Machine learning for LQCD: ensemble generation



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Generate QCD gauge fields

Generate field configurations $\phi(x)$ with probability $P[\phi(x)] \sim e^{-S[\phi(x)]}$



Burn-in time and correlation length dictated by Markov chain **'autocorrelation time'**: shorter autocorrelation time implies less computational cost

Generate QCD gauge fields

QCD gauge field configurations sampled via Hamiltonian dynamics + Markov Chain Monte Carlo





"Critical slowing-down" of generation of uncorrelated samples

Generate QCD gauge fields

QCD gauge field configurations sampled via

Hamiltonian dynamics + Markov Chain Monte Carlo



Machine learning QCD

Accelerate gauge-field generation via ML

Multi-scale algorithms: parallels with image recognition Shanahan et al., PRD 97, 094506 (2018)

Generative models to replace Hybrid Monte-Carlo *parallels with image generation* Albergo et al., PRD 100, 034515 (2019) [MIT + Google DeepMind, arXiv:2002.02428] Kanwar et al., MIT-CTP/5181



3.) Hybrid approaches

Consider only approaches which rigorously preserve quantum field theory in applicable limits

Machine learning QCD

Generative models for QCD gauge field generation



Sampling gauge field configs

Generate field configurations $\phi(x)$ with probability $P[\phi(x)] \sim e^{-S[\phi(x)]}$



Sampling gauge field configs

Generate field configurations $\phi(x)$ with probability $P[\phi(x)] \sim e^{-S[\phi(x)]}$

Parallels with image generation problem



unlikely (log prob = -6107)







likely



[Karras, Lane, Aila / NVIDIA 1812.04948]







likely

Machine learning QCD

Ensemble of lattice QCD gauge fields

- $64^3 \times 128 \times 4 \times N_c^2 \times 2$ ~10⁹ numbers
- \sim 1000 samples
- Ensemble of gauge fields has meaning
- Long-distance correlations are important
- Gauge and translationinvariant with periodic boundaries

CIFAR benchmark image set for machine learning

- 32 x 32 pixels x 3 cols
 ≈3000 numbers
- 60000 samples
- Each image has meaning
- Local structures are important
- Translation-invariance within frame

Machine learning QCD

Ensemble of lattice QCD in fields

CIFAR benchmark image set for machine learning

Out-of-the-box ML tools are not appropriate

- Need custom ML for physics from the ground up
 - Gauge and translationinvariant with periodic boundaries

Translation-invana. within frame

Flow-based models learn a change-of-variables that transforms a known distribution to the desired distribution [Rezende & Mohamed 1505.05770]



Flow-based models learn a change-of-variables that transforms a known distribution to the desired distribution [Rezende & Mohamed 1505.05770]



Choose real non-volume preserving flows: [Dinh et al. 1605.08803]

- Affine transformation of half of the variables:
 - scaling by exp(s)
 - translation by t

Z

r(z)

 s and t arbitrary neural networks depending on untransformed variables only

 $f^{-1}(z)$

 g_{i+1}^{-1}

Simple inverse and Jacobian



 ϕ

 $\tilde{p}_f(\phi)$



Training the model

Target distribution is known up to normalisation

$$p(\phi) = e^{-S(\phi)} / Z_{p(\phi)} = e^{-S(\phi)} / Z$$

Train to minimise shifted KL divergence: [Zhang, E, Wang 1809.10188]

 $f(\tilde{p}_{f}||p) = L(\tilde{p}_{g}) \neq D_{KL}(\tilde{p}_{f}||p) = D_{KL}(\tilde{p}_{f}||p) = S(f)$ shift removes unknown normalisation Z $= \int \prod_{j} d\phi_{j} \tilde{p}_{f}(\phi) \left(\log \overline{\tilde{p}}_{f}(\phi) \int_{j}^{d\phi_{j}} d\phi_{j} \tilde{p}_{f}(\phi) \left(\log \tilde{p}_{f}(\phi) + S(\phi) \right) \right)$ $L(\tilde{p}_{f}) := D_{\tilde{p}_{f}}(\tilde{p}_{f}) = D_{\tilde{p}_{f}}($

