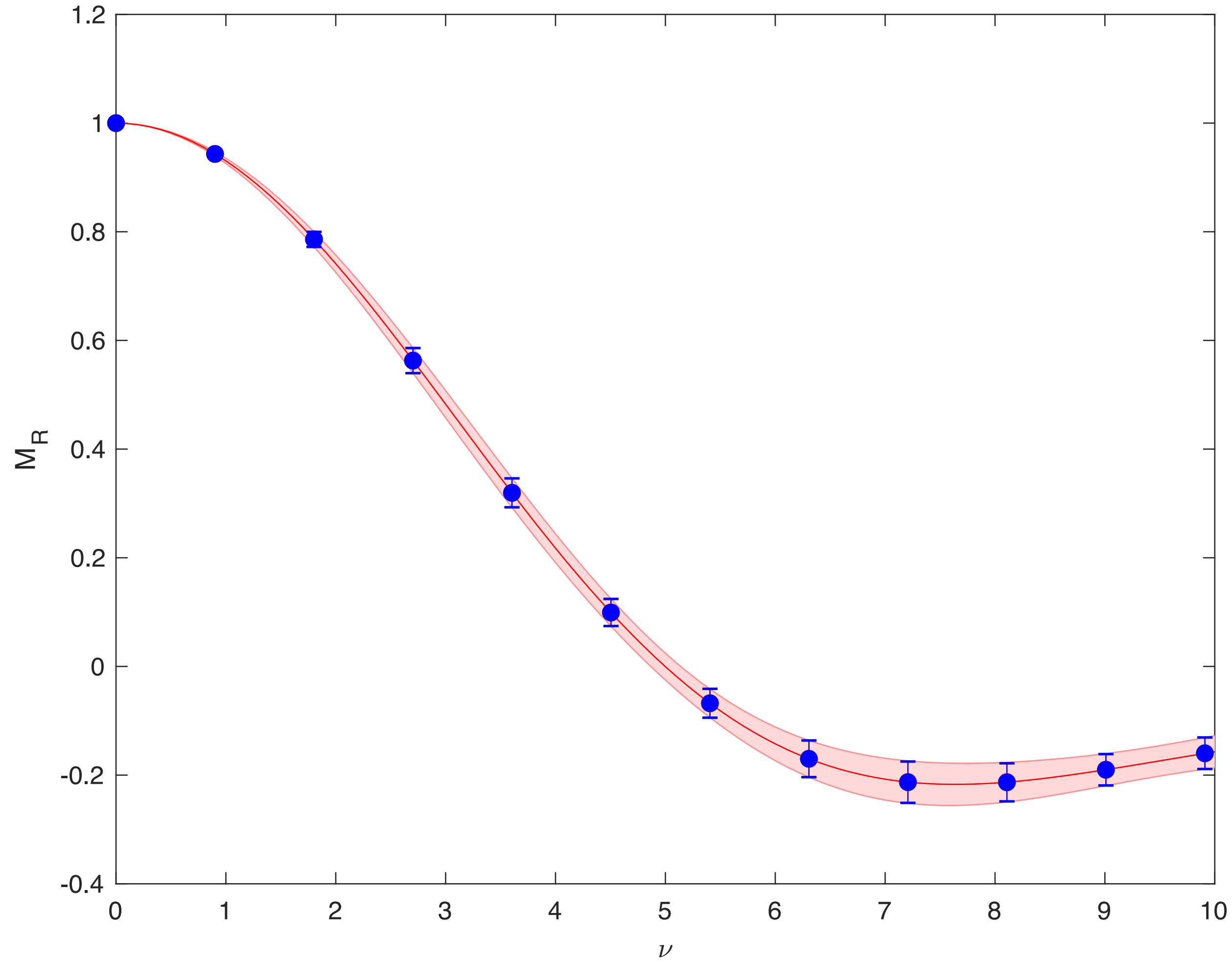
Methodology used by NNPDF

- Implementation in MATLAB
- No gradient search algorithm
- Start from an ensemble (replicas) of random guesses for the network parameters
  - Eliminate the worst performers and randomize sligthly the good onces
  - Repeat optimization cycle until a predefined number of replicas survive
- Typically obtain loss function very close to zero
- Final result is obtained as average over replicas
- Variation over replicas represent the uncertainty

