Modeling Nuclear PDFs with Neural Networks

[arXiv:1904.00018]

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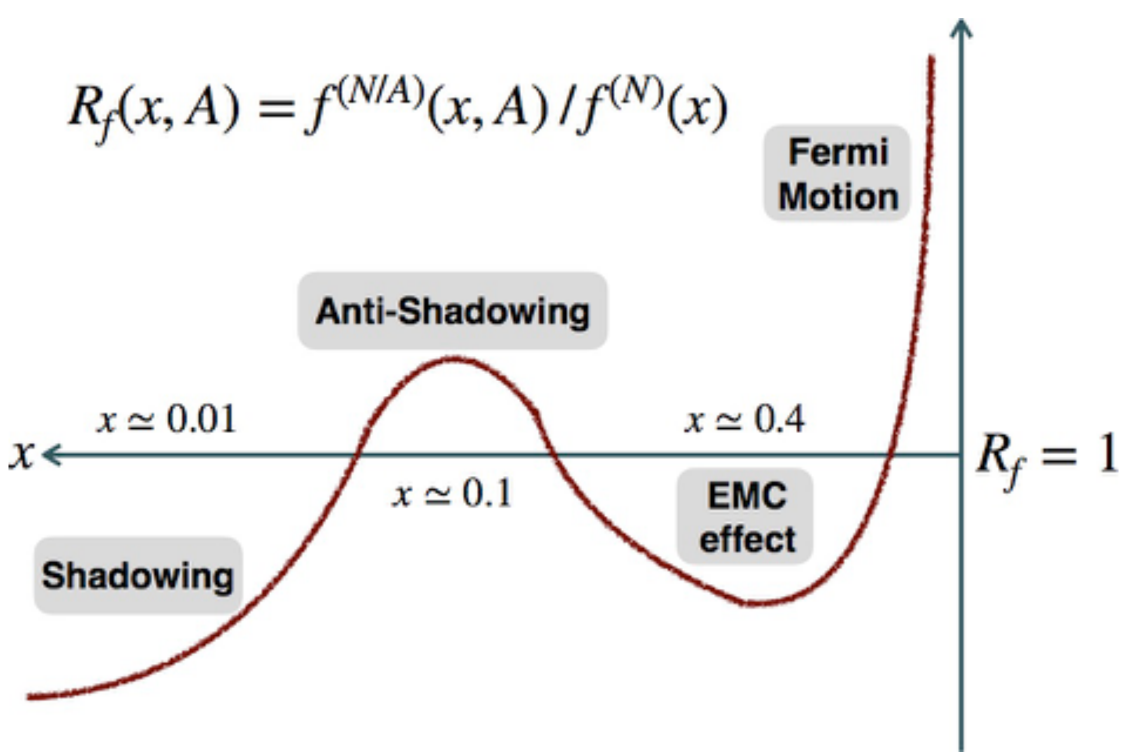
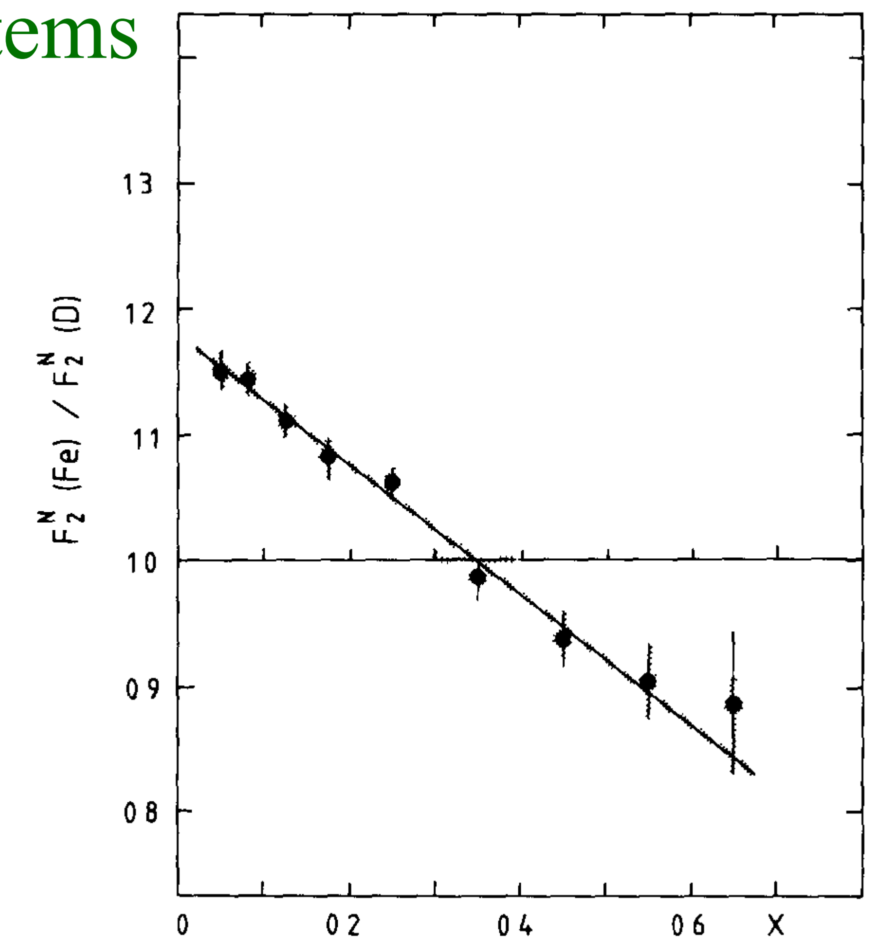
Motivation

- Deep-inelastic scattering of leptons from nuclei cannot be described by free nucleon formalism (EMC effect), e.g.

DIS structure functions: $F_2^A \neq Z F_2^p + (A - Z) F_2^n$

- Parton structure is modified in bound systems





- Mechanisms that originate nuclear effects not well understood

nPDFs from Global QCD Analyses

For a recent review see
JJE & E. Nocera [arXiv:2001.07722]

- Empirical determination of in-medium modifications to parton structure

- Relevant for interpretation of heavy ion collisions at RHIC and LHC

- Relies on collinear factorization formalism analogous to free proton fits

$$d\sigma^A(x,Q^2) \simeq \sum_f \int_x^{M_A/M} \frac{d\xi}{\xi} f^A(\frac{x}{\xi},Q^2) d\hat{\sigma}_f(\xi,Q^2)$$