Exactness via Markov chain

Guarantee exactness of generated distribution by forming a Markov chain: accept/reject with Metropolis-Hastings step

Acceptance
probability
$$\mathcal{A}(\phi^{(i-1)}, \phi') = \min\left(1, \frac{\tilde{p}(\phi^{(i-1)})}{p(\phi^{(i-1)})} \frac{p(\phi')}{\tilde{p}(\phi')}\right)$$

proposal independent
of previous sample



Exactness via Markov chain

Guarantee exactness of generated distribution by forming a Markov chain: accept/reject with Metropolis-Hastings step

Acceptance
probability
$$_{\mathcal{A}}A(\phi^{(i-1)}, \phi') = \min\left(1, \underbrace{\tilde{p}(\phi^{(i-1)}, p(\phi')}_{p(\phi^{(i-1)}, \tilde{p}(\phi')}, \frac{\tilde{p}(\phi')}{Model dist}, \frac{\tilde{p}(\phi^{(i-1)}, p(\phi')}{Model dist}, \frac{\tilde{p}(\phi^{(i-1$$



Fields via flow models



First application: scalar lattice field theory

One real number $\phi(x) \in (-\infty, \infty)$ per lattice site x (2D lattice)

Action: kinetic terms and quartic coupling

$$S(\phi) = \sum_{x} \left(\sum_{y} \frac{1}{2} \phi(x) \Box(x, y) \phi(y) + \frac{1}{2} m^2 \phi(x)^2 + \lambda \phi(x)^4 \right)$$

5 lattice sizes: $L^2 = \{6^2, 8^2, 10^2, 12^2, 14^2\}$ with parameters tuned for analysis of critical slowing down

	E1	E2	E3	E4	E5
L	6	8	10	12	14
m^2	-4	-4	-4	-4	-4
λ	6.975	6.008	5.550	5.276	5.113
$m_p L$	3.96(3)	3.97(5)	4.00(4)	3.96(5)	4.03(6)

 g_1

 $g_{\tilde{c}}$

g

 g_2^{-1}

First application: scalar lattice field theory

- Prior distribution chosen to be uncorrelated Gaussian: $\phi(x) \sim \mathcal{N}(0, 1)$
 - Real non-volume-preserving (NVP) couplings
 - * 8-12 Real NVP coupling layers
 - * Alternating checkerboard pattern for variable split
 - * NNs with 2-6 fully connected layers with 100-1024 hidden units
 - Train using shifted KL loss with Adam optimizer
 - Stopping criterion: fixed acceptance rate in Metropolis-Hastings MCMC

First application: scalar lattice field theory

Compare with standard updating algorithms: 'local', 'HMC'



ML model produces varied samples and correlations at the right scale

First application: scalar lattice field theory

Compare with standard updating algorithms: 'local', 'HMC'



Rejectance runs in the Metropolis-Hastings accept/reject step are comparable to those in Hamiltonian Monte-Carlo tuned to same acceptance

First application: scalar lattice field theory

Compare with standard updating algorithms: 'local', 'HMC'



Physical observables match computed on ensembles generated from ML model and from standard methods

Two-point susceptibility
$$\chi_2 = \sum_x G_c(x)$$

Ising limit energy
$$E = \frac{1}{d} \sum_{1 \leq \mu \leq d} G_c(\hat{\mu})$$

 $m_p = -$

 $G_c(x) = \frac{1}{2}$

First application: scalar lattice field theory

Compare with standard updating algorithms: 'local', 'HMC'



Uncertainties in physical observables follow statistical scaling as the number of samples is increased

red dashed curve: $\propto 1/\sqrt{N}$

First application: scalar lattice field theory

Success:Critical slowing down is eliminatedCost:Up-front training of the model



Next steps: ML for LQCD

Target application: Lattice QCD for nuclear physics

Scale number of dimensions
 Scale number of degrees of freedom
 Methods for gauge theories
 [MIT + Google DeepMind, arXiv:2002.02428]