Numerical experiments of reconstruction



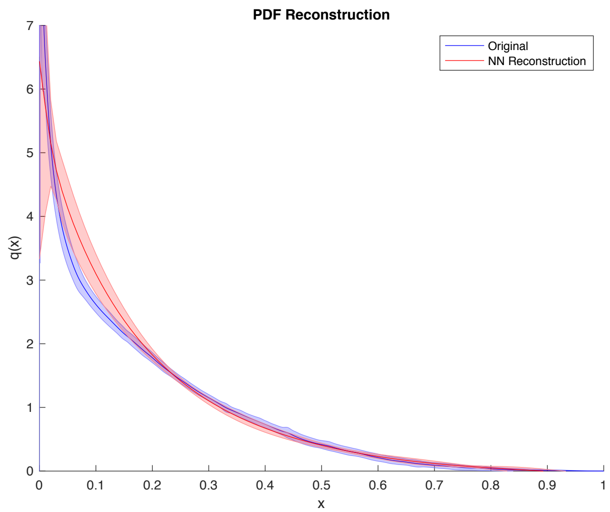
NNPDF 3.1

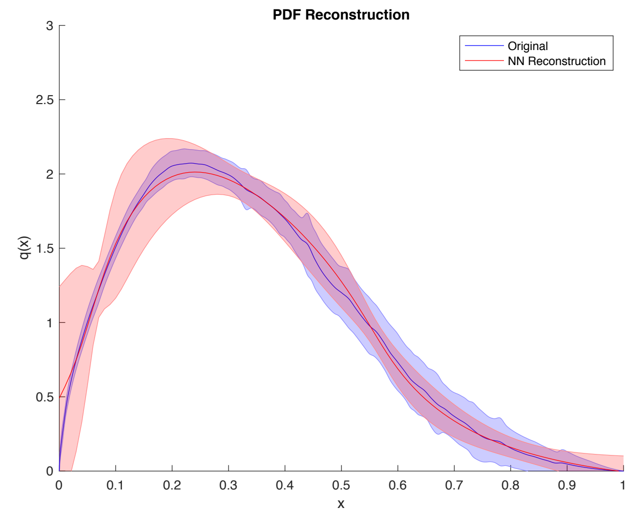


Unphysical model

Mock data sets

Network topology





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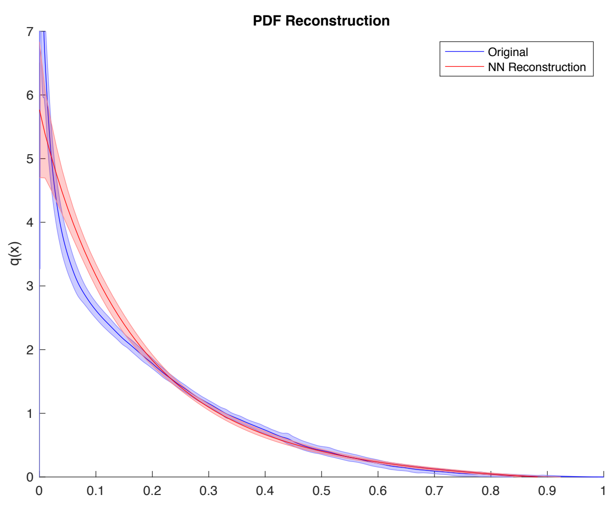

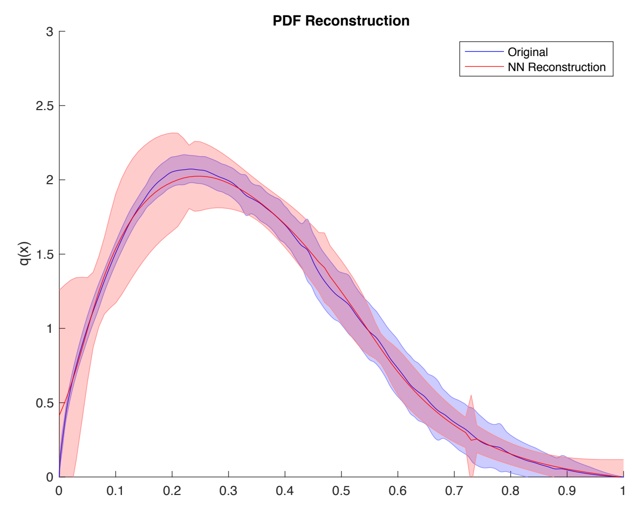

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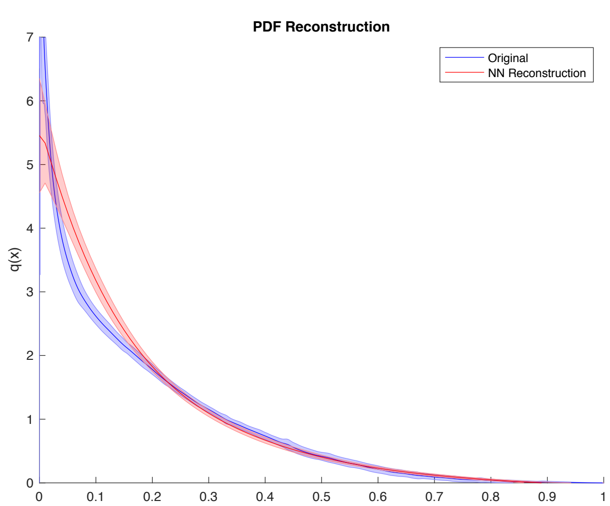

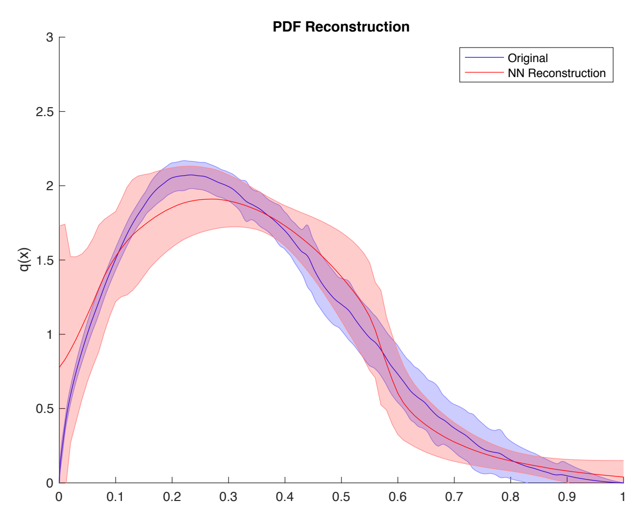

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Summary

- Methods for obtaining parton distribution from Lattice QCD have now emerged
- Obtaining the PDFs from lattice data amounts to a solution of an inverse problem similar to the one solved on the experimental data
- NNPDF approach applied on mock data produces interesting results
- Future: Collaborate with NNPDF to perform a detailed study for extracting PDFs from our lattice QCD data
    - Work in progress
    - Implementation of NLO curnel in TensorFlow
- Investigate other AI inspired methods for solving the inverse problem at hand