- Scattering from nuclei treated incoherently from single bound nucleon

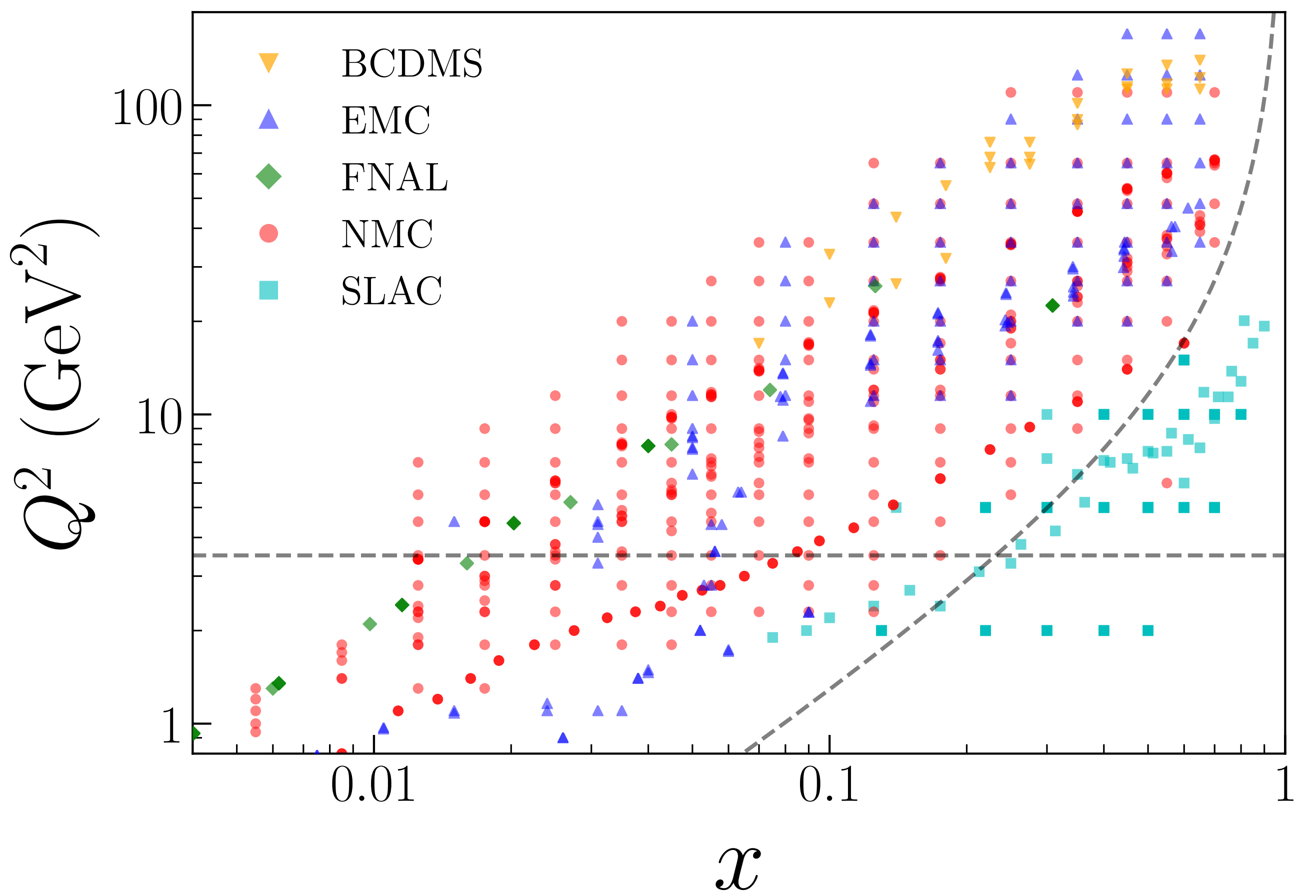
$$d\sigma^A(x,Q^2) \simeq Z d\sigma^{p/A} + (A-Z) d\sigma^{n/A}$$

$$d\sigma^{N/A}(x,Q^2) \simeq \sum_f \int_x^1 \frac{d\xi}{\xi} f^{N/A}(\frac{x}{\xi},Q^2) d\hat{\sigma}_f(\xi,Q^2)$$

- nPDFs are parameterized and fitted to global lepton-nucleus and hadron-nucleus scattering measurements

nNNPDF1.0 Analysis

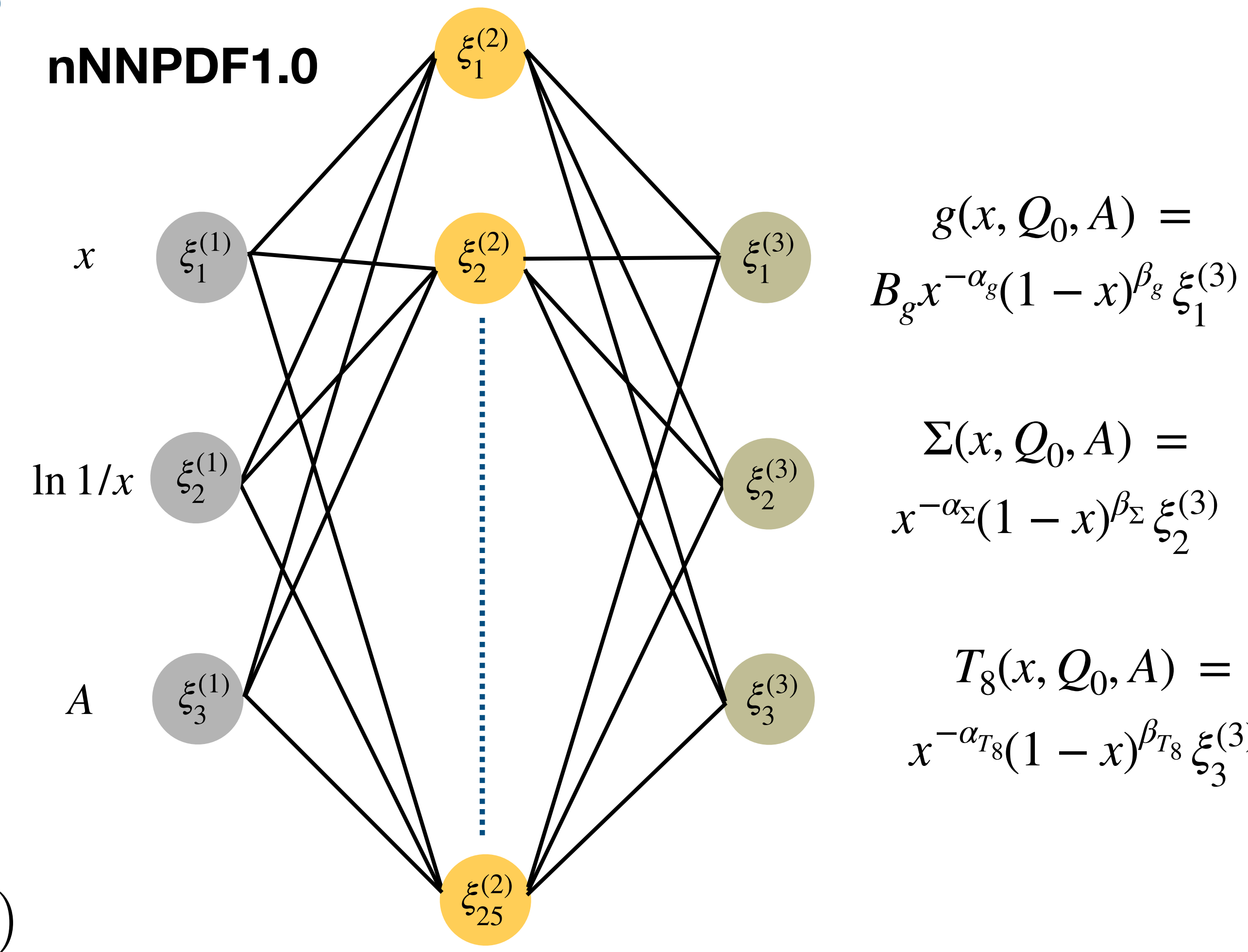
- Includes all available neutral current DIS data from CERN, SLAC, and FNAL experiments
- Kinematic cuts: $$W^2 > 12.5\ \text{GeV}^2$$ $$Q^2 > 3.5\ \text{GeV}^2$$
- 451 total data points