Aurora21 Early Science Project Awarded

Next steps: ML for LQCD



Incorporating symmetries

[MIT + Google DeepMind, arXiv:2002.02428]

Normalizing Flows on Tori and Spheres

Danilo Jimenez Rezende^{*1} George Papamakarios^{*1} Sébastien Racanière^{*1} Michael S. Albergo² Gurtej Kanwar³ Phiala E. Shanahan³ Kyle Cranmer²

NCP Möbius CS Target Arbitrarily flexible model architectures designed for compact θ (rads and connected manifolds e.g., physics data on compact domains OR robot arm positions $\frac{3\pi}{2}$ $2\pi 0 \frac{\pi}{2} \pi$ $\frac{3\pi}{2}$ $\frac{\pi}{2}$ $\frac{\pi}{2}$ 2π 0 $\frac{\pi}{2}$ $\frac{3\pi}{2}$ 2π 0 $\frac{3\pi}{2}$ 0 π π π ϕ (rads)

Interdisciplinary applications

Molecular genetics and drug design



RESEARCH ARTICLE SUMMARY

MACHINE LEARNING

Boltzmann generators: Sampling equilibrium states of many-body systems with deep learning

Frank Noé*†, Simon Olsson*, Jonas Köhler*, Hao Wu

1 Sample Gaussian distribution



2 Generate distribution 3 Re-weight Boltzmann distribution

Robotics



H. Application: Multi-Link Robot Arm

As a concrete application of flows on tori, we consider the problem of approximating the posterior density over joint angles $\theta_{1,...,6}$ of a 6-link 2D robot arm, given (soft) constraints on the position of the tip of the arm. The possible configurations of this arm are points in \mathbb{T}^6 . The position r_k of a joint k = 1, ..., 6 of the robot arm is given by

 $r_k = r_{k-1} + \left(l_k \cos\left(\sum_{i \le k} \theta_j\right), l_k \sin\right)$

where $r_{2} = (0, 0)$ is the position where the arm is affixed

Joint software effort

Our codes exploit and extend existing ML software frameworks

- Tensorflow
- Pytorch



TensorFlow

PYT⁶RCH

Active research projects into training protocols:

- Pruning
- Lottery tickets
- Initialisation frameworks

We run on

- CPUs
- GPUs
- TPUs



Targeting exascale hardware for nuclear physics projects



Outlook

IF a generative flow model can be trained for QCD

After the up-front cost of training the model, it is

- Cheap to generate an arbitrarily large ensemble
- No need to store configurations, only the trained model
- Volume scaling is tractable via hierarchical flow and transfer learning approach
- Cheap to re-train the model to move to nearby parameter values (quark masses, beta)

i.e., if possible, this approach would have significant advantages, even if initial training is expensive

CAIFI: The Center for Artificial Intelligence "X" and Fundamental Interactions



William Detmold Philip Harris Phiala Shanahan Kerstin Perez Tracy Slatyer Washington Taylor Jesse Thaler Mike Williams



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Edo Berger Cora Dvorkin Daniel Eisenstein Doug Finkbeiner Matthew Schwartz

Demba Ba Yaron Singer Todd Zickler



James Halverson Brent Nelson



Taritree Wongjirad



Senior Investigators:

Junior Investigators in steady state:

20 Physicists + 7 AI Experts

- \approx 20 FTE Graduate Students
- \approx 7 CAIFI Postdoctoral Fellows

CAIFI: The Center for Artificial Intelligence "X" and Fundamental Interactions

Nuclear physics @MIT



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CAIFI: The Center for Artificial Intelligence ^{"χφ"} and Fundamental Interactions

Our Vision: Advance fundamental physics and foundational Al Build strong multidisciplinary collaborations Close contact between early-career researchers Intellectual freedom fo Training and education Access to state-of-the Advocacy for shared s

★ Ab Initio AI for Theory Calculations
★ Ab Initio AI for Experiments
★ Ab Initio AI for Advancing AI

CAIFI: The Center for Artificial Intelligence "XP" and Fundamental Interactions



Modeled after nuclear/particle experimental collaborations: Dedicated coordinator for each major activity



Phiala Shanahan



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