Experiment	A_1/A_2	N_{dat}
SLAC E-139	^{4}He/^{2}D	3
NMC 95, re.	^{4}He/^{2}D	13
NMC 95	^{6}Li/^{2}D	12
SLAC E-139	^{9}Be/^{2}D	3
NMC 96	^{9}Be/^{12}C	14
EMC 88, EMC 90	^{12}C/^{2}D	12
SLAC E-139	^{12}C/^{2}D	2
NMC 95, NMC 95, re.	^{12}C/^{2}D	26
FNAL E665	^{12}C/^{2}D	3
NMC 95, re.	^{12}C/^{6}Li	9
BCDMS 85	^{14}N/^{2}D	9
SLAC E-139	^{27}Al/^{2}D	3
NMC 96	^{27}Al/^{12}C	14
SLAC E-139	^{40}Ca/^{2}D	2
NMC 95, re.	^{40}Ca/^{2}D	12
EMC 90	^{40}Ca/^{2}D	3
FNAL E665	^{40}Ca/^{2}D	3
NMC 95, re.	^{40}Ca/^{6}Li	9
NMC 96	^{40}Ca/^{12}C	23
EMC 87	^{56}Fe/^{2}D	58
SLAC E-139	^{56}Fe/^{2}D	8
NMC 96	^{56}Fe/^{12}C	14
BCDMS 85, BCDMS 87	^{56}Fe/^{2}D	16
EMC 88, EMC 93	^{64}Cu/^{2}D	27
SLAC E-139	^{108}Ag/^{2}D	2
EMC 88	^{119}Sn/^{2}D	8
NMC 96, Q^2 dependence	^{119}Sn/^{12}C	119
FNAL E665	^{131}Xe/^{2}D	4
SLAC E-139	^{197}Au/^{2}D	3
FNAL E665	^{208}Pb/^{2}D	3
NMC 96	^{208}Pb/^{12}C	14
Total		**451**

nNNPDF1.0 Analysis

- Single NN architecture with 3-25-3



- Input scale: $Q_0 = 1$ GeV

- NN output multiplied by preprocessing

$$x\Sigma(x, Q_0, A) = x^{-\alpha_\Sigma}(1-x)^{\beta_\sigma}\,\xi_1^{(3)}(x, A)$$
$$xT_8(x, Q_0, A) = x^{-\alpha_{T_8}}(1-x)^{\beta_{T_8}}\,\xi_2^{(3)}(x, A)$$
$$xg(x, Q_0, A) = B_g x^{-\alpha_g}(1-x)^{\beta_g}\,\xi_3^{(3)}(x, A)$$

- Momentum sum rule: $\int_0^1 dx x(\Sigma(x, A) + g(x, A)) = 1 \;\rightarrow\; B_g = \dfrac{1 - \int_0^1 dx x\Sigma(x, A)}{\int_0^1 dx x g(x, A)}$

- Parameters optimized by stochastic gradient descent in TensorFlow (open-source ML software library)

$$\Sigma \equiv \sum_q q^+ = \sum_q q + \bar{q} \qquad T_8 \equiv u^+ + d^+ - 2s^+$$

nPDF Optimization

- Minimizing the cost function

$$\chi^2 \equiv \sum_{i,j=1}^{N_{\rm dat}} \left(R_i^{(\rm exp)} - R_i^{(\rm th)}(\{f_m\}) \right) (\mathrm{cov}_{\rm t_0})_{ij}^{-1} \left(R_j^{(\rm exp)} - R_j^{(\rm th)}(\{f_m\}) \right)$$

$$+ \lambda \sum_{m=g,\Sigma,T_8} \sum_{l=1}^{N_x} \left(f_m(x_l, Q_0, A) - f_m^{(p+n)/2}(x_l, Q_0) \right)^2$$

nPDF Optimization

- Minimizing the cost function

$$\chi^2 \equiv \sum_{i,j=1}^{N_{\mathrm{dat}}} \left(\boxed{R_i^{(\mathrm{exp})}} - R_i^{(\mathrm{th})}(\{f_m\})\right) (\mathrm{cov}_{\mathrm{t}_0})_{ij}^{-1} \left(\boxed{R_j^{(\mathrm{exp})}} - R_j^{(\mathrm{th})}(\{f_m\})\right)$$
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- Artificial data replicas — experimental data smeared by Gaussian

nPDF Optimization

- Minimizing the cost function

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- Artificial data replicas — experimental data smeared by Gaussian

- Theoretical predictions (functions of parameterized PDFs)

nPDF Optimization

- Minimizing the cost function

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- Artificial data replicas — experimental data smeared by Gaussian

- Theoretical predictions (functions of parameterized PDFs)

- Covariance matrix — encodes all statistical and systematic errors

$$(\mathrm{cov}_{t_0})_{ij}^{(\rm exp)} \equiv \left(\sigma_i^{(\rm stat)} R_i^{(\rm exp)} \right)^2 \delta_{ij} + \left(\sum_{\alpha=1}^{N_{\rm add}} \sigma_{i,\alpha}^{(\rm sys,a)} \sigma_{j,\alpha}^{(\rm sys,a)} R_i^{(\rm exp)} R_j^{(\rm exp)} + \sum_{\beta=1}^{N_{\rm mult}} \sigma_{i,\beta}^{(\rm sys,m)} \sigma_{j,\beta}^{(\rm sys,m)} R_i^{(\rm th,0)} R_j^{(\rm th,0)} \right)$$

t_0 prescription: multiply correlated multiplicative uncertainties by central theory values from previous fit iteration

nPDF Optimization

- Minimizing the cost function

$$\chi^2 \equiv \sum_{i,j=1}^{N_{\rm dat}} \left(R_i^{(\rm exp)} - R_i^{(\rm th)}(\{f_m\}) \right) (\mathrm{cov}_{t_0})^{-1}_{ij} \left(R_j^{(\rm exp)} - R_j^{(\rm th)}(\{f_m\}) \right)$$

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- Artificial data replicas — experimental data smeared by Gaussian

- Theoretical predictions (functions of parameterized PDFs)

- Covariance matrix — encodes all statistical and systematic errors

- Perform many fits to obtain representative MC sample distribution

$$E[\mathcal{O}(\vec{a})] = \sum_k w_k \mathcal{O}(\vec{a}_k) \qquad V[\mathcal{O}(\vec{a})] = \sum_k w_k (\mathcal{O}(\vec{a}_k) - E[\mathcal{O}])^2$$

nPDF Optimization

- Minimizing the cost function

$$\chi^2 \equiv \sum_{i,j=1}^{N_{\rm dat}} \left(R_i^{(\rm exp)} - R_i^{(\rm th)}(\{f_m\}) \right) (\mathrm{cov}_{t_0})^{-1}_{ij} \left(R_j^{(\rm exp)} - R_j^{(\rm th)}(\{f_m\}) \right)$$

$$+ \lambda \sum_{m=g,\Sigma,T_8} \sum_{l=1}^{N_x} \left(f_m(x_l, Q_0, A) - f_m^{(p+n)/2}(x_l, Q_0) \right)^2$$

- Boundary condition (imposed in the data region)

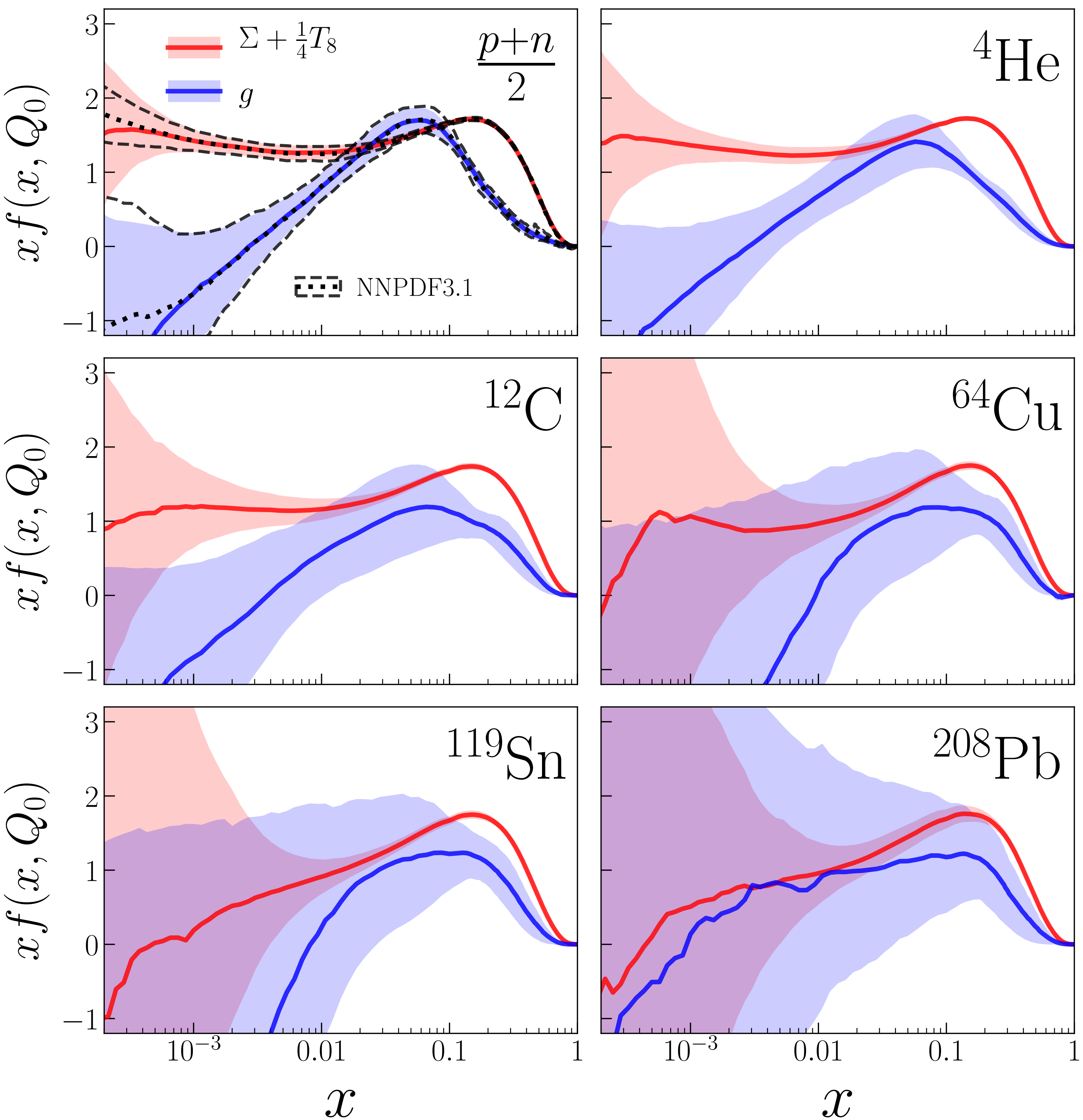
  - Free nucleon PDFs must be reproduced at A=1

$$f(x, Q, A=1) = \frac{1}{2} \left[f_p(x, Q^2) + f_n(x, Q^2) \right] \,, \quad f = \Sigma, T_8, g$$

  - NNPDF3.1 proton PDFs used as baseline

  - Central values and *uncertainties* reproduced at minimization level — "simultaneous" fit of proton and nuclear PDFs

nNNPDF1.0 Results

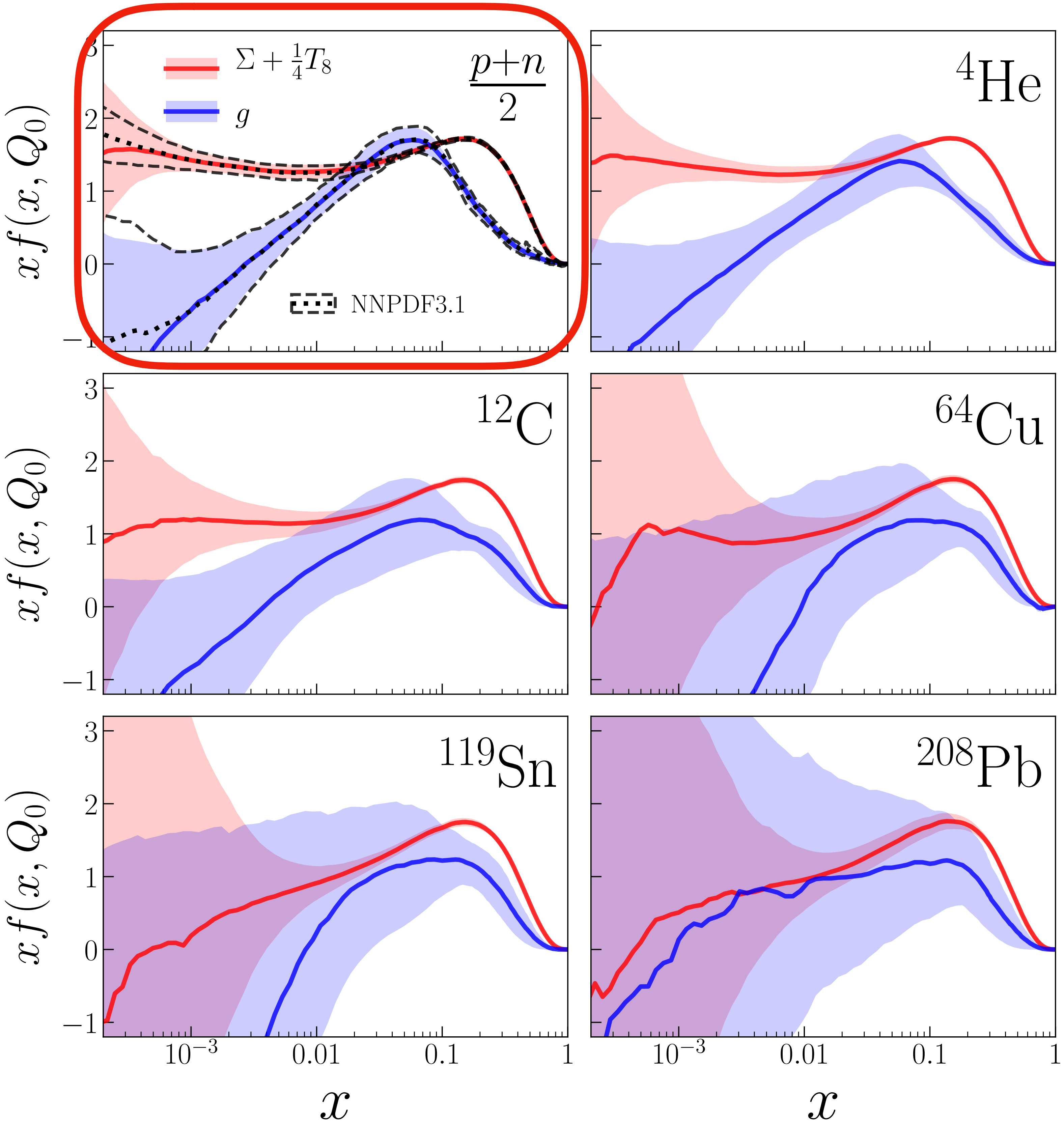


- Uncertainties computed as 90% CL range
- Only linear combination of quark singlet and octet constrained by NC DIS

$$\Sigma = \sum_i^{n_f}(f_i + \bar{f}_i) = \sum_i^{n_f} f_i^+$$

$$T_8 = u^+ + d^+ - 2s^+$$

nNNPDF1.0 Results



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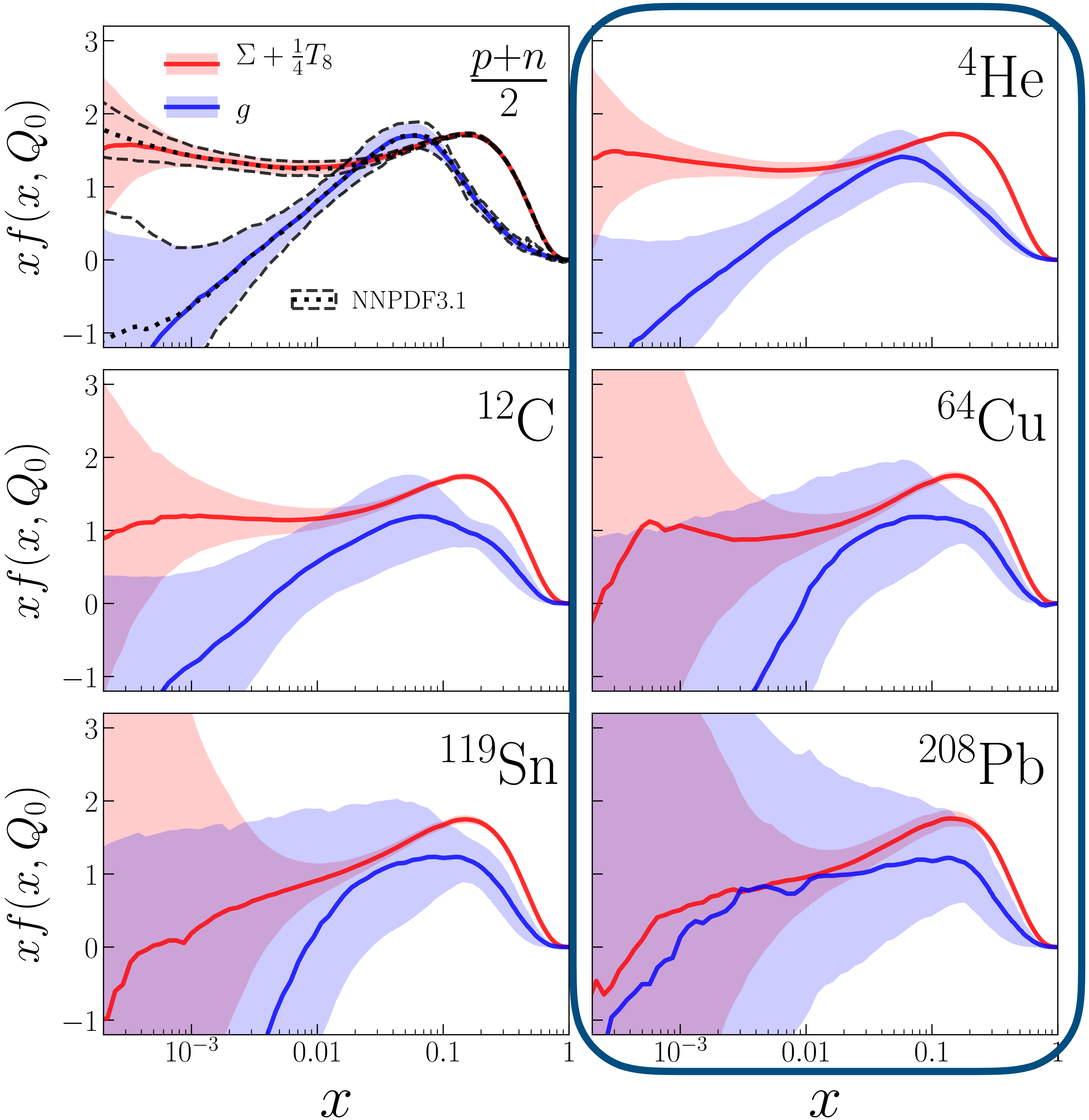
- NNPDF3.1 central values and uncertainties reproduced

nNNPDF1.0 Results



- Uncertainties computed as 90% CL range
- Only linear combination of quark singlet and octet constrained by NC DIS

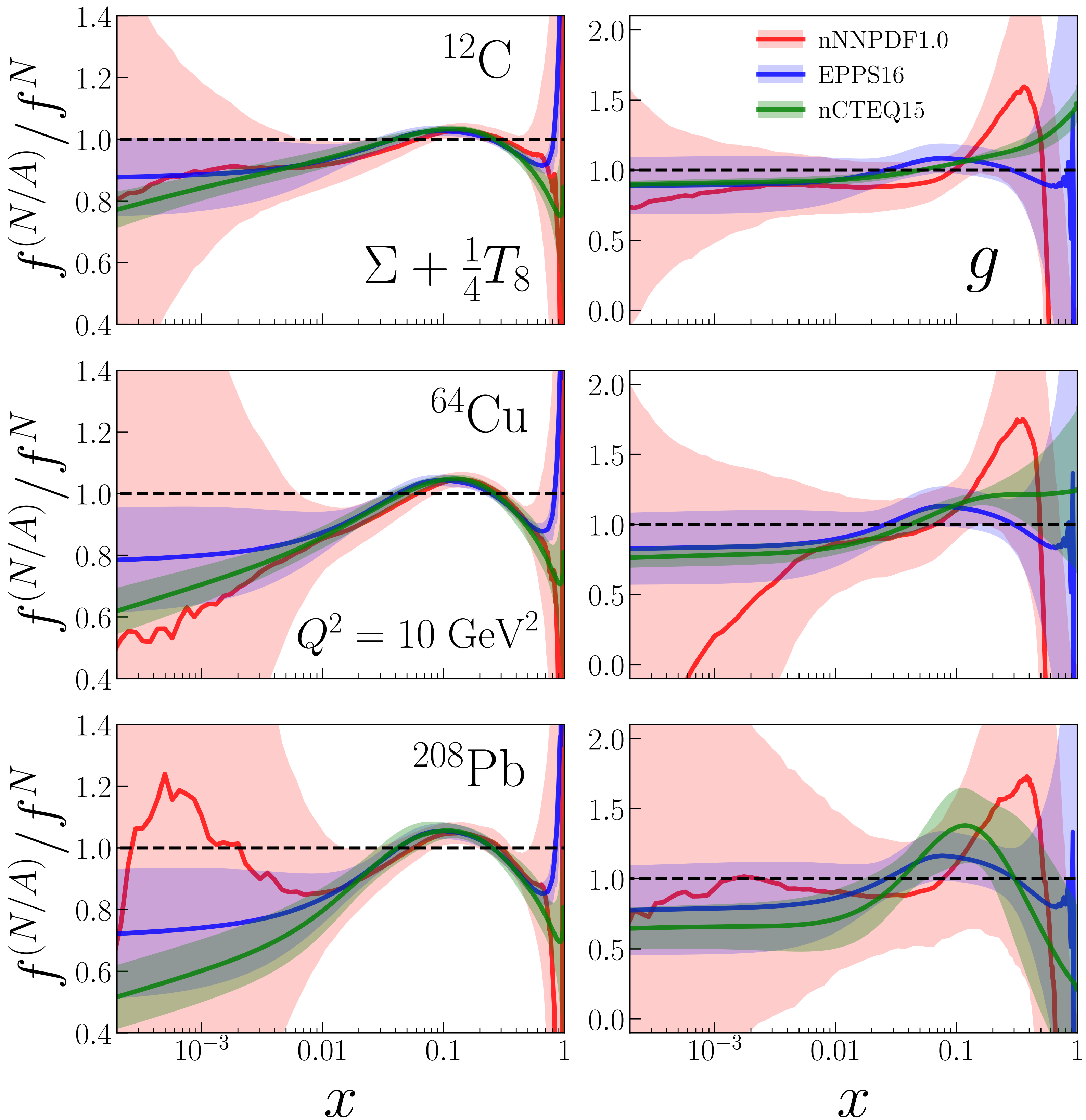
$$\Sigma = \sum_i^{n_f} (f_i + \bar{f}_i) = \sum_i^{n_f} f_i^+$$

$$T_8 = u^+ + d^+ - 2s^+$$

- NNPDF3.1 central values and uncertainties reproduced
- Increasing uncertainties with A — boundary condition effect

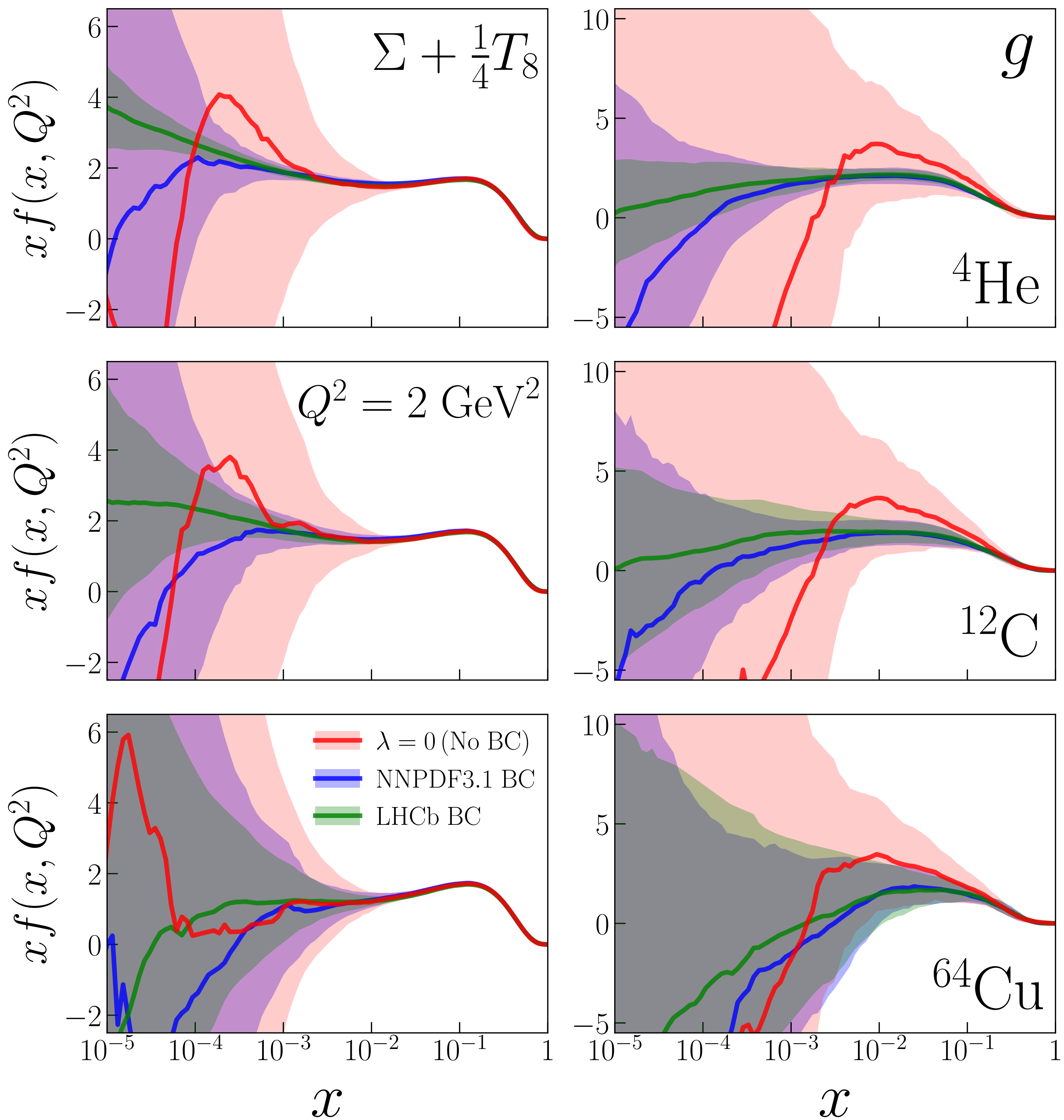
nNNPDF1.0 Results

- Distributions normalized by respective proton boundary conditions
- EPPS16 and nCTEQ15 show 90% CL ranges based on Hessian method
- Significant differences in uncertainties



nNNPDF1.0 Results

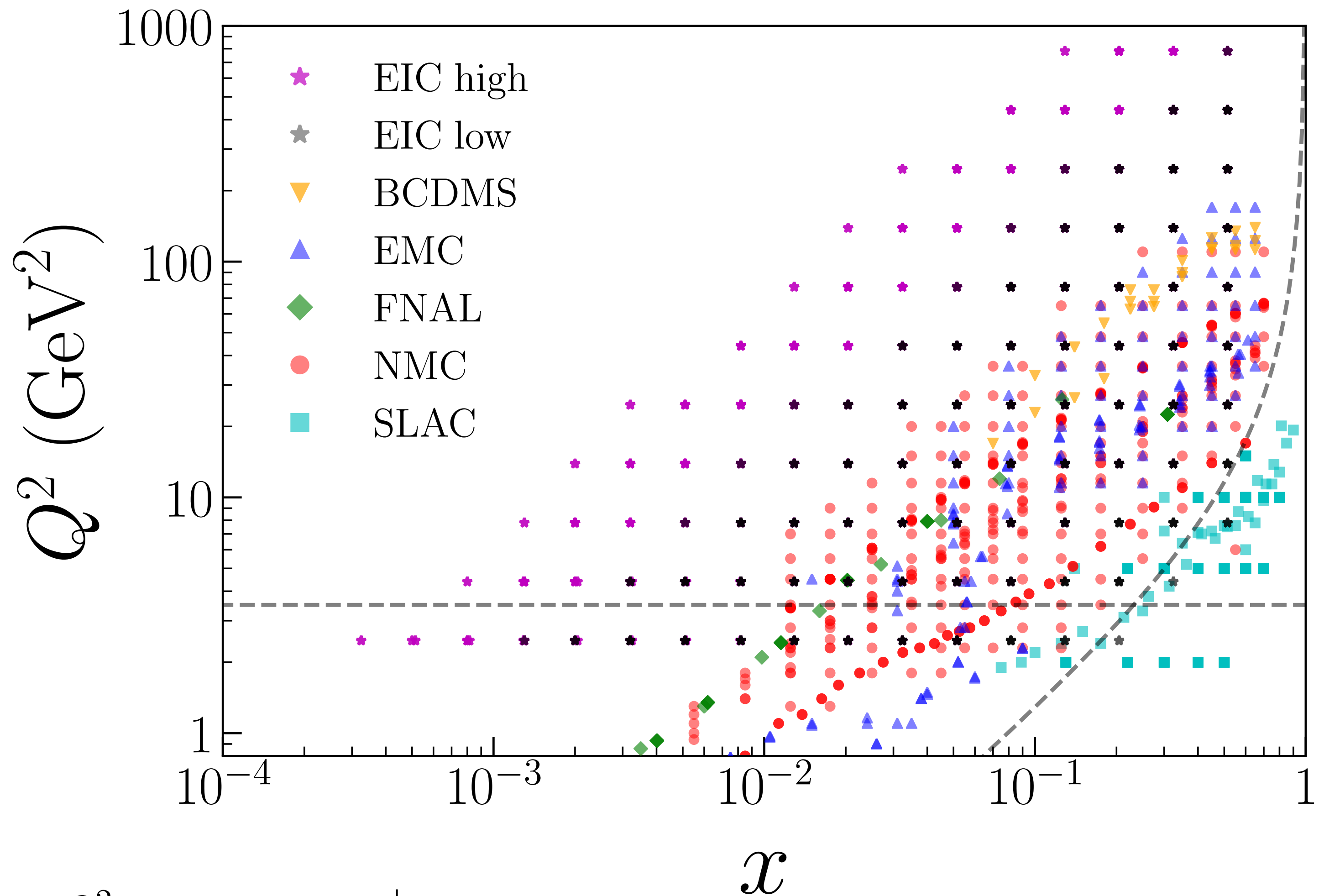
- Can test other boundary conditions — e.g. NNPDF3.0+LHCb PDF set with smaller uncertainties at low x
- Remarkable impact of boundary condition choice — proton PDF constraints relevant for low-A nPDF extraction!



Impact of EIC

- Analysis of EIC pseudodata — extended kinematic coverage

- Two scenarios: low energy (5 GeV) vs high energy (20 GeV) electron beam

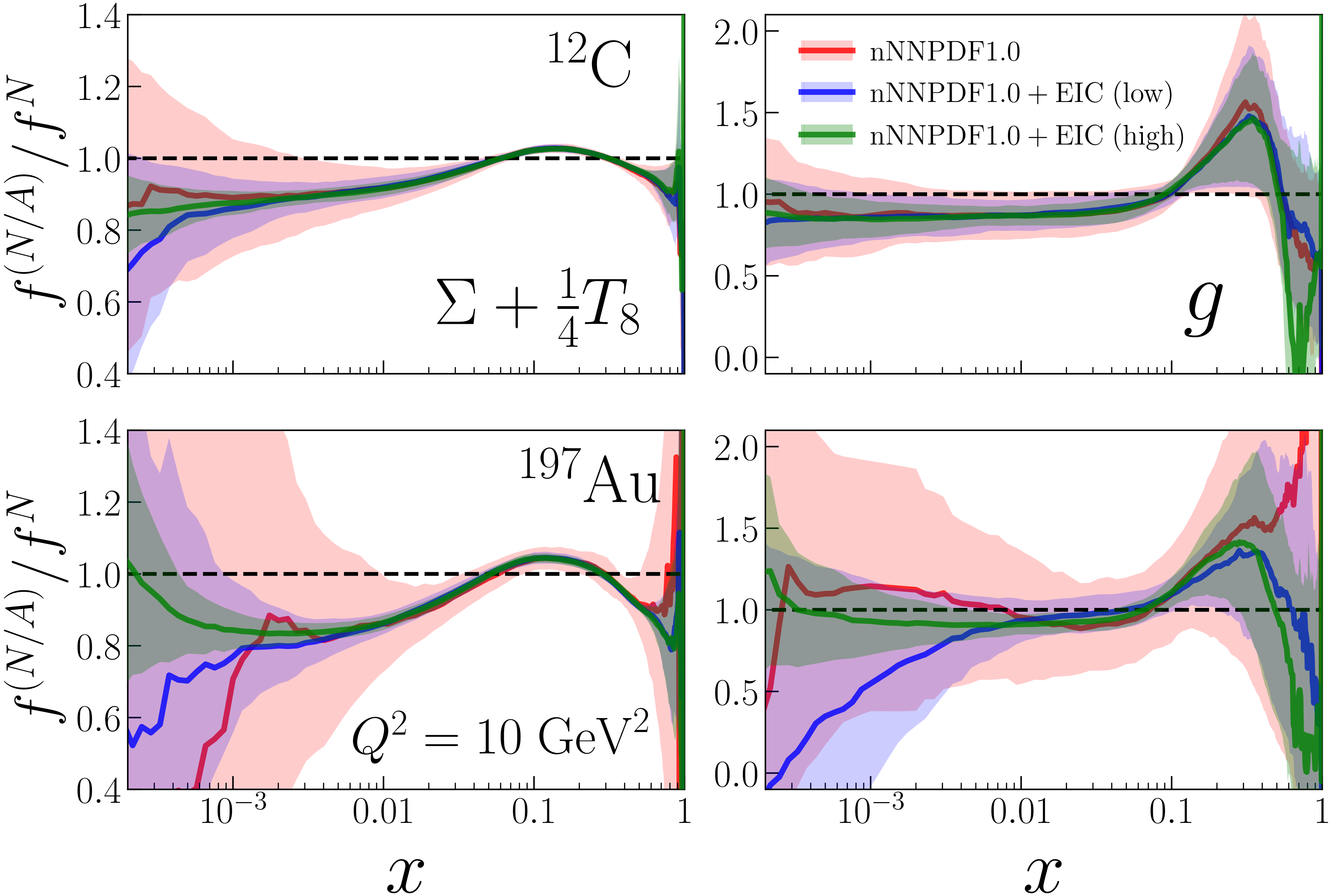


Scenario	A	E_e	E_A/A	$Q^2_{\rm max}$	$x_{\rm min}$	$N_{\rm dat}$
eRHIC_5x50C	12	5 GeV	50 GeV	440 GeV^2	0.003	50
eRHIC_5x75C	12	5 GeV	75 GeV	440 GeV^2	0.002	57
eRHIC_5x100C	12	5 GeV	100 GeV	780 GeV^2	0.001	64
eRHIC_5x50Au	197	5 GeV	50 GeV	440 GeV^2	0.003	50
eRHIC_5x75Au	197	5 GeV	75 GeV	440 GeV^2	0.002	57
eRHIC_5x100Au	197	5 GeV	100 GeV	780 GeV^2	0.001	64
eRHIC_20x50C	12	20 GeV	50 GeV	780 GeV^2	0.0008	75
eRHIC_20x75C	12	20 GeV	75 GeV	780 GeV^2	0.0005	79
eRHIC_20x100C	12	20 GeV	100 GeV	780 GeV^2	0.0003	82
eRHIC_20x50Au	197	20 GeV	50 GeV	780 GeV^2	0.0008	75
eRHIC_20x75Au	197	20 GeV	75 GeV	780 GeV^2	0.0005	79
eRHIC_20x100Au	197	20 GeV	100 GeV	780 GeV^2	0.0003	82

- Pseudodata constructed with nNNPDF1.0 results

- Uncertainty projections from E.C. Aschenaur et. al. [arXiv:1708.05654]

Impact of EIC

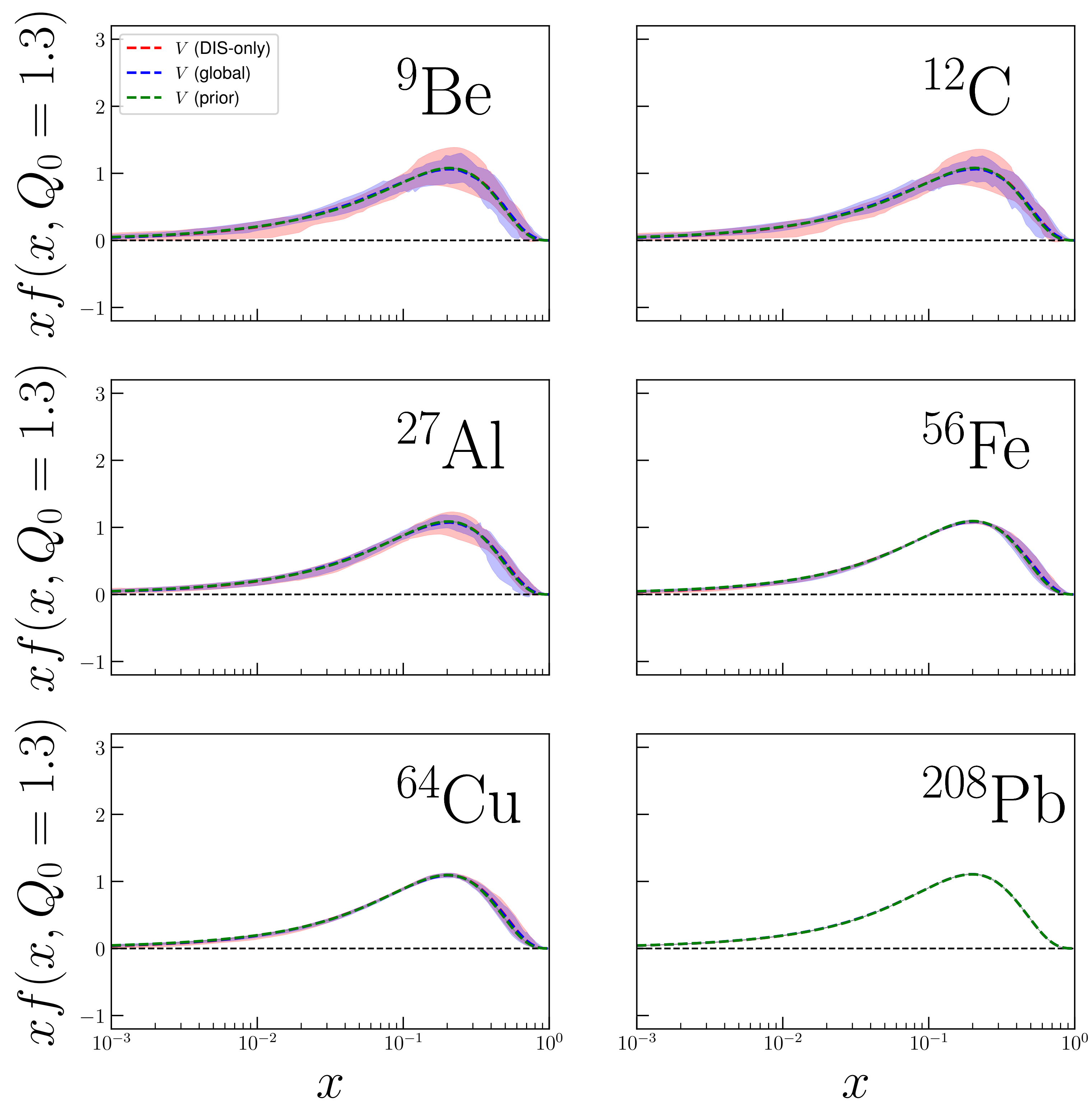


- Significant reduction of uncertainties for high-A, low-*x* — particularly for higher energy option

Towards nNNPDF1.5

- Experimental data: CC DIS + W/Z production at LHC
- Theoretical improvements: new BC, positivity, valence sum rules
- Closure tests successful! — full analysis to be completed soon

CT Level 0 (prior = nCTEQ15)



Summary and Outlook

- Machine learning + Monte Carlo methods are important for robust extractions of nPDFs and their uncertainties

- Methodology improvements in nPDF analysis
  - Neural network optimized by SGD in TensorFlow

- Highlights from first Monte Carlo nPDF fit
  - Significant impact of A=1 boundary condition for low-A nuclei
  - High energy EIC option constrains PDFs down to $x \sim 10^{-4}$

- Upcoming nNNPDF1.5 analysis (completing very soon!)
  - Experimental data: CC DIS + W/Z production
  - Theoretical improvements: new BC, positivity, valence sum rules